Bayesian/MMSE Estimation for MIMO/OFDM Wireless Communications

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Lecture – 01
Basics - Introduction to Bayesian Minimum Mean Squared Error (MMSE) Estimation

Hello welcome to this massive open online course on Bayesian or MMSE estimation for wireless networks. So, the title of this massive open online course is going to be Bayesian estimation.

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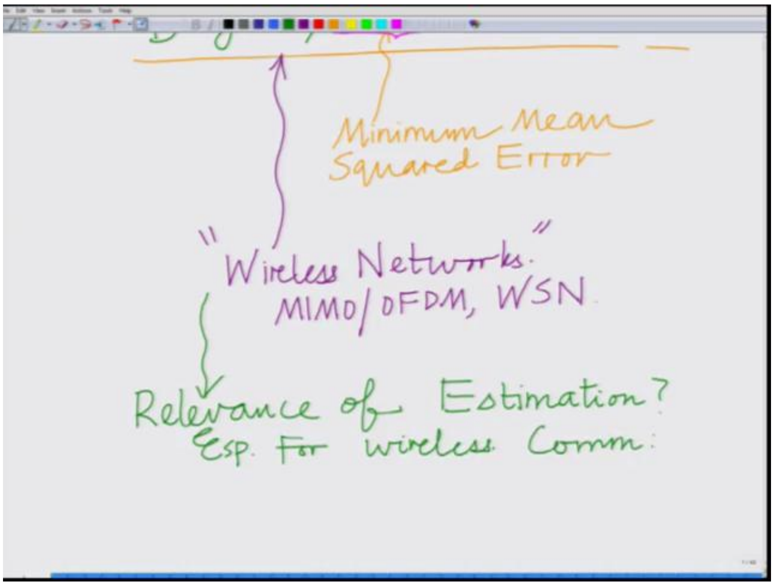

So, we are in this course we are going to look at different aspects of Bayesian or basically MMSE estimation. Where this term MMSE stands for minimum mean squared error. So, term MMSE stands for minimum mean squared error. So, let me right down because this is something that, we are going to keep using throughout the course it is essential that you are familiarising with the nomenclature.

This is the minimum mean squared error and we are going to look at both of these things we are going to look both of these things in the context of wireless networks, that is important that is you are going to look at examples or applications of both these in the context of wireless networks. Especially MIMO OFDM and wireless sensor networks correct. So, we were like to look at in this course we would like to look at Bayesian estimation or minimum mean squared error estimation especially in the context of wireless networks as applicable to problems involving MIMO, that is multiple input multiple output systems OFDM is stands for orthogonal frequency division multiplexing WSN, which stands for wireless sensor networks also some of you might already meet familiar with these terms based on your exposure to different concepts in wireless communications. So, now what I would like start with is I would like start by motivating the need for estimation or by motivate by motivating or illustrating the importance of the concepts of estimation through a simple example.

So, first we would like to motivate the relevance of these problems, what is the relevance what is the relevance of the estimation especially for especially for wireless communication what is a relevance of estimation?

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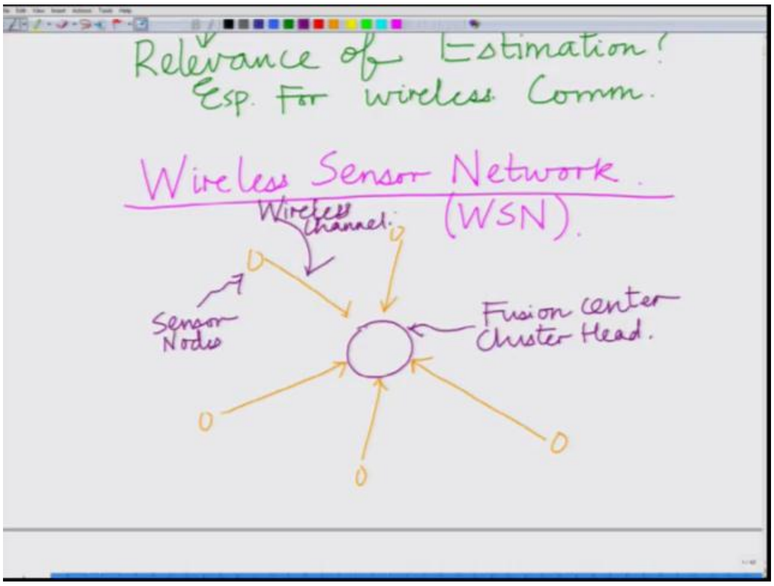

For that what we would like to know is, we would like to start by considering a simple example of a wireless sensor network which we can also abbreviate as WSN. WSN starts stands for wireless sensor network; we would like to motivate the relevance of estimation in the context of wireless by considering the simple example from wireless sensor network. That is you might already be familiar from a general understanding that wireless sensor network is consist of basically several sensors, several sensor nodes which communicate over the wireless channel through a central node or which is also termed as the cluster head. So, in a wireless sensor network what we have is basically we have a central node which is also termed as the head or the fusion centre or the cluster head. We can call this as a fusion centre or your cluster head and what we have other than this fusion centre or we have several sensor nodes which communicate over the wireless channels.

