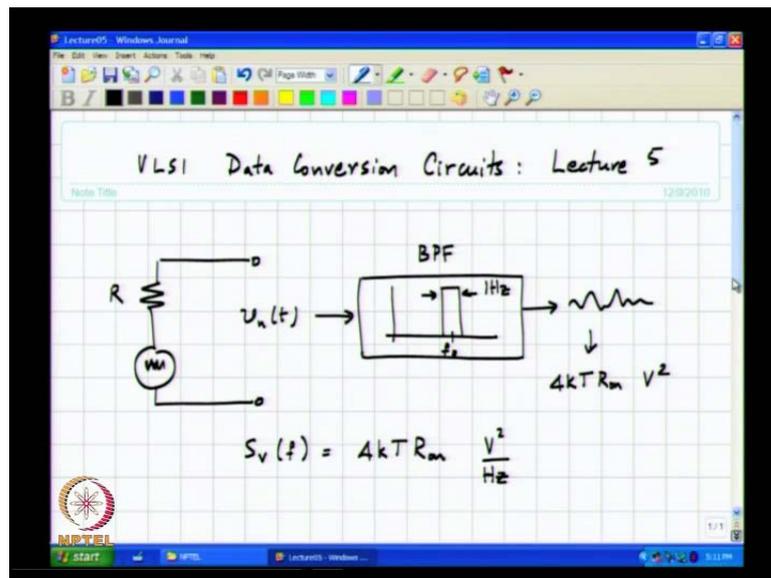


VLSI Data Conversion Circuits
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Lecture - 5
Noise due to Sampling

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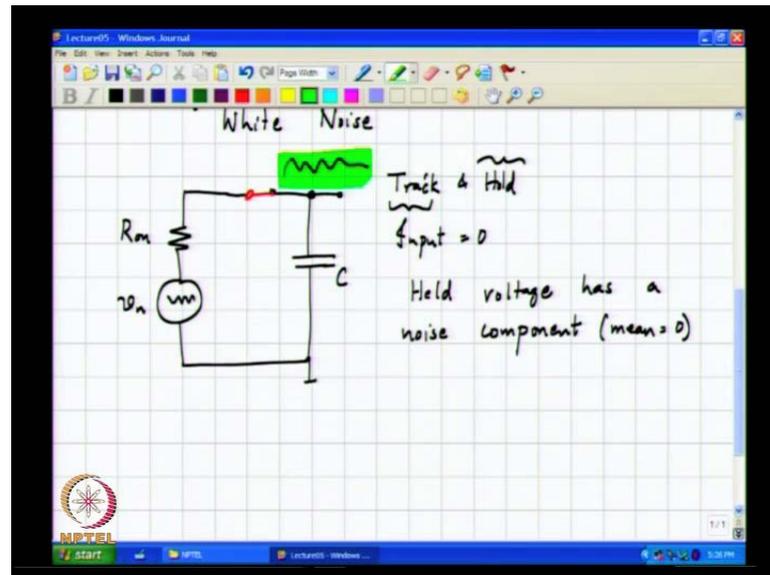
So, the last class, we were discussing what happens during the sampling phase and in that connection we were looking at the noise module of a resistor. So, it turns out that if you have a resistor in thermal equilibrium with surroundings at an absolute temperature T . Then there is a noise associated with it this is due to random motion of charge carriers inside the resistor. And the spectral density of this noise turns out to be $4kTR$ on Whole Square per Hertz. We will not get into the details of the math behind spectral density and such. But what I would like to mention here is that it is important get a physical field for what this means? And what this means is the following I took a resistor and monitored the voltage across it I would see some random waveform.

Now, if I took that random waveform and filtered it through an ideal band pass filter with a bandwidth of 1 Hertz and a center frequency of f_{naught} . There will be some waveform at the output of this band pass filter is not it, because you had taken a band pass filter you the input is some signal. The output is bound to be some noise like waveform, please note that a band pass filter has 2 attributes; one is the center frequency which in this case

we have denoted by f_{naught} and the other one is the bandwidth which is fixed at 1 Hertz. Now, it turns out that if you do this experiment and measure the mean square value of the waveform at the output of the band pass filter that mean square value will be $4 k T$ times R on volt square. The input is dimensions of volts the output of the filter also has dimensions of volts. So, the mean square value has dimensions of volt square couple of things that one should notice what is.

So, striking about this relationship the mean square voltage is $4 k T R$ on volt square and the key point to note is that this is completely independent of f_{naught} . The center frequency of the band pass filter you understand this means that regardless of what f_{naught} is as long as the bandwidth of the band pass filter is 1 Hertz. The mean square value at the output of the band pass filter is simply $4 k T$ times R on is this clear. So, you can think of this as you know a sinusoid with roughly this power at I mean you can think of spectral density. Therefore, in a very loose and you know non mathematical way as a sinusoid at that frequency with this with a power given by $4 k T R$ on. So, since this basically means that regardless of f_{naught} if the mean square value is $4 k T$ times R on volt square. It means that as far as the input signal here is concerned it means that it must have energy or power at the same concentration regardless of frequency. I mean finally, what is the band pass filter doing? It is taking a signal and only selecting those frequencies around f_{naught} in a narrow bandwidth of 1 Hertz. Now, regardless of what I make f_{naught} if I am getting the same power at the output. It means that the input process which is driving the band pass filter has a strength which is independent of frequency correct such a process is what is called a white process.

