Networks and Systems
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Lecture-4
Time Invariance and Causality

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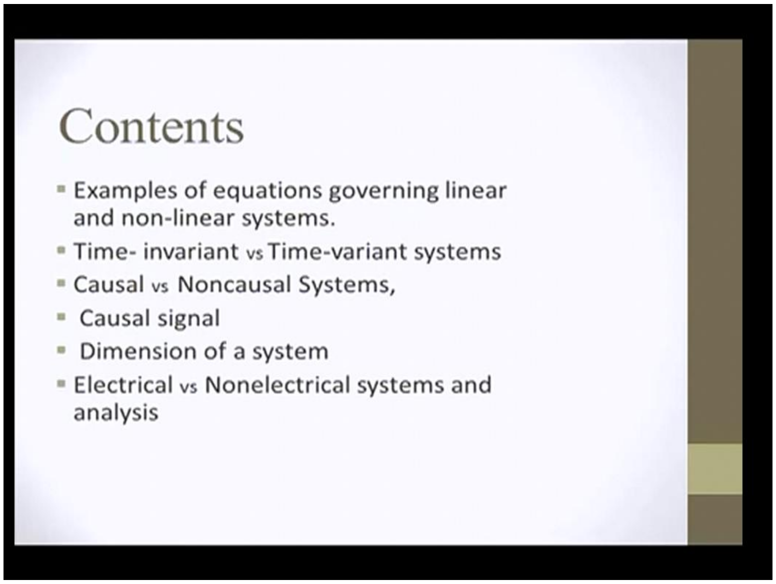

In the last lecture, we were considering the classification of systems and networks. We saw the difference between a static system and a dynamic system. The difference between a continuous time system and a discrete time system. We also observed the difference between a linear system and a nonlinear system.

Basically, we said a linear system obeys the principle of superposition that is a combination of additivity and homogeneity and we promised ourselves at the end of last lecture that we will look at some examples of the equations governing the performances of a linear system and a nonlinear system and this is what we propose to do to start with.

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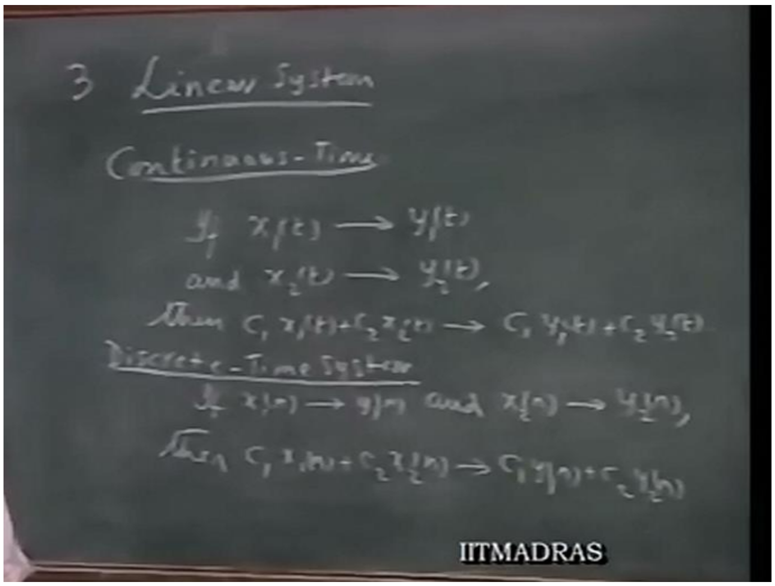

So you recall that we said a linear system satisfies the principle of superposition if it is a continuous time system then if x 1 of t is an input gives rise to an output y1 of t and x2 of t gives rise to an output y2 of t, then a linear combination of these 2 inputs c1 x1 of t plus c2 x2 of t for any arbitrary fair of constants c1 and c2 will give rise to an output c1 y1 of t plus c2 y2 of t.

On the other hand if we are talking about a discrete time system naturally now the signals will not be a function at the continuous variable t but their functions have a discrete variable. Let us say n then the same statement will be carried over in this domain as if x1 n gives rise to an output y1 n and another arbitrary input x2 n gives rise to an output y2 n then if the system is linear if the discrete system is linear then c1 times x1 n plus c2 times x2 n will give rise to a similar linear combination of the corresponding outputs.

Now the input, output relations of a continuous time linear system will be described by differential equation a linear differential equation and here the corresponding equation will be called a linear difference equation.