So, what are these is basically your sensor and this basically, denotes the wireless channels. So, we have these sensor nodes which communicate over their respective channel with a fusion centre they send their measurements to the fusion centre over the wireless channel. That is the basic set up of a wireless sensor networks which can basically collates allot of this measurements from a large number of sensor nodes about some ambient parameter such as the temperature pressure or other such parameters and then followed by some processing at the fusion centre to come up with an appropriate decision all right, that is the basic paradigm of a wireless sensor node, sensor nodes and measurements.

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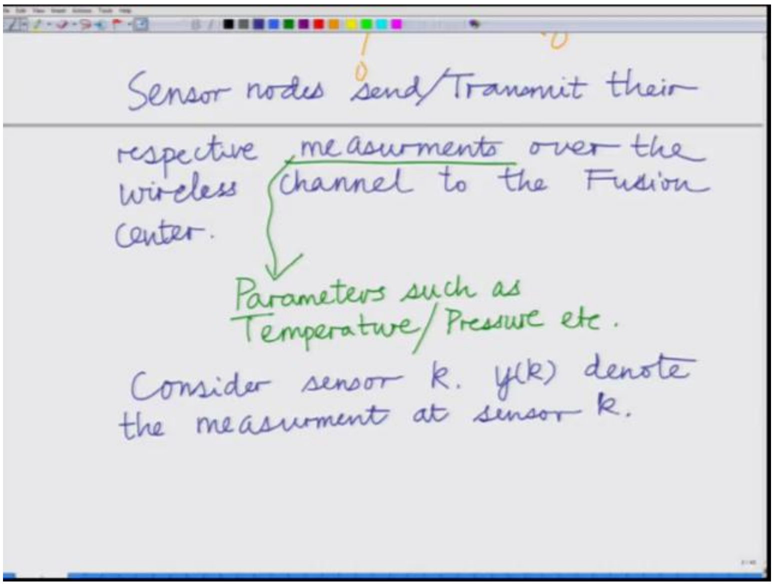

So, let us write that down sensor nodes basically send or transmit their respective they, transmit their respective measurements over the wireless channel to the to the fusion centre or basically your cluster head for instance it is measurement can be relating an ambient parameter measurements of parameter such has such has your temperature pressure etcetera.

Now, what you want to do is want to start by considering a simple sensor case. So, let us start by considering a kth sensor. So, consider sensor k let us model the measurement at sensor k let y k denote, let y k denote this measurement at sensor k.

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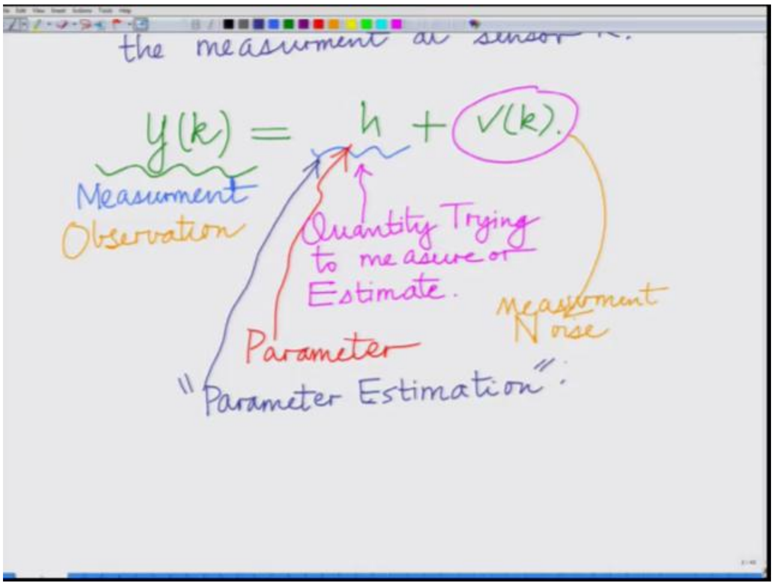

Now y k, y k can be modelled as and this is the model that is frequently employed y k or this is the simple model that can be used to model this measurement process y k equals h. Yes we can going to explain these terms subsequently equals h plus v k, where y k as we have mentioned this is the measurement at this is the measurement at sensor k this h is the parameter we are trying to measure such as your temperature or pressure this is the quantity that you are trying to measure or basically estimate or estimate and therefore, estimation. Estimation is nothing but more scientific term or one can look at.

