Probability and Random Variables/Processes for Wireless Communication
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Module No. 1
Lecture 5
Independent Block Transmission Example.

Hello, welcome to another module in this massive open online course on the principles of probability and random variables for wireless communications. So in the previous model, we were looking at the concept of independence and independent events. We had defined the concept, we defined mathematically what does it mean to say that the 2 events A and B are independent. Today let us continue to look at the other examples of independence. For instance, we had looked in the previous module.

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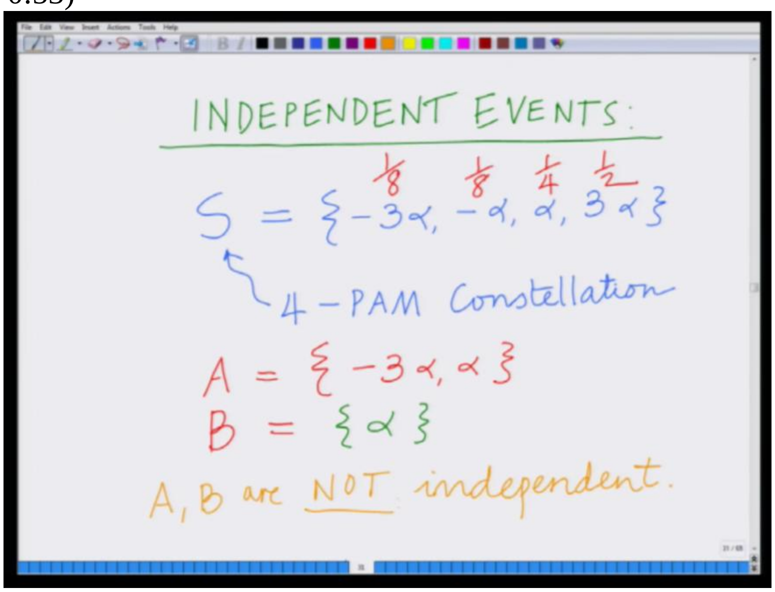

So we were talking about independent events. We are currently focusing on independent events and we had looked at this particular example where, we had looked at our sample space

$$S = \{-3\alpha, -\alpha, \alpha, 3\alpha\}$$

consisting of 4 PAM modulation. We had said that this is our 4 PAM, M-ary PAM, pulse amplitude modulation constellation. This is our 4 PAM constellation and we had considered the probabilities. 1 by 8, 1 over 4 and half and we had also looked at 2 events. Corresponding to the 1st event is A

$$A = \{-3\alpha, \alpha\}$$

and B which is

$$B = \{\alpha\}$$

And we had realized that, we had rigorously proved that the event A where the symbol is either -3α or α and this event B where the symbol is α , these 2 events, A and B are not independent. So we had established in the last module that these 2 events A, B are not independent.

Then one would like to ask, what is the relevance of independence in digital communication or wireless communication? What is the relevance of this concept of independence and independent events in a wireless communication system? The relevance is that we had looked previously at a single symbol. Remember, when we looked at the previous examples, we had talked in the context of a single symbol belonging to the set A that the event A and the event B in the context of a single symbol.

However now, we will start looking at the concept of different symbols generated. Because we have a wireless communication system, so the source is generating a stream of output communication symbols. So when we look at these different symbols which are generated at successive instance of time, these different symbols in the digital communication system or the wireless communication system are frequently independent.

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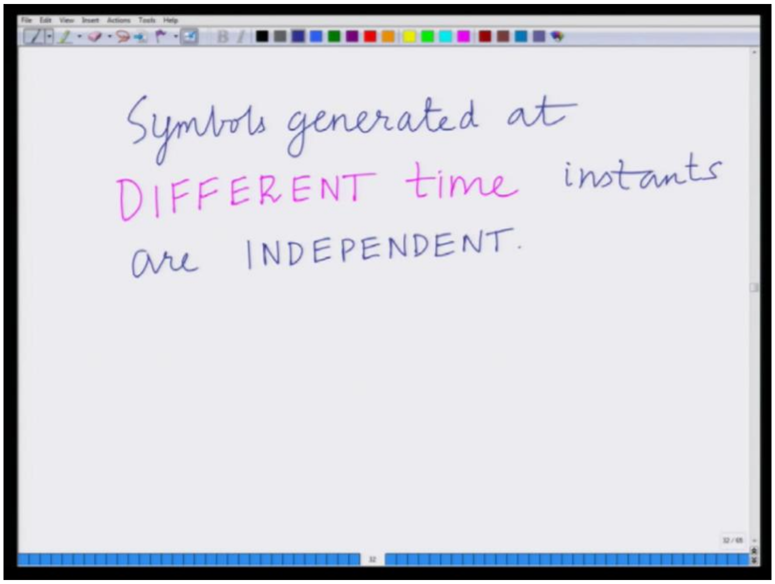

