Probability and Random Variables/Processes for Wireless Communication
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Module No. 1
Lecture 3
Conditional Probability-Mary-PAM Example.

Hello, welcome to another module in this massive open online course on probability and random variables for wireless communications. So today, let us start looking at a new topic, that is conditional probability.

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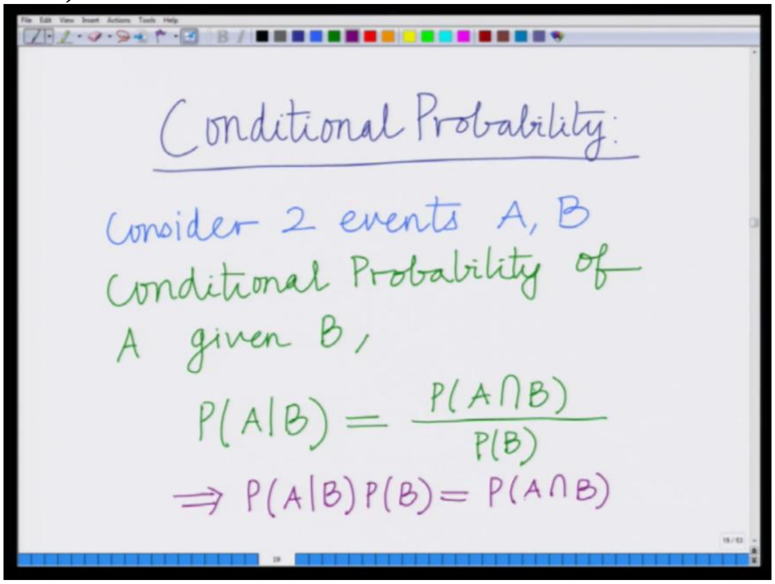

So today we will start our discussion on conditional probability. So Conditional Probability, Consider 2 events, A, B. Now, the conditional probability of A given B is defined as the probability of, is denoted by –

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

So we are now defining a new concept that is the conditional probability. Let us say, A and B are 2 events.

The conditional probability of A given B, that is the, what is the, how is the probability of A or how is the probability of occurrence of A affected given that the event B has occurred.

That is let us say, someone has told you that the event B has occurred. Then how does that affect the probability of occurrence of A? This is termed as the conditional probability. That is denoted by -

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

And therefore, naturally, this also implies –

$$P(A|B)\, P(B) = P(A \cap B)$$

Or in other words, $P(A \cap B) = P(A|B)\, P(B)$

And also, therefore we can write this as…

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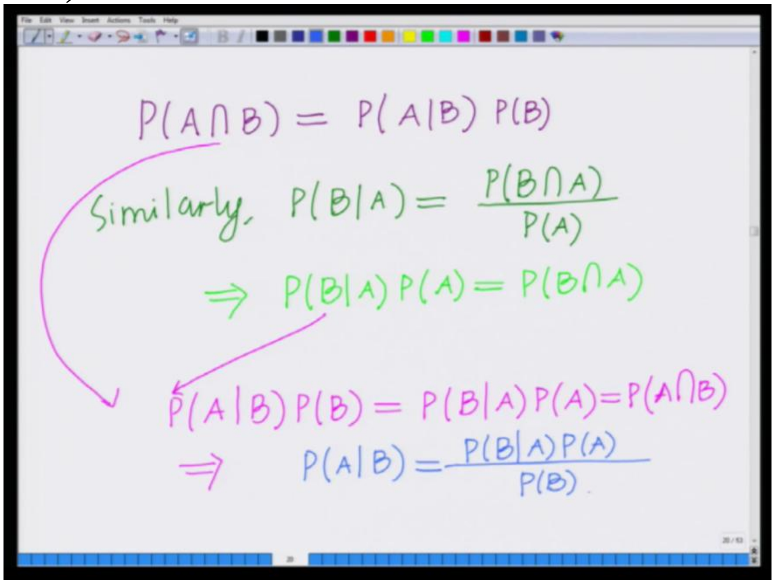

So, we said we have $P(A \cap B) = P(A|B)\, P(B)$. Similarly, we can write, we can define the probability of B conditioned on A as

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

which basically implies,

$$P(B|A)\, P(A) = P(A \cap B)$$

And now you can see,

$$P(A|B)\,P(B) = P(A \cap B)$$

Therefore from these 2, we can conclude that

$$P(A|B)\,P(B) = P(B|A)\,P(A) = P(A \cap B)$$

which basically implies that,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

So,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

All right? So we have derived an important property of conditional probability. This is also known as the ~~base~~ Bayes' result. We will explore more about this later. So 1st, we have said that

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Similarly,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Let us now look at an application of this concept of conditional probability in the context of a ~~Mary Pam~~ M-ary PAM modulation. Remember, or M-ary PAM stands for pulse amplitude modulation. So let us look at a simple application of conditional probability.

