Networks and Systems
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Lecture-2
Prerequisites: Exponential Function

The next type of functions that we will see are what are known as exponential functions.

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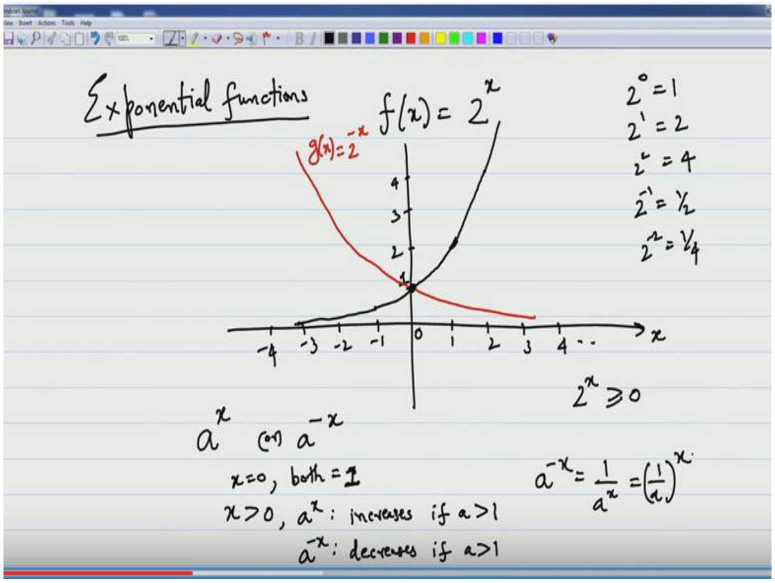

Okay these are very important exponential functions shows up quite often in circuit analysis and before we see a circuit situation in which it shows up naturally. I would like to show you how to plot an exponential functions and how to think of an exponential functions etc. Okay, so lets begin with a very simple example.

Lets say, let us look at the function f of x equals 2 power x okay. This is a very simple exponential function and let us try to plot it to see what happens if you try to sketch it. Okay, so may be this is x and started 0 1 2 3 4 etc may be - 1 - 2 - 3 - 4 etc. Okay, so the easiest point to evaluate 2 power x is that x equals to 0 and an x equal to 0 you actually get 1.

Okay, so we know that 2 power 0 is going to equal to 1. So let us look at 2 power 1 that is going to 2 you know that’s easy and then 2 power 2 on the other hand is going to be 4 okay and that is 3 that is 4 okay and so on. It goes it increases quite fast 2 power 3 is going to be 8 so the way it increases on the positive side is going to be like this. Okay it will go up like that and it will be a smooth function.

Okay think about why it would be smooth function it increase smoothly like that there is no jumps. Okay, so what happens for x less than zero again it is going to be smooth 2 power minus 1 is going to be 1 by 2, half 2 power minus 2 is going to 1 by 4 which is quarter etc and if you extrapolate and plot like this it will once again be a smooth function and look like that okay and it will go of like this. Okay, thats how 2 power x looks.

Okay, so few properties that are very easy to see 2 power x is always greater than or equal to 0. In fact it is actually greater than 0 its never equal to 0 right when does it becomes equal to 0 only when x equals minus infinity it is not going to happen so 2 power x is greater than 0 always it is positive always okay. Another interesting function which you might see is let us say may be I should use red color for this you might want to look at let say g of x which is 2 power minus x.

Okay, how will 2 power minus x look you can go through the calculation it is basically going to be flipped version of this 2 power x right whatever happened in the negative values of x for 2 power x would happen for positive values of x for 2 power minus x. Okay and it will just be a flipped version and you can plot it and it will go like this way and hopefully like that.

Okay you see that g of x which is 2 power minus x. Okay, so hopefully you see this point whatever value I put over here 2 power minus x, 2 power x, 3 power x, 4 power x, at x equal to 0 it will always passed through 1. Okay any a power x it will pass through 1 at x equal is 0. Okay this is something you know and for if you have 2 power minus x that is going to decrease like this decrease very very fast as x increases positively.

