Networks and Systems
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Lecture-1
Prerequisites: Constant, Sinusoidal Functions

Hello and welcome once again to this course on Networks and Systems. This is a this lecture is about the prerequisites for the course. We mentioned in the introductory video and you might be aware yourself that you need some basic calculus for doing well. In this course and you also need to know ac and dc circuit analysis. So in this brief lecture what I am going to try is to give you a glimpse of what the prerequisites are.

Mention a few of the important details and after that you will have to be after that you will be able to go out and may be look at some textbooks that we point you to or look at some material on the internet that we point you to and learn the prerequisites on your own.

So once again this lecture is not the substitute for the prerequisites. It is only a pointer or a refresher of the basics that you will need for doing this course. So, let us see how it goes okay. So like I mentioned the prerequisites for this course are basic calculus. In particular you will need to know about functions of one variable.

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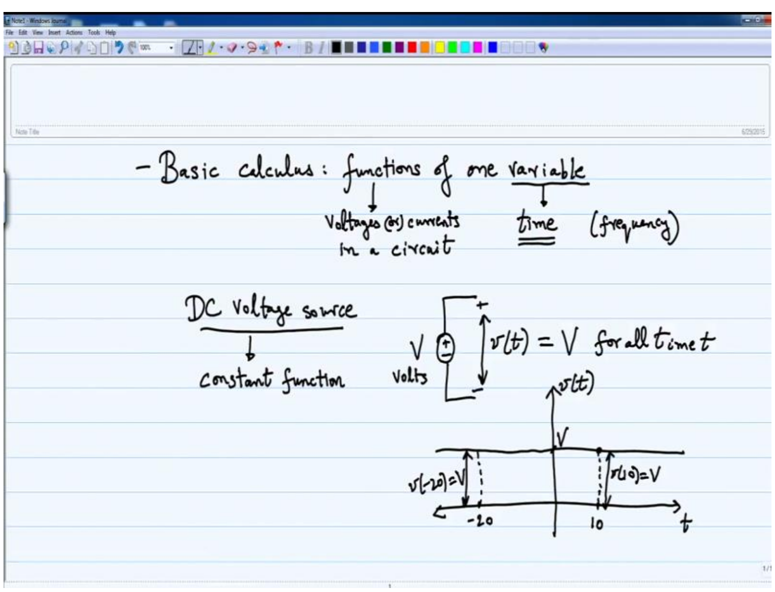

Now you might wonder why in an electrical networks and systems course functions of one variable are very important. The answer is quite simple the reason is any electrical circuit or network the variables are voltages and currents and these vary over time okay. So, in our in our course this variable in most cases will be time okay. Later on, we might also have variable such as frequency etc.

But to start of you can think of the variable that we have as time and these functions in our case will typically represent voltages or currents in a circuit okay. So you might have studied about circuits in your previous courses circuits are made up of circuit elements and the circuit elements are basically voltage sources, current sources, resistors, capacitors, inductors.

And these components when connected together result in voltages and currents in different parts of the circuit and these vary overtime depending on the situation. So let me give you a few examples from the circuit world and point you to corresponding functions that you might see. Okay, so once again the style of this lecture is to give you a refresher of the basic ideas.

So, I want to be very formal I will not introduce or define things very formally. But simply point to you to different aspects which possibly give you a glimpse of the prerequisites needed. Okay, so lets us take one of the simplest of circuit component out there which is a voltage source.

Okay, so this is typically denoted like this may be put plus minus and then you have a v. Okay, so I am going to say this is dc voltage source. Okay, this is the typical notation and this v in volts is the value of the voltage across these 2 terminals. So what does this mean? This means that if you measure the potential difference between these two points you are going get v volts as the answer and that’s going to be a constant for all time that is the definition of the source.

So, let’s say if I call the voltage between these two points as a function v of t. Okay, so notice once again this notation I have used the voltage cross the voltage source I am calling as a function of v of t and this t is my variable which is going to be time and v is the function which denotes the voltage across the these two points.

In this particular case this is a very simple function you know that answer is going to be this v of t is going to be equal to v for all t. Okay, so for all time t this v of t is going to equal to v and that's the definition of the voltage source. So now you might want to plot this function and plotting function is very important it gives a you clear idea of how things work. So to plot functions of one variable you need axis. You need the independent axis the t axis and the y axis which plots values of this function.