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So, this is white noise and the reason for calling it white is that it consists of all frequencies. All frequencies just like white light which consists of all frequencies. Now, let us see what happens when I take the track and hold. It is in the track phase and the input is 0. So, track and hold in the track phase when the input is 0 v_n represents the noise of the resistor. Correct now during the track phase therefore, the output voltage even though the input is 0 the output is not 0. Why? Basically as far as the noise source is concerned it sees an RC network and the output voltage is the voltage across the capacitor. So, as far as the output voltage is concerned it is nothing but a low pass filtered version of whatever noise there is in the resistor, you understand. And this noise was often also called thermal noise, because of the origin of this effect is because of random motion of electrons which depends on temperature.

So, physically speaking this voltage is now no longer 0 even though the input is 0 there will be some voltage fluctuation there. And that voltage fluctuation is nothing but the noise waveform v_n filtered by a first order low pass filter formed by the RC network. Before we get into the math of what happens with the output. Physically let us try and understand what happens when we open the switch. So, what do you think will happen during the hold phase? So, the switch was closed it is in the track phase and the noise waveform on the resistor is something. And correspondingly the output is also doing something given that, if we know this waveform we can compute the waveform at the output. Now, suddenly what are you doing?

Opening the.

You are opening the switch. So, what do you think happens at the output?

Disturbance noise by the hold.

So, if you suddenly open the switch which is what will happen when you go from the track phase to the hold phase. Whatever voltage was there on the capacitor just before you opened the switch will tend to.

Retain that way.

Stay that way correct and you think that voltage will be 0.

No.

No because that depends on the noise waveform. So, what is this telling us that even if there was a no signal, because of the noise of the resistor the output voltage is not 0 it is some random quantity because it is a function of the noise waveform you understand. So, what I want to impress on you is the fact that even if there was no input the held voltage on the capacitor has got a noise component, does it make sense. And given that the average of the input is 0, can you comment on the average of the noise component which is stored on the capacitor. The average value of the resistive noise is 0 correct first of all why does this make physical sense or does it sounds strange that the average value of v_n is 0. Or it is not strange at all and this is. In fact, what you must expect.

Sir randomly noise is.

Pardon.

Noise is coming due to the motion of the electrons and volts.

All that is fine coming because of you know motion of electrons. Let us say it moves one side if it has to come back otherwise normally. Is there a more tongue itching argument you can give.

We are not giving any input sir.

So, what I mean. So, so as to the battery you are not giving any input still the output of the. At T is equal to minus infinity plus infinity it is non 0. No I did not understand.

Since the magnitude is not tending to 0 at any point.

No, my question is the following if I plot if I record this waveform it turns out that the average is.

0.

0; now, the question is does it seem reasonable that the average is 0 or does it seem unreasonable? Of course, it is a scientific fact. So, this we cannot dispute it, but does it make sense or does it not make sense it makes sense. Because if it was not then there is no need for a battery if the output average value is not 0, I will hook up hundred resistors in series. And the average values across the resistance will be some finite number from which I can generate.

Power .

Power. So, since; obviously, that is not possible it must mean that the average value must be 0 notwithstanding whether electrons are moving one way or the other inside you understand. Now, given that the average value of v_n is 0 can you now comment on the average value of the noise quantity that is sampled on the capacitor. Please note that v_n appears in series with the input correct. Just like the input is sampled when you open the switch the post switch does not know what is input and what is noise when you open it samples. Whatever is there on the capacitor is simply the voltage across it at that instant in this particular example it happens to be simply v_n , because v_n happens to be 0 correct. So, given that the average value of v_n is 0 and the output voltage is nothing but a filtered version of v_n . Can you comment on the average value of this noise voltage which is sampled on the capacitor?

It has to be 0.

0 is that clear? So, in other words what we have seen now is that the held voltage has a noise component whose value is mean is 0 correct. So, if the mean is 0 I mean the other thing we would like to be interested in is what happens to the mean square value. Just like, we are interested in the mean square value of v_n . We are also likely we are also

going to be interested in the mean square value of the noise component which is sampled on the capacitor you understand. So, in other words the average value of the noise voltage sampled on this capacitor here is got to be the same as the average value of the waveform. That exists across the capacitor when the switch is closed correct, because the switch is closed, you suddenly open the switch whatever existed at that node before you opened the switch is what is stored in the capacitor. So, the mean square value of the voltage sampled on the capacitor due to the noise is simply equal to the mean square value of the voltage waveform. That exist across the capacitor when the switch is closed. So, now how do you figured out what the mean square value of this waveform is.

Transfer function.

