Probability and Random Variables/Processes for Wireless Communication
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Module No. 1
Lecture 6
Independent Events-Multiantenna FadingExample.

Hello, welcome to another module in this massive open online course on probability and random variables for wireless communications. So we are looking at independence, the concept of independence and independent events and let us look at one final example to understand it. We have already looked at several examples in this context to better understand the concept of independence and more importantly, its relevance in the context of wireless communication. Remember, in the beginning of this course itself and in the introductory module also, we have said that this is not simply an abstract course on probability and random variables. This is not a course, this is simply the mathematics of probability and random variables.

So one of the important aspects of this course is to explore and examine the impact or examine the relevance of probability and random variables, random processes in the context of communication, especially digital communication systems and wireless, modern wireless communication systems. So that it bridges or basically builds a bridge between these abstract concepts in probability and random variables and the practical applications of these concepts in the context of modern communication systems, both high-speed digital communication systems as well as high-speed wireless communication systems, 3G and 4G wireless communication systems. So in this module, let us briefly at a very high-level look at another, very important impact or relevance of this concept of independence for independent events in a wireless communication system.

And to draw this example from the context of 3G, 4G wireless communication system, some of you who have seen previous courses of wireless communication systems, might have already seen this. This is a concept in the context of what is known as diversity.

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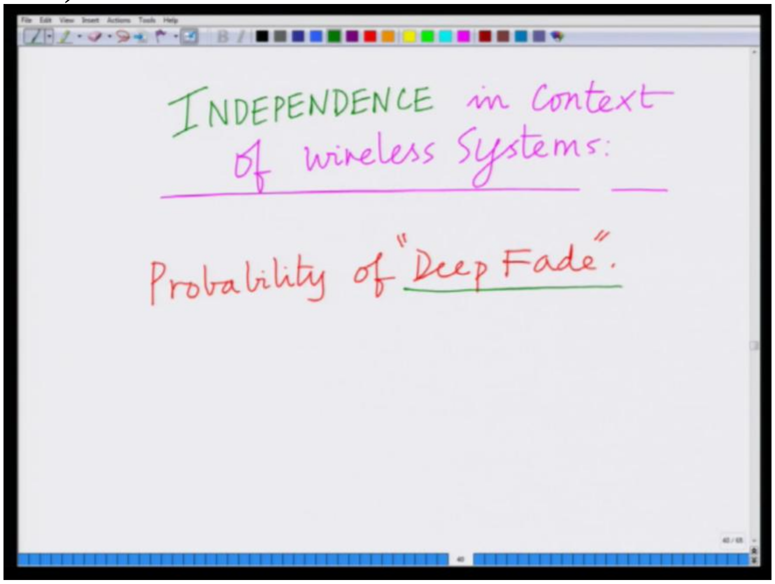

So let us look at an example of independence in the context of wireless communication systems. So I am looking at independence in the context of wireless systems. And especially in the context of 3G and 4G wireless communication systems. And what we would like to look at? We would like to look at what is known as the probability of a deep fade event? Some of you might already be familiar with this concept of a deep fade. But if you are not familiar, let me explain this to you briefly at a high level.

The wireless communication signal that is received at the receiver in a wireless communication system is the result of superposition of multiple signal copies that arise for instance from a direct signal that is coming from the base station or also multiple copies that are coming from the reflections from various buildings, various trees and other objects in the wireless government. So when these signals superpose at the receiver, there is interference. And this interference can be either constructive in nature or it can be destructive which causes fading. So the interference in the wireless communication environment, that results in fading. That is, which is basically either an amplification of the received signal power or the dip in the received signal power. This process is known as fading.

And when the received signal power dips significantly below a certain threshold, that is below a certain threshold, below the noise threshold at the receiver so that communication is not possible, that is known as a deep fade event.

