Networks and systems
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Lecture 5
Signals Elementary Continuous Signals

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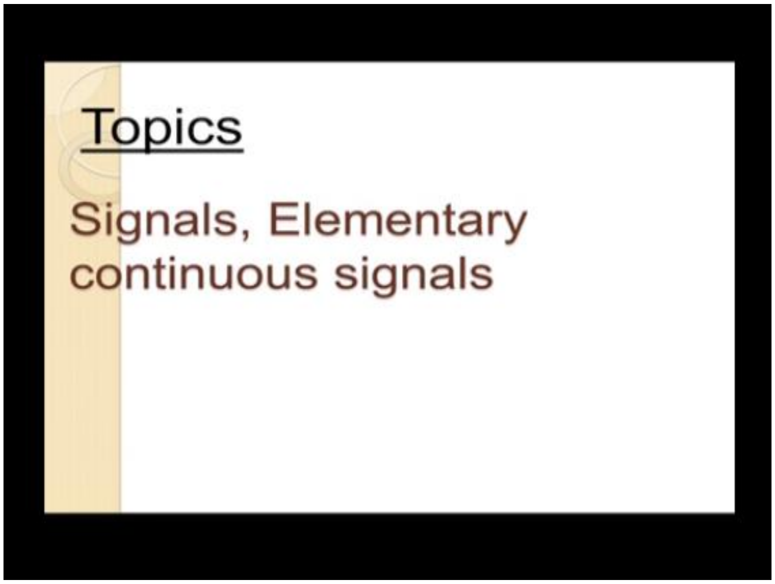

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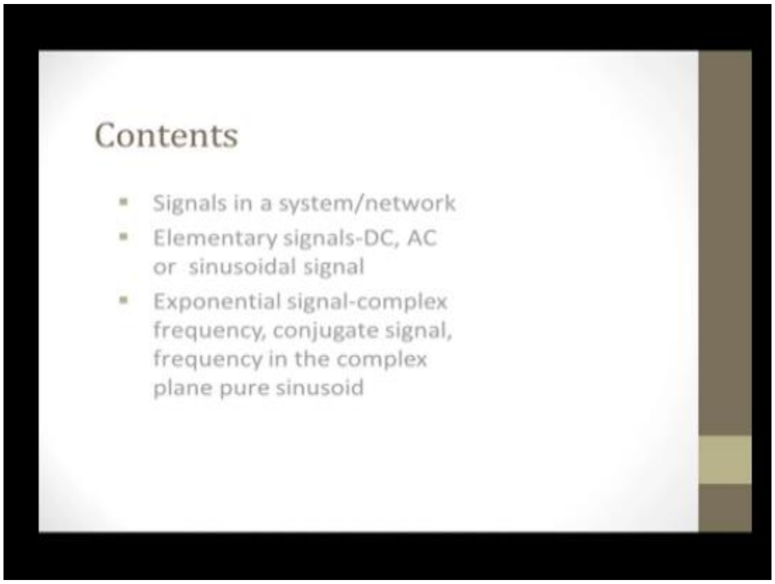

Literally a signal is a means of conveying some information but in the context of systems we take the meaning of a signal to collectively indicate the various variables which describe the status of the system at any particular point in the system or at any particular point of time. As far as electrical networks are concerned the variables or the signals that we deal with are the voltages and currents.

It is the signals as I mentioned earlier which show to say give the breadth of life to a system or a network because in the absence of signals the network or signal or system is completely lifeless and therefore an analysis of signals and a knowledge of the various kinds of signals that one comes across is important for us and when we are dealing with the dynamic of systems and networks the signals are functions of time.

We would have naturally an infinite variety of a signal possible which are functions of time. However, a few special kinds of signals are important for us because of their simplicity for one thing.

Later on we will also argue that any composite signal, any general signal can be decomposed in terms of this elementary simple signals and therefore a study of these signals is of the simple signals is important for us and we will see we will first of all review some of the signals which are familiar to us already and introduce new types of signals which will be found useful in our study.

