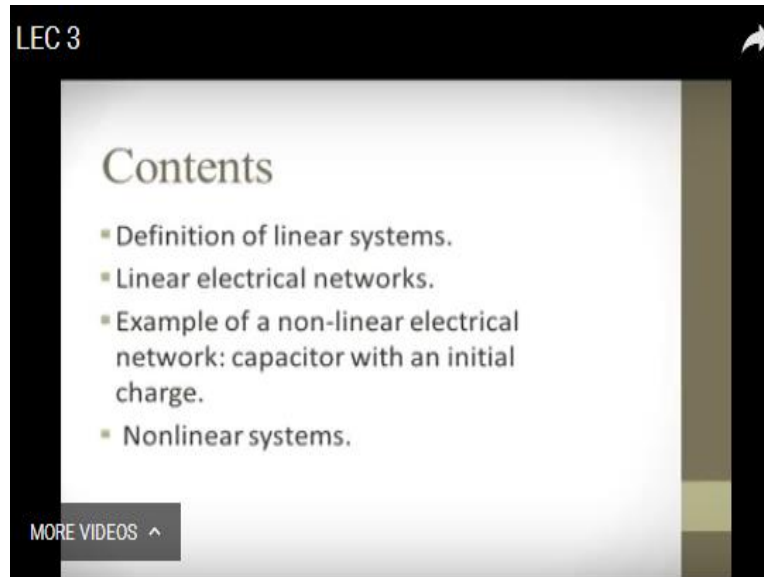


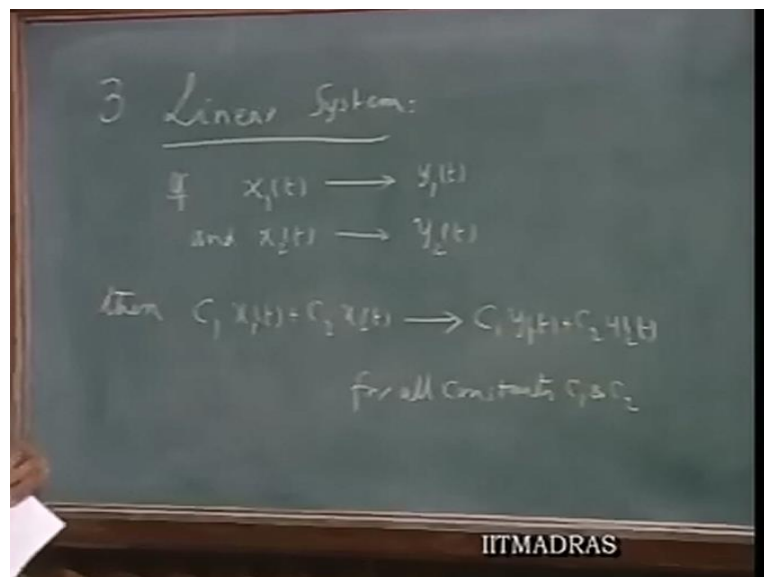
Networks and Systems
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Lecture-3
Linear Systems

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A third important classification is the concept of linear system and non linear system. In a linear system if an input x_1 of t I am talking about a continuous time system. Now this is our discussion

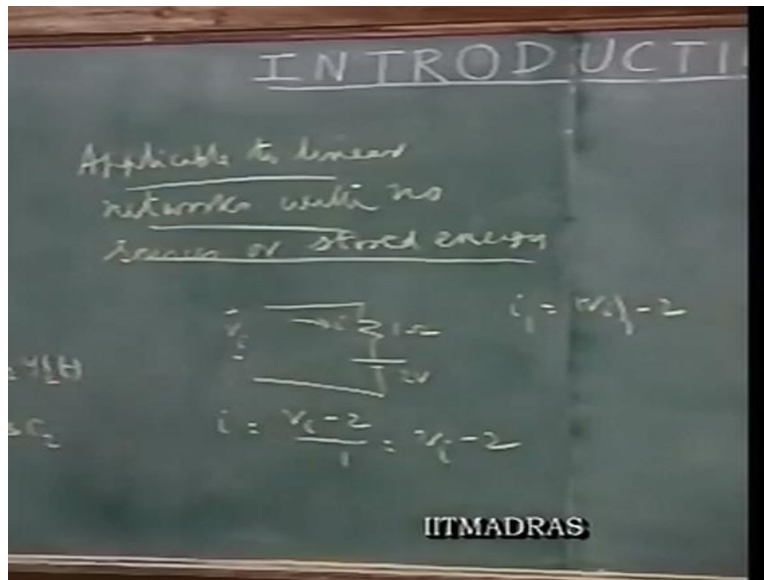
will be mostly in terms of continuous time system at later point in time of time in this course we will talk about the discrete time system in a specially but wherever examples are given we talk in terms of continuous time system now for the most part.