As a technical term that we are going to describe this process of measurement any measurement that we make is basically towards estimation of some quantity such as the pressure temperature humidity moisture so on and so forth. And this process I mean the process of this measurement all right the process of this measurement is towards the estimation of a certain quantity. And that quantity beat the temperature or pressure etcetera is denoted by this parameter h and this quantity v k is another important aspect which has to be explaining this is the noise in the measurement. This is the measurement noise because right because none of the measurement are ideal none of the measurements none of the measurement none of the measuring instruments are infinitely precise. So, what we have is whenever, even for the same quantity if you measure it several times the measurement readings are going to be different because of the noise in the measurement process.

So, therefore, this noise in the measurement process which basically leads to error or this deviation of the parameter that deviation from the parameter h that we are trying to measure this is basically your measurement noise all right and frequently this is modelled as a Gaussian noise. We will come to that later. So, every measurement process, every measurement model has 3 key components one is the measurement for the observation which is denoted by this quantity y k the parameter h that is we are trying to estimate that is the aim of this measurement process or this observation process and the noise v k, which is basically trying to corrupt this measurement process. So, this y k is also known as a measurement. We will also frequently call this as an observation because that is in this is the observed quantity right. So, this is your observation.

This is observation and the h is the quantity or that we are trying to measure this quantity h that we are trying to measure this is also known as the parameter. This is the parameter that we are trying to measure hence the process of estimation of this parameter that is when we trying to find out what this parameter is that is the temperature or pressure this is also known as parameter estimation. And the frame work that we are going to use in this course is the Bayesian or the MMSE frame work for parameter estimation. That is to basically evaluate this quantity or basically reconstruct this quantity h based on these observations or measurements y k.

So, this frame work is termed as parameter estimation and that is basically, that is basically the key to understand, what we are doing that is we are doing try to estimate this parameter or parameter estimation. Now what we have is we have considered the single sensor.

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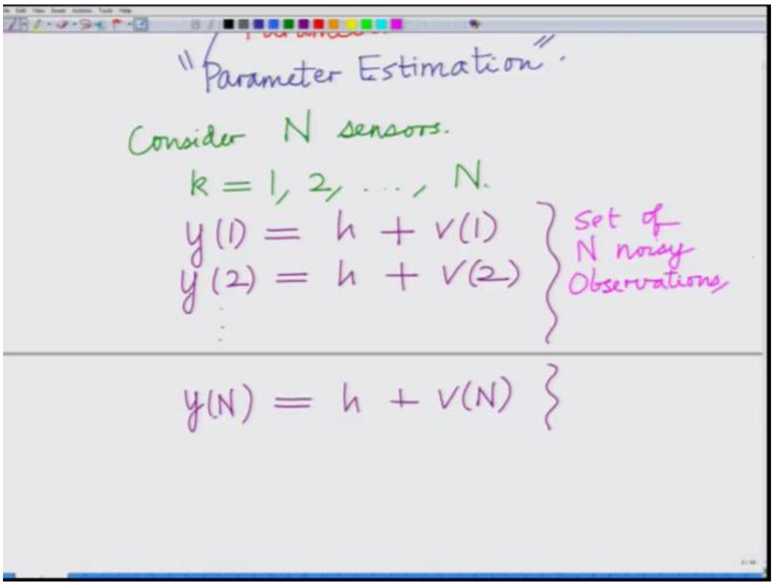

So, now consider n sensors consider n sensors which means your index k is equals 1 2 up to n. So, we have n sensor scenario you, what we have is basically we have a wireless sensor network in which several sensors n sensors trying to measure the same parameter such as the temperature or pressure. So, naturally each sensor is going to come up with it is individual measurement, which it is going to convey over the wireless channel to the fusion centre, these measurements can be modelled as so, the measurement. So, naturally I can replicate the model at each sensor.

So, at sensor one I have measurement one equals h plus v1 because they are trying to measure the same parameter y 2 equals h plus v 2 which is a different noise v 2. So, on y n this is the n sensor equals h plus v n. So, we have n sensors naturally, we have n observations or n measurements or more precisely set of this is a set of n noisy this is set of n noisy observations. This is your set of n noisy observations from which, we have to estimate this parameter h and that parameter h is basically that process of estimating this parameter h is basically termed as parameter estimation.