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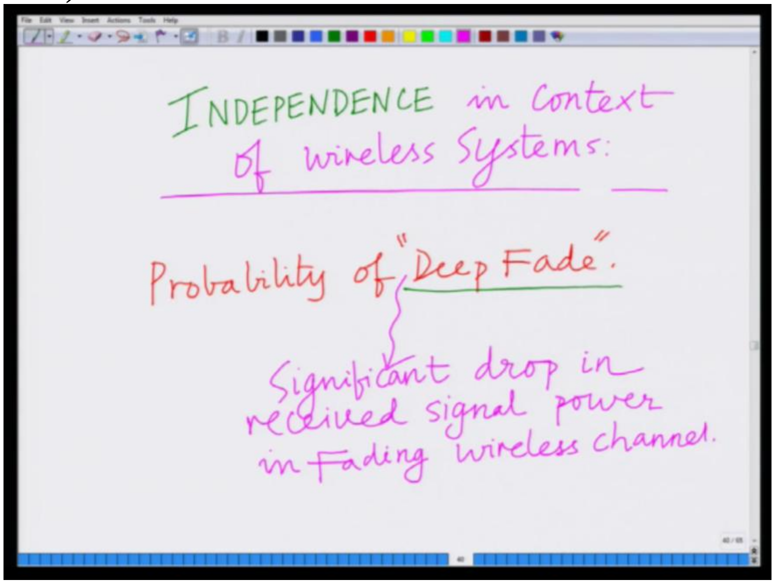

So a deep fade event, to put it simply that is a significant drop in the received signal power in a fading wireless channel. So the significant dip in the received power in a fading wireless channel or in a fading wireless system, this is known as a deep fade event. Obviously, we would like to avoid a deep fade event because whenever the signal is in a deep fade, then communication is not possible between the transmitter and the receiver because simply the received signal power is very low. And one way to avoid a deep fade is through what is known as multiple antennas. By increasing the number of antennas at the transmitter and receiver, one can significantly reduce the probability of deep fade.

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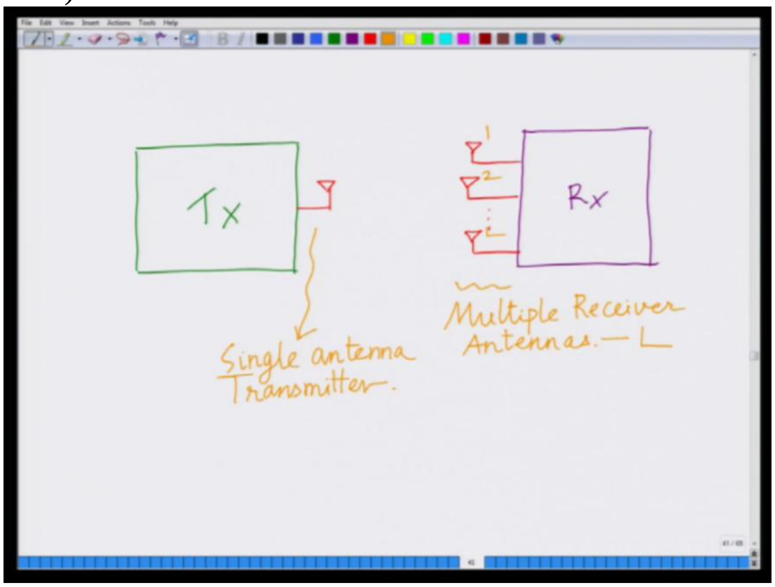

Let us look at how this happens. Let us say, I have a transmitter which is let us say something like my base station. So let us say I have my transmitter, I have my receiver. Let us say my transmitter has a single antenna but my receiver, let us say has multiple antennas. So my receiver has antennas 1, 2, up to L antennas. So these are multiple antennas at the receiver. These are L antennas. I have a single antenna at the transmitter. Although in the most general case, one can have multiple antennas both at the transmitter and the receiver. For the purpose of this simple example, we are looking at a single antenna at the transmitter and multiple antennas, capital L antennas at the receiver.

This is known as a SIMO system. This is a single input multiple output system because I have a single antenna at the transmitter, that is the input, multiple antennas at the receiver, that is the output. This is known as a SIMO system or a single input multiple output system.