On the other hand if you have 2 power x it is going to increase very very fast. Okay, so in the exponential function you have to pay attention to a couple of things. One is okay so lets look at a general exponential function if you look at a power x or a power minus x. Okay at x equal to 0 both of them are equal to 0 both equal to 1 I am sorry if the execute is 0 both of them are equal to 1 and as x increases for positive x a power x. Okay lets first a power x and a power minus x behave in different fashion.

Okay a power x increases exponentially are very quickly if a is greater than 1 and a power minus x decreases very quickly falls of to 0 very quickly if a is greater than 1. Okay, so 2 is clearly an example where I took is greater than 1 so this is what is going to happen. On the other hand if a is less than 1 you can also quickly see the opposite would happen if the a is less than 1 a power x is going to decrease for a for x positive and a power minus is going to increase for a positive.

So you can think about why these things are true and the basic reason is a power minus x is the same as 1 by a power x which is the same as 1 by a whole power x. Okay, so it is depends on whether or not a is greater than 1 or less than 1 if a is greater than 1, 1 by a is going to be less than 1, if the a is less than 1, 1 by a is going to be greater than 1 and the same behavior (()) 05:56.

Okay, so this is the exponential function and you might want to sketch it and look it at and how it behaves for various values for a and x and ah. And in typical examples you might have something slightly more complicated you might have a power say cx. Okay, now there are few constants here first of all a is thrown in and c is also thrown in now whether or not so whatever happens to a or c is x is 0 a power cx is 0. Let me know very easily so let me may be I should take this example in more detail this is an important function to get your ahead around.

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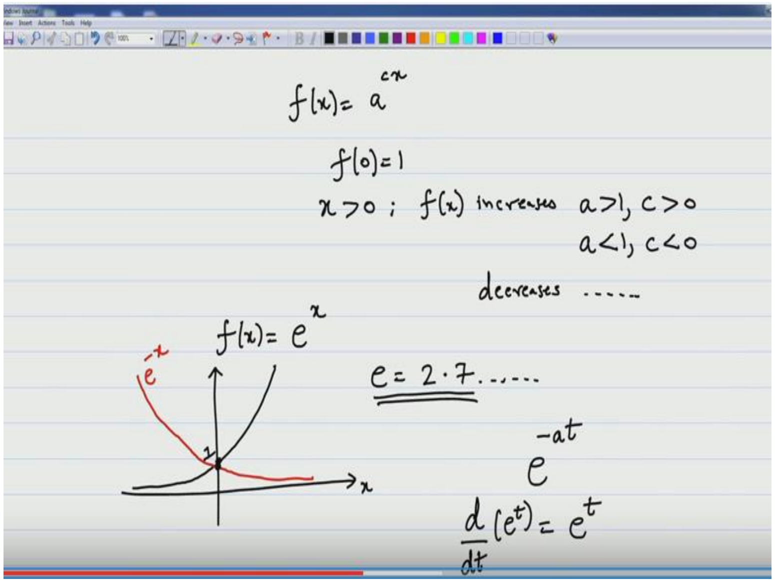

So lets look at this function f of x equals a power cx. Okay, now f or 0 is going to be 1 whatever value of a whatever value of c is going to be equal to 1. Now for x greater than 0 f of x increases under various conditions for instance if a is greater than 1 and c is greater than 0, if a is greater than 1 and c is greater than 0 f of x is going to increase and there are so many other situations for instance a power could be less than 1 and c could be less than 0.

So if c is negative and a is less than 1 it again starts to increase. So all these kind of arguments we have to carefully make and you might want to list conditions under which f of x decreases for what combination of a and c and you will get an idea of how exponential functions behave. Okay in particular we will be interested in the case of f of x equals e power x and what is this e this e is this number which is something like 2 point 7 something.

Okay you might know the definition for e e has a very simple series definition even if you don't know just think of e some constant which is greater than 1 okay 2 point 7 whatever. Okay, so if you have a function e power x how is that going to behave I know e is greater than 1 so at 0 is going to be 1 and then for positive its going to decrease for negative it is going to go like that.