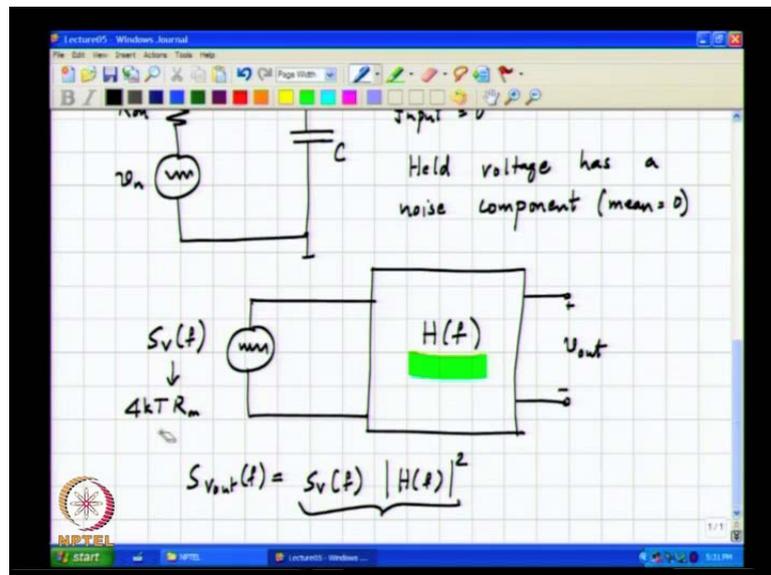
Pardon.

4 k T R into.

Transfer function whole square.

Transfer function whole square and what is the intuition behind that.

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So, what is happening? The answer is indeed. So, we have a noise source with some spectral density S_v of f which happens to be $4 k T R$ on hold square per hertz. Now, all I have done is taken this noise source and passed this through a filter which has a transfer

function of H of f . And I am interested in finding the mean square value of this waveform clearly this is a filter a linear filter being driven by a random waveform. So, the output is also a random waveform which depends on the input as well as the impulse response of the filter or the transfer function of the filter. So, if we now go back to the intuition, we developed with regard to what this S_v of f means that will help us get an understanding of how to compute the mean square value of the output voltage. So, what is the meaning of S_v of f ? What did we just discuss?

What S_v of f means is that if you take this noise voltage conceptually pass it through a band pass filter centered at some frequency f naught with a bandwidth of 1 Hertz. The mean square value at the output of this filter is going to be $4 k T R$ ohms. Now, if I took a sinusoid. So, basically if I look at this voltage and pass it through a band pass filter this is going to be the variance the mean square value. Now, if I took this a sinusoid with this power and pass this through a filter with a frequency response H of f . What is the meaning of H of f ? It means that if I take a sinusoid and pass it at a frequency f , the output sinusoid will be at an amplitude.

Modulus of H of f .

Modulus of H of f times the input.

Sinusoid.

Sinusoid or the power at the output of the sinusoid at the output of the filter will be.

Square.

Mod H of f the whole square times the

Input power.

Input power correct and what is the input power in a narrow frequency range around f naught around some frequency f ?

$4 k T$ into

In general it will be S_v of f is the power in 1 Hertz around a frequency f correct. Now, if I took this and pass this through an amplifier or a transfer function with the gain H of f the power at the output will therefore, be S_v of f which is the input power multiplied by.

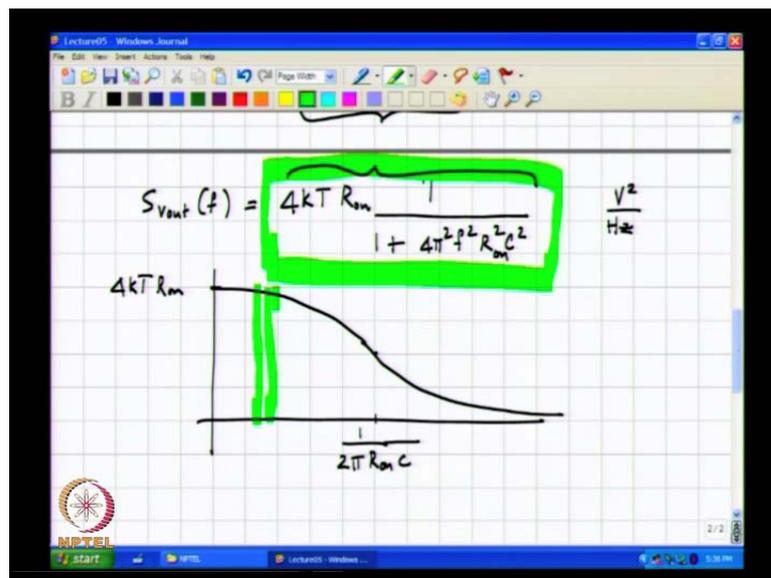
H of f (()).

$\text{Mod } H$ of f the whole square correct. So, if you look at the output power spectral density. And this is simply the power of the output waveform in a 1 hertz bandwidth around.

Center frequency.