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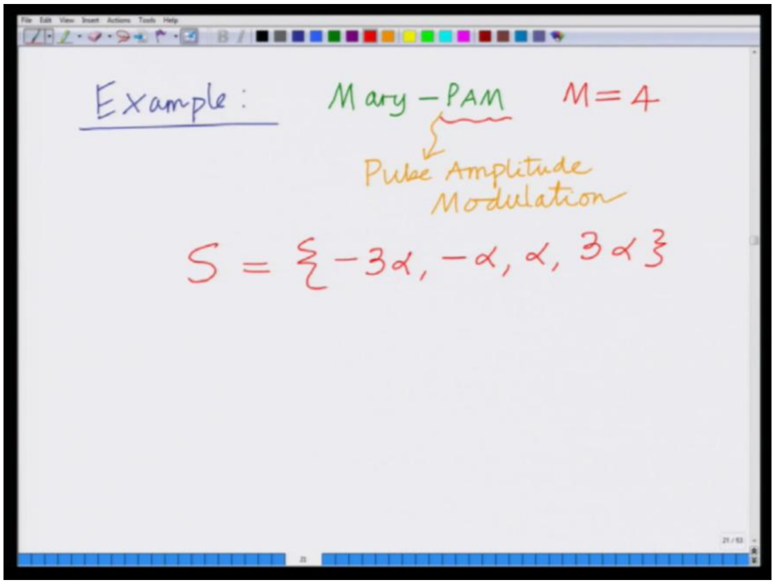

So let us look at a simple example of conditional probability in the context of M-ary PAM. Remember, we are considering M-ary PAM for **M = 4** symbols. And remember, PAM denotes a pulse amplitude, ~~population~~ modulation, which is also basically an amplitude shift keying. And remember, for **M = 4 symbols**, our sample space,

S = { -3α, -α, α, 3α }

We are considering 4 PAM. That is M-ary PAM, pulse amplitude modulation which is a digital modulation. That is this is a digital constellation from which the digital transmission symbols are drawn..

Further, we are considering, M is equal to 4 symbols. That is, we are considering, M equal to 4 symbols and the sample space for this is **-3α, -α, α, 3α**.

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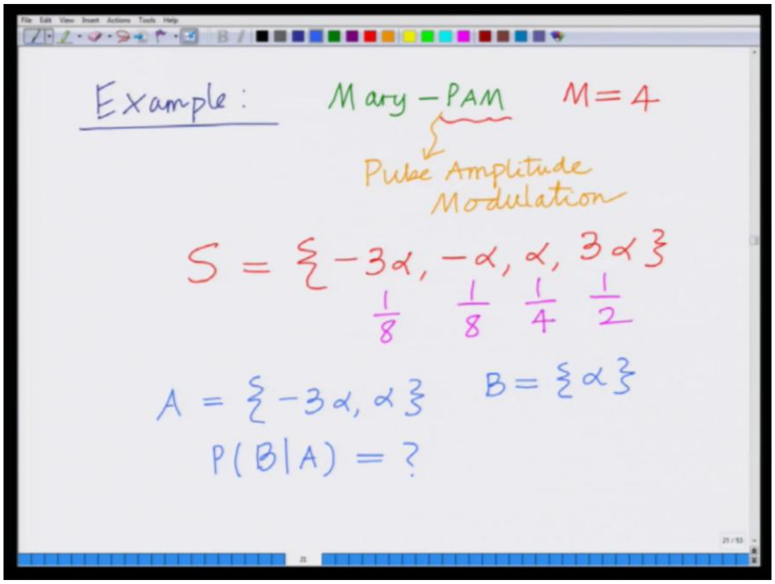

And we also consider an example in which the various probabilities are given as 1/8, 1/4 and 1/2. And let us take A as before, let us take A to denote the set of symbols **-3α, α** and B to denote the symbol **α**.

A = { **-3α, α** } and B = { **α** }

Now what we are asking is what is **P(B|A)** ?. So we are saying, A ~~common descent~~ , that's it, A denotes the PAM symbols, **-3α and α**, B denotes the PAM symbol **α**.

Now we are asking, what is the conditional probability of B given A? That is, given that event A has occurred, that is the observed symbol belongs to the set, **-3α, α** which means to say that is the PAM symbol is either **-3α or α**. What is the probability of event B conditioned on a given A? What is the probability of B? That is what is the probability that the PAM symbol is alpha?

Remember, B is the PAM symbol **α**. Right? So we are asking, what is the probability that the observed symbol is **α** given the observed symbol is either **-3α or α**. And this can be found as follows.

