REAL ANALYSIS II

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TYPE OF COURSE : New | Elective | UG/PG
COURSE DURATION : 12 Weeks (26-Jul' 21 - 15-Oct' 21)
EXAM DATE : 24 Oct 2021

INTENDED AUDIENCE : Anyone Interested Learners

PRE-REQUISITES : Real Analysis I, Linear Algebra, Multivariable Calculus

INDUSTRY SUPPORT : DeepMind, Microsoft Research, OpenAI

COURSE OUTLINE :

This is the follow-up course to Real Analysis I. This time we deal with differentiation and integration of functions of several variables. First, we set the stage by studying metric spaces with special emphasis on normed vector spaces. Even here we will encounter several deep theorems like the existence of the completion of metric space, the Arzela—Ascolli theorem as well as the famous Stone—Weierstrass theorem. We will then study the derivative as a linear map and prove the famous implicit and inverse function theorems. These theorems will naturally lead on to the definition of a manifold. We will use the language of manifolds to make precise the method of Lagrange multipliers for constrained optimization. Finally, we will take an elementary approach to the Lebesgue integral that bypasses the more abstract and set-theoretic construction via measures. We will prove all the famous convergence theorems. We will also briefly see how our elementary construction can also be quickly obtained using the completion theorems we studied in metric spaces. The final theorem of the course is the difficult Jacobi transformation formula commonly known as change of variables for which we will give a geometric proof. This course is designed for ambitious undergraduate students as well as beginning graduate students in mathematics. Knowledge of the content of Real Analysis I is assumed as well as content of a basic course in Linear Algebra at the undergraduate level. I will also assume the basics of an undergraduate level course on multivariable calculus as typically done in the first year of BSc./B.Tech.

ABOUT INSTRUCTOR :
Jaikrishnan J is an Assistant Professor at IIT Palakkad. He specializes in Complex Analysis.

COURSE PLAN :

Week 1**:** Metric spaces and normed linear spaces

Week 2**:** Compactness and completeness in metric spaces

Week 3**:** The Arzelà–Ascoli and Stone--Weierstrass theorems

Week 4**:** The derivative in several variables as a linear map I

Week 5**:** The derivative in several variables as a linear map II

Week 6**:** The inverse and implicit function theorems

Week 7**:** Manifolds, Tangent spaces and Lagrange multipliers

Week 8**:** A precis of curves and surfaces

Week 9**:** The definition of the Lebesgue integral

Week 10**:** Convergence theorems for the Lebesgue integral

Week 11**:** Multiple Lebesgue Integrals

Week 12**:** The Jacobi transformation formula for Lebesgue integrals