So what you would like to say is that the symbols generated at different time instants are independent. I am not saying that this is always the case but frequently, the successive symbols, the information symbols, the successive digital modulation symbols that are generated by the communication system, these are frequently independent.

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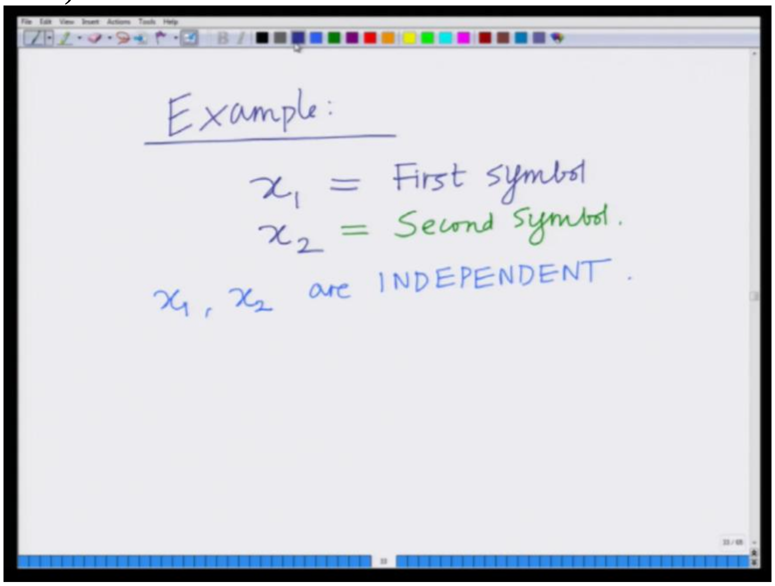

So let us see what this means. For example, let

X_1 denote the 1st symbol that is generated at time interval 1. Let X_2 be the 2nd symbol. Then X_1, X_2 are independent. Right? We are saying that X_1 is the symbol that is generated at time instant 1 and X_2 is the symbol generated at time interval 2. Both of these can be drawn from the same digital modulation constellation. For instance, in this example, let us say, X_1 and X_2 are both symbols that are drawn from the 4 PAM constellation.

$$\text{i.e.} \quad X_1 \in \{-3\alpha, -\alpha, \alpha, 3\alpha\}$$

and X_2 also belong to same set,

$$\text{i.e.} \quad X_2 \in \{-3\alpha, -\alpha, \alpha, 3\alpha\}$$

and we are saying that these two 4 PAM symbols, X_1 and X_2 are drawn independently from this sample space corresponding to the M-ary PAM or the 4 PAM, the 4 pulse amplitude modulation constellation.

So we are saying. This X_1 and X2 are symbols generated independently from the 4 PAM constellation.

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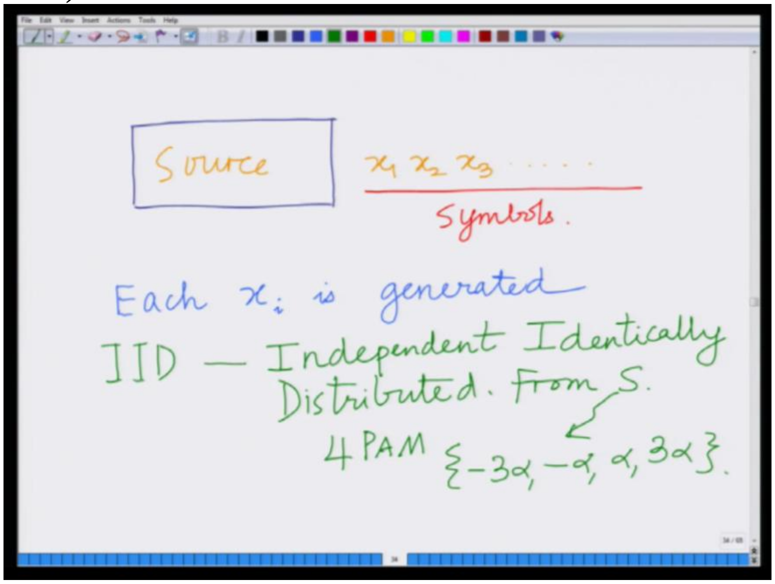