A frequency f . Now, clearly the input the S_v of f that is the noise process is white which means that S_v of f is independent of frequency.

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And that turned out to be $4kTR_{om}$ volt square per hertz and what is $\text{mod } H$ of f whole square for s ?

1 by 1 plus omega square.

Omega square $R_{om}^2 C^2$ and please note this is H of f . So, what should it be $4\pi^2 f^2 R_{om}^2 C^2$. So, in other words S_v out of f is this and let us not lose sight of the units volts square per hertz, does it make sense to you people. The input noise source had a uniform power as a function of frequency you pass that through a filter whose gain is varying as a function of frequency. So, it follows that the

output power is the input power multiplied by square of the gain at that frequency. So, the output spectral density will be the input spectral density multiplied by $|H(f)|^2$. So, now what do you think would be the mean square value of the output waveform. See what this means is that if I took the, what does it mean? Let us recollect again if I took the output waveform pass this through a band pass filter at a frequency f and a bandwidth of 1 Hertz. The mean square value I get will be this quantity let us plot it to get some more understanding. And see how will this look like at DC what will it be?

$4kTR$.

$4kTR$ at will decrease at $1/(2\pi R)$ times c what will happen to the power?

It become half.

It become half I will do this. So, why does this shape make intuitive sense? How is it that even though the input has a frequency content which is constant with frequency? Why is it that the output frequency content seems to be decreasing?

Sir.

Pardon.

Filter in there.

There is a filter which is selectively letting only low frequency components get through. So, it makes sense that only the low frequency components of the noise are passed through with a gain of 1. As frequency increases the gain is reducing correct now we can think of. So, you can think of this as many sinusoids closely spaced together each at 1 Hertz spacing and each sinusoid has got a power which is $4kTR$. R times $1/(2\pi f)^2$ plus $4\pi^2 f^2$ does it make sense. So, you can think of this as a whole bunch of sine waves all separated by 1 hertz where each sine waves mean square value or power for a sine wave means square value is same as the I mean as the power. So, the mean square value of each sinusoid is this quantity here.

So, now, when you add all these sinusoids all of these are at different frequencies mind, you when you add many sinusoids have different frequencies. So, what can you conclude

about the power of the composite waveform or the mean square value of the composite waveform. You understand question I am asking you have many sinusoids all at different frequencies. Each one of these sinusoids has some power when you add when you when you construct a waveform which is the sum of all these sinusoids. Can you comment on the power of this composite waveform?

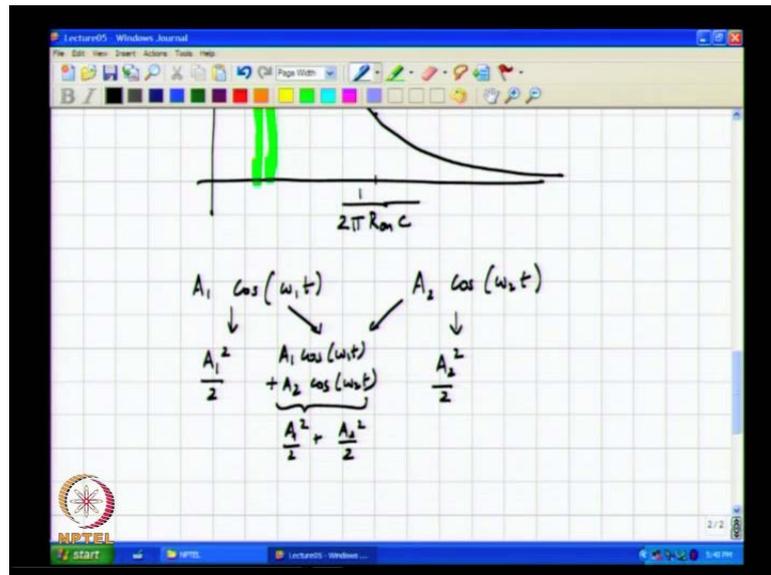
They will add up.

They will simply add up why what happens if all the sinusoids were of the same frequency can you add up powers? Sinusoids are orthogonal to each square root of square no simply divided, we have to add the roots and squares. No if they are no if they are same frequency and same phase then the amplitudes will add up. So, why is it that now you say that I can simply add the powers. Otherwise if it is not and simply a Fourier series frequency multiply in the orthogonal. I mean see you have a waveform $v_1 v_2 v_3$ and so on . So, all I am saying is mean square I mean v_1 square plus average value of v_1 square plus v_2 square plus v_3 square is not the same. As in general is not the same as average value of v_1 plus v_2 plus v_3 the whole square when will it be the same.

When all the cross terms become 0

They are 0 and if you have sinusoids when will all the cross terms becomes 0. If its multiplied when there is. When there 2 of when the 2 waveforms are of different frequencies. The cross terms all the average value of all the cross terms becomes 0 you understand which is why you can add the powers in general. If we just add many waveforms the power of the combined waveform is simply not the same as the sum of the individual powers is this clear. No let me just what I was trying to say.