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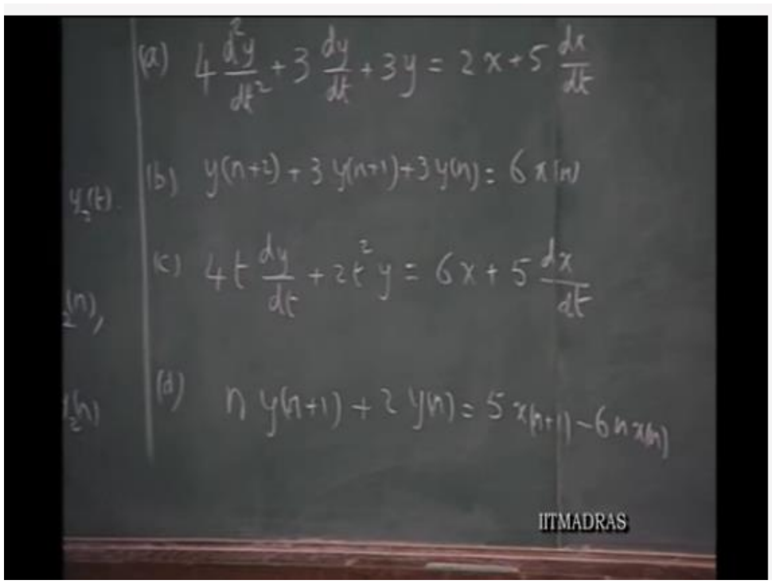

Let us look at with some examples, examples of equations pertaining to linear systems a) 4 d square y by dt square plus 3 dy by dt plus 3y is 2 x plus 5 dx by dt. So x is the input and y is the output all the coefficients and the derivative terms are constants.

So this is certainly a linear differential equation of order 2 with constant coefficients therefore this is an example of a linear the equation corresponding to a linear continuous time system. b) y of n plus 2 plus 3 y of n plus 1 plus 3 y n let us say is 6 x n. This is an example of the system equation characterizing a discrete time system which is linear again the coefficients of the various output terms and the input terms are constants and therefore this is again a linear difference equation with constant coefficient.

So this is a characteristic of a linear discrete time system. Let us take another example 4t dy by dt plus 2 t squared y equals 6 x plus 5 dx upon dt. Now here we have the coefficients of the various derivative terms has the functions of time 2t 2 squared and so on. This also is an equation pertaining to a linear system.

One can easily manipulate this equations find out the y1 which satisfies this of the given input x1 a y2 which satisfies this of the given input x2 and combined these two to show that if x1 of t gives rise to y1 x2 of t gives rise to y2 of t this condition is satisfied even here.

In other words if we have a linear system it does not mean that the differential equation should have only constant coefficients even if the coefficients are functions of the independent variable which is t in this case even then it corresponds to a linear system. So this is also an example of a linear system.

A corresponding equation for a discrete time system would be something like this n y n plus 1 plus 2 y of n equals let us say 5 x n plus 1 minus 6 n x n. So you see here the coefficients are not constant but functions of n this at least this location you have got n times y n plus 1. So this corresponds the independent variable here is t, the independent variable here is n.

Therefore, if the coefficients are functions of the independent variables still it is a linear system it satisfy this kind of relationship and therefore these are examples of systems equations which pertained to linear systems.

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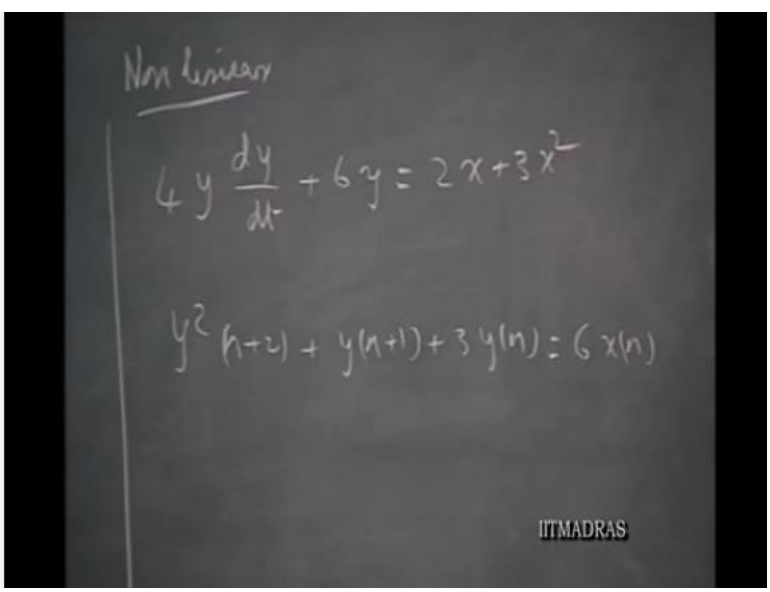