Alright ? In fact, to extend this analogy further, we can consider a source S and frequently this is the case. That is we can consider a source. This is generating the symbols X_1, X_2, X_3 and so on. So the source is generating various symbols. Each X_i, that is the i^{th} symbol, symbol generated at the i^{th} time instant, each X_i is generated we say IID with stands for independent identically distributed from capital asset is our 4 PAM constellation,

i.e. $\{-3\alpha, -\alpha, \alpha, 3\alpha\}$

So what we are saying is, not only X_1 and X_2, all of these symbols, in communication system, it is continuously generating a stream of information symbols. Right? Continuously generating a stream of modulated symbols. We are saying these symbols, $X_1, X_2, X_3 \ldots X_i$, all of these symbols are independent. And not only that, they are identically distributed.

Each is generated independently and has the identical distribution as the other in the sense that they belong to the 4 PAM constellation that is the set S,

$S = \{-3\alpha, -\alpha, \alpha\ 3\alpha\}$.

And they have the same probability distribution. So each of these symbols are generated and these are independent and identically distributed according to the 4 PAM, from the 4 PAM constellation. So these symbols are generated independent identically.

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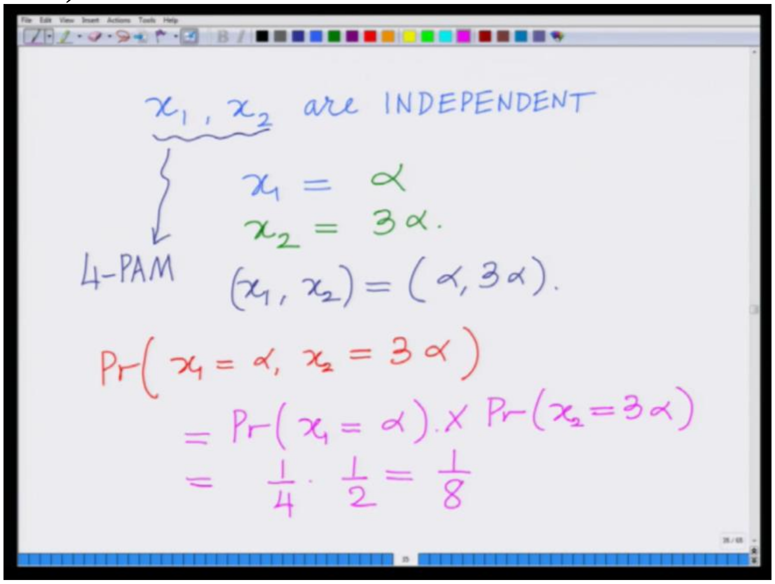

Let us take a simple example to understand this. Let us look at a simple example to understand this thing. Let us ask the question, if X_1, X_2 are independent,

What is the probability that

$$X_1 = \alpha \quad \text{and} \quad X_2 = 3\alpha\ ?$$

So we are asking the following question, we are looking at a simple example, we are saying, X_1 and X_2 are 2 symbols that are generated by this 4 PAM source. So we are asking this, given that X1 and X2 are generated independently what is the probability $X_1 = \alpha$ and $X_2 = 3\alpha$?

This answer is as follows for evaluating the probability that $X_1 = \alpha$ and $X_2 = 3\alpha$.

Since we said that these 2 symbols are generated independently, now we can use the property of independent events, i.e. the joint probability or the probability of the intersection is the product of the individual probabilities.