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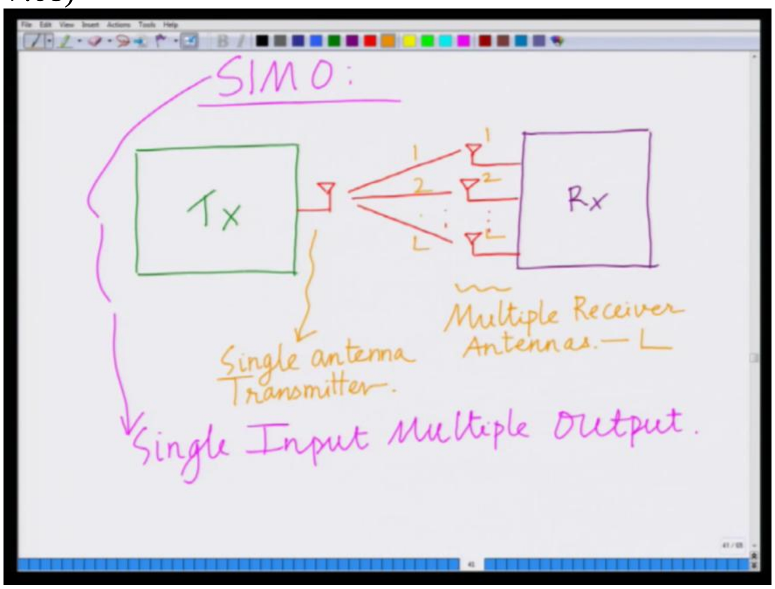

So this is known as a SIMO system where SIMO stands for single input multiple output. And therefore, in this system, we have capital L links. We have 1, 2, up to L links. So we have link 1, 2, up to L links. So what do you mean by L links? We have a link between the transmit antenna and each receiver antenna. So in this system what we are saying is we have basically L different links. Because I have a single transmit antenna and L receiver antennas, one can consider the link between, one can consider the channel between the single transmit antenna and each of the receiver antennas as a link. So we have a single transmit antenna, L receiver antennas. So there are L links in this system.

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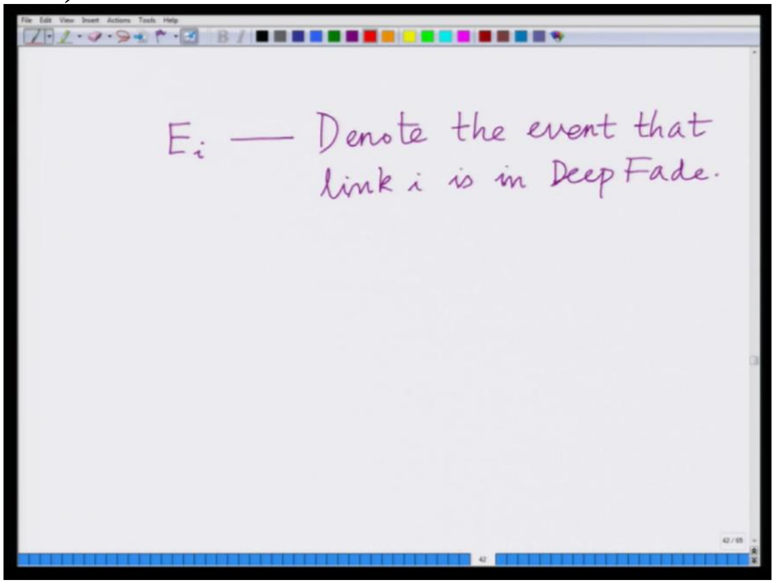

And now therefore, the probability that each, let Ei, let this denote the event that link i is in deep fade. What we are saying is let Ei denote the event, let this quantity Ei denote that the link i is in deep fade. So each of these links can be in deep fade . That is the SNR received over this link is below a certain threshold or below the noise threshold and we can say that then the link i is in deep fade and let this Ei denote the event that link i is in deep fade. Now therefore one can ask the question, when is this wireless communication system, when is the whole system in a deep fade? The whole system is in a deep fade that is no communication is possible naturally if all the links are in a deep fade. In this system with L links, the communication between the transmitter and receiver is not possible if all these links that is each of these L links is in a deep fade.

