Fiber Optics
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Lecture - 08
Transmission Characteristics – III

This is the third part of the module transmission characteristics of an optical fiber. In the first and second modules we had seen that the attenuation and intermodal dispersion they limit the repeater less length of the link. In this lecture we will look into what would be the implication of the wavelength content of the source, the flow of the lecture is something like this we will talk about chromatic dispersion, then what is an optical pulse, what our faith and group velocities, then material dispersion then since the fiber is telecom fiber is made of pure silica glass.

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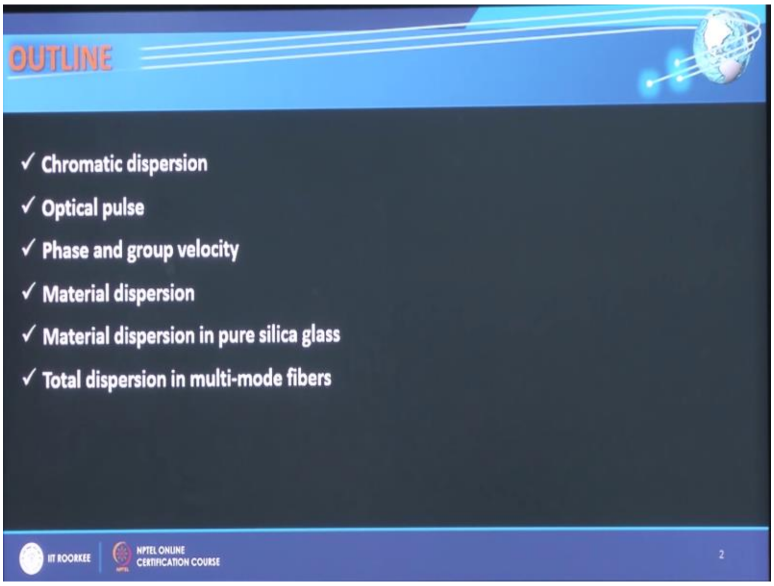

So, we would look into the material dispersion of pure silica glass, and then what is the total dispersion in a multimode fiber.

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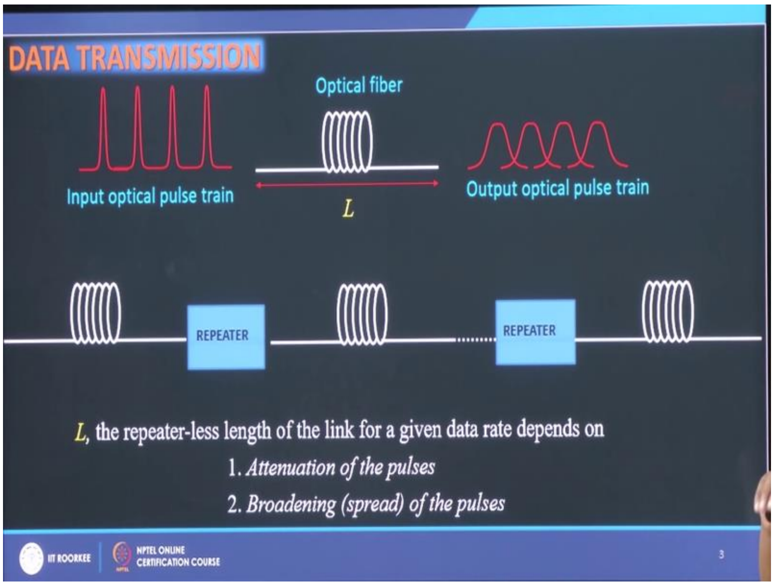

So, I come back to this slide again, where we had seen that the repeater less length of the link for a given data rate, depends upon attenuation and broadening of the pulse. Broadening of the pulse we had already seen in the last lecture, the material dispersion where light is coupled into various ray paths, and these ray paths take different times to reach at the output end of the fiber. So, light coupled into these various ray paths takes different times to reach at the output end of the fiber and these causes, what is known as the intermodal dispersion.

Today we would see if we have a source and which has certain wavelength content, what would be the implication of this on the broadening of pulses.