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So, let us say I had a waveform $A_1 \cos(\omega_1 t)$ another waveform $A_2 \cos(\omega_2 t)$ the mean square value of this chap is $\frac{A_1^2}{2}$ the mean square value of this guy is $\frac{A_2^2}{2}$. Now, let us say I add these 2 together then I will get $A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$, what is the mean square value of this character $\frac{A_1^2}{2} + \frac{A_2^2}{2}$? Only if ω_1 is not equal, ω_1 is not the same as ω_2 if ω_1 equals ω_2 , what is the mean square value? It will be $\frac{A_1^2 + A_2^2}{2}$.

Whole square by 2.

Whole square by 2 you understand you are clear. So, even though a lot of you said simply you can add up the powers, that is only possible because all these different tones are at different frequencies. Now, therefore, the mean square value of the waveform at the output of the filter can simply be got in by adding the powers in individual each infinite signal bandwidth.

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$$v_{out}^2 = \int_0^{\infty} 4kT R_{on} \frac{1}{1 + 4\pi^2 f^2 R_{on}^2 C^2} df$$

Choose $2\pi f R_{on} C = u$

$$v_{out}^2 = 4kT R_{on} \int_0^{\infty} \frac{du}{2\pi R_{on} C (1 + u^2)}$$

So, v_{out} square average is simply integral 0 to infinity of the output power spectral density integrated over all frequencies. Of course, you must bear in mind that none of this is I mean none of what I have told you is really rigorous from a random processes slash communication point of view. But in this course; obviously, it is not possible for us to get into mathematical details of spectral density and so on. All the time interested in giving you is some feel for why those formulae make sense you understand this. Now, why the mean square value is simply the integral of the power spectral density? Now, what is the power spectral density? It is nothing but $4kT R_{on}$ which is the input power spectral density times $\frac{1}{1 + 4\pi^2 f^2 R_{on}^2 C^2}$ from 0 to infinity and we use a change of variable. So, choose for example, $2\pi f R_{on} C$ as u . Then this integral simply becomes the mean square value of the output is $4kT R_{on}$ times df becomes du by $2\pi R_{on} C$ times $1 + u^2$. And this integral transforms 0 to infinity is this clear.

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Chosse $2\pi f R_m C = u$

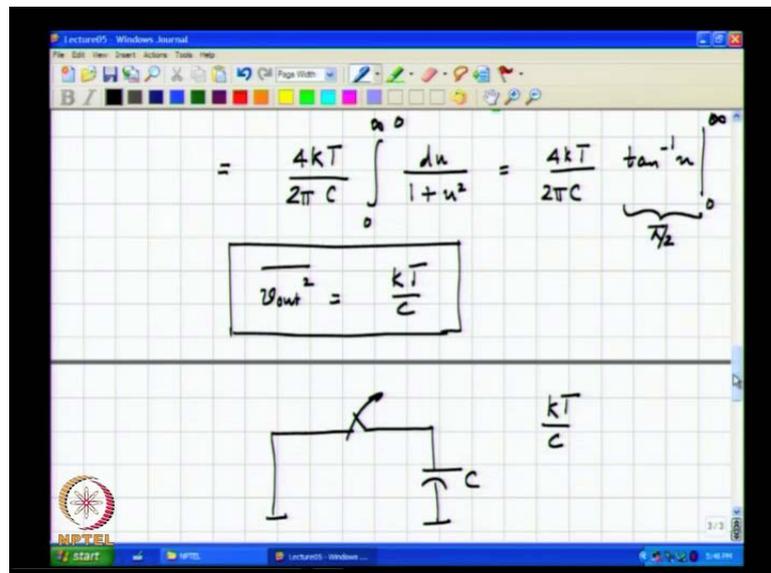
$$\overline{v_{out}^2} = 4kT R_m C \int_0^{\infty} \frac{du}{2\pi f R_m C (1+u^2)}$$

$$= \frac{4kT}{2\pi f C} \int_0^{\infty} \frac{du}{1+u^2} = \frac{4kT}{2\pi f C} \left[\tan^{-1} u \right]_0^{\infty}$$

$$\overline{v_{out}^2} = \frac{kT}{C}$$

And magically we see that this is $4kT$ by 2π into C , because the R on goes away. And this is integral 0 to infinity of du by $1 + u^2$ which is $4kT$ by $2\pi c$ $\tan^{-1} u$ evaluated from 0 to infinity. Now, what is this π by 2 ? So, the grand result is that v out square is kT by C .

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So, in English what this results means is that if I took a track and hold with 0 input and a capacitor the switch can have an arbitrary resistance. And because of the noise associated with the resistor of the switch when I open the switch when I transition from the track

phase to the hold phase the voltage stored on the capacitor is not 0 is not 0 which is what I would expect for a noiseless situation. There is some random voltage on the switch that random voltage is because of noise. And the mean square value of that random voltage which is stored on the switch on the capacitor is $k T$ by c or else spend a couple of minutes staring at this. And see why this might make sense one thing is very intriguing is that the mean square noise sampled on the capacitor is independent of.

