INTRODUCTION TO INTERFACIAL WAVES

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PRE-REQUISITES : Introductory Transport Phenomena / Fluid Mechanics

INTENDED AUDIENCE : Chemical & Mechanical Engineering students

INDUSTRY SUPPORT : Industrial personnel working on two phase flows

COURSE OUTLINE :

The course is an introduction to the rich field of interfacial waves. The first half of the course prepares the student for studying wave phenomena by introducing discrete mechanical analogues of wave phenomena in fluid systems. The basic principles of normal mode analysis are introduced through point-mass systems connected through springs. The exact solution to the (nonlinear) pendulum equation is used to introduce the notion of amplitude dependence on frequency of the oscillator. The Kapitza pendulum is introduced as a discrete analogue for Faraday waves. Basic perturbation techniques are then introduced for subsequent use. The second half of the course introduces basics of interfacial waves viz. shallow and deep-water approximations, phase and group velocity, frequency and amplitude dispersion etc.. Capillary as well as capillary-gravity waves in various base state geometries (rectilinear, spherical (drops and bubbles), cylindrical (filaments) are taught and the corresponding dispersion relation derived. The Stokes travelling wave is derived using the Lindstedt-Poincare technique and the amplitude dependence in the dispersion relation is highlighted. Side-band instability of the Stokes wave is discussed. Introductory ideas in resonant interactions among surface gravity waves are discussed. The fundamental aspects studied in the course will be related to various engineering applications continuously.

ABOUT INSTRUCTOR :

Prof. Ratul Dasgupta is an Associate Professor at the Chemical Engg. Department at IIT Bombay. He completed his Ph.D. at the Jawaharlal Nehru Centre for Advanced Scientific Research in Bangalore and was a postdoctoral fellow subsequently at the Weizmann Institute of Science in Israel. He has been on the faculty at IIT Bombay since 2014. He works on interfacial waves, hydrodynamic stability and the mechanics of amorphous materials employing a combination of theoretical and computational tools and occasionally simple experiments.

COURSE PLAN :

Week-1: Introduction to waves and oscillations, Normal modes of linear vibrating systems with finite degrees of freedom, Eigenmodes (shapes of oscillation) and frequencies, continuum limit
Week-2: Normal modes of a string with fixed ends, a clamped rectangular and circular membrane, Introduction to elliptic functions
Week-3: Nonlinear pendulum: exact solution using elliptic integrals, amplitude dependence of frequency, intro. to perturbation methods, non-dimensionalisation
Week-4: Perturbative solution to projectile equation, regular perturbative solution to the non-linear pendulum, Lindstedt Poincare technique, Damped harmonic oscillator, regular perturbation, method of multiple scales
Week-5: Multiple scales solution (contd..), Duffing equation, Parametric instability and the Kapitza Pendulum, Introduction to Floquet theory
Week-6: Mathieu equation stability tongues, Introduction to inviscid, irrotational surface gravity waves in deep water, boundary conditions, non-dimensionalisation and linearisation, dispersion relation
Week-7: General solution for surface gravity waves, linearised standing and travelling waves, phase and group velocity, Cauchy-Poisson problem for surface waves in deep water: rectilinear geometry, waves in cylindrical geometry
Week-8: Cauchy Poisson problem in cylindrical geometry, Cauchy-Poisson problem for delta function at origin and group velocity, similarity solution, stationary phase approximation, capillary-gravity waves
Week-9: Waves on finite depth pool, shallow and deep water approx., group velocity and energy propagation, axisymmetric Cauchy-Poisson problem with a Gaussian, engineering applications, Rayleigh-Plateau instability
Week-10: Waves and instability on a coated cylinder, waves and instability on a cylindrical air column, physical interpretation, shape oscillations of drops and bubbles, physical interpretation of zero frequency
Week-11: Applications of shape oscillations, Faraday instability on a fluid interface, subharmonic response, Floquet
analysis, atomization from Faraday waves, engineering applications, waves on shear flows & Kelvin-Helmholtz
Week-12: Stokes wave in deep water, stability of Stokes wave (sideband instability), comparison of deep and shallow-water theory, non-linear Schrodinger equation and KdV equation, Resonant interactions among water waves