Suppose, I take a nonlinear system then the coefficients are not necessarily constants or functions of the independent variable the characteristic of the differential equation corresponding to a nonlinear system is that the coefficients are functions of the dependent variable y. Examples 4 y dy by dt plus 6y equals 2x plus 3x square.

So, you see that y is multiplied by dy by dt and also you have an x square term here and these are the two factors which destroy the linearity of the equation. Similarly you have y squared n plus 2 suppose I have plus y n plus 1 plus 3 yn equals 6 x at n. y n plus 2 is multiplied by y n plus 2 so you think of this the coefficients being y n plus 2 multiplying y n plus 2 so this the square term that is involved here so this fails to be a linear difference equation.

So, essentially to summarize what to we have is in the case of a linear system whether it is a differential equation or the difference equations the coefficients of the various terms the dependent variable or its derivative or its incremented terms like this should be either constants or functions of the independent variable t or n at the case may be. But if the coefficients turn out to be functions of the dependent variable then it fails to be linear it belongs to the category of nonlinear difference or differential equations so we will leave it at that.

The next category of, the next classification that we will talk about is the difference between time invariant system and time variable systems.

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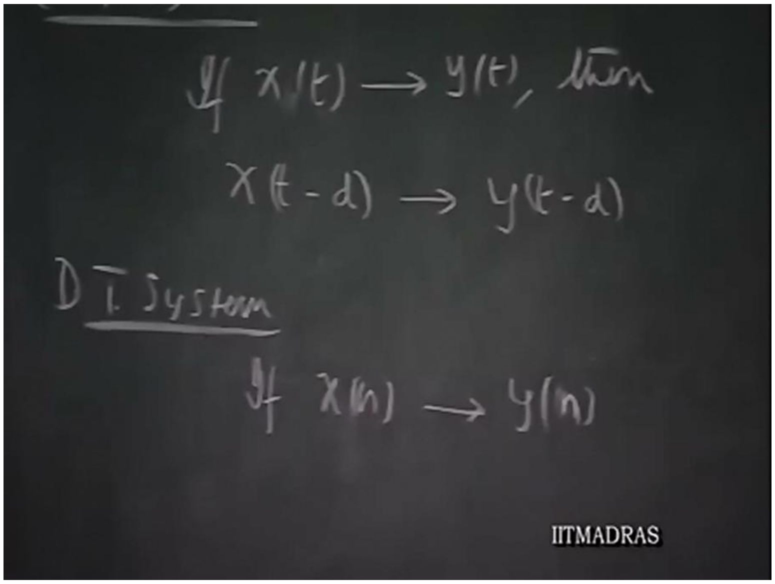

So we have the classification constant parameters versus variable parameter systems. Another name for constant parameter system is time invariant system these 2 are equivalent. Another words the parameters which characterize the system the parameters in various components which constitute the system are constant with respect to time.

Take the case of an electrical network if r l and c are fixed in respect to time with respect to time is called a constant parameter system. On the other hand suppose you have a device in which an inductance and resistance is continuously changing with respect to time then it becomes a time variable system.

For example, if I have a carbon microphone and depending upon the input signal that you are having the resistance of the microphone may changing so that becomes a variable parameter system or a time variable system. So, variable parameter system is also sometimes referred to as time variable system.

Generally, we will be interested in talking about time variable system or time invariant system with respect to linear systems that is because that is the main focus of our work. So if you look at these 2 equations here before equations here a and b how the derivative terms with constant coefficients and this is characteristic of a time invariant system or a constant parameter system.

On the other hand, in these 2 equations we have coefficients functions of time or n as the case may be these are time variables systems. So, these describe the operations of some time variable systems. So, both are linear, but one these first two equations belong to the category of time invariant systems the later two belong to the category of time variable systems.

