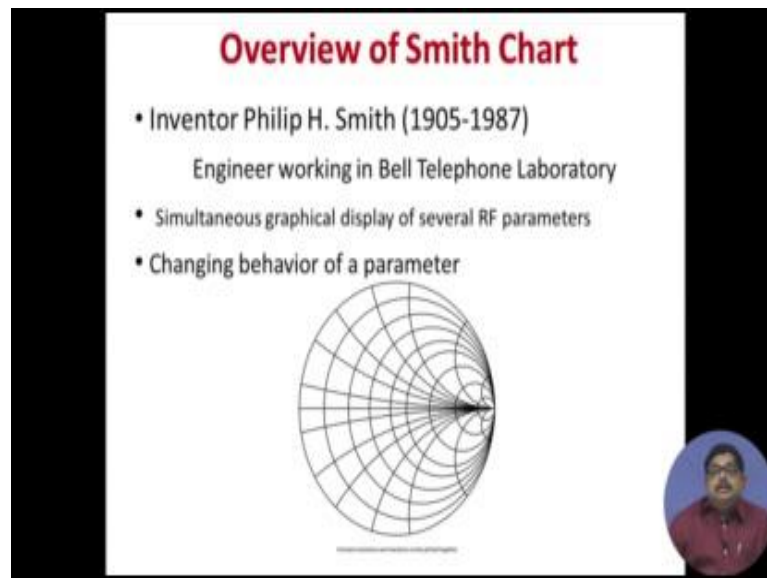


Basic Tools of Microwave Engineering
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Lecture – 02
Introduction to the 1st Tool: Smith Chart

Welcome to the 2nd Lecture, Introduction to the 1st of the Tool. Actually, in this course we will be seeing 3 tools which are heavily used for microwave design or RF design. And they are; Smith Chart, Scattering Parameter and Signal Flow Graph. The first tool will be the topic of this 2nd lecture Smith Chart.

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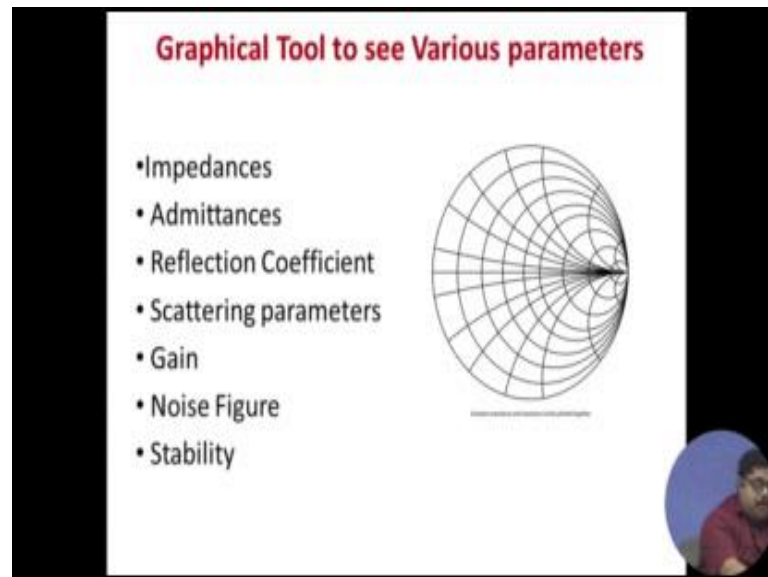


Smith chart was invented by Phillip Smith; a brilliant engineer was working in Bell laboratory. You see that only in the last century it was invented at the middle of the century roughly like 1940 type he gave that smith chart. Actually, it is a graphical tool, it is a very simple tool like one important equation it is just displaying graphically and so several array parameters are graphically displayed in this tool. And also if you change a parameter how the other parameters are behaved that is described here. You see unless and until you have simultaneous display of at least two parameters then when you are changing one how the other is changing you cannot display.

So, smith he understood this problem that, if I am changing one part of the story or one part of a circuit how the other part is or other parameters are behaving I need to have a

graphical tool to do that and that is why he invented that. But if you look like it looks a bit complicated some circles etcetera. So basically, in this lecture we will try to unravel those complications.

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You see that the first parameter that is put or displayed in this smith chart is Impedance. You know I have already in my first lecture emphasized the need of impedance for an RF designer. So, I need to calculate impedances at various places of my network, so impedance is the one which I always try to see. And when I connect to networks if they are in series then I need to find out impedances of them so that I can find out what is the total or equivalent impedance of that series connection.

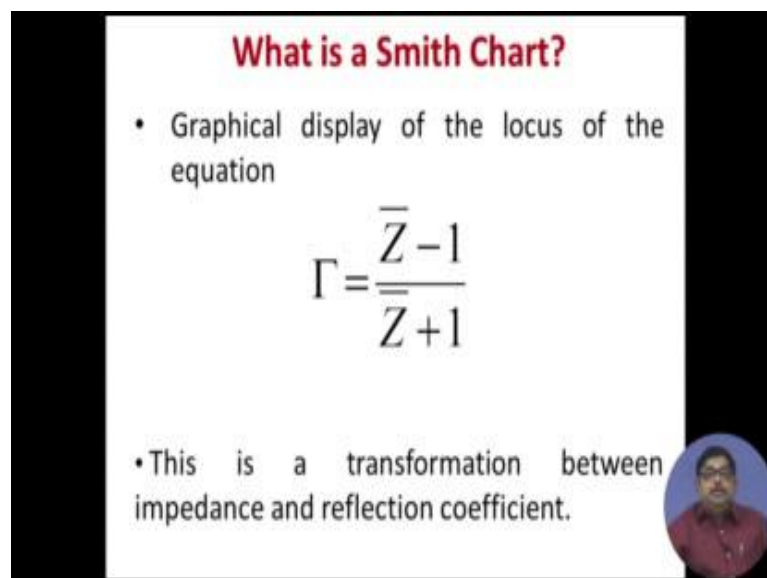
Sometimes you know that we will also have parallel connections and that time instead of impedances you know admittances they get added. So, admittance also can be displayed on the same smith chart. Then you know that at RF frequency or actually at any frequencies starting from 50 hertz there is a wave that goes, because energy from any source which is having any time variation that means any ac source that always radiates energy, it always radiates but that radiation may be sustained may not be that is why we do not always get radiation from it.