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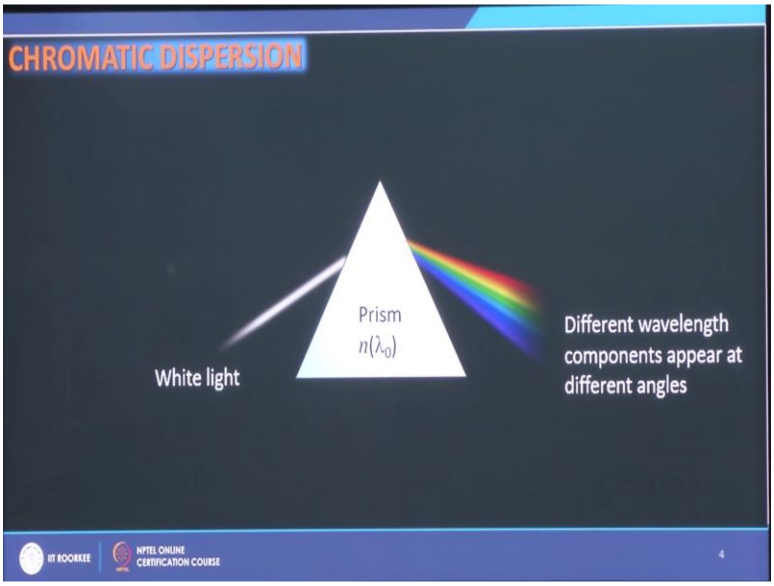

I know that if I launch a beam of white light into a prism then I see dispersion, I see various colors appearing at various angles, and this is purely due to the refractive index wavelength dependence of refractive index of the material of the prism and the geometry of the prism enables these different colors coming out at different angles, this is known as chromatic dispersion.

Now, if I have a fiber then made of glass, then fiber material also has this characteristic, the refractive index of the material at different wavelengths is different, what would be the implication of that. So, for that let me first look at how do I transmit data in an optical fiber, I transmit data in the form of pulses and when I switch on a laser and switch it off I generate a pulse.

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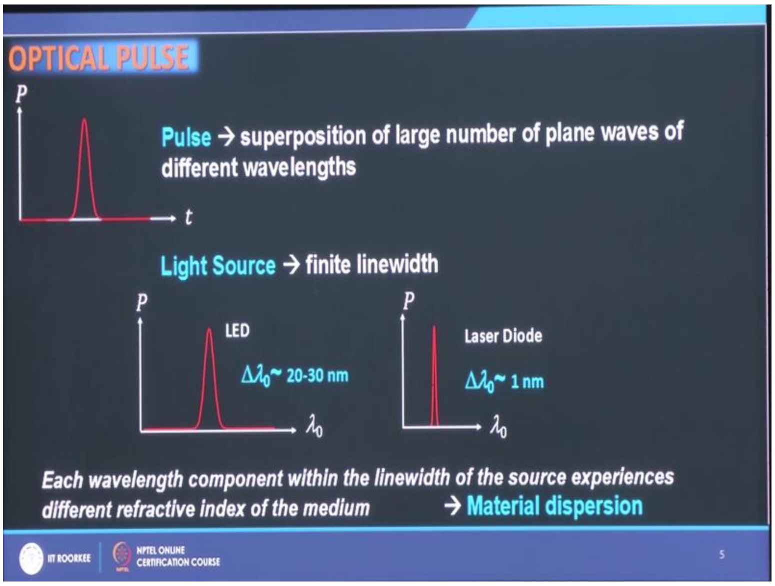

If I look into this pulse, then as I will see a little later that this pulse is essentially a superposition of large number of harmonic waves of different wavelengths or slightly different frequencies, and when I use a light source like LED or a laser diode, for creating these pulses to send data into an optical fiber, then these light sources have finite line width.

For example an LED has a typical line width of 20 to 30 nanometers, while a laser diode although it is highly monochromatic it also has a certain line width. Its line width can be of the order of a nanometer and if it is very highly chromatic and very good laser source then it can be 0.1 nanometers or so, but it has certain line width. So, all the wavelengths components which fall in the line width of these light sources, would now experience different refractive index of the material and they will travel with different velocities and that should give rise to dispersion.

To understand that and this kind of dispersion is known as material dispersion. To understand that let us first find out it with what velocity this pulse travels in an optical fiber or in a material. Even if we forget about optical fiber even if it is infinitely extended materials with what velocity this pulse travels in the fiber; does it travel in the same way as a continuous waves a harmonic wave let us look into it.