Okay, so now this v of t is going to equal to v for all time t. So you get what is known as a constant function whose value is v for all t. Okay, so how do I read this function this function v of t is equal to v for all t and I have drawn a flat line at v parallel to the x axis. So what does that means suppose I go to t equals 10 and asked you what the corresponding value of v of t is and that will tell you that this height which is v of 10 is in fact equal to v.

On the other hand lets say I go to minus 20 which would roughly be here in time and then asked what is the value of the voltage that answer is again this height which is again going to be v. Okay, that is this constant function. Okay, so constant functions are the simplest functions and like you saw here and across the dc voltage source the voltage is a constant function. So this is an example of a constant function.

Okay, so now there are other constant function functions that would show up an a circuit and you can imagine if I connect this voltage source to let say a resistor. Okay, I will show you that it is quite simple to see that.

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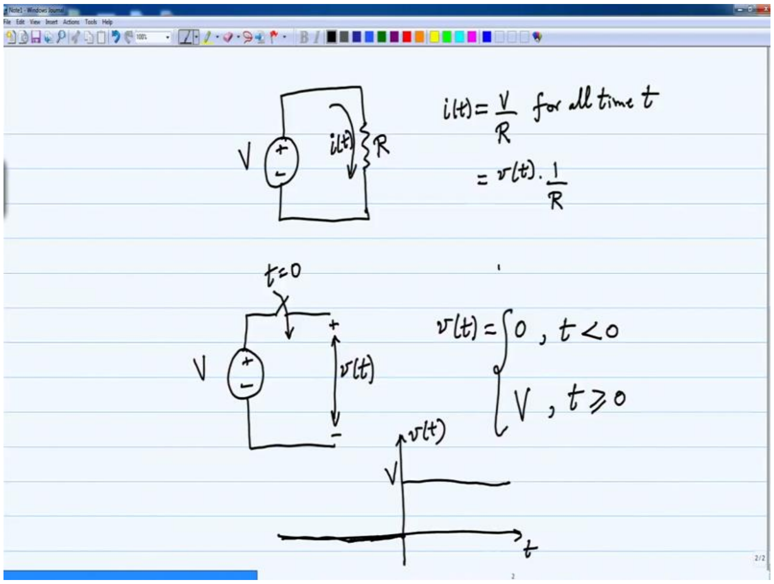

If I connect this voltage source a dc voltage source of value v to a resistor in this circuit like this and may be this resistor has resistance r ohms and in that case, let us look at the current that is going to flow through the resistance. Okay, that is going to be denoted as a function i of t. Okay, once again i of t is the current through the resistance and that function you know from your basic circuit theory is going to be v divided by r.

Okay, so once again this is equal to v divided by r for all t for all time t which is again a constant function and you can see quite readily that this is the constant function v of t multiplied by 1 over r. Okay, so you see the constants functions in different parts of the circuit are related to each other in a simple resistive circuit in this fashion.

Okay, so once again constant functions are the simplest functions out there and one can see the show up obviously in a very easy way in simple dc circuits. Of course, if you complicate your resistive circuit here you put a lot of resistances in series parallel etc you might have to do lot of solving to figure out the currents in the different resistors are going to be.

But nevertheless the currents are always going to be constant over time the voltages are going to be always constant over time. Okay, so that is the very simple example of a constant function. Okay, so let me let me change things around a little bit make it a little bit more interesting. Let say we consider a voltage source like before okay exactly like before except that I am going to add a little switch here.

Okay, I am going to say I will throw the switch or basically turn the switch on at t equals 0. So let's say we turn the switch on at t equals 0. Okay, now I am going to ask the same question once again. Okay v of t is the voltage across these 2 points okay and so now in this case things are little bit different from before in previous case we had a voltage source and I said v of t is the voltage across the 2 ends of the voltage source for all times.

So now I have a switch thrown in and that switch is off for t less than 0 for negative time and from t equal to 0 onwards it becomes on. Okay, so in this case one can justify this quite easily how is v of t going to be. My function v of t okay when the switch is turn off there is may be no potential difference between these two points okay and that will tell me that v of t is 0 for t less than 0. Okay, so once again there is lots of notation going on here.

I am saying the function v of t equals 0 for t less than 0 and then what happens for t greater than or equal to 0. I might want to say that the value of v of t equals v. Okay, once I throw the switch or once I turn the switch on the potential difference between these two points is going to be equal to v volts. Okay, so this is the function now and you see this is not a constant function anymore but its quite close to a constant function.