But whatever it is even if it sending the power it is sent in the form of wave, that means with space the energy gradually goes and then if it sees some discontinuity either a material discontinuity or geometric discontinuity etcetera that means, if it sees a

impedance discontinuity then the wave gets reflected. So, we need to find out how much wave got reflected that parameter is called Reflection Coefficient that also is displayed on the same smith chart. Then I said scattering parameters they are very important we will see later, they also can be displayed on this smith chart.

Then gain of any amplifier etcetera that can also be displayed there. I explained the importance of this point how much noise is there one parameter noise figure characterizes that that also can be put on this smith chart. Then I said that RF amplifiers have stability problems that stability also can be put that way that a amplifier is stable or not that we can plot on this. So, you see various RF parameters of importance they can be simultaneously displayed in the smith chart.

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What is a Smith Chart?

- Graphical display of the locus of the equation

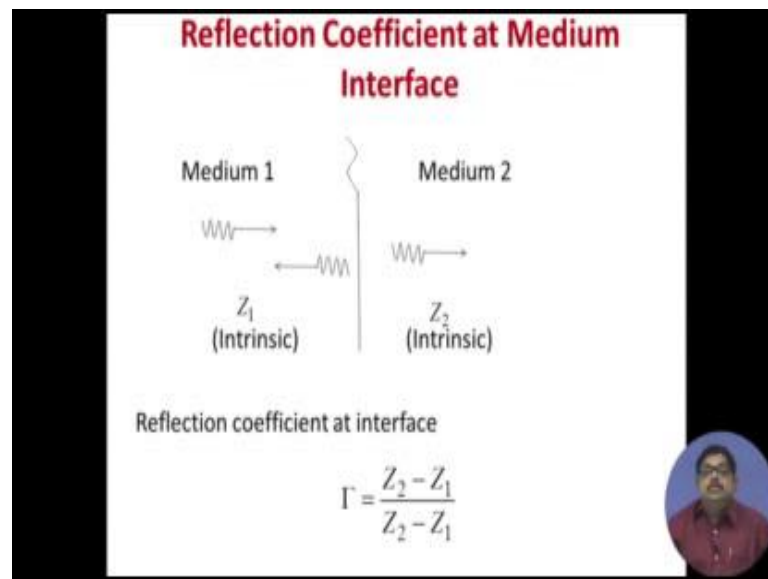
$$\Gamma = \frac{\bar{Z} - 1}{\bar{Z} + 1}$$

- This is a transformation between impedance and reflection coefficient.

Now, let us come to what is a smith chart. Basically, you know this famous equation that gamma reflection coefficient is equal to Z with a bar that is normalized impedance minus 1 by Z bar plus 1. This is an equation I think you have done transmission line course and they are when there is a forward wave there is a reflection from a load, so that reflection coefficient and the impedance they are always related by this formula. Now this formula if we plot that and now one of this either reflection coefficient or impedance if we vary how other is varying that displays when one of the parameters of any display is varied we get various points, so if they are joined by that that is called Locus.

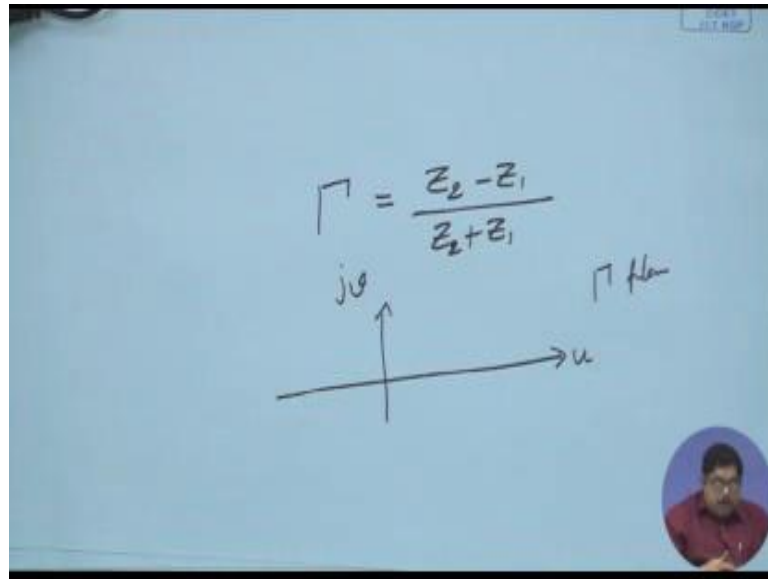
So basically, smith chart is a locus of this equation and this is basically a transformation. We can also look it as a transformation that reflection coefficient suppose I have a reflection coefficient description I am by this equation transforming that same description to impedance. So we can call that smith chart is also a transformation between impedance and reflection coefficient.