If x_1 of t gives rise to an output y_1 of t and x_2 of t gives rise an output y_2 of t then for a linear system no matter what x_1 and x_2 are a constant time x_1 of t plus another constant times x_2 of t will give rise to an output which is $c_1 y_1$ of t plus $c_2 y_2$ of t for all constants c_1 and c_2 . This is what is refer to the principle of super position some times people refer to this break this up into two parts homogeneity and additivity.

What they say is if x_1 of t gives rise to y_1 of t then $c_1 x_1$ of t gives rise to $c_1 y_1$ of t that is called the principle of homogeneity. Then the second part is if x_1 of t gives rise to y_1 of t and x_2 of t gives rise to y_2 of t then x_1 plus x_2 will give rise to y_1 plus y_2 that is called additivity. But now we will combine these together and we will say $c_1 x_1$ of t plus $c_2 x_2$ of t will give rise to $c_1 y_1$ of t plus $c_2 y_2$ of t for all constants c_1 and c_2 .

And this is the class of systems which are called linear systems one which obey this principle of super position. And most of the electrical networks that we have sawed earlier belong to this class. But you must keep in mind that this particular super position principle will be valid for example in the case of a electrical circuit with zero initial stored energy. If a capacitor has got some stored energy in that this principle of super position will be not valid.

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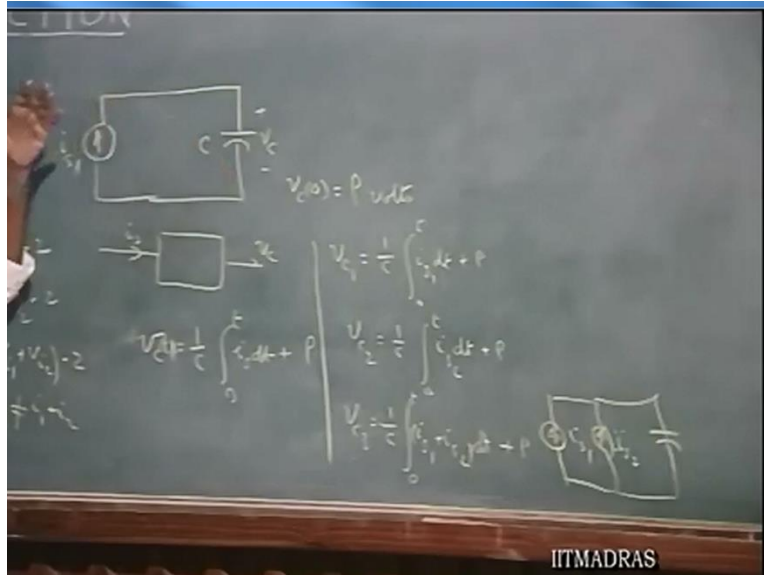


Let us an taken example for example I will say this applicable to linear networks as a special in of course with no independent sources or stored energy important. For example this is a linear network resistance suppose I have a source here 2 volts 1 ohm then if I have an output an input here v_i and I will look to take the current to be by response quantity i equals v_i minus 2 divided by 1 or v_i minus 2 and if I have 2 different voltages v_1 and v_2 .

And find the currents i_1 and i_2 and if I say if I have v_1 plus v_2 then I will not get i_1 plus i_2 as you can see for example i_1 here is v_{i1} minus 2 for one particular excitation. For second excitation i_2 equals it is v_{i2} minus 2.

But suppose I have a single excitation which is equal to v_{i1} plus v_2 then the current would be v_{i1} plus v_{i2} minus 2 that will be the current when I have a single excitation v_{i1} plus v_{i2} but i_3 is not equal to i_1 plus i_2 , why because of the source here i_3 is not equal to i_1 plus i_2 . So when you apply this linearity principle as superposition principle you must make sure that you are applying it to a network which does not have sources or stored energy for that matter.

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Let us take a second example to illustrate the second point which I have in mind, suppose I have a capacitor which has got an initial stored v_c this is c and I have a current source as excitation. So I think of these as the system with i_s as the input quantity and v_c as a output quantity. Okay, and let me assume that v_{c0} equals some ρ ρ volts that is initial capacitor voltage is ρ volts.