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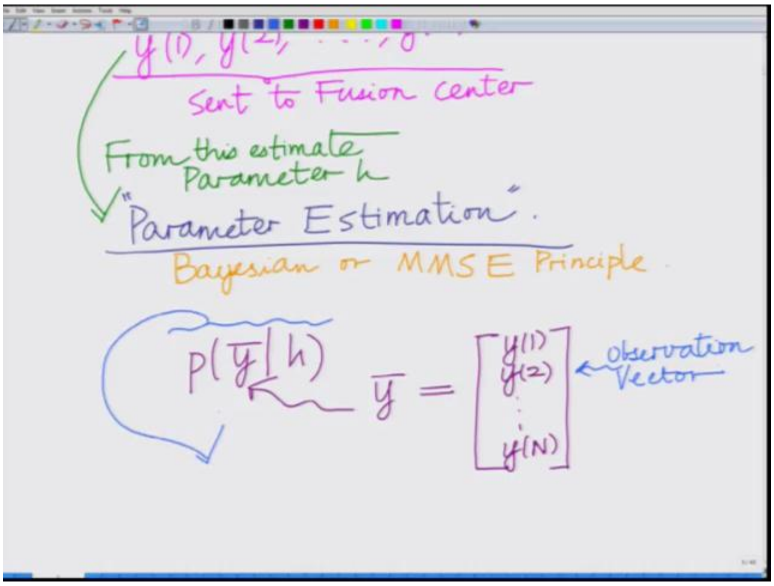

So, these noisy measurements y one y 2 y n, are now basically sent by the sensor. So, the fusion centres. So, these y 1, y 2, y n these are sent to the fusion centre or your cluster head and from this we have to estimate parameter h and this process is termed as parameter estimation this process of estimation of the parameter h from this measurement is termed as parameter estimation. Now there are various techniques for parameter estimation. One such technique is basically known as, the maximum likelihood estimation principle all right and this is about the principle seen in one of the previous course. In this course we are going to focus on a separate technique and equally important and powerful technique which is known as the Bayesian or minimum mean squared error estimation. That is basically the title of the course which is the Bayesian frame work that is how we use a Bayesian principle or the minimum mean squared error principle for parameter estimation. So, for parameter estimation in this particular course we would like to use basically the Bayesian. Bayesian we would like to use the Bayesian or the minimum mean squared error principle.

Now, what is the Bayesian principle? Now the Bayesian principle, now the any estimation process depends on the probability and probability density of functions. So, we have 2 important components in this process of Bayesian estimation these 2 different components are what I am going to list. Now the first component that is very important in any estimation process is p y bar given h what is this y bar is basically your vector of observations y one y 2 y n this is known as the vector of observations or simply as your observation, simply known as the observation vector. Of course, h is the parameter therefore, what is p y bar given h this is the probability density function of the observation vector y bar conditioned on the parameter h this is the probability density function of the observation vector given the parameter h.

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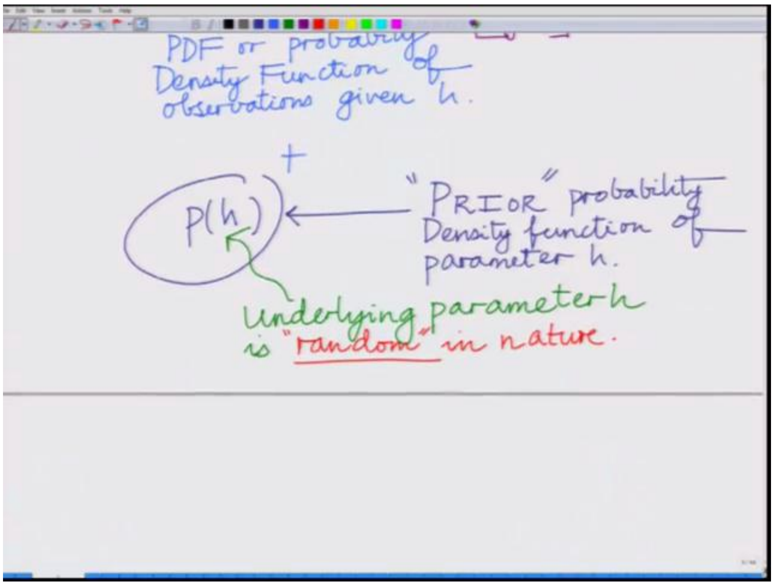

So, this is the p d f or the probability density function of this is the probability density function of the observations observation vector y bar given h or conditioned on the parameter h plus we need another important quantity. Which related to the which is related to the parameter h this is the probability density function all the parameter h this is known as the prior probability density of function. So, we need 2 quantities one is p y bar given h plus we need another important quantity which is p of h which is the prior and this is an important quantity this is the prior probability density this is the prior density function of the parameter h or the probability density of function.