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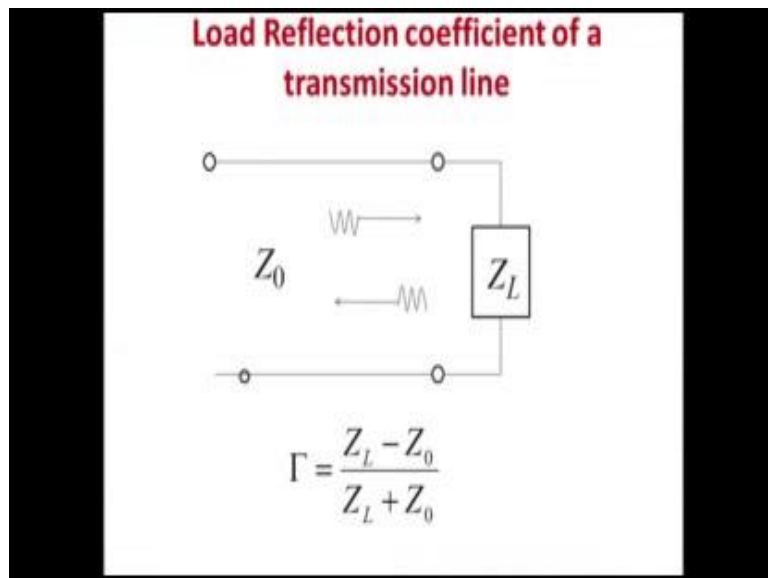
Now, what I said that if you have a medium 1 with intrinsic impedance Z_1 and then it falls on a medium 2, so there is an interface between the medium 1 and 2 where they are joining. So, medium 2 is having some other intrinsic impedance Z_2 . Then from the interface there will be a reflection back to the medium 1 and that reflection coefficient of interface is given by Z_2 minus Z_1 sorry, there is a mistake Z_2 minus Z_1 by Z_2 plus Z_1 .

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Please note this; the slide is wrong here gamma is equal to $Z_2 - Z_1$ by $Z_2 + Z_1$.

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Now, if I have a load in a transmission line and suppose the characteristic impedance of the transmission line is Z_0 then if the transmission line was carrying power towards the load. Then since it is also the case of the previous slide that I have an impedance Z_0 characteristic impedance of the transmission line, so from that impedance level suddenly I am seeing a new impedance level Z_L there will be a

reflection and then the reflection coefficient will be Z_L minus Z_0 by Z_L plus Z_0 . So, here the equation is correctly written the denominator is plus. So this is called Load Reflection coefficient of a transmission line.

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Normalisation of Impedance

- To make the chart independent of characteristic impedance

$$\Gamma = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{\bar{Z} - 1}{\bar{Z} + 1}$$

- Locus of this transformation is Smith Chart

$$\bar{\Gamma} = u + jv$$

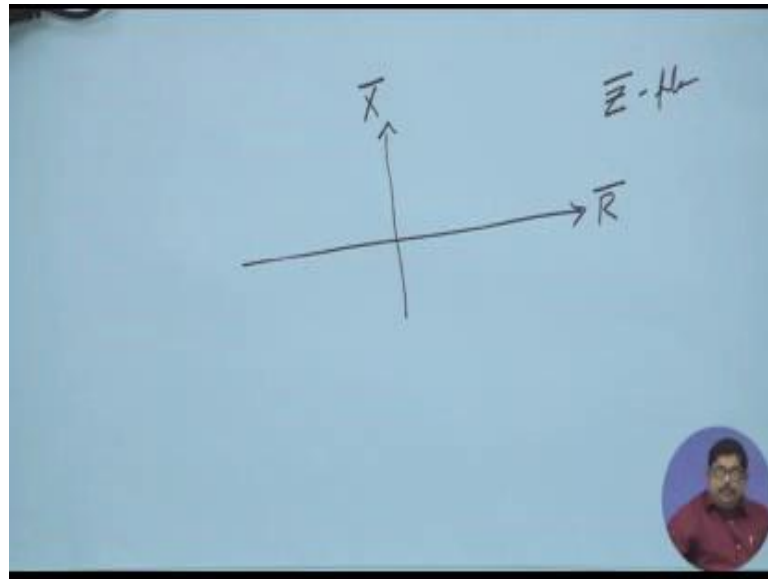
$$\bar{z} = \bar{R} + j\bar{X}$$

$$\bar{\Gamma} = u + jv = \frac{(\bar{R} - 1) + j\bar{X}}{(\bar{R} + 1) + j\bar{X}}$$

Now, to make the chart independent of characteristic impedance because I can use various types of transmission lines, someone is using some with characteristic impedance 50 ohm, someone else may use it with 75 ohm etcetera. So the chart becomes a general, what is done in this whole chart is the whole equation on the right-hand side that means the impedance side of the equation is divided by the characteristic impedance, so that is called Z bar. So, Z_L by Z_0 that we are calling Z bar, it is normalized impedance. And so that equation which was a Smith chart fundamental equation Γ is equal to Z bar minus 1 by Z bar plus 1 we are getting from this.

Now both this Γ reflection coefficient and Z bar the normalized impedance these two are complex quantities. So, any complex quantity I can write it as a plus j beta real part plus imaginary part, so here we take that $\bar{\Gamma}$ is equal to $u + jv$; that means u and v are real numbers they can be plotted on a $u-v$ scale like this. I have this side u , real side, this side v , this side, so this is a Γ plane I can say. Complex Γ plane can be described by this $u-v$ this is your imaginary side.

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Now also Z, if I look the same story in the Z plane then I will get that this side will be my R bar and this side is my X bar. This you know that we can write any complex impedance as R bar and X bar, so this is our Z plane. Now basically smith chart is a conversion from them, so in that equation this gamma is equal to Z bar minus 1 by Z bar plus 1, we can put these values so that we get the gamma bar gamma in terms of this all real values. If we do that equation then we land with gamma is equal to R minus 1 plus j X bar by R plus 1 plus j X bar.

