Networks and Systems
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Lecture-6
Complex Frequencies of Signals

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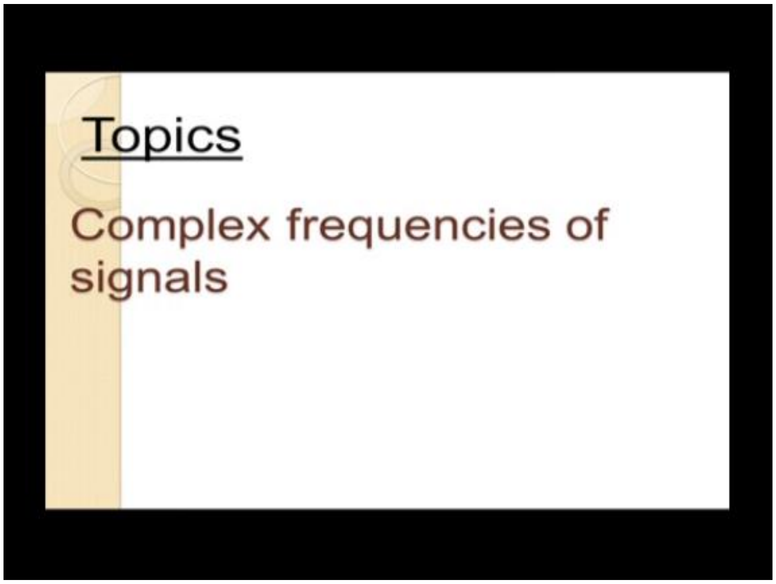

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We were looking at, some of the standard signals which we come across in the study of networks and systems and their waveforms.

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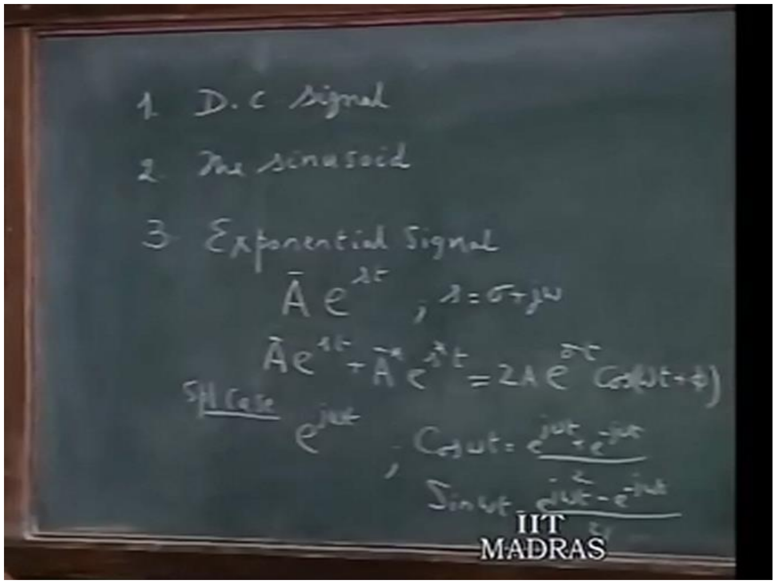

We recall among these standard signal waveforms we mentioned the D.C Signal, then the sinusoid a cos omega t plus theta being its general expression. We also mentioned that the Exponential Signal A e to the power of s t where s is a complex number in general sigma plus j omega plays also an important role.

We term this quantity s the complex frequency, simply for the reason that it has the dimensions of 1 over time something per second and something per second is called frequency and since this is general complex we call it complex frequency. We should not read more meaning into that, we should not interpret this as indicating a repetitive phenomenon like a periodic phenomenon.

We also mentioned that any signal like that in general will yield a complex value for real values of time. So, in order to for us to have a real function of time every such signal is accompanied A e to the power of s t is always accompanied by a conjugate e to the power of s conjugate t. In fact, every time every time is a physical situation where you have such signal this corresponding signal with conjugate coefficient and conjugate of the complex frequency will be present simultaneously.

These 2 will yield a real signal 2A e to the power of sigma t cosine omega t plus phi where A is the magnitude of this complex coefficient a bar and phi is the angle associated with its complex number k. So, this is the and we saw for different values of sigma and omega different natures of their values positive or negative as the case may be we saw how the waveforms look like and so.