So, there are 2 important components p of y bar given h probability density function of the observations conditioned on the parameter h p of h which, is and this is unique to the Bayesian estimation that is the prior probability density of function of the parameter h which means we are treating the unknown parameter h has been random in nature and that forms a key of the Bayesian frame work. That is the underline parameter is the random in nature, if you are aware of the maximum likelihood estimation principle the unknown parameter there is treated as a deterministic parameter. So, hence there is no there is a in the maximum likelihood frame work.

There is no concept of a prior probability density of function of the parameter h; however, if we are not familiar with that concept it does not matter in the Bayesian frame work, we assume that the underlying parameter h is a random in nature hence it is associated with the probability density of function that is p of h. Therefore, p of h this is what this denotes is that this this underlying parameter h this underline parameter h is it denotes the very fact the important fact that this underlying parameter h is random in nature and this is something that is important to realise because this is what distinguishes the Bayesian frame work from other similar frame works that is the underlying parameter h is random in nature.

Now we would like to use this 2 quantities p y bar given h and p of h to basically determine an estimate of this parameter h.

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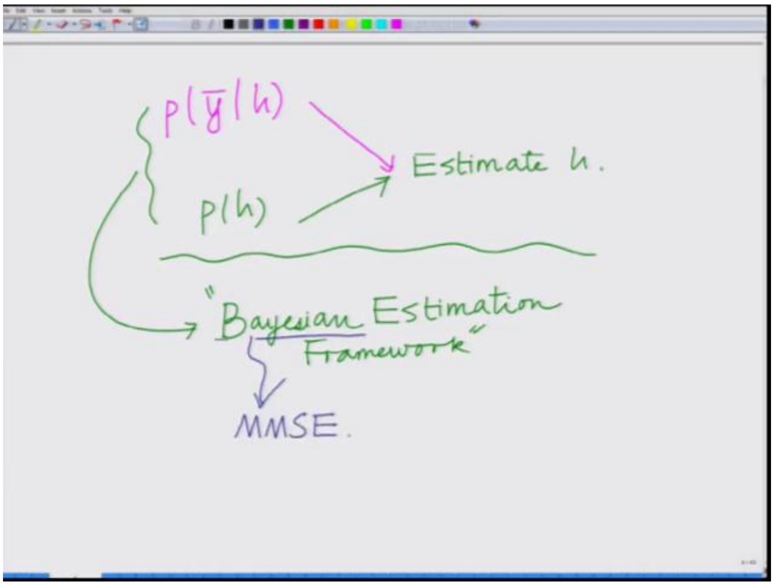

So, what we would like to do is we would like to basically use these 2 quantities the probability density function of the parameter given h p of y bar given h this is the observation this is the conditional probability density of function of the observation the prior o k and then, what we would like to do is we would like to basically use these to estimate our parameter h. So, these are the 2 quantities we would like to use to estimate the parameter h and how do we do that? That is basically that basically forms that basically and how do we do that this process basically using these 2 process, how do we that that forms the core of your Bayesian estimation frame work that basically forms the core of your Bayesian estimation frame work which we already also said that we are going to consider a specific version of this estimation frame work that is your MMSE or minimum mean squared error frame work.

So, what we have done in this point will basically, we have introduce this concept of parameter estimation. So, we have motivated this in this concept or this idea of parameter estimation as being relevant in the context of wireless communication this is of course, we have seen a simple example in the context of a wireless sensor network.

We have a several sensor nodes which are making noisy measurements of this parameter h and they are conveying this noisy measurement to the fusion centre. The fusion centre then has to estimate from these noisy measurements come up with an estimate of this parameter h and it basically realise on 2 quantities; one is the prior probability density function p of h on the parameter h and the conditional probability density function p of y bar given h of the observation vector y bar given the parameter h basically using these 2 quantities, we would like to come up with estimate of this unknown parameter h and this is termed as the Bayesian frame work or MMSE specifically the MMSE. The MMSE part of the Bayesian frame work and this is the unique aspect of the Bayesian frame work is that it treats the underlying parameter h as a random parameter as being random in nature with a certain probability density function p of h.

So, we will stop this module here and we will continue with other aspects in the subsequent modules.

Thank you.