It takes 2 different values one value for t less than 0 and one other value for t greater than equal to 0. And if you were to sketch this or plot this or graph this function it would look like this. Okay, you have the t axis here we have the v of t here for t less than 0 its going to be equal to 0 and this line will be right on the x axis and it might be difficult for you to see and then for t greater than 0 it is going to be equal to v.

Okay, that's your v of t and this function you can see is clearly different from the constant function and things start becoming very interesting very quickly with functions like this. Okay, so you quickly see the utility of thinking of voltages across points in a circuit as functions of time. This lets you deal with the practical notion of turning the switch on a circuit. Okay, when you turn the switch on what happens in the circuit.

When you turn it on how do the voltages across difference components in the circuit change overtime. That's a very relevant question when you design or build an electrical circuit on network and you should know the answer to that question. So in this course you will learn things like that.

When the voltage across two points in a circuit varies in this fashion as supposed to be a constant if it varies from 0 suddenly to be what happens in the circuit? The circuit is going to react in a dynamic fashion it will do something depending on the components in it and you should be able to study that and that's one of the things we will do in this class and you can see why functions are useful to know for a course like this.

Okay, so this function is called a step function in our parlance. Okay, so you would call it step function and the step at 0 is equal to v. It jumps so you can see why I call it a step function right because a step at 0 and the height of the step is equal to v.Okay, so you learn very nice notation for functions like this these are very important functions and they show up time and again in many of our analysis.

Okay, so we will come back and visit this function once again later on when I show you slightly more complicated circuit and show you how to analyze that. For now lets keep continuing ahead and go beyond these kind of constants and simple functions and start looking at ac sources.

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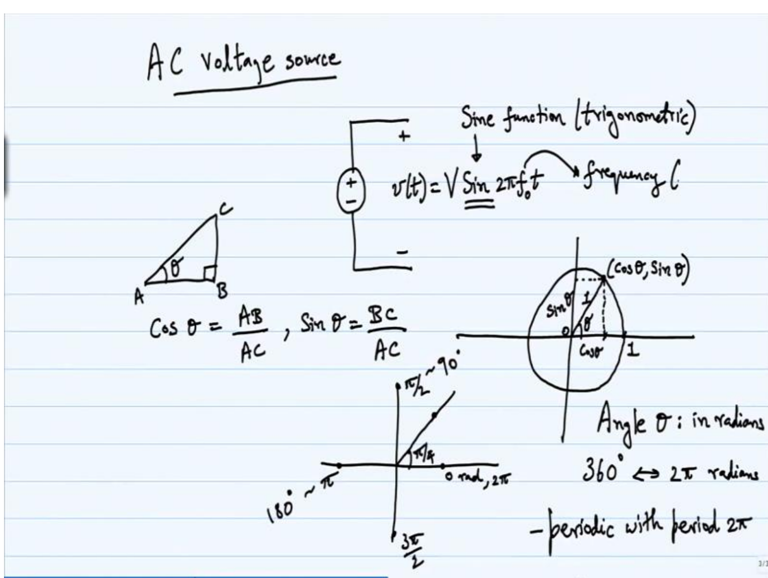

Okay and ac source you might have seen ac voltage source is a little bit more interesting than a dc voltage source. The notation probably a similar to before you might use different notation here but this is the notation and in fact the voltage across the these two the two ends of the ac voltage source is not constant overtime. On the other hand it takes a sinusoidal shape and its very common to write at in this fashion.

Okay, so the voltage v of t across an ac voltage source could be something like I don't know v lets say sin2 pi f 0 t. Okay, such a lot of stuff I wrote down there. So lets first try and think about what this function is okay. So the most interesting thing I wrote here sin and then I guess 2pi f 0 t. If you have not seen these things before this could be very new to you.

Basically, this is the sinusoidal function the sin function which is a trigonometric function. Okay, so lets just quickly see definitions of the trigonometric function and how will think of the argument of the trigonometric function etc etc. Okay, so the most common definition you might have seen from your high school is if you take a right angle triangle. Okay this is a right angle triangle and if this angle is theta okay and lets say we call these things abc.