Associated with this exponential signal we have special cases e to the power of j omega t. This is a very important signal in that; a sinusoid is closely related to this. This is a special case of e to the power of st. This is also an exponential signal, but the value of the complex frequency is purely imaginary.

And if you have cos omega t this is always e to the power of j omega t plus e to the power of minus j omega t by 2 and similarly, sine omega t is e to the power of j omega t minus e to the power of minus j omega t divided by 2 j. So, sinusoidal functions of time cosine omega t and sine omega t can always be expressed in terms of e to the power of j omega t and this is a special case of a exponential signal with complex frequency.

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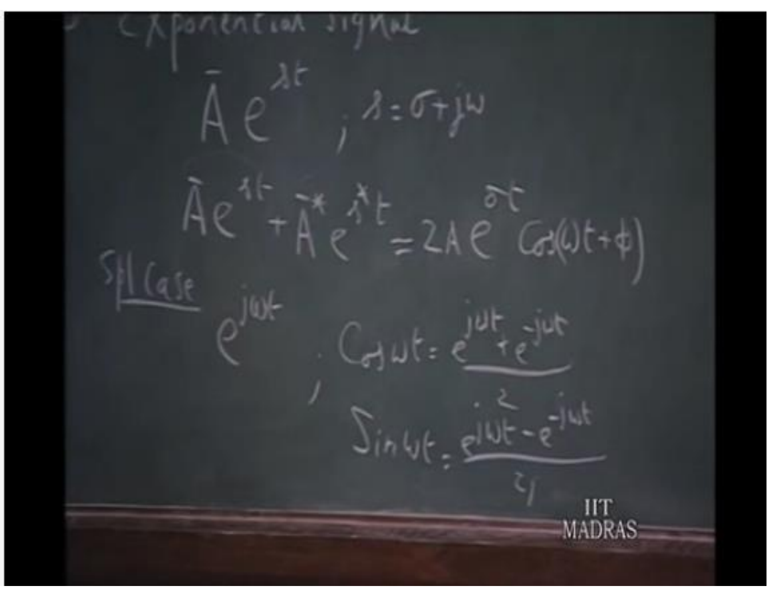

In fact, e to the power j omega t can be thought of as almost synonymous with a sinusoidal quantity. Because after all the real part of that yields cos omega t the imaginary part of this sine omega t and it is always convenient for whenever we have to deal with excitation functions of the type cos omega t or sine omega t we replace them immediately by e to the power of j omega t.

Because, we recognize the cos omega t as the real part of e to the power of j omega t and work with e to the power of j omega t and finally, take the real part of the solution to get the resultant that you would get if indeed excitation had been cos omega t. More about this we will learn later, but we can always say that e to the power of j omega t we can always use that whenever we have to use a sinusoidal functions of time.

Very often that turns out to be case. And In fact, manipulation of e to the power of j omega t is easier than the manipulation of trigonometric functions because you have only 1 function to deal with and differentiation and integration of e to the power of j omega t exponential function is convenient than differentiation and integration of trigonometric functions because you alternately between sign and sign with plus sign and minus sign and so, forth.

It is much more convenient to handle e to the power of j omega t. Now, the characteristic of all these functions are that you can their derivatives exist up to an infinite order. So, D. C Signal sinusoid you can go on taking the derivatives the successive derivatives. They are all smooth functions and all the derivatives exist. Before, we proceed further let me take examples of signals and say what complex frequency terms are present in these signals? So, let us as an exercise let us take an example.
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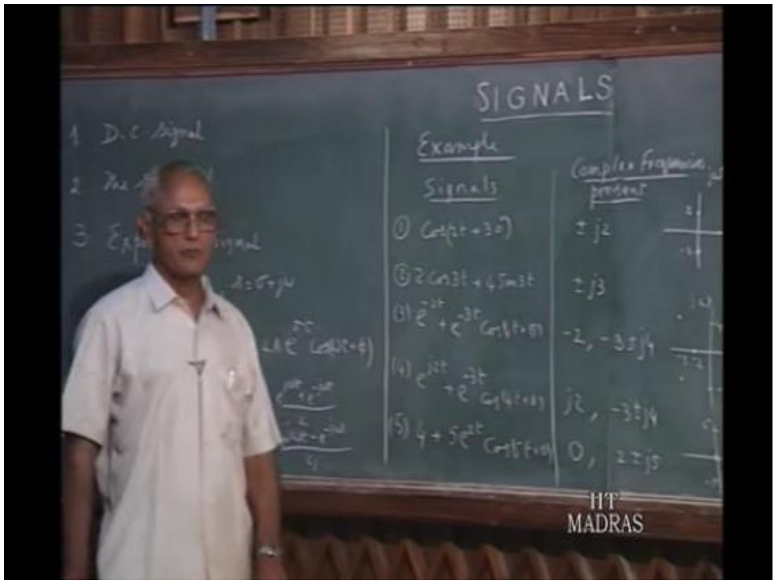

So, i will write down a series of signals which are not the elementary kind that we have been talking about, but then they are composed of signals of this type. e to the power of st complex frequency signals.