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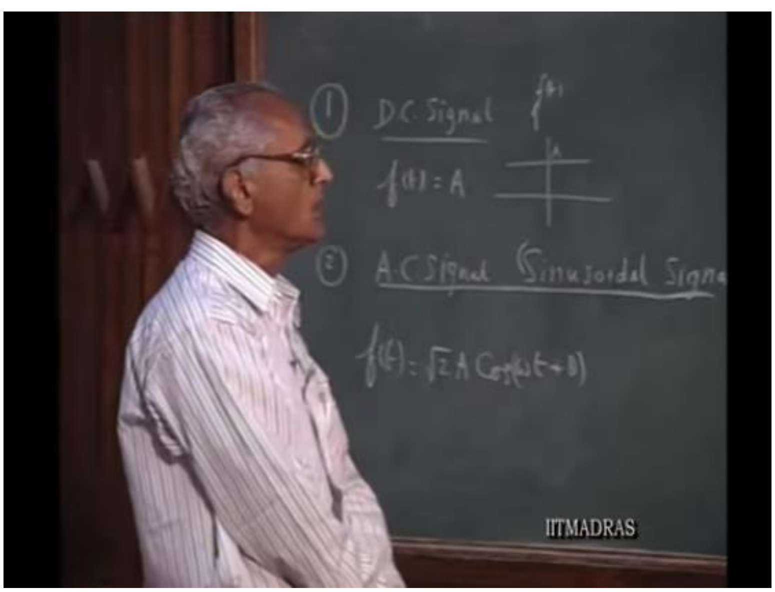

Let us now talk about some various types of signals elementary of signals that we are interested in 1 a D.C. signal, so f of t is a constant a is independent of time okay so nothing be further need to be said about this so we will leave it like that. Then we have an A.C. signal what we mean by that a sinusoidal signal. So a general form of the sinusoid f of t route a cos omega t plus theta and the whole A.C. circuit analysis is based upon signals of this type and you know the importance of sinusoids in a circuit analysis because this sinusoid has got a very distinctive property.

This is the only periodic signal which retains its shape, which retains its general waveform under the linear operations of additional of 2 sine waves of the same frequency subtraction of one sine waveform another the same frequency, of course differentiation of a sine wave will lead to another sine wave of the same frequency, integration of a sinusoid will lead to another sinusoid of the same frequency.

So under the linear operations of addition, subtraction, differentiation and integration the sinusoid retains it character the same wave shape retains is retained. No other periodic function has got this wonderful property. The result therefore is that if you have a whole system with sinusoidal sources of the same frequency distributed all over then when a sinusoidal current passes through an inductor it produces a voltage which is sinusoid of the same frequency.

When a sinusoidal voltage is applied a capacitor it produces a current a sinusoidal same frequency. Therefore all these currents and voltages in the entire system as for example in power system where we have got different sources of the same frequency. All currents and voltages ideally never point in the system are sinusoidal at the same frequency.

If we did not have this wonderful property then a sinusoidal current in a inductor will produce a non-sinusoidal some kind of. If you have non-sinusoidal current in an inductor you will have a waveform of the voltage which is completely different and if we have all these different types of waveforms mingling with each other in a complicated system we will certainly it will drive 1 to insanity if 1 has to analyze this because the waveforms so unpredictable and so complex.

So sinusoid has got a wonderful property and it stands out as I said this is the only periodic signal which has got this kind of property. And we have phasor notation which deal with sinusoidal signals analysis of systems driven by sinusoidal signals. So we know all the about this from the A.C. circuit analysis so we will leave it at that except to remark at this stage that this is really a very wonderful signal.