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Locus of a Complex number

- Smith Chart plots the locus of Γ for constant \bar{Z}
- Both $\bar{\Gamma}$ and \bar{Z} are complex numbers
- Two families of locus, one for constant \bar{R} one for constant \bar{X}

Now, just we will have to plot the locus of gamma for constant Z. That means, smith chart is the locus of gamma for constant Z, but as we have seen gamma and Z both are complex numbers so there will be generally we break whenever we come across complex numbers always we try to break these complex numbers into two orthogonal real numbers a plus j b. So, here also we will see that we generally try to make this in terms of real thing so a complex Z complex impedance constant that will break into that both its real part and imaginary part. That means, R and X or R and X bar they are constant.

So, that will lead us to two families of curves that is why since two families of curves are plotted on the same chart smith chart looks a bit complicated. But if you understand then you will unravel this; Two families of locus - one for constant R bar, one for constant X bar.

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Equations for Smith Chart

$$\bar{Z} = \frac{1+\Gamma}{1-\Gamma}$$

$$\bar{R} + j\bar{X} = \frac{1+u+jv}{1-u-jv}$$

- Equating real and imaginary parts,

$$\bar{R} = \frac{1-u^2-v^2}{(1-u)^2+v^2} \quad \bar{X} = \frac{2v}{(1-u)^2+v^2}$$

So, let us see the equation again Z is equal to 1 plus gamma Z bar. Now we are inverting this because we are saying that we will make smith chart is plot of or locus of gamma when constant Z. So, that is why we are writing that equation which previously we have written in terms of gamma, now we have changed it to Z bar is equal to 1 plus gamma by 1 minus gamma. Then you put it in terms of the real and imaginary part and you come here, now you equate the real part and imaginary part. Then you get this two (Refer Time: 13:13) equations that R bar is something. So, you see that the right hand side of

this equation \bar{R} is equal to $1 - u^2 - v^2$ by $1 - u^2 + v^2$, there right side does not contain any \bar{R} or X .

Similarly, \bar{X} also can be written as that sum $u + v$ things u and v both are real quantities, so now we are having an equation that there are no complex thing in this equation all the left hand side and right hand side they are real quantities. Now we know this is a second order equation not a first order equation So, let us see what is this equation if we that \bar{R} is equal to 1.

As I said that when we are saying smith chart is for constant impedance how the reflection coefficient vary. Constant impedance means constant \bar{Z} that means, both constant \bar{R} and constant \bar{X} , now we have separated it because unless and until we get it into real and imaginary parts we cannot handle it that is why these two.

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Constant resistance locus

For constant \bar{R} ,

$$u^2 + v^2 + 2\left(\frac{-\bar{R}}{\bar{R}+1}\right)u + \frac{\bar{R}-1}{\bar{R}+1} = 0$$

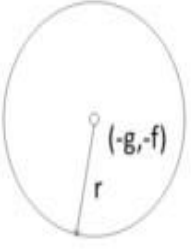
- Equation of family of circles in Γ plane
 - centre at $\left(\frac{\bar{R}}{\bar{R}+1}, 0\right)$
 - radius $\rightarrow \frac{1}{\bar{R}+1}$

The slide contains two diagrams. The left diagram shows a horizontal axis with several vertical lines at different positions, representing constant resistance values. The right diagram shows a family of circles in the Γ plane, all centered on the horizontal axis and tangent to the vertical axis at the origin.

Now for constant \bar{R} , you see that in the left side diagram the constant \bar{R} means all vertical lines with various values \bar{R} may be \bar{R} may be 0, \bar{R} may be 0.5, \bar{R} may be 1, \bar{R} may be 3, etcetera infinite. That means, this is an infinite line extend vertical lines. Now, each of them you see the mathematics that for constant \bar{R} we can write it as this, so if we make \bar{R} constant then this becomes an equation of the form $u^2 + v^2 + 2u$ something into plus $2u$ something plus this. This you know that equation of a family of circles, so you can easily find out take the help of this.

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General Equation of a circle



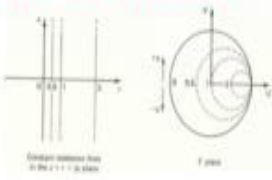
$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f)$
Radius $= \sqrt{g^2 + f^2 - c}$


This is just for refreshing your class 12 knowledge that if I have a circle with center at minus g minus f and radius R. I can write it as and the equation of the circle is x square plus y square plus 2 g x plus 2 f y plus c is equal to 0 then I have minus g minus f is the center and R is the radius, where R is given by root over g square plus f square minus c. From this if you put in that equation you get the family of circles, so center is at R bar by R plus 1 and a is the 0. That means, all the centers they are on this real axis of the graph and the radius is given by this.

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Constant resistance locus (Contd.)



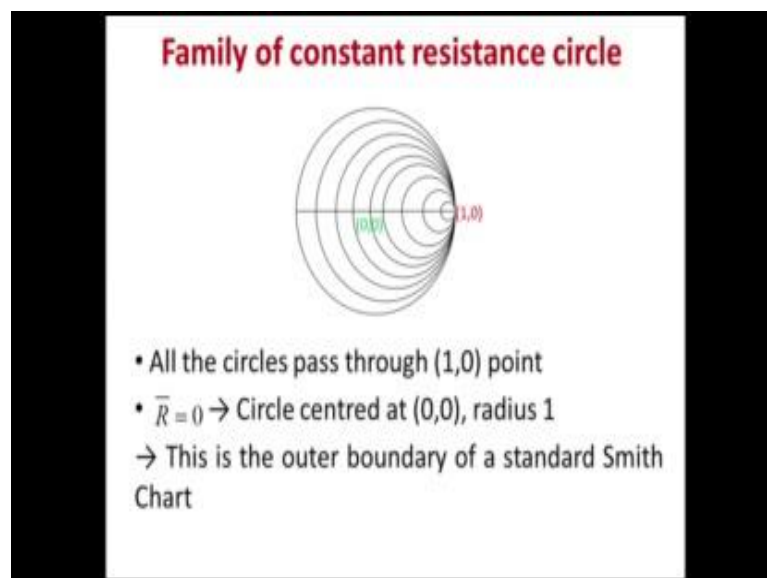
- Constant \bar{R} locus is
 - family of vertical lines in impedance plane and
 - family of circles in Γ plane.
- When \bar{R} negative, radius $(\frac{1}{R+1}) > 1$
→ used for oscillator design, stability analysis.