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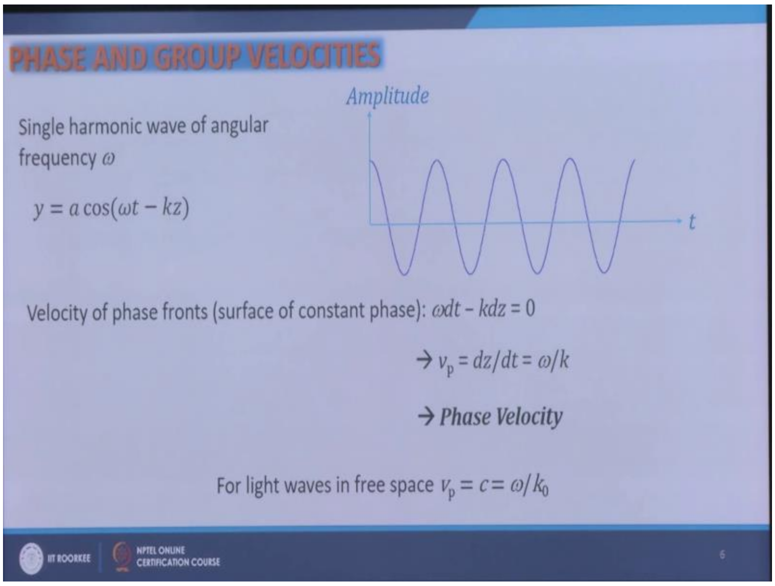

So, if I first consider a single harmonic wave of angular frequency omega which is propagating in z direction, then I can write the displacement of this wave as $y = a\cos(\omega t - kz)$ where a is the amplitude, ω is angular frequency, k is the wave vector. And if I plot it at certain z if I observe the amplitude of this the displacement of this at certain z for all the times, then it will vary with time something like this.

Now, the velocity of phase fronts I can find out from here, since what is the velocity of phase fronts what are phase fronts, phase fronts are the surfaces of constant phase, and here the phase is $\omega t - kz$.

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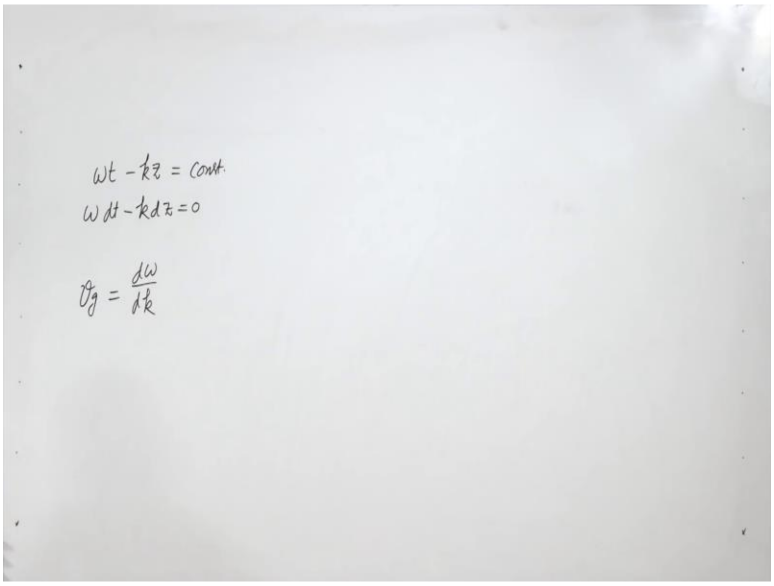

So, surface of constant phase would be given by $\omega t - kz = const.$, now I can find out the velocity of phase fronts from here I just differentiate it, then $\omega dt - kdz = 0$ that is what I have there, and this will give me $dz/dt = \omega/k$, and this dz/dt is nothing, but the velocity of the phase front and this is known as the phase velocity.

So, when a single harmonic wave travels in a medium, then it goes with this velocity. For light waves in free space this is nothing, but c and can be given by ω/k_0 where k_0 is the wave vector in free space now let us consider 2 harmonic waves a very close angular frequencies ω_1 *and* ω_2 and very close wave vectors k_1 and k_2.

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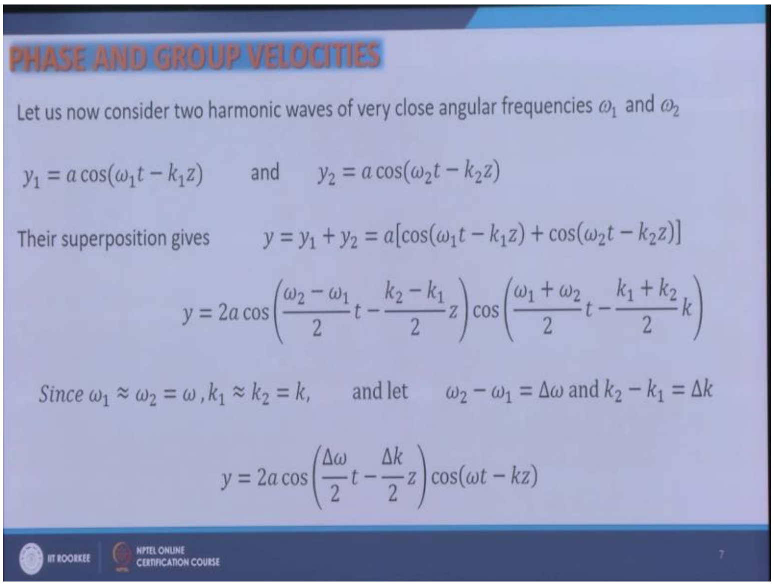