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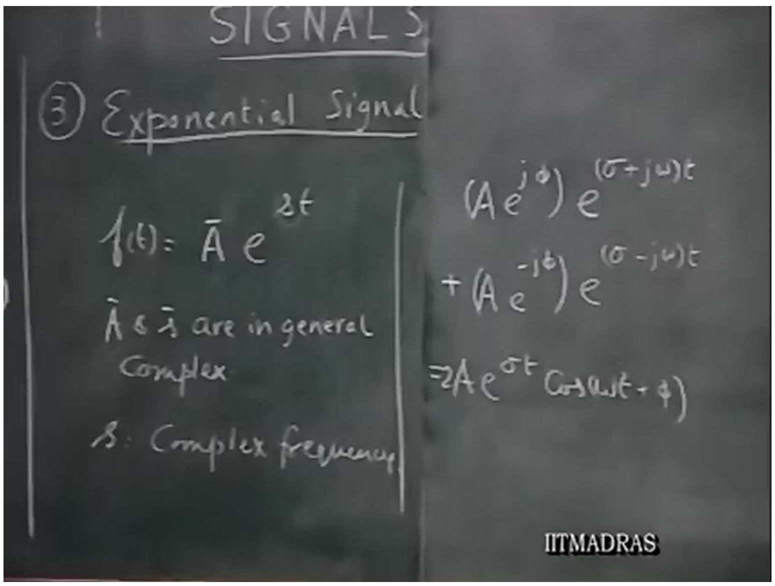

Then we have the exponential signal. So the general form of an exponential signal would be f of t is a I will put say okay a is a complex number e to the power of st where s is also could be in general complex a and s are in general complex. Now mathematically therefore when you substitute a value of t here because s is complex and the coefficient a is complex you get a complex value for this so this is a complex exponential signal.

Now what about the dimensions of s. You know from your may be some of your earlier courses that whenever you have a physical equation in which an exponent appears the argument of the exponent this exponential function that is st must be dimensionless.

Similarly, sine omega t whatever it is that argument must be dimensionless. So st must be dimensionless otherwise you can't match the dimensions on both sides of an equation. If st had a dimension then e to the power of st has 1 plus st plus s square t square by 2 so on and so forth. The dimensions of that you do not know what they are because if it is st has the dimension of some x some say m or l or whatever it is your m square l square and so on so because of these reasons this must be dimensionless.

If this is dimensionless then the dimension of s must be something per second st is dimensionless therefore s must be something per second. And what is the quantity which is something per second it is called frequency therefore we termed this quantity s as frequency and because it is in general complex we called this a complex frequency.

So s is termed to be termed a complex frequency. It is not frequency in the sense something is repetitive in character like in a periodic function but because its dimensions are something per second therefore it is called frequency this is called complex frequency.

So this is the most general kind of exponential signal. Then if it is going to yield a complex value for f of t what is the use for this that is the question that naturally arises. As it turns out that whenever we have to deal with real system real signals any complex of this type will be always be matched by its mate such that the some of these 2 will always be yield a real term.

For example if I have A for example suppose is a complex number Ae to the power of j phi that is complex number A supposed I put it in this form and e to the power say s is equal to sigma plus j omega t such a complex signal this is a the same signal I am writing for s sigma plus j omega and for complex number a i am writing a e to the power of j phi I have written this.

This will always be matched by its counter part which we have the coefficient its conjugate and like wise the complex frequency also will have its conjugate usually these

2 come together. So if you have Ae to the power of st you will also have a star e to the power of s conjugate t these 2 will always occur together. So when you combine this two you observe that you have Ae to the power of sigma t and e to the power of j omega t plus 5 and e to the power of minus j omega t minus 5. Therefore you will have 2A cos omega theta so this what ___fields so this is the real signal.

So we don’t have to the particularly alarmed about the fact that this is going to be yield the complex number because we can be sure that in the in an analysis of any system that we going to take up this will always be matched by a conjugate quantity of the entire mate like this. So the some of these 2 will always be this type.