So, you see the constant R locus they are in the tau plane, that means the smith chart plane they are all circles. What happens so you see there are various circles, all circles are passing through the right hand portion 1 0. And you see that what happens if R is negative? You see our expression that radius is $1 + \bar{R}$. So, if \bar{R} is negative what happens, radius becomes more than 1.

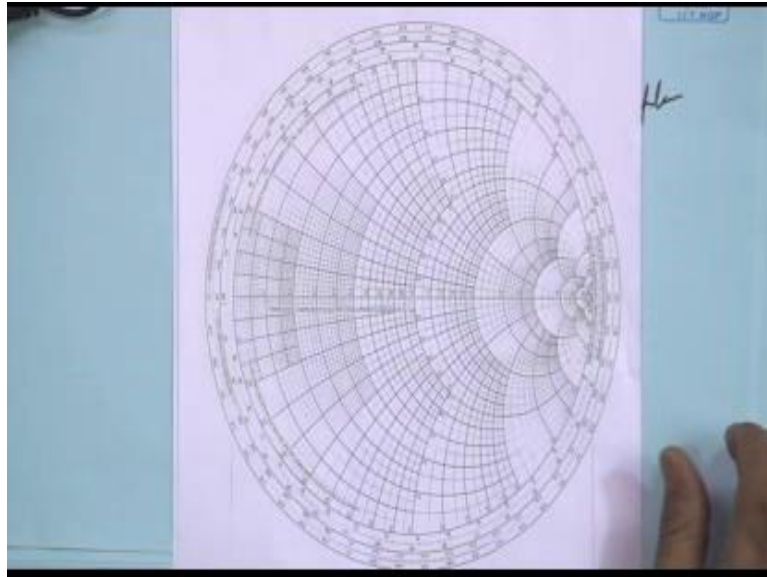
Now can we have \bar{R} negative that means resistance negative means a is in active devices we have resistance negative, so in that case that point who has got \bar{R} negative that means, the resistance negative means it is an active device it is instead of dissipating it is giving power so that the radius falls outside is more than 1 it is used for oscillator designs stability analysis etcetera.

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So, you see if we plot the family of constant resistance circles it looks like these, all these circles are passing through the point 1 0, all the centers of the various circles they are on the real axis of the thing. And there is one important point \bar{R} is equal to 0 that is the outer boundary of the whole smith chart. Generally we deal with passive circuits in this smith chart, so smith chart you can find out it will be uploaded in your site also.

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So, this chart is smith chart you see that all constant resistance circles they are this and this outer boundary that is corresponding to the R/Z_0 is equal to ∞ circle, if center is at 0 and its radius is 1 that means, from the center to the right most point this is considered as 1. It may be some other millimeter etcetera, so you can measure this but that is in the smith chart calculations normalized to 1.

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Negative Resistance

→ For negative resistances one needs to go outside of the standard boundary.

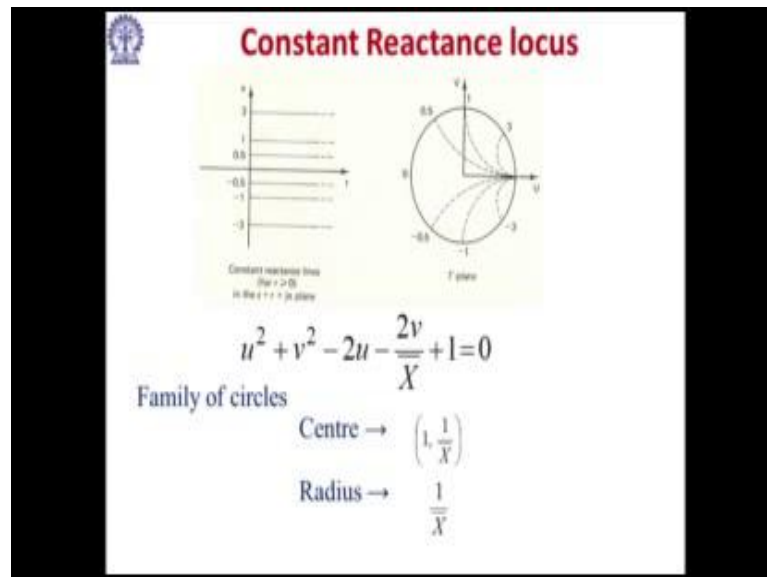
→ If at any point, one is outside of this boundary, there are oscillations in the circuit.