So, I can write them as $y_1 = a\cos(w_1 t - k_1 z)$, and $y_2 = a\cos(w_2 t - k_2 z)$, for simplicity I have taken the same amplitudes of these waves.

Now, I superpose them when I superpose them then $y = y_1 + y_2 = a[\cos(w_1 t - k_1 z) + \cos(w_2 t - k_2 z)]$ and if I simplify this what do I get? I get $y = 2a\cos(\frac{\omega_2 - \omega_1}{2}t - \frac{k_2 - k_1}{2}z)\cos(\frac{\omega_2 + \omega_1}{2}t - \frac{k_2 + k_1}{2}z)$. Now since ω_1 is very close to ω_2 , and k_1 is very close to k_2 let me write them down as ω and k, and also assume that $\omega_2 - \omega_1 = \Delta\omega$, and $k_2 - k_1 = \Delta k$, then I can write this down as $y = 2a\cos(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}z)\cos(\omega t - kz)$.

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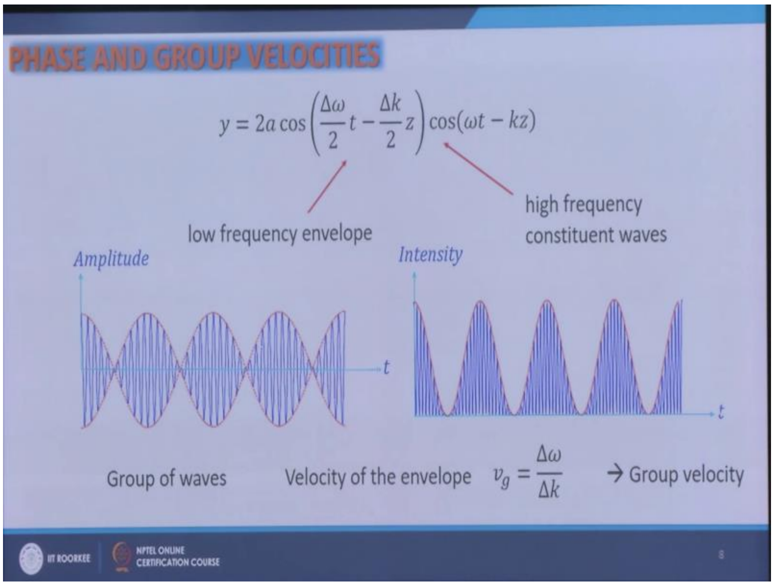

Now, let us examine this if I examine this. This is nothing, but effect a term which contains a frequency ω while this term contains a very small frequency $\Delta\omega$. So, this is nothing, but high frequency constituent waves and this is the envelope which is low frequency. If I plot them if I plot this y now as a function of t for any given z what do I see? The amplitude both something like this. So, I have this red one is the envelope low frequency envelope, and blue one is the high frequency constituent waves. And if I plot the intensity of this which is proportional to y^2 , then it comes out like this.

So, what I have observed that if I superpose 2 waves I make groups a train of group, I generate a group of waves. And these groups of waves are travelling with certain velocity, which is the velocity of the envelope from here and if I find out the velocity of the envelope from here it comes out to be $\Delta\omega/\Delta k$. So, this is group velocity. So, this group of wave travels with velocity v_g which is $\Delta\omega/\Delta k$. However, the constituent waves individual constituent waves they travel with velocity ω/k, which is the phase velocity of the constituent waves.

So, this is a series of group, but if I have a pulse and isolated pulse then it is a group of very large number of such harmonic waves with continuous frequency variation in ω/k. So, instead of taking the superposition of 2 ways, I take the superposition of 100 ways and then 1000 waves then I can find out that these groups are separated ok.