R on

R on, but what does the mechanism that is producing noise.

R.

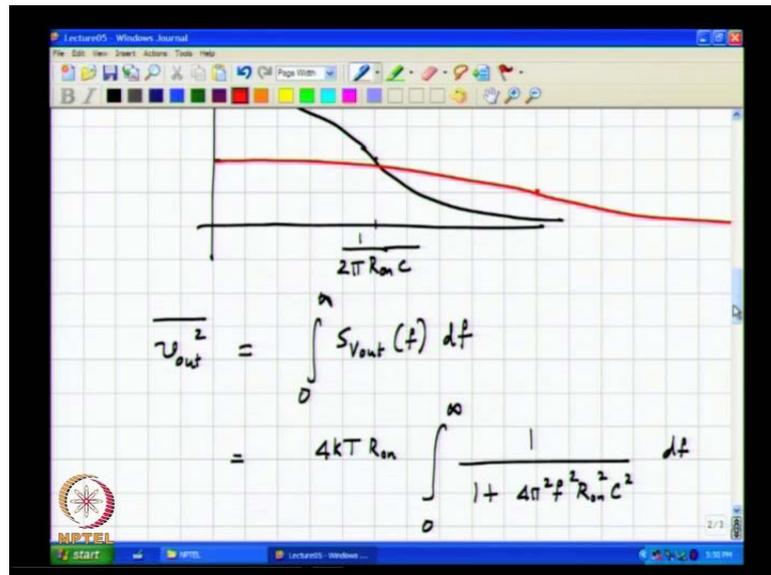
It is R. So, at first sight it seems somewhat strange that the resistor is the 1 which is producing noise. But magically the mean square value sampled on the capacitor is independent of the resistor it only depends on the capacitance. So, why do you think this makes sense? Because that noise source is dependent on temperature. But clear the final expression for noise has temperature in it. So, that is.

Sir this we have taken when the switch is open. So, any noise not coming

Good argument, but no. So, please note that the value of we are sampling which is held on the capacitor has something to do with the resistor is not it. So, why do you think this makes sense?

R is constant with frequency.

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So, what? So, in other words the mean square value is simply the integral of the noise spectral density across all frequencies which is which means that this is simply the area under this curve. So, if I reduce resistance what is happening?

Bandwidth.

Two things are happening one.

Bandwidth peak.

The peak spectral density is reducing; however, what other effect is the bandwidth is.

Increasing.

Increasing; so, the bandwidth is now become twice as it was before. So, if I reduce R by a factor of 2 this is what happens. And on the other hand if I increased R, what would happen? The bandwidth will come down the spectral density at DC will increase, but in all 3 cases the area under the curve remains the same. And why it makes sense is that if I increase resistance it is true that the noise spectral density of the resistor is is higher or lower.

Higher.

It is higher, because the noise spectral density of the of the corresponding to the resistor is $4 k t$.

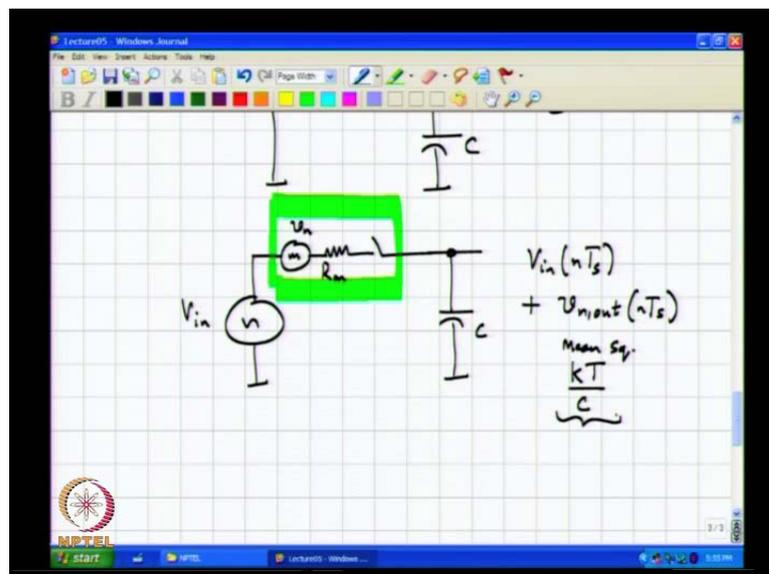
R.

R is directly dependent on the resistance. However, the voltage across the capacitor is not simply the noise of the resistor it is that which is being filtered by an R c filter. So, if you increase the resistance while it is true that the spectral density of the noise source of the resistor increases simultaneously the bandwidth is..

Reducing.

Reducing the R c filter's bandwidth is proportional to 1 by R c. So, if the R increases it is accompanied by reduction in the bandwidth of the track and hold. And these 2 are effects which are working in the opposite direction. And it seems I mean it does not seem surprising that they cancel out or rather you know while being it being an absolutely constant is perhaps surprising. You can at least see that these are 2 opposing effects and luckily they cancel out making the mean square value of the noise voltage held on the capacitor independent of the resistance of the switch it only depends on the capacitor.