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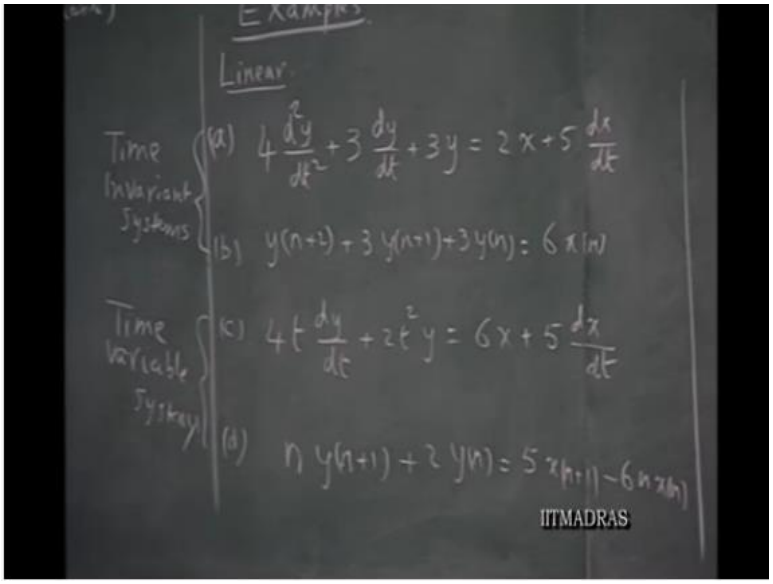

Now, the importance of distinguishing between these 2 class of systems. First of all the solutions of these equations is much more simpler than the solutions of this equations because the time factor t is involve here and further there is a very useful property of time invariant systems which I will discuss presently and that makes you that makes analysis of that systems for simpler than what it would be in the case of time variable systems.

It goes like this if you take a continuous time system. If you have an input say xt which used as output yt then suppose this x of t is delayed by translated in time by certain interval x t minus d then such an input will give a response which is similarly delayed in time by d units of time no matter what t you take.

For discrete time system we have if x of n these rise to an output yn and if you consider another input signal which is same as xn but delayed by d or k instance or n instance n minus capital n variant is a fixed number. So, whatever is occurring at n equals 1 now is occurring at n plus 1 units then the corresponding output will be y n minus n.

So, that means the response will be shifted by the same amount but the shape of the response will not be changing at all. Let we illustrated this by means of some figures.

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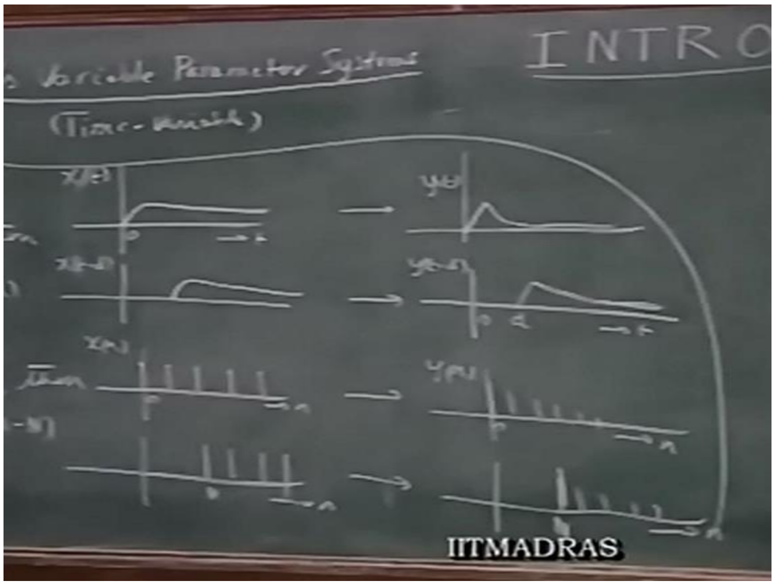

Suppose I have x of t and this gives rise to a response yt like this. Then suppose delayed this signal by d units so instead of 0 it starts here so this is x of t minus d. Then if it is a time invariant system you are sure that the output will be the same as this but starts little later. This is y of t minus d so the same response same well shape except that it is delayed by the same amount as the input is delayed.

So, this is a feature of time invariant systems and this is a very useful feature as we will see later on when we talk about how use impulse responses to characterize the find out the input for find out the response for general input. In the case of a discrete time system let us save we have an xn a signal like this xn is a function of discrete value n and let us say this gives rise to response yn like this.