Okay, so that's how e power x going to behave. How is e power minus x is going to behave? e power minus x is going to behave like okay this is going to be e power minus x. Okay, so this e power x is a is this the function that we will obsess over in this course it will come up again and again and so many different forms it will take sometimes real arguments, complex arguments, positive arguments, negative arguments and you should be really really comfortable with e power x.

Okay, so there is series expansion for it there are various ways of thinking about it and computing it but to be the simplest notion is to simply think of e as a constant 2 point 7 whatever and then e power x is simple exponential function like 2 power x instead of 2 you have 2 point 7 something this number e.

Okay there is also the logarithm function which is the inverse of the exponential and I will let you read it upon your own I want to describe that in the class but it is the inverse of this e power x and think about what that means. Okay, so this is the exponential function in particular we will look at functions of this form e power minus at.

Okay where a is a constant and you should be able to sketch this very easily. If a is positive this is going to start at 0 and decrease as time goes forward if a is negative this is going to start at 0 and increase as time goes forward. Okay, so this kind function you should be very comfortable with and the exponential function will play crucial role again and again in this class. Okay, why is the exponential is important?

Why is that it show up several analysis? The basic reason is the derivative of the exponential is itself. Okay, so if you take the derivative of e power at or derivative of e power t we know from basic calculus that is equal to e power t. Okay, so this ends up a being very crucial in solving differential equations and all that and that's why this function shows up again and again and its an important function to know very well.

Okay, so let me see where we are we started with the constant function we looked at the step function and then we looked at sinusoidal functions, then we looked at a combination of the step function and the sinusoidal function and then we looked at the exponential function.

And I want to sign off this little refresher with a discussion of a simple ac circuit which I will try to solve and when I solve that you will see some differential equation show up and this exponential will show up as a solution for those differential equations and these are important for solving circuits you might to learned before but it might be good for you to see it real quick in this refresher.

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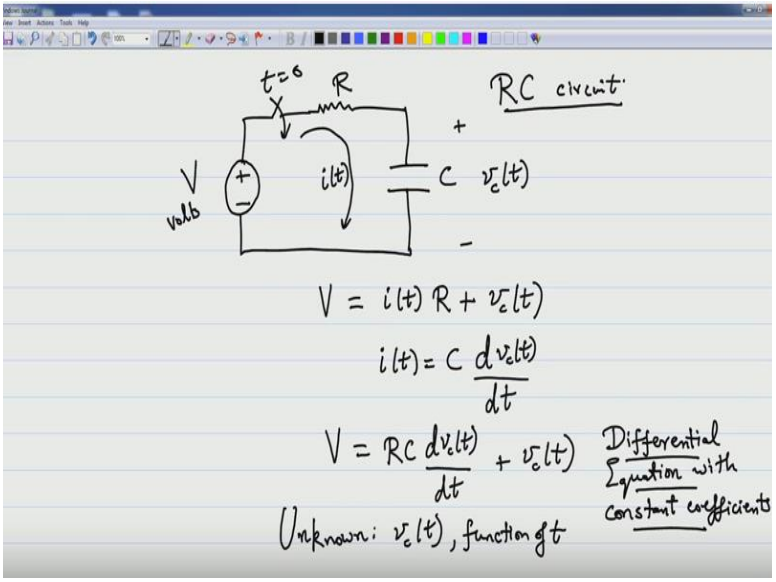

Okay, so here is the simple circuit I have a dc voltage source v volts and there is a switch which is thrown at t equal to 0 and there is a resistance r and a capacitance c connected to it in a series in series. Okay, this is a very simple circuit it is also called on rc circuit because it has r and c series rc circuit.

Okay, so now you should know a few facts about the components of a circuit we know a voltage source behaves in a very simple fashion the voltage cross its terminals always v with respective what happens now the resister behaves a little bit differently let says the current in the circuit is i of t okay i of t is the current that’s flowing in this circuit.