Therefore, this is equal to

$$p(X_1) . p(X_2)= p(\alpha) .p(3\alpha)=1/8$$

So this is a simple problem where we are saying what is the probability that we have 2 symbols X_1 and X_2 and these are generated independently and we are saying, what the probability that $X_1 = \alpha$ and $X_2 = 3\alpha$?. We are saying, the probability of this joint event is equal to the product of the probabilities since these 2 symbols are independent.

So this is a simple problem.

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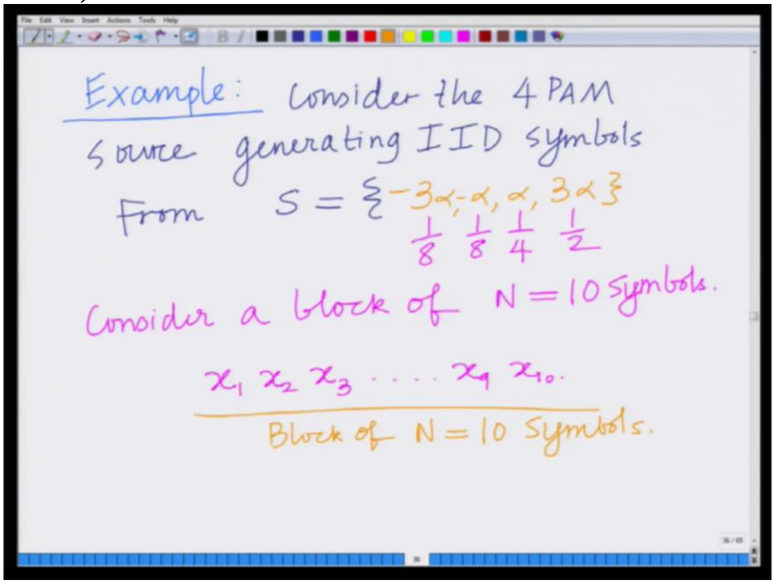

Now let us look at a slightly more refined problem to understand this better. Let us look at this example. Consider the 4 PAM source, as usual, our 4 PAM source generating IID symbols that this independent identically distributed symbols from the modulation, the constellation S is,

$$S= \{-3\alpha, -\alpha, \alpha\ 3\alpha\}$$

and we set the probabilities as

$$p(-3\alpha)= 1/8$$

$$p(-\alpha)= 1/8$$

$$p(\alpha)= 1/4 \text{ and}$$

$$p(3\alpha)= 1/2$$

This is the distribution. And we consider a block of $N=10$ symbols. That is we are considering symbols $\{X_1, X_2, X_3 \ldots X_9, X_{10}\}$. So basically we are considering a block of N is equal to 10 symbols. And we would like to ask the following questions.

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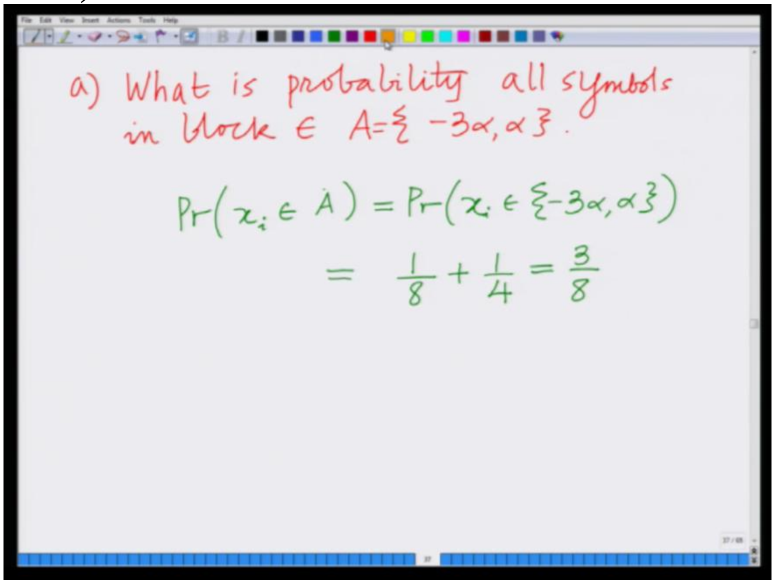

What is the probability that all the symbols in the block are elements of the set? So the 1st question that we would like to ask is what is the probability that all the symbols in the block belong to the set A,

$$A = \{-3\alpha, \alpha\}.$$

So the 1st question. So what we are saying is the following thing. Understand this example. We are considering a source which is generating IID 4 PAM symbol. That is, each 4 PAM symbol, that is each symbol belongs to the 4-ary PAM constellation, pulse amplitude modulation constellation. And each symbol is generated independently. And it is identically distributed according to the distribution of the 4 PAM constellation.