Of course, discrete time signals we have values only at a discrete points along the time axis and so the interval between 2 points may be any arbitrary value depending upon the system. So let us say for interview values of certain sequence of values xn and corresponding values of yn.

Now, suppose I delay the signal by n units so that means I starts here. So instead of 0 it starts at n that is your xn otherwise it is the same and then the corresponding y of n would be starts not at 0 but at point n here and then it is as if the whole output is translated along the x axis by n units.

So that is how the response will be occurring for a shifted input signal these particularly as you can see this particular property will be very useful in the case of when you want to superposed the responses due to different inputs as we see later this is all for time invariant.

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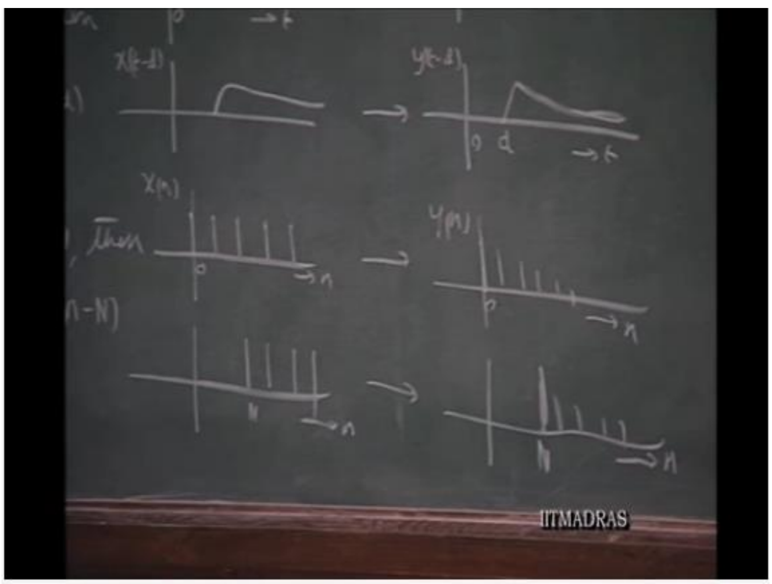

In the time variable case we cannot have this kind of property because as you can appreciate when this x of t is on applied to the system the parameters are certain values and therefore there is certain response. But if we accept t has been delayed by this amount and you are talking about the application of this time t equals d at this time the parameters of the system might have undergone some change there are no longer the same values that are adjusting at this point.

Therefore, even though the input has a same shape the output need not follow the same shape and that is the reason why this such property will not be valid for a time invariant systems time variable system?

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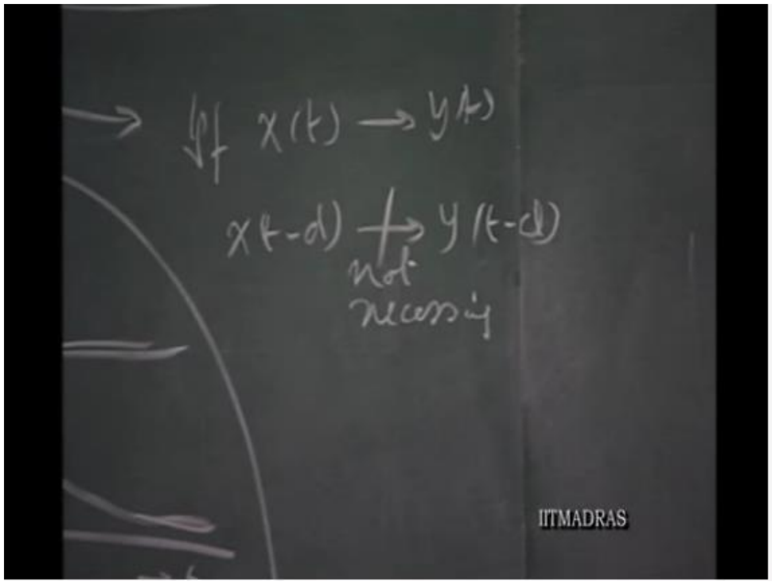

Time variable system if x t gives rise to yt then x t minus d not necessarily give rise to y of t minus d that means it is not necessary that it is just gives rise to that particular response. So, you cannot say anything about it unless you know the actual behavior at the variable with respect to time.