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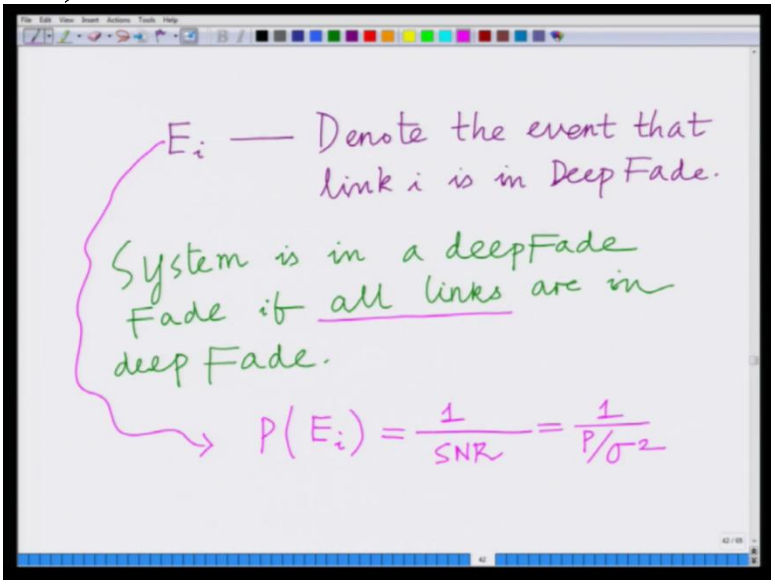

So the system, the entire wireless system is in a deep fade if all the links , this is the system, a wireless communication system, multiple antenna wireless communication system is in a deep fade if all the links are in a deep fade. Now the probability that each of these individual links is in a deep fade is equal to 1/SNR. This is the result from wireless communication which I am not deriving here. The probability that each of these, the probability that link Ei, the probability of this event is Ei that link i is in deep fade is

$$P(E_i) = \frac{1}{SNR} = \frac{1}{P/\sigma^2}$$

where P = transmitted power and σ^2 = noise power at the receiver.

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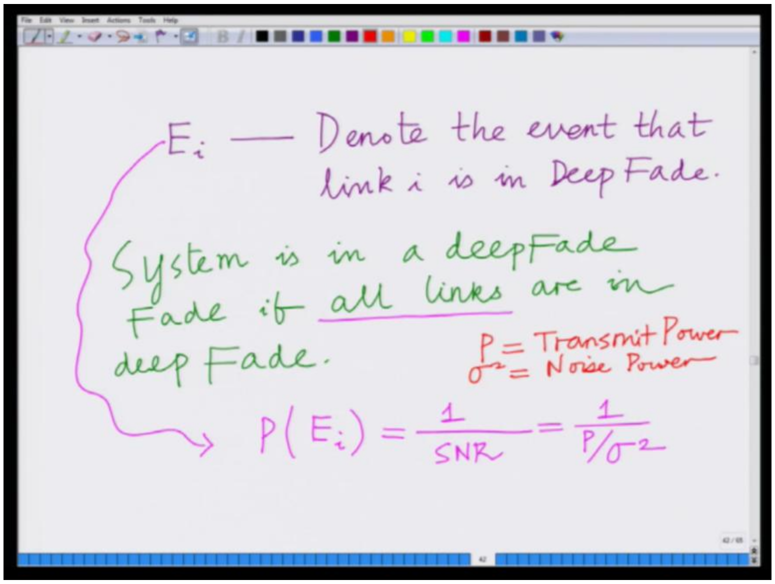

So this P, let me also clarify, this P = transmitted power and $\sigma^2 =$ noise power.

$$\frac{P}{\sigma^2} = \text{SNR}$$

P over Sigma Square is the SNR and the probability that link Ei,

$$P(E_i) = \frac{1}{SNR}$$

and we are asking the question that what is the probability that the entire system is in deep fade?

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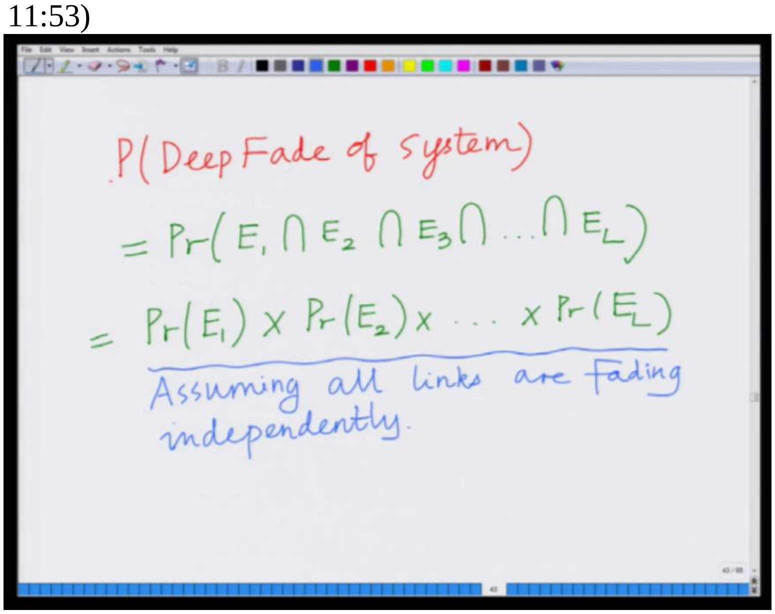