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Now, what implications does this have on the track and hold design? What this means is couple of things? So, now, if there was an input voltage in addition to the noise of the switch which I? So, let us denote model the switch as the noise source, it is on resistance

and an ideal switch. So, this is a real switch and I have here the sampling capacitor. So, the voltage sampled on the capacitor is v_n of kT plus. What is the voltage sample of the capacitor?

Is small very small

Plus some v_n out of whatever kT where the mean square value of v_n out is independent of the resistance. So, in other words the very act of sampling has already, what is this? What I mean? Do you think this a good thing or a bad thing ideally what you want to store on the capacitor V in of kT . You want v_n in of kT I mean please do not mistake this kT for Boltzmann constant and temperature this case the index in time. And T is the sampling rate I mean your sampling period. So, perhaps oh I think this is a bad idea to I will call this m times yeah I mean n times T sub S T sub S .

Sir.

Yes.

This R on cancelation it is happening, because it is a first order system R on s .

Yes.

Suppose it is becoming second order $10 R v 1$.

Well. So, it turns out that it does even if it is I mean what you are saying is if you had a higher order system you know does this cancel out it turns out that it does you can prove it you know. In fact, it is not very difficult to prove, but it can be from what I have discussed today you should be able to prove that that is indeed the case. So, if I take a general Rc network and find the mean square value across the output of some capacitor. If I increase all the resistors by the same factor it will turn out that if you measure the mean square noise everything will the increase in the spectral density will be compensated by the decrease in the bandwidth and you will get the same mean square value.

So, what I wish to point out therefore, is that the very act of sampling the input on the switch I mean using the switch. And the capacitor has already corrupted the input you understand and the degree to which it is corrupted is dependent on is only dependent on

kT of course, and c . So, if you have to reduce the noise due to the sampling action of the switch and the capacitor. There are I mean there is only 1 choice in practice I mean you are often not do not have freedom to change the temperature at which the system operates and due to cannot change Boltzmann constant. So, the only thing left is to increase c . And this makes intuitive sense as we just discussed before we went into this discussion. We said that if the sampling capacitance is small you are now signing up for getting easily disturbed by noise external disturbance.

You know extraneous disturbances and therefore, this does not seem very surprising at And what this is telling you is that if you make the capacitance bigger the noise associated with sampling will be will be small. So, this makes intuitive sense. So, now, can we answer the question? There are any number of choices of R and c which will give us the same tracking bandwidth correct. So, is there. So, now, can we revisit that question we had earlier we said that they may be we can choose a very bad switch with a large lot of resistance and a very small capacitance. So, what comment do you have now given that we have had this discussion?

Some reasonable value is.

So, if you get too greedy and choose a use a very bad switch and a very small capacitor claiming that any way the Rc times constant is what matters for a given tracking bandwidth. What you will end up with is that?

Noise component.

The noise component which is held on the capacitor after opening the switch will be large since you are now added the since you are now added noise to the input signal. It now makes sense to talk of the degree to which you corrupted the signal by adding this noise and a number. I mean what kind of measure do you is commonly used? I mean how, what do you think you can characterize signal to noise ratio. So, you can use a signal to noise ratio as a number to characterize the degree to which the sampling operation has corrupted the input. Before I get there 1 minor point that I want to add is that let us say you are sampling; you close the switch; you opened it once; you open the switch. There is some noise voltage on the capacitor in addition to the signal. The next time again you close the switch to get into the track mode and you open the switch again. Now you will have another value for the noise, can you comment on the correlation

between the noise you had earlier in the previous cycle with the noise you had in this cycle.

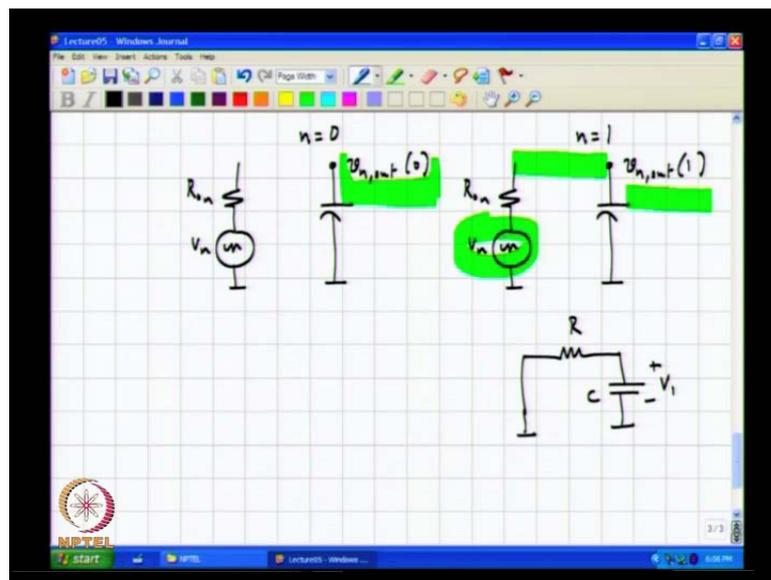
Uncorrelated.