And we are considering a block of 10 such symbols that is $\{X_1, X_2, X_3 \ldots X_9, X_{10}\}$ where each of these symbols are generated in an IID fashion. And we are asking the question, the 1st question that we are asking is, what is the probability that each of these symbols, $X_1, X_2, X_3 \ldots X_9, X_{10}$ belongs to the set $A = \{-3\alpha, \alpha\}$. That is basically, we are asking the question, probability that each X_i belongs to the $A = \{-3\alpha, \alpha\}$.

Now let us 1st compute the probability that a given symbol Xi belongs to A. This is equal to the sum of the probabilities corresponding to the symbols -3α and α.

This is basically equal to

$$p(-3\alpha)+p(\alpha)= 1/8 + 1/4= 3/8$$

This is something that we have already seen.

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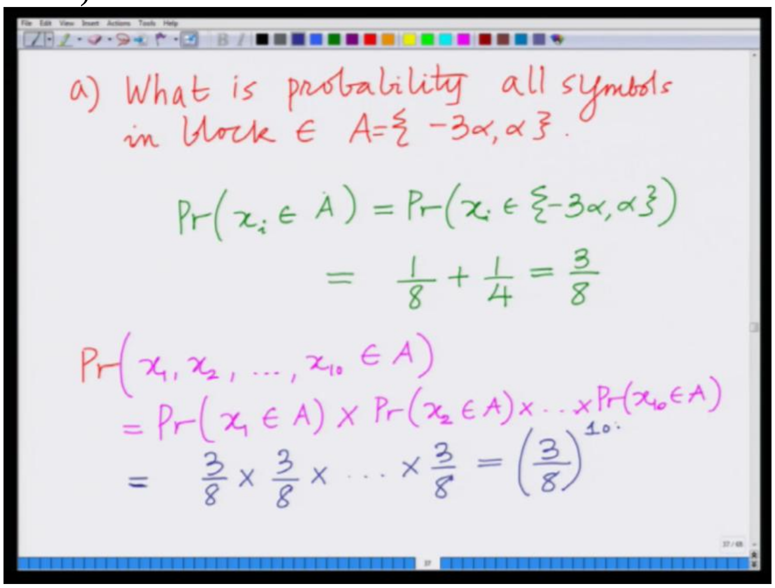

Now let us ask the question, what is the probability that all the symbols belong to this, all the symbols that belong to the set A, that is all the symbols, either -3α or α. We are asking the question, what is the probability that $\{X_1, X_2, X_3 \ldots X_9, X_{10}\}$ belong to the set A when all these symbols are independent. This is now basically the product of the probability X_1 element of A times the probability X_2 element of A times the probability X_{10} belongs to A which can now be seen to be given as, each of these probabilities, probability $X_1 \subseteq A$ is

$$3/8 \,.\, 3/8 \ldots 3/8 = (3/8)^{10}$$

So therefore using the independence and identical distribution property, we have computed the probability that all of these symbols, $\{X_1, X_2, X_3 \ldots X_9, X_{10}\} \in A$. That is each of these X_i is, X_1, X_2, upto X_{10} is either -3α and α. The probability of that is basically $(3/8)^{10}$ and we have used the independent identical distribution property of these various 4 PAM symbols to calculate that.