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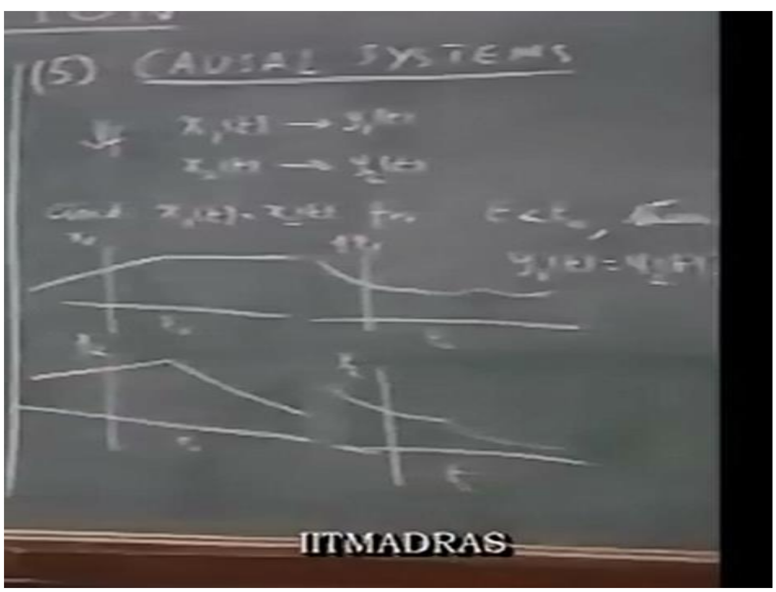

The next property that we will talk about next classification of system or is causal systems and those which are not causal call non-causal systems. A causal system is one which is defined as follows if you have two inputs x1t which gives rise a to response y1t and x2t which gives rise to response y2t.

And further let us say x1t equal x2t for t less than some point t0 that means up to some point t equals t0 both are inputs are the same. So, you may have x1 of t up to point t0 and afterwards may be it goes like this and you have another x2 which also far as the same wave form up to top point t0 but afterwards may be deviates from the values with x1t. Then if this so then for a causal system we can expect that y1t equals y2t for t less than 0.

So, the corresponding outputs here if y1 had some output like this up to t0 and afterwards may be it goes like this. We can say that y2 also we have the same response may be from here onwards that means up to the point t0 the same input the inputs are the same the outputs will be the same such systems as said to be causal system.

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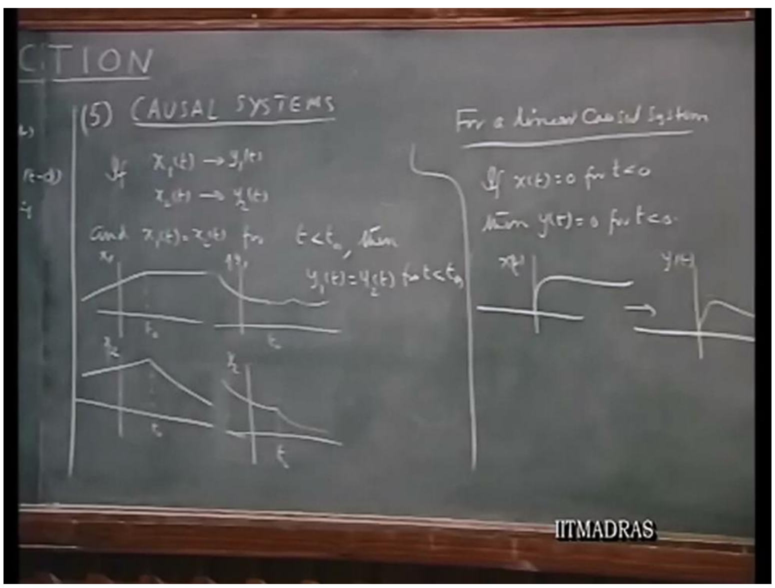

Now, if as a this is general definition of causal system in particular for a linear causal system if the system is not only causal but also linear then you can easily see if I have an input which is x1 minus x2 which means up to time t0 x1 minus x2 is 0 then the output must also be 0 because of superposed these 2 this also will be 0 or we can take t0 is equal to 0 as a general case we can say if xt is 0 of t less than 0 then the output yt is 0 that is the consequence of a linear causal system.

That means the whatever input you had if it is 0 to this point and this is your xt your output must be 0 up to this point and later on whatever output you get have it but it must less than be 0 for t less than 0 that means before you apply the input you cannot get an output which is seems to be quite reasonable for physical optimist and therefore we believe that all physical systems follow this causality principle.