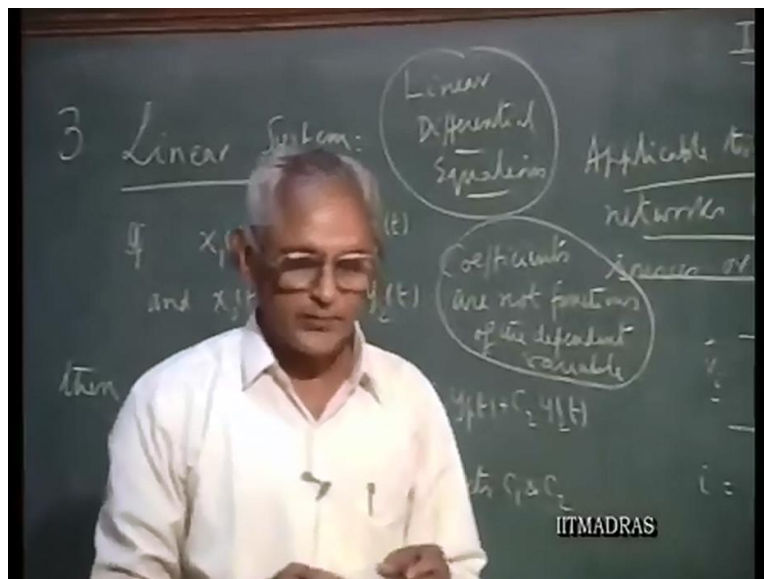
Then you have the relation i_s equals v_c equals $\frac{1}{c} \int_0^t i_s dt$ that is the initial charge that is convert to that capacitor in the integral from 0 to t , this is v_c of t plus the initial voltage that is across the capacitor ρ that is this initial voltage plus the increment in the voltage due to the additional charge that has been conducted to the capacitor in the integral 0 to t . This is the equation between the input quantity i_s and the output v_{ct} .

Now let us take two situations where I have i_s is this 1 is 1 then it you will get v_{c1} equals $\frac{1}{c} \int_0^t 1 dt$ plus ρ 0 to t of course. And let us take a second excitation which is 0 to t is 2 dt plus ρ that is v_{c2} . So i_{s1} and i_{s2} respectively will give rise to this voltages v_{c1} and v_{c2} . Suppose I had a single source which i_{s1} plus i_{s2} both of them are active in parallel simultaneously i_{s1} and i_{s2} together charging the capacitor then the third time the voltage v_{c3} will be $\frac{1}{c} \int_0^t (i_{s1} + i_{s2}) dt$ plus ρ again because the initial is the capacitor charge ρ volt.

Now you can see v_{c1} plus v_{c2} which are the response to the i_{s1} and i_{s2} will not add up because v_{c3} is not equal to v_{c1} plus v_{c2} what's spoil is this initial charge in the capacitor because when you add this 2 up you get 2 rho but it only one rho which means that the argument is that this principle of superposition is valid only for systems which are purely linear in the sense that there should not have any sources inside the independent sources are initial charges in the capacitors or initial currents in the inductors which are always consider to be equivalent resources.

So this is an important principle and to keep in mind when you talk about linearity of systems. A system which is not linear is of course called non linear. The difference between the linear system and a non linear system is a linear system is governed by linear differential equations and a non linear system by non linear differential equation. So what is the meaning of a non linear differential equation, linear differential equation is one in which the coefficients of the various derivative terms coefficients are not functions of the dependent variable.

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I will take some examples later on so if I have a term like b squared by dt squared its coefficient should be either constant it could be function of time but it cannot be a function of another derivative of y or y itself. So linear differential equations are the one which govern the operation of the linear system and in the system if the i equation fails to be linear then it becomes non linear and that the super position principle will no longer valid.

We will take a some examples of these a equations which governs the performance of the difference kinds of systems in the next lecture. But let me stop at this time and briefly summarize what has been discussed so far. Starting with the scope of this course which is mainly the dynamic performance of linear time invariant systems and networks. We had a look at the number at the difference and text books that you may use for following this course.

Then I mentioned the background material I assume that you have had before taking up this course then we looked up looked at the meaning of a system broadly speaking we said system is a collection of objects. A system components united to some form of interdependence among the various consequent components. We said a network is a special kind of system in which all the variables ill be of a particular kind a one kind in a electrical network and mechanical network given as examples and then we also talked about the modeling of a system.

Modeling of a system is done in the form of ideal components put together the constituted the system interconnected fashion and we do this idel idealization keeping in on one hand keep in mind on one hand that the complex the system should be not too large to ficiate convenient handling at the model at the same time it is should be succinctly realistic so that there is all obtained for the model are generally realistic in practice.

Otherwise the model have to be find once again also mention that the modeling system we you use the two terms inter changeably almost as if the model itself is a real system which is of course not true. Then we looked at the representation of the system and we talked about the multiple input, multiple output system and a single input, single output system and then we went on to discuss the various kinds of classification of systems because as I mention we are talking about a linear time invariant system therefore we must know what they mean.

So for that with that in mind we looked up at the we were trying to find out the classification of the important classification of systems. We talked about the difference between static systems and dynamic systems. We talked about the difference between continuous time systems and discrete time systems and lastly we have talked about the difference between linear system and non linear systems.

The discussion on the linear and non linear systems is not complete because I would like to give some examples of the governing differential equation for both the type of systems and we will pick up at this point in the next lecture and continue our discussion from that point onwards.