That is, probability of deep fade of the system is basically we said the probability that all the links are in deep fade which is –

$$P(\text{Deep fade of system}) = P(E_1 \cap E_2 \cap \dots \cap E_L)$$
$$= P(E_1) \times P(E_2) \times \dots \times P(E_L)$$

That is the probability of the joint event. Therefore the probability that the system is in deep fade is the probability that all the links are in deep fade. Now I am going to use the independent assumption to write this as $P(E_1) \times P(E_2) \times \dots \times P(E_L)$ and this is the key. This is using assuming all the links are fading. So what we are saying is the probability that the system is in fade is the joint event that is $E_1 \cap E_2 \cap \dots \cap E_L$.

That is the joint event that all the links are in a deep fade and the probability of this joint event we are saying is the product of the individual deep fade events. That is, $P(E_1) \times P(E_2) \times \dots \times P(E_L)$, given that all the links are fading independently. And previously, we had seen that the probability that each of the links, each of the individual links is in a deep fade is 1/SNR.

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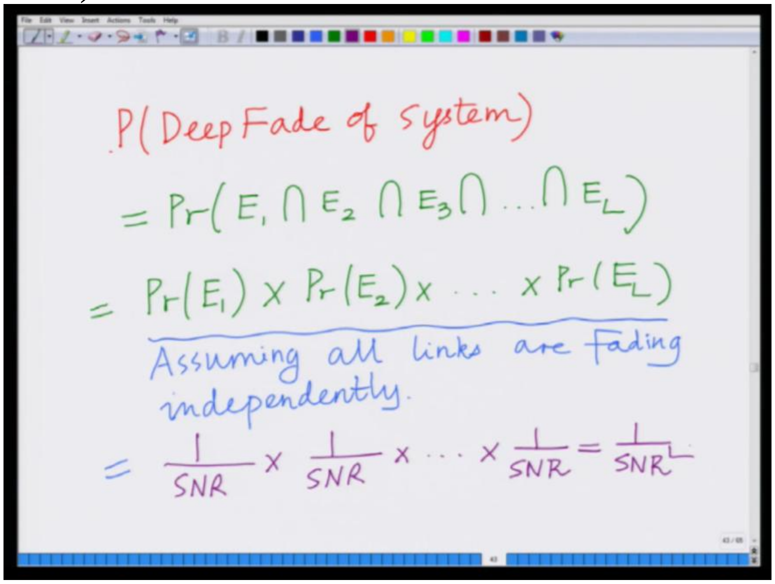

Therefore the probability of this joint event is basically simply given as –

$$P(E_1)\ X\ P(E_2)\ X\ \ldots\ldots.\ X\ P(E_L) = \frac{1}{SNR}\ X\ \frac{1}{SNR}\ X \ldots.X\ \frac{1}{SNR} = \frac{1}{SNR^L}$$