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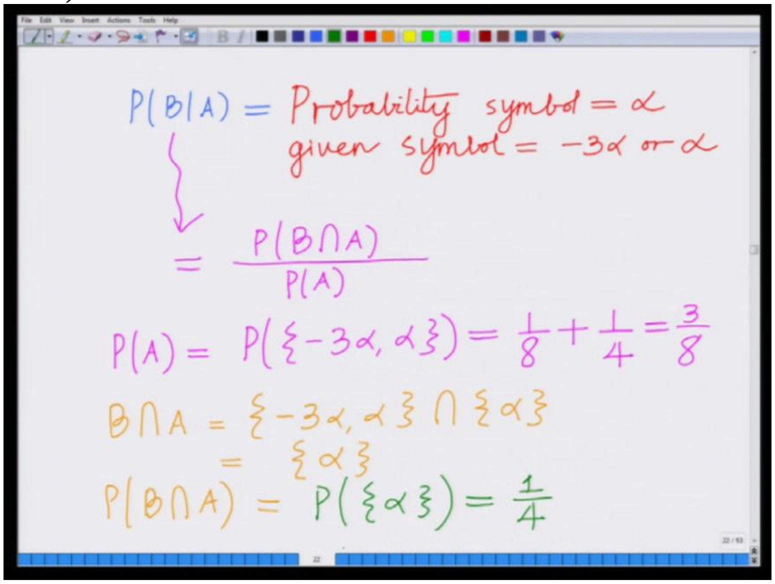

We have that is what is probability of B given A? Probability of B given A equals probability this is equal to, the probability symbol equals alpha given A that is symbol equals **-3α or α**. And what is the probability of B given A? Remember, from the definition,

$$P(B|A) = \frac{\mathrm{P(A \cap B)}}{\mathrm{P(A)}}$$

And $\quad \mathrm{P(A) = P(}\mathbf{-3\alpha\ ,\ \alpha\ ) = \frac{1}{8} + \frac{1}{4} = \frac{3}{8}}$

Now what is B intersection A?

$$\mathbf{B} \cap \mathbf{A} \ = \ \mathbf{\{-3\alpha\ ,\ \alpha\ \}} \cap \{\ \boldsymbol{\alpha}\ \} \ = \ \{\ \boldsymbol{\alpha}\ \}$$

Therefore, we have

$$\mathrm{P}(\mathbf{B} \cap \mathbf{A}\ ) = \mathrm{P}(\{\ \boldsymbol{\alpha}\ \}) = \mathbf{\frac{1}{4}}$$

So we have found 2 things. We have found P(A) which is 3/8 and we have also found P(**B** ∩ **A**) which we are saying is basically 1/4.

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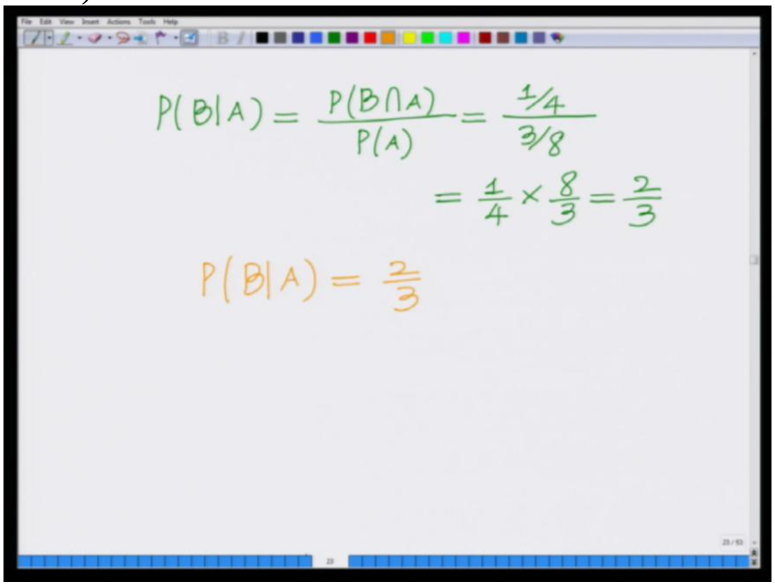

And therefore,

$$P(B|A) = \frac{\mathrm{P(A \cap B)}}{\mathrm{P(A)}} = \frac{1/4}{3/8} = \frac{\mathbf{2}}{\mathbf{3}}$$

Therefore we have derived $P(B|A)$ equals 2/3. Therefore the conditional probability of B given A or the probability of the event B conditioned on the event A is two third that is 2/3. Now let us find the other conditional probability. What is $P(A|B)$?