Now depending upon the location of the complex frequency here in the complex plane you have different kinds of behavior as for the time dependence of the signal is concern.
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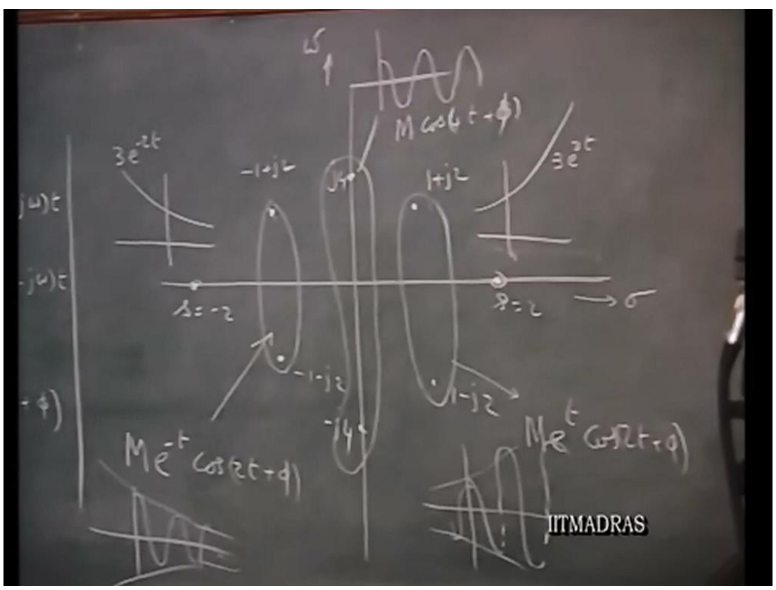

Let we illustrate this here. So the complex value of s can be represented in the complex plane like this with x axis representing sigma and the y axis representing omega. So suppose if the s value of s equal to minus 2 let us say a particular location s equal to minus 2.

That means the value the general value of s now takes a real value this represents a signal which will be say Ae to the power of minus 2t. So you will have something like this. So the time dependents of that let suppose let us take A to be a real number because it doesn’t have a this real quantity therefore you cannot have a conjugate of this complex frequency. Therefore A has to be real so for example a signal like this may be 2e 3e to the power of minus 2t.

On the other hand if I have sorry s is equal to 2 s is equal to sorry sorry I made mistake here if you s equal to 2 here. This represents an increasing signal like this for example of that means exponentially growing. If I take x equal to minus 2 then this is exponentially decreasing signal may be this is 3e to the power minus 2t.

On the other hand if I have 2 complex frequencies which are conjugates sufficient are like this say this value equals minus 1 plus j2 and this is minus 1 minus j2. This 2 frequencies together this 2 terms like this will give rise to a response which is at the form some m e to the power of minus t minus sigma is now minus 1 cos 2t something like this so that means you have a decaying sinusoid.

That’s how the quantity will vary with reference to time. On the other hand if I have 2 conjugate complex frequencies with positive real parts 1 plus j2 and 1 minus j2 such 2 terms will yield a function of time which is m e to the power of t cos 2t plus v and that would yield a time function which is growing in time something like this.

On the borderline between the left half plane and right half plane suppose I have a two frequencies let say j4 and minus j4 this two terms will yield a pure sinusoid and will have a say typically m cos 4t plus theta the mn theta or m n phi depending upon the coefficient complex coefficient either that you are having.

So that means your complex frequencies exponential signal like this n can passes in its generality the whole lot of time functions of this type. It can be decaying exponential signal where s happens to be real and negative it can be growing exponential where s happens to be real and positive and if s happens with purely imaginary a pair of such frequencies will give rise to a sinusoidal signal pure sinusoidal signal.

On the other hand if s have a negative real part it indicates a decay in oscillations and if s real part of s represent positive think it has a increasing oscillations. So we have this e to the power of st has a special case the our sinusoidal signals. That means some of 2 such exponential functions will give rise to a pure sinusoidal.
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So we have take a now a look at 3 different kinds of signals which have which are namely the D.C. signal, the sinusoid and an exponential signal e to the power of st and I mentioned that even though exponential signal in general is used as a complex value for real time t but 2 such signals can combine and will give rise to real function of time. So the value at any t earlier point of time t will turn out to be real.

We also have acquainted ourselves with the meaning of the term complex frequency it is mainly because that particular coefficient of t in the exponential representation that exponential signal has a dimensions of frequency we call it a frequency it does not necessarily mean that it give sized a periodic quantity cause none of these signals are purely periodic.

Of the characteristic of all these 3 signals that were talked about so far is that there smooth functions of time that means not only they do not have discontinuities but you take can take derivatives of this signal as I continued to have continues at there for this a characteristic of the signals.

So in the next lecture we will consider some examples bearing on this concepts particularly exponential signal in the complex frequencies and we will also go on to discuss some signal wave forms which are not continuous but which are important in our further study.