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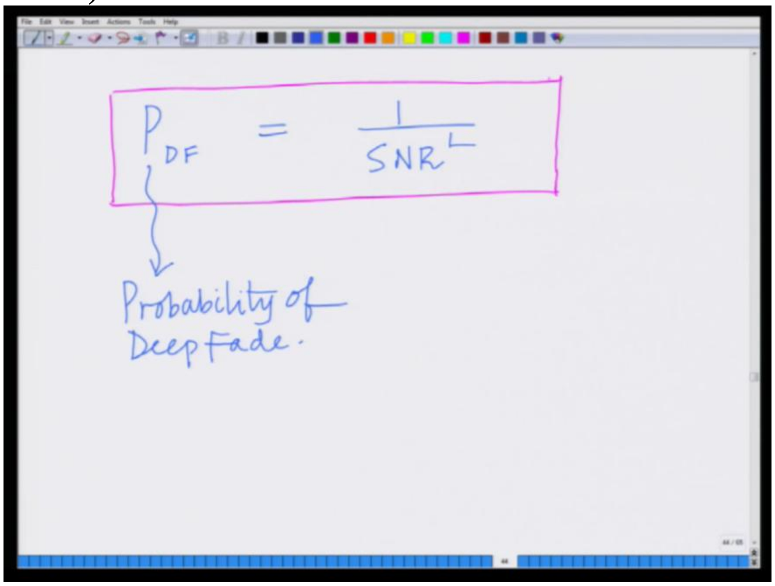

Therefore the probability of deep shade, if I call this as –

$$P_{DF} = \frac{1}{SNR^L}$$

And this is an important property in wireless communication systems. Especially in modern 3G, 4G wireless communication systems, this P_{DF}, this is the probability of deep fade. This is the probability of deep fade which is basically $\frac{1}{SNR^L}$. So we are saying that as the number of antennas at the receiver increases, the probability of the fade that is the probability that the system is in a deep fade or the signal power at the receiver is below the noise threshold decreases as $\frac{1}{SNR^L}$.

And therefore, having a large number of antennas helps improve the efficiency of communication or helps decrease the error rate, helps decrease the bit error rate of communication because the probability of deep fade decreases as we have seen in this. We are employing the assumption of independently fading channel. This property is known as diversity.

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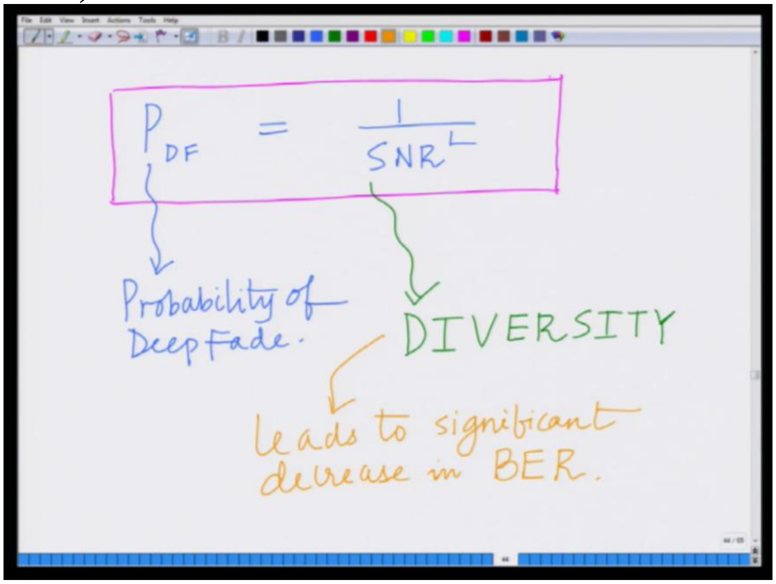

The probability where the bit, the probability of is decreasing as $\frac{1}{SNR^L}$, this is known as diversity. And this leads to, diversity leads to a significant decrease in what is the bit error rate or the BER of wireless communication. So what we are saying is, having multiple antennas, the probability that the system is in a deep fade decreases as $\frac{1}{SNR^L}$.

So the larger the number of receiver antennas, the probability of deep fade becomes correspondingly lower and therefore this leads to a significant decrease in the probability of deep fade which in turn leads to a significant decrease in the bit error rate of wireless communication. This is an important property in 3G or 4G wireless communication systems very imply multiple antennas at both, the base station, either the base station or the handset and if possible, both at the base station and also the handset.

The main reason is because having multiple antennas leads to a significant improvement in reliability of wireless communication. This is achieved through the property of principle of diversity and the principle of diversity is fundamentally based on the assumption of these independently fading place.

So this is a key impact or a key relevance of the concept of independence and independent events in the context of wireless communication, especially modern 3G, 4G wireless communication systems. So with this, let us stop this module here and I hope I have explained an important aspect of independence in the context of wireless communication. Thank you very much.