And a system which is not causal is referred to as non-causal or anticipating systems. This is the opposite of the causal system we believe as I said all physical systems that we can built follow the principle of causality. Very often one describes causality principle in these steps but that is not quite correct in the sense. If I have a system like this and i is the response and your vi is the input so vi is the input and vi is the response even vi is 0 you are having a kind of current there I in the circuit.

Therefore, we cannot say that this particular system follows this.

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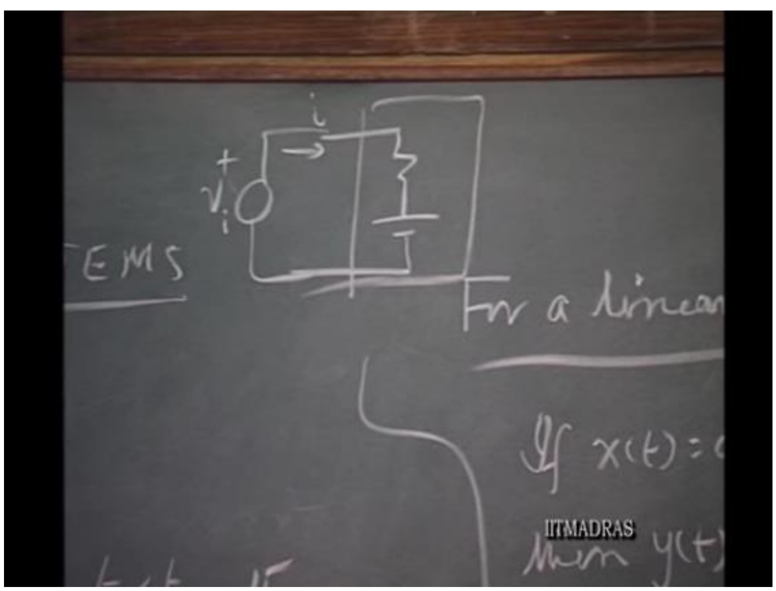

If x of t is the input of the vi voltage and current i is the yt i will existing if y vit is 0. Therefore, if you start a signal vi of t from t equal 0 onwards it does not mean the current will be 0 because the current will be delivered little bit. But this is not a linear system because that the presence of the source you are not have a proportion and relation between b and i that is not a linear system the resistance part is linear but the source destroys the linearity of this.

So, this particular property is not valid for this. But on the other hand, this part this property will be valid for the system. If you have two voltages vi1 and vi2 that will give the same response as long as vi1 is equal to vi2 up to t equals t0. So this is the more general definition of causality and for a particular special case of linear causal system we can simply say the input is 0 for time t less than 0 the output will likewise be 0 for time t less than 0.

Now another terminology that we can introduce at this stage is convenient to use such as term causal signal is 1 which is 0 for t less than 0. So a causal signal is 1 which is normally defined as 1 which is 0 for t less than 0 that means here the except t is the casual signal it is 0 for t less than 0. So one can say that the property is that even linear causal system as causal signal as the input the output will also be a causal signal right.

Because when you talk about transient linear systems and so on it would be nice to have a term which describes all the signal which are 0 for t less than 0 you would call such signals causal signal. So we can say a causal signal given as an input to a linear causal system will produce a response which is also a causal signal.
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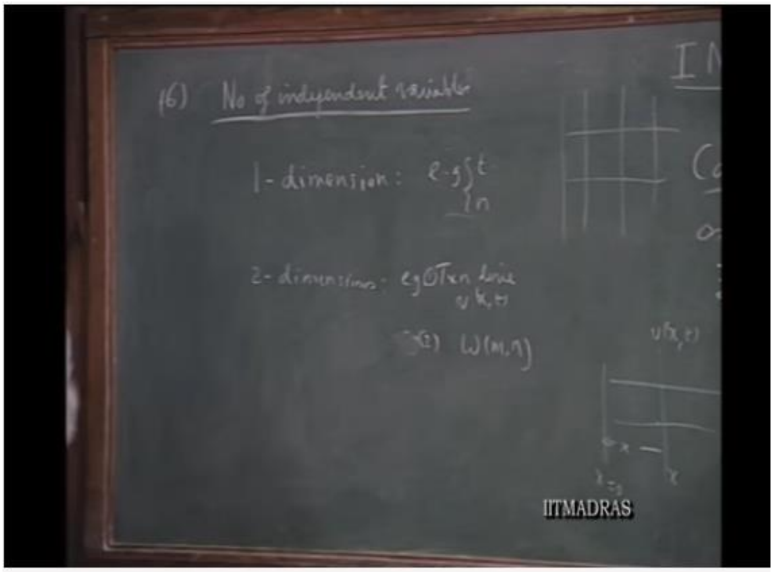

We will have one more property classification which we will discuss that is the number of independent variable. Systems can be classified depending upon the number of independent variable that we have to reckon with in the system and this number of independent variable is also referred to often at the dimension of the system.