Now for negative resistance one needs to go outside of the standard boundary as I explained. If at any point 1 is outside of this boundary then you be careful because

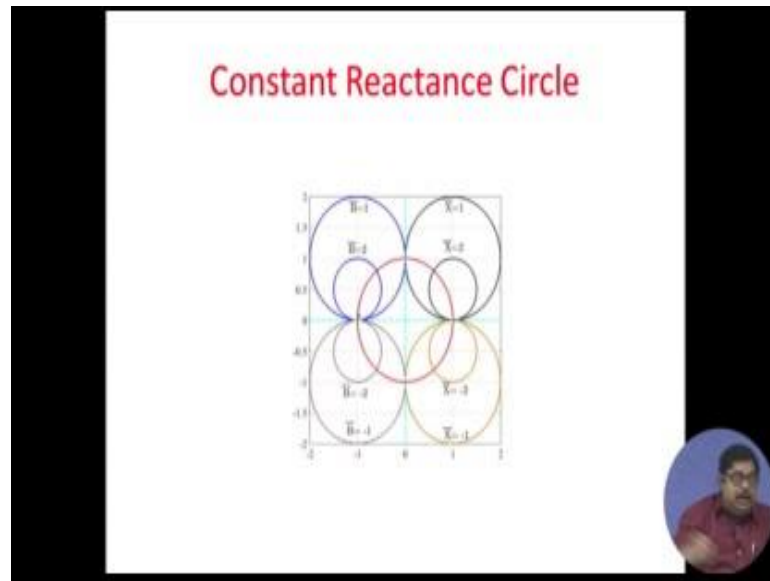
oscillations are starting in the circle. So, if by doing something you find out that you are going outside of this outer boundary that means your R bar that is becoming negative then your oscillations have started, so it is no more a passive circle. This is a very good question RF engineer's get.

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Now, let us see the other family of circles constant reactance circle. Because as I said that Z bar has two parts impedance as resistance as well as reactance, so reactance means X bar. What is a constant reactance in the impedance plane, these are all horizontal lines. So, infinite horizontal lines of various values again if we do that equation for constant X bar. So you see this is again a circle equation and the center and radius you can always work out j. So this is the center from that same formula and radius is this.

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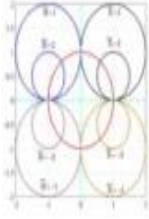
Again these circles are plotted here only the red marked one that is the outer boundary of the smith chart. If you see again the figure you will see that all these circles most of them are falling outside of that standard smith chart, so their centers are all lying outside the smith chart. But one point is still true that if you see the first and fourth quadrant that all the circles of constant reactance circle they are again passing through the right most point 1 0.

So, the same point through which all the constant resistance circles passed the constant reactance circles also passed through the same point. But, unlike in this constant reactance circles they are mostly going outside this standard smith chart.

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Constant Reactance Circle (Contd.2)

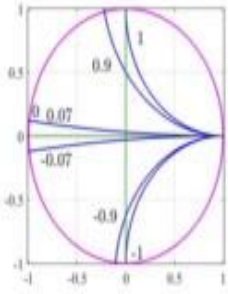
- All the circles again pass through (1,0) point
- Red Circle ($\bar{R} = 0$) is standard smith chart outer boundary
- In standard smith chart, constant reactance circles look like arcs




So, red circle is the standard smith chart outer boundary. From this we can say that in a standard smith chart which is smith chart where this \bar{R} is equal to 0 is the boundary. So, there the constant reactance circles will be forming as if arcs.

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Constant Reactance Circle (Contd.3)

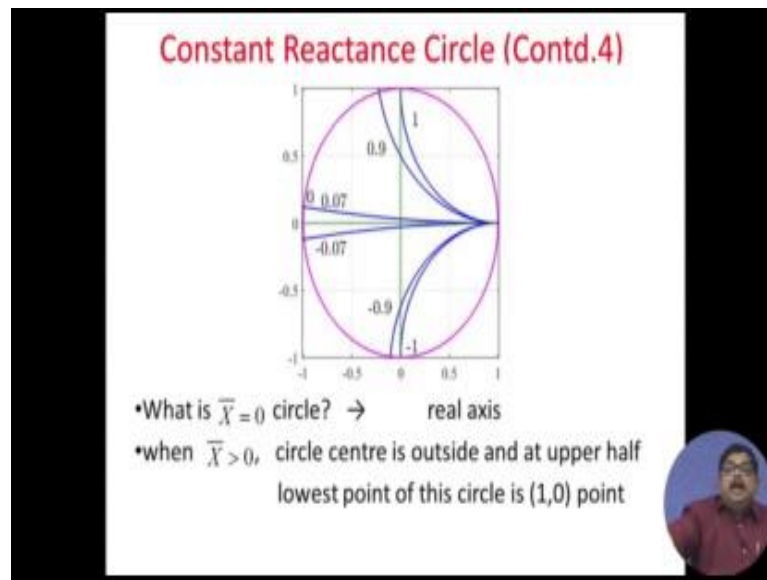


Constant Reactance circles are arcs in Standard Smith Chart



On the standard smith chart if you plot the constant reactance circles they will be part of the circles arcs. So, you see various values we have shown 1, 0.9, etcetera, so constant reactance circles are arcs in standard smith chart.

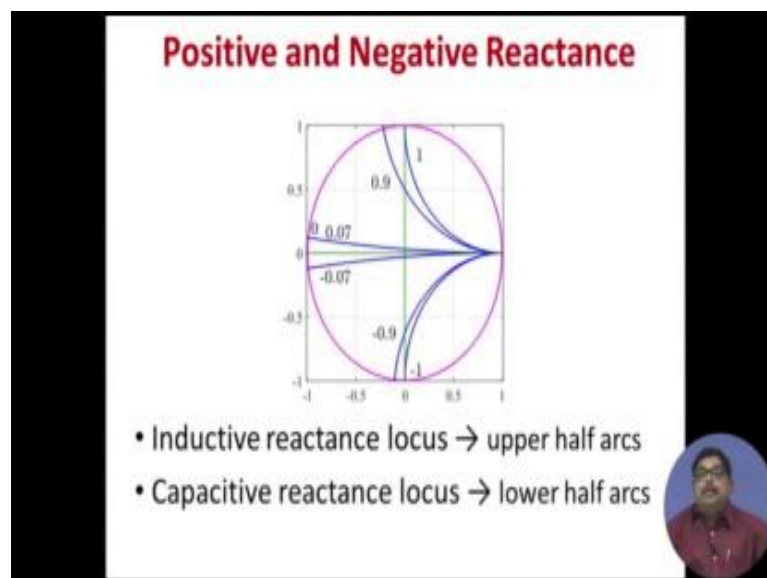
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Now, what is an X bar is equal to 0 circles? X bar is equal to constant X bar may be 0. So, you see if you do that actually degenerates to a straight line infinite line. In a circle for X bar is equal to 0 that degenerates to a real axis of the thing, so we have shown that it is the 0. So, the central real axis that corresponds to X bar is equal to 0.