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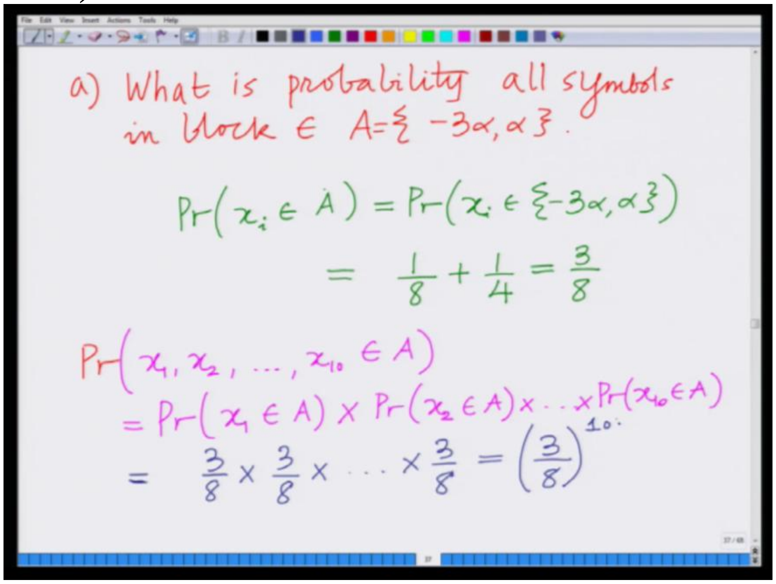

Let us now extend this example further. Let us now ask the question, what is the probability that none of the symbols belong to A? So now we are asking the reverse question.

1st we answered the question, what is the probability that all the symbols belong to A? Now, what is the probability that none of the symbols belong to A? And this answer is also fairly straightforward. If none of the symbols belong to A, then all the symbols must belong to A complement.

Therefore, the probability that a given symbol does not belong to A is simply the probability that a symbol belongs to A complement. That is, the probability that the symbol belongs to the set minus alpha, 3 alpha. And this is also the probability Xi belongs to A complement

i.e. $p_r(X_i \in \bar{A}) = 1-(3/8) = 5/8$

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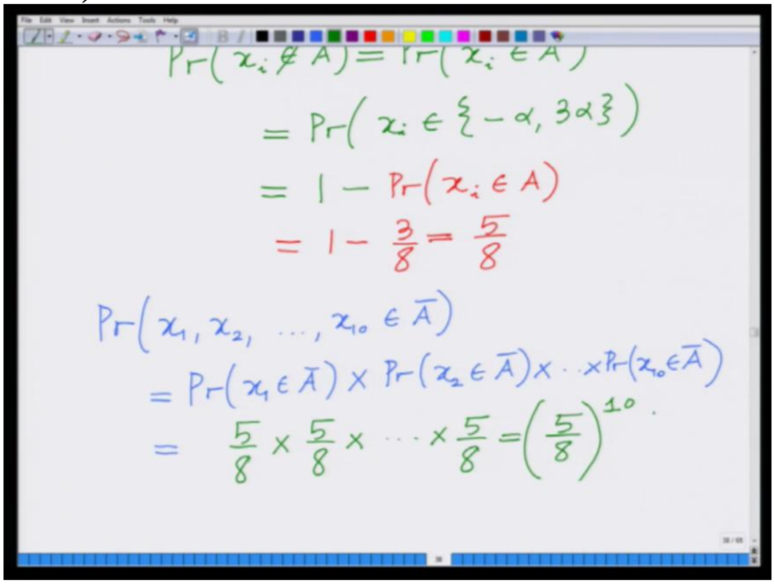
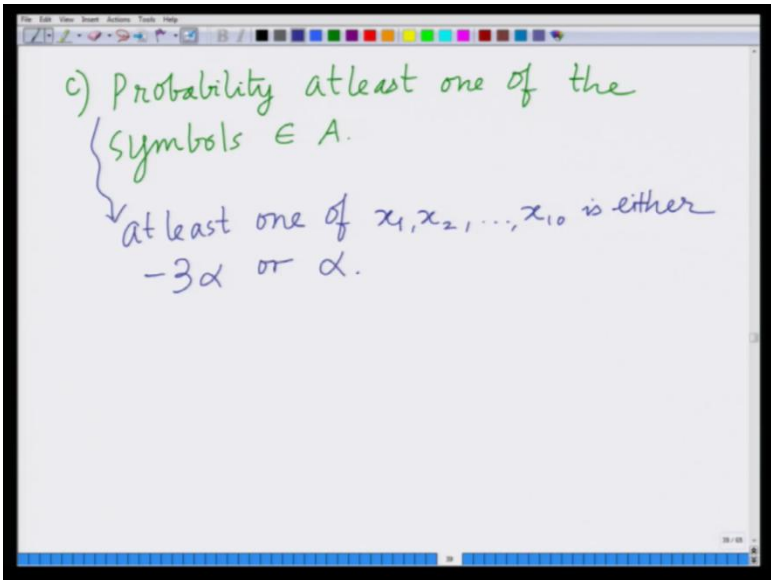