Okay you have a right angle triangle abc and the angle here is theta then you might have learned that cos theta is the adjacent side divided by the hypotenuse and sin theta is the opposite side divided by the hypotenuse ac. Okay, you might have learned it like this is your basic trigonometry but for the purposes of this class it helps to think of the sin and cosine in the two dimensional plane and to think of the unit circle on it okay so lets look at that.

Okay, so here is the two dimensional plane and here is the unit circle. Okay, so I am not drawing it very perfectly but you can imagine that this is the unit circle. Okay, unit circle meaning that the radius of the circle this one okay and how do I think of sin theta and cosine theta. If you draw if you take any point okay. If you take any point on the unit circle lets say which is at an angle theta.

What is cosine theta in terms out if you drop a line like this that points x coordinate is exactly equal to cosine theta. And if you drop a line on the y axis that points y coordinate is exactly equal to sin theta. Okay this is the same definition as what I have done here with the triangle you can quickly see that right. You can see that this length is 1 so the length of the hypotenuse is 1 and this is the angle theta so clearly cosine theta is that adjacent side divided by the hypotenuse which is exactly cost theta.

So this adjacent side itself is cos theta and that's the x coordinate of this point. So this point is actually cosine theta, sin theta okay x and y coordinate of that point are cosine theta sin theta. Now so this is quite important to have a good idea of what this trigonometric functions are the unit circle is quite powerful and you can use many of these properties for instance quickly you see that cosine squared theta plus sin squared theta is going to be equal to 1 etc etc all those things might be able to see here.

And there's one more thing I want to point out the angle theta is typically measured in radians. In a serious mathematics courses and serious electrical engineering courses we will always measure the angle theta in radians. Okay, so if you might if you want me to remind you of what radians are 360 degrees equals 2pi radians. Okay, what is 360 degrees you start from here on the x axis and if you go 360 degrees you have gone one full circle around the circle right.

So one full complete circle you have finished around this and that's 360 degrees and that is equal to 2pi radians. Okay, so if you want me to show it once again. Okay, so this is the angle here the 0 radians the angle here is pi by 2 radians the angle here is pi radians the angle here is 3 pi by 2 radians and what is 2pi radians? Its back here again okay. What about the angle let us say here if you want the angle pi by 4 that's going to be 45 degrees. Okay pi by 2 is 90 degrees, pi is 180 degrees.

Okay, so its good to know these things and 60 degrees ispi by 3 and 30 degrees is pi by 6 okays 120 degrees is 2 pi by 3 etc etc. Okay, so you should have this comfort of going between radians and degrees if you are not used to it and start thinking in terms of radians from now on because radians is what you really need to understand what's going on here.

Okay, so now you can quickly see that as I increased theta starting at 0 as I increase theta cos theta starts at 1 and slowly starts decreasing when i get 2pi by 2 cos theta becomes 0. Look at what happens to sin theta when I start at theta equal to 0 sin theta starts at 0 and then as I increased to pi by 2 sin theta becomes 1. Okay and how does that increase happen it happens in a nice smooth way and then what happens beyond pi by 2?

Beyond pi by 2 sin theta starts decreasing from 1 and it goes to 0 at pi. What happens to cos theta beyond pi by 2? It starts going negative and at pi cos theta becomes minus 1.

Okay, so all these things are nice and easy to study if you remember the unit circle. And another interesting thing is suppose you go through one full circle 2 pi and then start of again what would happen? If you go through one full circle of 2 pi and then look at 2 pi plus theta you come back to the same theta right.

One full circle is back to the same place 2 pi is the same as 0 so after 2 pi sin theta and cos theta start repeating themselves. So both of these functions are periodic with period theta. So, all these things are nice observations one can make periodic with period 2 pi I am sorry is a period theta periodic with period 2 pi. Okay, so these are nice thinks to know about the trigonometric functions is a crucial functions we will see them again and again in a class.

So lets try and plot them versus t. Okay, so for instance I have written here of sin 2 pi f0t. Okay, so now this is a very standard way of writing sinusoids for ac sources when you write it like this t is of course the time variable that changes from minus infinity to infinity any time is possible negative time, positive time t equals 0 and this f0 is called the frequency.

Okay and it has the unit of hertz or second inverse. Okay, so typically your ac source high voltage ac source, power lane source is 50 hertz and you might have other oscillators in your circuits which oscillated kilo hertz or mega hertz or giga hertz so f0 is the frequency of that.

So if you want to sketch sin 2 pi f0t versus t let see how it looks based on what we know about sin and cos.