They are uncorrelated why?

By random 1 twice it is.

So, what in auto correlation function you will have an impulse set. Yes I mean I agree that this noise source successive samples of v_n are uncorrelated that is absolutely correct. What I am asking for is not that what I am saying is let me draw the diagram when there is no input v_n R on.

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So, at time instant n equal to 0 this was the state of the network and I open the switch. So, some voltage v_n out of 0 is held on the capacitor then what do I do for n equal to 1 again I close the switch and I open it again. And I get v_n out of 1 can I comment on the correlation between this voltage and this voltage. In other words can we say anything specific about v_n out? In the next sample given that we know what it was in this sample? Yes, no.

No.

No why?

Capacitor voltage is it is like sampling the output of an RC filter whose input is a thermal noise.

Correct.

So, if you are sampling then there is if the correlation has to be there between T and $T + \Delta t$.

Correct.

That is that would be the successive samples here.

Correct.

Because the thermal noise is white and its auto correlation function is a delta function.

Yes.

$X(t)$ into $x(t + T)$ that correlation will be 0 that is for the input.

Input.

Correct.

No, but actually the time separated between T and $T + \Delta t$ equal to 0 and T equal to T they are the successive samples here. So, not quite please note. So, n equal to 0 we open the switch a voltage the noise trapped on this capacitor is v_n out of 0. Then what is happening? We close the switch then again what happens the noise I mean the input noise waveform is connected to the capacitor what happens on the. So, what is? So, v_n out of 0 now behaves like the initial charge on the capacitor correct. And so, it is basically like having an initial charge on the capacitor and it is being driven by a noise source And as he pointed out between T and $2T$ or between 0 and T the waveform v_n of T is is completely.

Uncorrelated.

Uncorrelated and independent of what happened in the previous clock? Previous clock cycle correct, but what happens to v_n out of 0 which was stuck on the capacitor when you began it might discharge at point. It might or will might.

Sir depends upon R on.

It depends on R on correct.

It might discharge.

Might.

It can charge or discharge they charge or discharge. See you can think of this R c circuit as having 2 components there was if you have an R c network. And there was some voltage v_1 on this capacitor and there was some input. You can deal with the initial conditions and the input as using superposition correct this is you are all in agreement with. So, let us see since you are only interested in what happens to be 1. I am going to say I am going to not worry about the input for the time being. So, if you have an R c network like this what you think will happen to v_1 .

Discharge.

It will discharge.

Definitely discharge it is not it might discharge, it will discharge and what is the time constant associated of the discharge?

R c.

R c correct; now can you comment on R c versus a clock period. R c is very very less than T R c is very very less than t . $T v_1$ it should be given. Pardon.

Clock period should be much much higher.

Very high or so, the clock period must be much much large than.

R c R c time constant.

The R c time constant and why is that chosen that way?

It discharge.

What happens if a, if I mean why should the clock period be much larger than

Discharge otherwise it would not discharge (()).

Please note that we want we are interested in having a high tracking bandwidth. The tracking bandwidth I mean if you are interested in sampling a signal at a rate f_s . Then the signal bandwidth is the maximum signal bandwidth you would be interested in is $f_s/2$ correct. So, in other words if you want to have to be able to track a signal with the frequency as large as $f_s/2$. It must follow that the tracking bandwidth must be much higher than $f_s/2$ which is equivalent to saying that the RC time constant must be much smaller than.

Clock period.

The clock period; because the clock period is related to the maximum signal period and if you want to track a high frequency corresponding to $f_s/2$ the RC time constant must be much smaller than. I mean give or take a constant must be much smaller than the clock period. Now, if the RC time constant is very very small compared to the clock period, what do you think will happen to v_1 between? It will weakly discharge weakly discharge. Weakly discharge which means that at the end of the next cycle v_1 is completely discharge. And the new voltage held on C_1 on the sampling capacitor is largely a function of only the input waveform during that next cycle, which as we all understand is completely uncorrelated. Because it is a white noise it is completely uncorrelated from period to period which is why there is no correlation between or virtually there is almost no correlation between successive samples of the output. Output is this clear please note that will not be the case if the RC time constant is large larger.

Is too large if I deliberately choose the RC time constant to be very large then what happens? There are 2 things happening v_1 is decaying and the input noise voltage is trying to charge up the capacitor correct. So, if the time constant is too large then that decaying will not happen which means that at the end of the second clock cycle. The voltage on the capacitor has got some part of v_1 plus something else which basically means that the previous held noise. And this noise are noise voltage are related which means they are I mean there is correlation does it make sense? But in practice, we do not have to worry about it, because by design we would always make RC times the RC time constant to be much smaller than a.

Clock period.

Clock period; so, in the next class, we will evaluate signal to noise ratios. And then get to implementation of a practical switch which uses devices which you know very well the mosfet.

Thank you.