Now we are asking the question, what is the probability that all of these X_i belong to A complement. Which is basically again, since X_1, X_2, up to X_{10} are independent. Therefore it is simply the product of the probabilities $p_r(X_1 \in \bar{A})$ times the $p_r(X_2 \in \bar{A})$ times and so on till $p_r(X_{10} \in \bar{A})$ which is equal to $(5/8)^{10}$.

Therefore now, we have answered the question, what is the probability that none of the symbols belong to A? Which means all of the symbols must belong to A complement and the probability of that is 5 by 8 raised to the power of 10. All right? Now let us ask another question which is the last question. (Refer Slide Time: 21:55)



This is part C of the same question which is basically the following thing. Let us ask the question, what is the probability that at least one the symbols belongs to A? So we are asking the question that we have this block of symbols, X_1, X_2, up to X_{10}. What is the probability that at least one of these X_1, X_2, up to X_{10} belongs to the set A? That is at least one of these X_1, X_2, up to X_{10} is either -3α or α. That is at least 1 of X_1, X_2, up to X_{10} is either -3α or α and this probability is also very simple. The probability that at least one belongs to A is basically 1 minus the probability that none of them belong to A.

And we have already computed in the previous part what is the probability that none of them belong to A.

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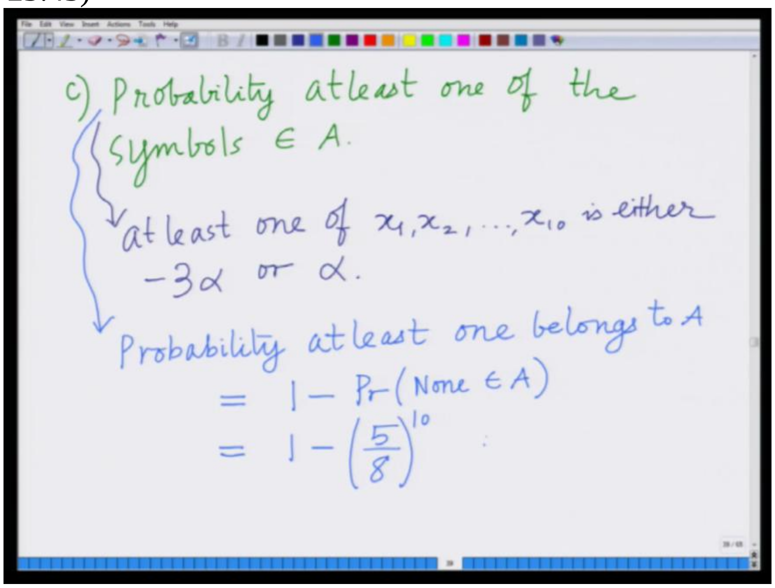

And therefore the probability of at least one of them belonging to A is

$$1 - P_r(\text{none} \in A)$$
$$= 1 - (5/8)^{10}$$

So we have calculated.

So what have we seen in this module? In this module, we have seen something interesting. We have seen that the concept of Independence as applied to the context of digital communication system or wireless communication systems. We have said that although, the previous examples, we had seen that the 2 events were not independent. When we looked at the 2 different symbols, the 2 different symbols generated at different time instant by a communication system, these are independent. And therefore, one can calculate, given a block of this, we have looked at an example about the various properties that is given a block of these symbols that is X_1, X_2, up to X_{10} that is given a block comprising of 10 symbols, what are the various properties?

What can we say about the probability that none of the symbols belong to a certain set A? What is the probability that one of the symbols belong to a certain set A? All of these, exploiting basically the independent identical distributed nature of these various symbols generated by the source, generated by the digital communication source or the source in the wireless communication system. So this is an important aspect or this is an important, key impact of we can say the relevance of this property of independence or independent event in the context of wireless communication systems. So we conclude this module here. Thank you very much.