

INTEGRAL EQUATIONS, TCALCULUS OF VARIATIONS AND ITS APPLICATIONS





TYPE OF COURSE: Rerun | Core | PG **INTENDED AUDIENCE**: M.E/M.Tech, M.S, M.Sc

EXAM DATE : 28 Oct 2018

COURSE DURATION: 12 weeks (30 Jul'18 - 19 Oct'18)

PROF. D.N. PANDEYDepartment of Mathematics
IIT Roorkee



COURSE OUTLINE:

This course is a basic course offered to PG students of Engineering/Science background. It contains Fredholm and Volterra integral equations and their solutions using various methods such as Neumann series, resolvent kernels, Green's function approach and transform methods. It also contains extrema of functional, the Brachistochrone problem, Euler's equation, variational derivative and invariance of Euler's equations. It plays an important role for solving various engineering sciences problems. Therefore, it has tremendous applications in diverse fields in engineering sciences.

ABOUT INSTRUCTOR:

Dr. P. N. Agrawal is a Professor in the Department of Mathematics, IIT Roorkee. His area of research includes approximation Theory and Complex Analysis. He delivered 13 video lectures on Engineering Mathematics in NPTEL Phase I and recently completed Pedagogy project on Engineering Mathematics jointly with Dr. Uaday Singh in the same Department. Further he has completed online certification course "Mathematical methods and its applications" jointly with Dr. S.K. Gupta of the same department.

Dr. D. N. Pandey is an Associate Professor in the Department of Mathematics, IIT Roorkee. Before joining IIT Roorkee he worked as a faculty member in BITS-Pilani Goa campus and LNMIIT Jaipur. His area of expertise includes semigroup theory, functional differential equations of fractional and integral orders. He has already prepared e-notes for course titled "Ordinary Differential Equations and Special Functions" under e- Pathshala funded by UGC.

COURSE PLAN:

- **Week 01 :** Definition and classification of linear integral equations, Conversion of IVP into integral equations, Conversion of BVP into integral equations, Conversion of integral equationsinto differential equations, Integro-differential equations.
- **Week 02 :** Fredholm integral equation with separable kernel: Theory and Examples, Solution of integral equations by successive substitution, Solution of integral equations by successive approximations, Solution of integral equations by successive approximations: Resolvent kernel.
- **Week 03 :** Fredholm integral equations with symmetric kernels:Properties of eigenvalues and eigen functions, Hilbert Schmidt theory, Examples. Construction of Green's function.
- **Week 04 :** Green's function for self adjoint linear differential equations, Green's function for non homogeneous boundary value problem, Fredholm alternative theorem, Fredholm method of solutions.
- **Week 05 :** Classical Fredholm theory: Fredholm first theorem, Fredholm second and third theorem; Method of successive approximations, Neumann Series and resolvent kernels.
- **Week 06 :** Neumann Series and resolvent kernels, Equations with convolution type kernels, Singular integral equations. Cauchy type integral equations
- **Week 07 :** Solution of integral equations using Fourier transform, Solution of integral equations using Hilbert-transform, Calculus of variations: Introduction, Basic concepts.
- Week 08 : Calculus of variations: Basic concepts and Euler's equation
- **Week 09 :** Euler's equation: Some particular cases, A particular case and Geodesics, Brachistochrone problem and Euler's equation.
- Week 10 : Euler's equation, Functions of several independent variables, Variational problems in parametric form
- **Week 11 :** Variational problems of general type, Variational derivative and invariance of Euler's equation. Invariance of Euler's equation and isoperimetric problem, Isoperimetri c problem, Variational problem involving a conditional extremum, Variational problems with moving boundaries.
- **Week 12 :** Variational problems with moving boundaries; One sided variation, Variational problem with a movable boundary for a functional dependent on two functions, Hamilton's principle; Variational principle of least action.

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