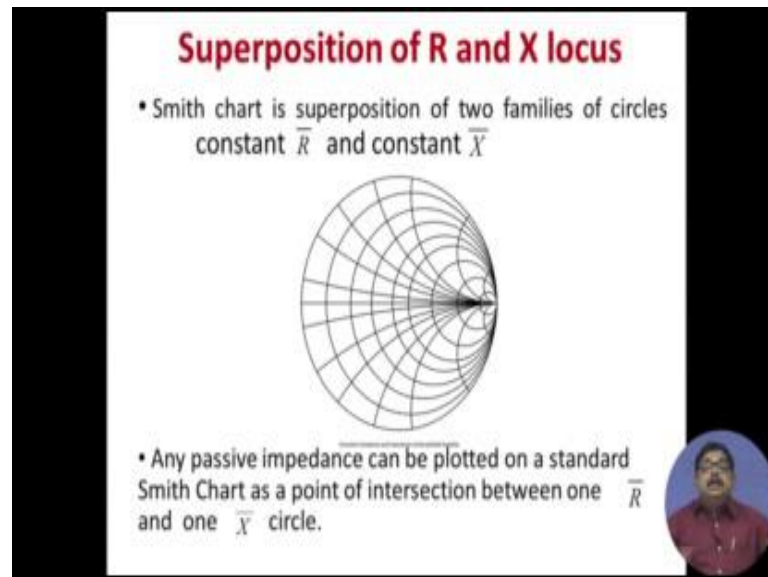
And when X bar is positive then circle center is outside so it shows that the arc is the upper half portion. And that upper half portion is still passing through the 1.0, the right most point of the standard smith chart and lowest point of this circle is 1.0.

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So, you can see the positive and negative reactance. From this we can say that inductive reactance locus are upper half arcs as you can see clearly and capacitive reactance locus they are the lower parts.

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Now, smith chart is super position of both these R bar and X bar locus. So, smith chart is superposition of two families of circles. Any passive impedance can be plotted on a standard smith chart as a point of intersection between one R Fixed R circle and one fixed X circle. Two circles you see some full circles inside the smith chart some are arcs. One family is full circle another family is arc, so there will be always a meeting point that is the impedance.

Now what is the advantage? Advantage is you see impedance can vary that means, the R and X they can vary from minus infinity to plus infinity or for passive devices they can vary from 0 to infinity. But that whole thing smith chart is plotting into a finite zone, this outer boundary of this circle. So all the impedances are present here, so you tell any passive impedance they are here that is the beauty. That instead of any infinite domain it is confined to a finite plane.

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Rotation of 180° in Smith Chart

$$\bar{Z} = \frac{1+\Gamma}{1-\Gamma}$$
$$\bar{Y} = \frac{1}{\bar{Z}} = \frac{1-\Gamma}{1+\Gamma}$$

In the Smith chart, radius vector of a point is rotated by 180°
Reflection coefficient of the new point is $\Gamma e^{j\pi}$

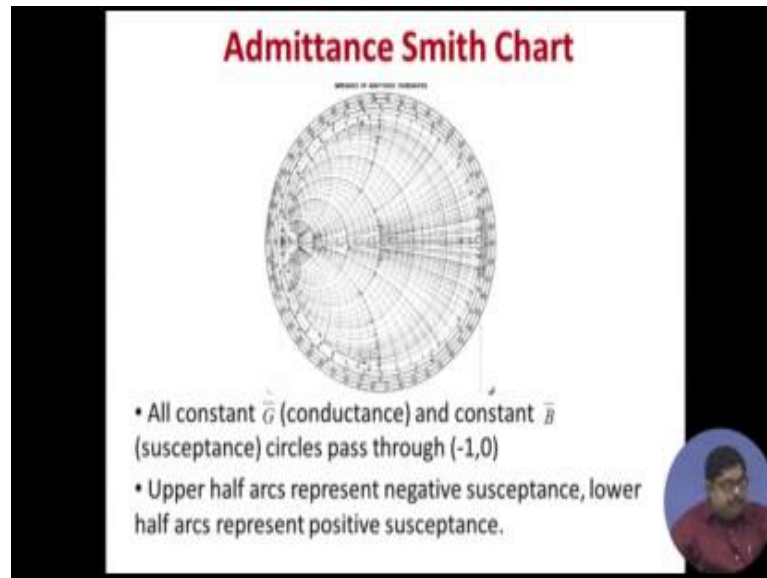
$$\bar{Z} = \frac{1+\Gamma e^{j\pi}}{1-\Gamma e^{j\pi}} = \frac{1-\Gamma}{1+\Gamma} = \bar{Y}$$

Rotation of 180° in impedance Smith chart transforms impedance point to admittance

Now, what happens if I rotate the smith chart by 180 degree? So this is my basic equation the first one \bar{Z} is equal to the first equation. Now what is the admittance for that, simply reciprocal of that. Now in the smith chart radius vector of a point is rotated by 180 degree, so reflection coefficient of the new point will become instead of gamma it will be e to the power j pi you put it into that first equation you see \bar{Z} becomes y.