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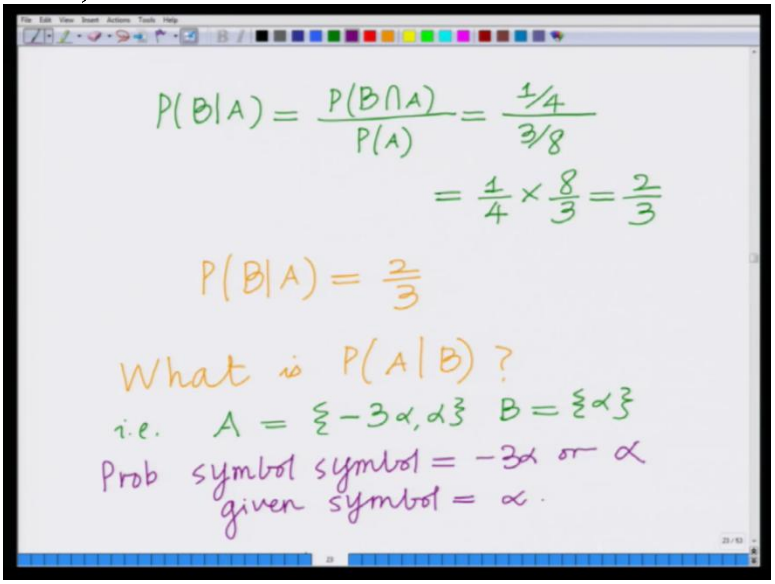

So now let us find, what is, so let us ask the question, what is probability of A conditioned on B? That is, remember, A = { **-3α, α** } and B = { **α** } .That is, we are asking, P(**-3α , α |B)**. That is given symbol equals **α** and therefore what is the probability of A conditioned on B? We are saying that the A = { **-3α, α** } and B = { **α** }.

Therefore $P(A|B)$ is the probability, we are asking the question, what is the probability that the observed symbol is **-3α or α** when it is already given that the symbol observed is B that is **α**. And clearly, this probability should be 1. Because given the symbol is **α**, this symbol is **α**, therefore the symbol is definitely either **-3α or α**. Therefore, $P(A|B)$ should be equal to 1. But let us check that,

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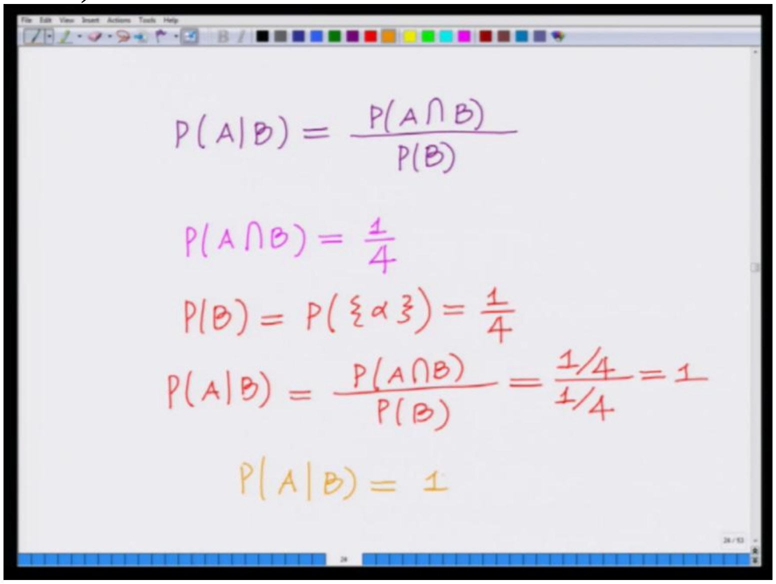

We have,

$$P(A|B) = \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}}$$

We have already derived $\mathrm{P(A \cap B)} = \mathbf{\frac{1}{4}}$.

Observe,

$$\mathrm{P}(\mathbf{B}) = \mathrm{P}(\{\ \boldsymbol{\alpha}\ \}) = \mathbf{\frac{1}{4}}$$

and indeed,

$$P(A|B) = \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}} = \frac{\mathbf{1/4}}{\mathbf{1/4}} = \mathbf{1}$$

Therefore,

$$\mathrm{P(A|B)} = \mathbf{1}$$

And also observe something very interesting.

$$\mathrm{P(A|B)} \neq \mathrm{P(B|A)}$$

Remember, we derived $P(B|A)$.

$$\mathbf{P(A|B) = 1}.$$

So,

$$P(A|B) \neq P(B|A)$$

So probability of A given B is 1 which is not equal to the probability of B given A which we basically derived, as which, if you see previously we basically derived, $\mathbf{P(B|A) = 2/3}$. So this is the concept, what we have seen in this module is, we have seen the concept of conditional probability. That is, given 2 events, A, B, what is the conditional probability of A given B? Or in other words, how does observing event B affect the probability of the occurrence of event A. That is the conditional probability of A given B.

Similarly we have also seen the conditional probability of B given A and we have seen a simple example to clearly illustrate this concept of conditional probability. And we will look at the other aspects in the subsequent modules.. Thank you very much.