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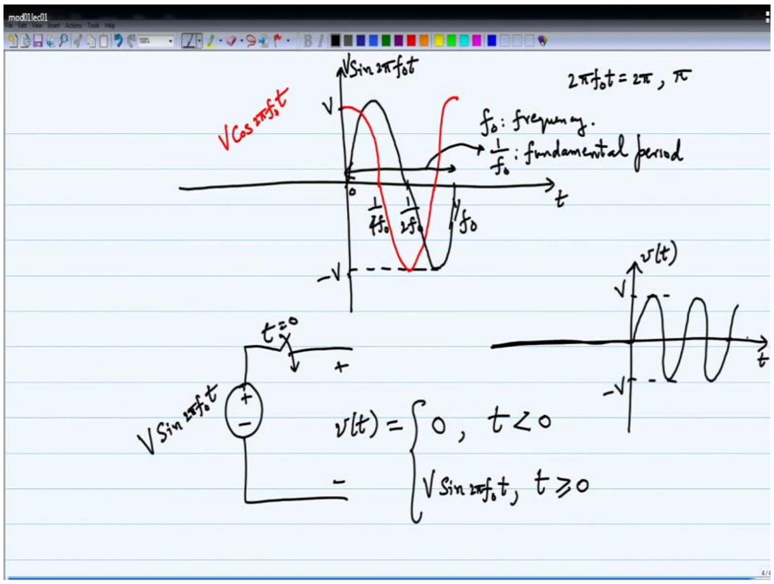

Okay, so this is an important and easy plot to remember you might know this I am sorry for repeating that but it good to see it once so that we are all on the same page. Okay, so v times sin 2 pi f0t and let me draw this axis here 0 so lets try and plot v sin 2 pi f0t versus t. Okay, so the crucial points here are when 2 pi f0t equals 2 pi okay and from there you see p is going to be equal to 1 by f0.

Okay and the other crucial point is when 2 pi f0t equals pi and that's going to be 1 by 2 f0. Okay and then 1 by 4f0 corresponds to 2 pi f0t equals pi 2 and we know from the property of the sin function sin is going to increase to 2 pi v then fall down to 0 at pi and then do the exact same thing on the other side. Okay and this is going to be minus v and that's how the plot for sin 2 pi f0t versus t looks.

Okay this f0 is called the frequency and this 1 by f0 is called the time period fundamental period of this sinusoidal. Okay, so one interesting exercise you might to want to try is you might want to plot lets say for instance vcos 2 pi f0t on the same diagram if you do that how would that workup the red plot is going to start at v is going to be 0 down here is going to be minus here and then it is going to be back up like that.

Okay, so in fact if you know about these function sin and cos they are actually shifted versions of each other you take the cos in and then shift to the right by pi by 2 or 1 by 4f0 you will get the other function. Okay, so you might know these things from your knowledge of sinusoids. Okay, so this is the sinusoidal function and you see it clearly shows up in a circuit. So here is an example of a ac voltage source where the sinusoidal function shows up exactly.

Okay, so you might once again have a voltage source an ac voltage source okay which is turned on switched on at time t equal to 0 that might happen. Okay, so may be this is v sin 2 pi f0t but it is turned on only a t equal to 0. In that case what will happen to the what a potential difference between these 2 points. Its going to be vft equals 0 for t less than 0 and is going to be equal to vsin 2 pi f0t for t greater than or equal to 0.

So now this is a typical occurrence in many circuits that you might build. There is going to be ac voltage source which you turn on at a particular time t equal to 0 and you want to be able to analyze it and for things like that you need to deal with slightly more complicated functions like this.

How will this function look if you want to try and plotted it is going to be 0 for t less than 0 like before but then at t equal to 0 t greater than 0 it starts oscillating and you get this sign right and this is going to be v this is going to be minus v and x axis you can plot similar to before. So this is how this function vft is going to look. Now if you have some circuit theory connected to this voltage source on this side what happens? what is going to happen?

How are the voltages across the different components going to behave? Now that is important to study and we will do that as part of this course and you can clearly see you need to be comfortable with functions of this sort okay. How do these functions behave? How do they affect a circuit components?

These are important questions that we need to answer. Okay and so hopefully you see the importance of knowing about functions, knowing about trigonometric functions and being able to plot them, being able to think of them and that is going to be useful now in a circuit okay.