So, the take away from this slide is if you rotate the impedance point on the smith chart by 180 degree impedance point get transferred to an admittance point. This will have a very useful implication in our later problem solving.

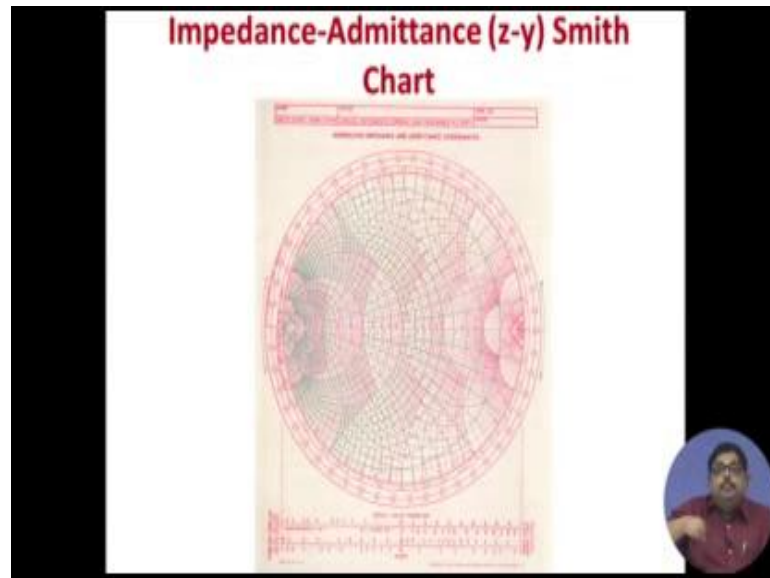
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Now, also there are admittance smith charts available where instead of having that 180 degree transform people have done that for you, so you will be getting it we will be uploading admittance smith chart also where the all the things are in terms of y . We will see that here all the points are passing through all the constant conductance and constant susceptance circles passed through minus 1.0. Last time it was right side for impedance smith chart, for admittance smith chart it is left side. And here the upper half arcs represent negative susceptance, lower half arcs represent positive susceptance.

So, these are the only changes you should note if you have admittance smith chart and admittance. At same point you can find out either by admittance smith chart or by impedance smith chart, we will see an example in the tutorial how to do this.

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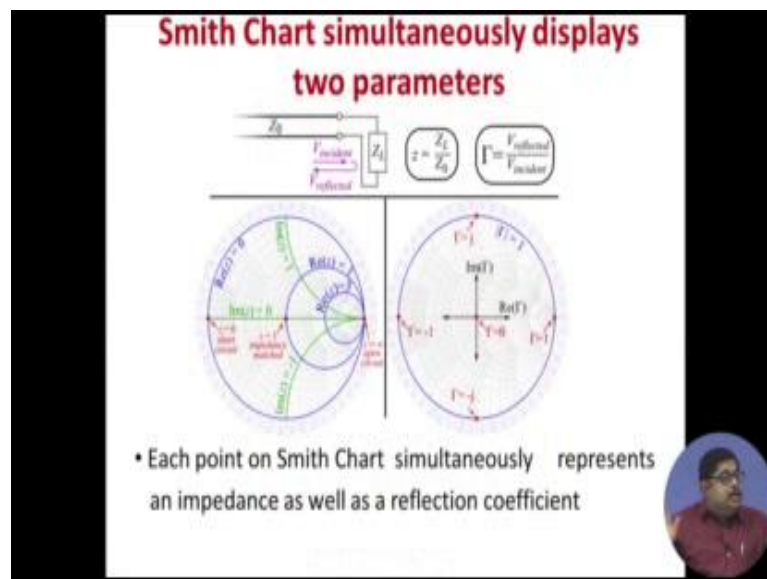
Then in various cases suppose in a network we will have combination of all these, that suppose some series connections of various RF components then a parallel combination etcetera. Sometimes we need to have impedance; sometimes we need have to stay in the admittance plane. Now there is a chart available it is called z-y chart or impedance admittance chart, where the red circles and arcs are for impedance and the green arcs and circles are for admittance.

Here, you can always go any point from because any point you can read it in terms of the red ones or green ones. So any point you have located let us say in y plane that means, you have located in terms of green values then the same you can read in terms of red and that will be the impedance that is the beauty this is called impedance admittance smith chart. This we will also be uploading this will be helpful in that.

And you will see that the outer most portion of the smith chart there you have shown that if you move by so and so distance in terms of lambda then where you are going. So there are two things; one is you can go either to a source which they call generator and also you can go towards load. From any point if you are in a transmission line you are going to source you can find out where you are going in the outer boundary. So, any impedance point if you move along the transmission line towards the generator you can find out where you are landing up.

Similarly if you go towards load you can find out where you are landing up also the same information is given in terms of angle. So, you know that any distance travelled by a wave that can be expressed either as a distance in terms of lambda or in terms of angle, because if you travel by lambda then you are basically travelling an angle of 360 degree the angle equivalent of that. By this which is in terms of lambda distance that can be also said in terms of how much phase you are changing.

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So, this is the last slide. You please remember this slide you look into carefully and remember that whenever any trouble you remember the smith chart simultaneously displays two parameters; the right side we are seeing always you can find out at any point what is the imaginary part of that and real part from that you can find out a reflection coefficient for that, left side shows it also is showing you in terms of intersection of two families of circles so you can find out impedance from that.

So, each point on smith chart simultaneously represents impedance the left side as well as a reflection coefficient. So whichever is convenient for you, whichever is convenient for a particular design problem, whichever is convenient for a particular analysis problem, you can revert to that is the beauty of smith chart. We will see more on application smith chart in the next 2 lectures.

Thank you.