Then complex frequency is present in this signal. What are the components which have certain complex frequencies present in the signal? So, we will take 1 by 1 cos 2t plus 30 degrees. Now, this is a pure sinusoidal signal, but as i said cos 2t can always be expressed as e to the power of j 2t and so on; therefore, the complex frequencies present here are plus or minus j2 e to the power of j2t e to the power of minus j2t together those terms will be present with appropriate coefficients a and a conjugate and these are the 2 complex frequencies present.

In the complex frequency plane, this is the complex frequency plane with the x axis representing sigma and the y axis representing omega you have therefore, 2 frequencies at plus 2 and minus 2. Second, suppose I have 2 cos t cos 3t plus 4 sine 3t again you have a sinusoidal. Naturally you can combine these 2 into cos some a cos 3 t plus theta.

But, even if as they are you have the complex frequency present here plus or minus j3 and so, are the complex frequencies here. So, plus or minus j3 or the complex frequencies present in this particular signal and once again you can portray them in the complex frequency plane in the same manner as for the first example. As a third example, suppose I say e to the power of minus 2t plus e to the power of minus 3t cos 4t plus t.

Here, as far as the first signal is concerned first part of this signal is concerned e to the power of minus 2t; therefore, there is a complex frequency there omega is 0 and sigma equals minus 2. So, that is the complex frequency associated with this component.

The complex frequency associated with this component is minus 3 corresponding to sigma you recall, that if you have A e to the power of s t sigma plus j omega e to the power of sigma t plus cos omega t plus p. Therefore, minus 3 is the value of sigma and corresponding to the cosine term you have plus or minus j4. So, as far as this signal is concerned there are 3 frequencies complex frequencies present here minus 2 and minus 3 plus or minus j4.

So, and the complex plane here 1 frequency at this location and another conjugate frequencies at minus 3 this is minus 2 this is 4 this is minus 4 sigma omega. Suppose i have, e to the power of j 2t plus e to the power of minus 3t cos 4t plus theta again as before. Now, corresponding to this you have j2 and corresponding to this as before minus 3 plus or minus j4.

Notice here, that this e to the power of j2 plus j2 there is no minus j 2. That means, this particular signal is not a real function of time. So, very rarely you will come across this you should not come across this for any as long as the signals are real, real functions of time. Therefore, just to indicate that this is not a realistic signal that you are likely to come across I just gave an example.

So, if you substitute a particular value of t you get an imaginary term. So, this is not a type of signal you are likely to encounter. Whenever you have e to the power of j 2t, you also should have e to the power of minus j 2t. So that, those 2 together will give rise to a real function of time. But mathematically if you are looking at this hypothetical signal these are the frequencies present there.

Lastly, suppose I have 4 plus 5 e to the power of 2t cos 5 t plus theta. Now this constant terms, D. C can be thought of as a special case of this where both sigma and omega are 0 when s equals 0 this is a constant therefore, a D. C corresponds to s equals 0. So, you have frequency corresponding to s equals 0 and corresponding to the second term you have plus 2 plus or minus j 5.

So, only in the complex plane you have a D. C term corresponding to a point sitting right at the origin and now 2 points corresponding to 2 plus or minus j 5 and what it means is these 2 frequencies together will give rise to a term which is e to the power of 2t cos 5t plus theta or whatever it is.

Therefore, it is, a growing oscillations a sinusoid which is growing in its amplitude and generally, those are signals which are you are likely to comes across as far as a physical situation is concerned because they can you can come across such signals for a limited period not they cannot go on indefinitely because they reach infinite proportions with respect to time.

So, as long as you have a sustained signal which can last for a long time or forever then you need to have a negative real part because you have seen from the waveforms of the complex exponential signals for different values of sigma and omega that we plotted in the last lecture that if you have the real part of the complex frequency positive that represents a growing signal.