When you through the switch some current it starts flowing and you might knowing from circuit theory that ultimately it will die down to 0 so the current is going to change over time and that function is i of t. Okay, now from your loop equation one can write a very nice equation here the voltage v you know is going to be equal to the voltage drop across the resistance which we know from ohms law is simply i of t times r plus the voltage across the capacitance.

Now the voltage across the capacitance also varies with time and I am going to call that vc of t. Okay, so hopefully you are you understand why functions are one variable important when solving circuits to understand circuits and networks.

Clearly, they show up all the time because voltage is in currents in your circuits are functions of time and when you turn the switch on when you start your start exciting the circuits components the current flowing through them, the voltage across them varies over time and it should be able to deal with functions of time to be able to solve for the voltages and currents that is the simple fact of life.

So now there are other equations relating i of t and vc of t what is that? That's the current through the capacitance that we know the i of t is c times d vc of t by dt the derivative of the voltage. Okay, so now if you plug the back end you get v equals rc d vc of t by dt plus vc of t. Now this is an example of a differential equation with constant coefficients.

Okay, so clearly to be able to solve for the voltages and currents in the circuits I should be able to solve this differential equation. In this differential equation what is unknown in any equation there is unknown which would try to solve in this differential equation the unknown is the voltage vc of t which as I told you is a function of t. So you have one equation which tells you which gives you okay we have one equation which involves an unknown vc of t which is a function of t and you have to solve for it.

So now one critical thing to remember is that this equation (()) 14:49 for t greater than or equal to 0 right. So of course, of t is equal to 0 t less than 0 then there is no current i of t 0 there is no voltage across the capacitance nothing happens so that is always understood here only for t greater than equal to 0 this is true.

Okay, so now one needs to have some general techniques for one needs general techniques for being able to solve these kind of equations and I am not going to go into great details here but you will see these techniques as part of this course. You might have learned it before in your circuit course but if you have not in this course you see it slightly more advanced way.

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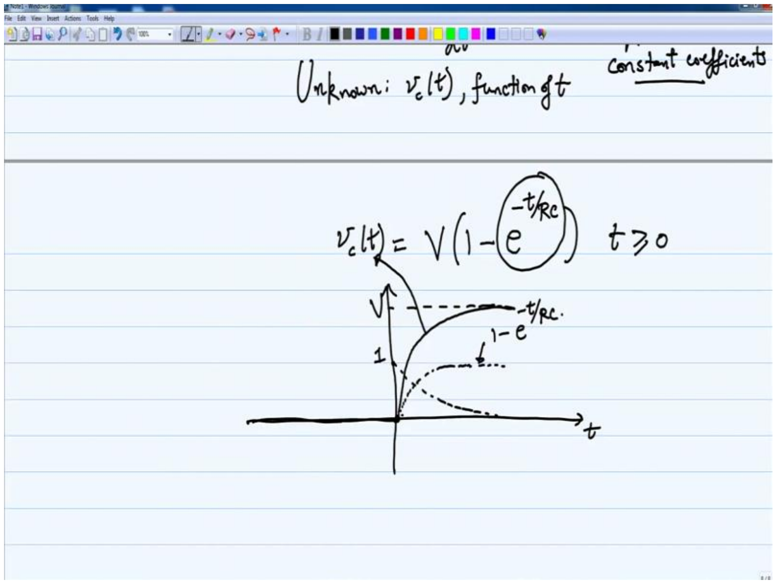

But lets try and write down the answer it is not very hard to write down the answer. The answer if you can guess that will involve the exponential function and it will be this form v times 1 minus e power minus t divided by rc. Okay, so now I know this answer from past experience but even if you don't know you might be able to solve it. But particularly if you know that this is the answer one can check whether this answer is valid or not.

Okay, so its not very hard, How do you check that valid or not? You plug in vc of t into this equation and you see the question will be satisfied. Okay vc of t will come here you have the derivative here. In fact if you take the derivative we will see the rc will cancel and you will get t by rc out there and then they will be a v term coming in and it will exactly cancel with the other guy on the side and you will get v.