So one dimensional systems the transient in electrical circuits are one dimensional systems because t is the independent variable for example t or l in the case of a discrete time system. Let us on the other hand think of the transient that arise a transmission line you have a transmission line and therefore at any point x you have the voltage between these 2 is a function of x and the time t. So x is 0 from staring sending a let us this is x therefore the voltage is here depends upon 2 independent quantities x and t. So along the line the voltage will change and not only with the distance but also with respect to time.

So, you may have 2 dimensions example transmission line where the currents and voltages of functions of x and t. Similarly you may have a grid like structure and you are describing some parameter here as a function of the x coordinate and y coordinate therefore some quantity w which is a function of the m and n 2 dimensions. These are called 2 dimensions next course we can continue with other dimensions also for example if we have a field problem electromagnetic field or electrostatic field so it is a function of x y z and perhaps if it is a dynamic you also have function of time.

So it may be 4 dimensions and when you deal with such multidimensional problems in the case of continuous system you have partial differential equations to contain with and in the case of a discrete time system you have multidimensional discrete difference equations that you could deal with.

So after having looked at the classification of the various systems you would like to say that as far as this course concern we will be talking about linear time invariant systems which are causal of course and which are 1 dimensional because we are talking about only one variable at a time t or n as the case may be and however we will and also dynamic. Dynamic, linear, time invariant, causal,1 dimensional systems this is the main focus of our course.

We will also have we will be talking both about continuous side time systems as well as discrete time systems perhaps where 70% of the course material we have to do with continuous time systems and 30% with a discrete time systems which we will take up at the end of this course.

Even though we are talking about the systems in general we would be taking specific examples from electrical networks as examples of systems and the variables therefore will be voltages and currents and the independent variable will be time.

In the matter of analysis of dynamic systems electrical engineers have all at always head start over others because they developed powerful tools for the analysis of linear systems under various excitation patterns the impedance concept, the phasor notation and the early application of operational methods for dealing with dynamic systems are all due to electrical engineers.

And furthermore electrical engineers had also an access to very sophisticated experimental techniques or measurements of the various parameters under dynamic conditions, high speed recording and a waveform observation. Not only for verifying the results from theory but also to gather data which will be useful to supplement the theory.

With the result that the methods developed by for the solution of electrical networks can be profitably employed by for the solution of other class of networks as well. So we often find that systems and networks of other kinds like mechanical systems or mechanical networks, acoustical networks or acoustical systems, hydraulic systems and so on are usually are sometimes modeled in terms of electrical circuits.

The analogous as a electrical circuits for a given mechanical system is setup and you analyze this electrical system with all the gamut or the techniques that are available for the electrical circuit analysis and once you have that you translate the results back to the original domain interpret the results suitably and then you get the solution for the non electrical system.

For experimentation also this is convenient so instead of having to do with a large masses, springs and dash parts and so on, You can set up a r l c circuit which is a replica which simulates the actual mechanical system and carry out all these experimentation in terms of voltages and currents which are easy to measure in the lab and then interpret the results suitably in the original domain,

So when we study electrical networks and the dynamic performance of the electrical networks in this course. We have the assurance that whatever techniques we employed and whatever methods we used here can also be profitably employed for other kinds of networks and other kinds of systems.

With this we close our discussion of the introductory remarks for this course. Next we will take up the consideration of the different signals that we come across in our discussion of linear systems and networks.

Literally a signal is a means of conveying some information but in the context of systems we take the meaning of a signal to collectively indicate the various variables which describe the status of the system at any particular point in the system or at any particular point of time. As far as electrical networks are concerned the variables or the signals that we deal with are the voltages and currents.

It is the signals as I mentioned earlier, which show to say give the breadth of life to a system or a network because in the absence of signals the network or system is completely lifeless.