INTRODUCTION TO ALGEBRAIC GEOMETRY AND COMMUTATIVE ALGEBRA

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PRE-REQUISITES : Linear Algebra ; Algebra – First Course

INTENDED AUDIENCE : BS / ME / MSc / PhD

INDUSTRIES APPLICABLE TO : R & D Departments of IBM / Microsoft Research Labs SAP /TCS / Wipro/ Infosys

COURSE OUTLINE :

Algebraic geometry played a central role in 19th century math. The deepest results of Abel, Riemann, Weierstrass, and the most important works of Klein and Poincar/'e were part of this subject. In the middle of the 20th century algebraic geometry had been through a large reconstruction. The domain of application of its ideas had grown tremendously, in the direction of algebraic varieties over arbitrary fields and more general complex manifolds. Many of the best achievements of algebraic geometry could be cleared of the accusation of incomprehensibility or lack of rigour. The foundation for this reconstruction was (commutative) algebra. In the 1950s and 60s have brought substantial simplifications to the foundation of algebraic geometry, which significantly came closer to the ideal combination of logical transparency and geometric intuition. Commutative algebra is essentially the study of the rings occurring in algebraic number theory and algebraic geometry. In algebraic number theory, the rings of algebraic integers in number fields constitute an important class of commutative rings — the Dedekind domains. This has led to the notions of integral extensions and integrally closed domains. The notion of localization of a ring (in particular the localization with respect to a prime ideal leads to an important class of commutative rings — the local rings. The set of the prime ideals of a commutative ring is naturally equipped with a topology — the Zariski topology. All these notions are widely used in algebraic geometry and are the basic technical tools for the definition of scheme theory — a generalization of algebraic geometry introduced by Grothendieck.

ABOUT INSTRUCTOR :

Prof. Dilip P. Patil received B. Sc. and M. Sc. in Mathematics from the University of Pune in 1976 and 1978, respectively. From 1979 till 1992 he studied Mathematics at School of Mathematics, Tata Institute of Fundamental Research, Bombay and received Ph. D. through University of Bombay in 1989. Currently he is a Professor of Mathematics at the Departments of Mathematics, Indian Institute of Science, Bangalore. He is also a Visiting Professor at the Department of Mathematics, IIT Bombay, and at Ruhr-Universität Bochum, Universität Leipzig, Germany and several universities in Europe and Canada. His research interests are mainly in Commutative Algebra and Algebraic Geometry.

COURSE PLAN :

Week 1: Algebraic Preliminaries 1 — Rings and Ideals

Week 2: Algebraic Preliminaries 2 — Algebras and Polynomial algebras

Week 3: The K -Spectrum of a K -algebra and Affine algebraic sets

Week 4 : Noetherian and Artinian Modules

Week 5 : Hilbert's Basis Theorem and Consequences

Week 6 : Rings of Fractions

Week 7 : Modules of Fractions

Week 8 : Local Global Principle and Consequences

Week 9 : Hilbert's Nullstellensatz and its equivalent formulations

Week 10 : Consequences of HNS

Week 11 : Zariski Topology

Week 12 : Integral Extensions