Okay this is the simple exercise which will urge you to try that this solves this equation and of course this is true for t greater than equal to 0. Okay, so the main reason why I showed you the solution is to tell you that the exponential function is very important to understand circuit analysis. It shows up naturally as the solution of even the simplest of circuits out here.

Okay a simple rc circuit if you throw the switch at t equal to 0 the exponential function is involved. So now if you want to plot this its interesting to plot this guy. Okay, now remember all the actions is for t greater than equal to 0 for t less than 0 you just have 0. Okay and what happens at t equal to 0 t equal to 0 we know the exponential function takes the value 1 and 1 minus 1 is going to be 0 so the voltage across the capacitances is 0.

So now you might know this has to be true because this the voltage across the capacitances cannot jump cannot be a discontinuity in the voltage across the capacitances the reason is the current is actually the derivative of the voltage and if there is a discontinuity things are going to be block in the circuit so that is not very nice in our model.

So that wont happen too much so its nice to be see the samples and the other things that's going to happen is you have one minus e power minus t over rc. This is slightly more complicated plot than what we saw before. We simply saw e power minus t by rc how does e power minus t by rc look? This alone would look like this it is going to start at 1 okay and it is going to start decaying for positive t.

The reason is e power minus t by rc this 1 by rc is actually positive is nothing negative there. So we have a minus overall so the coefficient becomes negative. So as t increases this is actually going to fall. Okay, so now we know this guy is just this plot by 1 this alone is this plot. Okay, now that is not the vc of t is actually 1 minus this and then multiplied by the v what is going to be 1 minus this 1 minus this will give you a picture which looks like this.

Okay it look like this okay this is 1 minus e power minus t by rc and then and I am going to multiplied by v so i get an equation like that may be I do not know just showing it here. Okay, so it will never exceed v and that is like look okay so become little messy so may be I should erase this a little bit and show you how it looks. Okay, so that is the plot and you see that this guy is actually vc of t.

Okay it reaches v as t as becomes very very large and that you know happens to the capacitance. Okay, so this is an example of slightly more complicated function it involves the exponential but then there other manipulation you had to figure out 1 minus exponential and then multiplied by v etc.

Okay, so this level of comfort and dealing with functions of one variable it is quite important there will be some examples of this as we go along in the lectures. But I will urge you to refresh yourself with these functions how to plot them? How to think of them? etc.

Okay, so in the prerequisite part of the course we have we are pointing you to several websites and several additional material which discuss information like this in great detail function of 1 variable, what are derivatives, what is meant by continuity of a function, what is the discontinuous function, what is the jump discontinuity, what is an exponential function, what are trigonometric functions, how to define them, how to think of them, how to sketch of them etc

All of this are provided in additional material which we will point you to as per in the course pages. I will urge you very strongly to go through those pages read them at your own pace make sure you get familiar with these functions. Because the rest of the course involves these functions very heavily and if you not used to these things you will be short changed and you may not be able to understand lot of things that happened.

So I will encouraged you to go through the preliminary material very very closely. Okay that's the circuit concept and hopefully you see why functions of single variable are very crucial for being able to solve circuits. If you don't know them very well it is very hard to solve circuits. Okay, so for instance in this problem in this very same rc circuit if I change this v if I replace with the ac source what happens.

Okay that's an interesting question and it is not very easy to solve that the way I did. Okay you might be able to write down write a differential equation which will be actually a very similar to this wont change very much compared to this. But solving the differential equation it is going to be a bit more challenging. You need to know some additional tools and after solving the differential equation being able to sketch that the differential equation is also going to be a little bit more challenging.

And in this course you will see generic techniques which help you solve such problems. Once you master the networks and system course when someone gives you a problem like this you will be able to quickly get to the answer even without doing complicated calculations. For instance I know now I know because I studied this course before that if you replace this dc source with an ac source right.

Initially there will be some variations but eventually after a while the study state solutions to speak is actually sinusoidal voltages and currents in the circuit and fact if possible to even compute that you might be able to quickly figure out what the transient and initial behavior will be as well. All these things are very important in circuits and you will get tools generic powerful tool which will solve these kind of problems as part of this course