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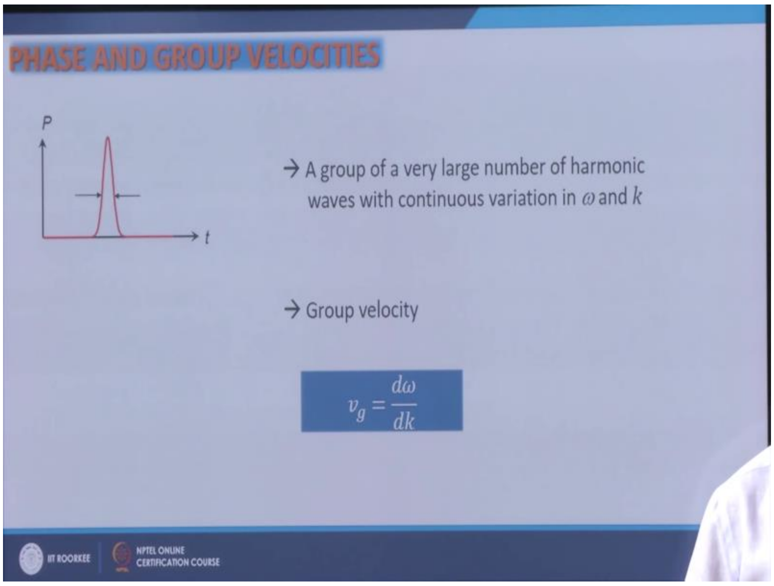

And I can create an individual group or wave packet is also known as wave packet. If I have this very large number of harmonic waves, with continuous variation in ω / k, and in such a situation $\Delta\omega / \Delta k$ would now become $\Delta\omega / \Delta k$, and the group velocity will be given by $\Delta\omega / \Delta k$.

So, now if I send this pulse through a material that material can be the material of optical fiber, then it is a group of waves.

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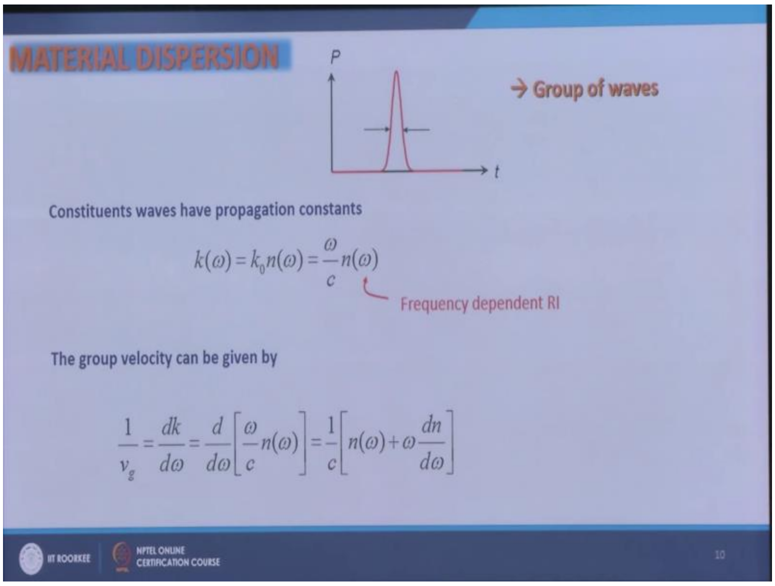

And now I want to find out how the different frequency components travel in this material and what is the implication of that. I know the constituent waves of this wave packet or the pulse, will have propagation constant given by $k(\omega) = k_0 n(\omega)$, where $k_0 = \omega / c$ and $n(\omega)$ is frequency dependent refractive index of the medium.

Now, I can find out the velocity from here group velocity, because the pulse will travel with the group velocity v_g. I know $v_g = d\omega / dk$ from the previous slide. I have $v_g = d\omega / dk$, but remember that ultimately what I want to do is, I want to find out the transit time to *L* length of the fiber and that would be given by *Ly* v_g. So, instead of working out the expression for v_g let me work out the expression for $1/v_g$ because that is how it will appear ultimately.

So, I find out $1/v_g$ from here, $1/v_g$ would be $dk/d\omega$. So, I take differential. So, I take differential of *k* with respect to ω, I take derivative of *k*. So, it comes out to be $\frac{1}{c}[n(\omega) + \omega \frac{dn}{d\omega}]$, and since I work with wavelength instead of frequencies in practical situations I always work with wavelength. So, let me obtain this v_g in terms of wavelength λ_0, $\lambda_0 = 2\pi c / \omega$, and I have v_g in terms of omega like this.

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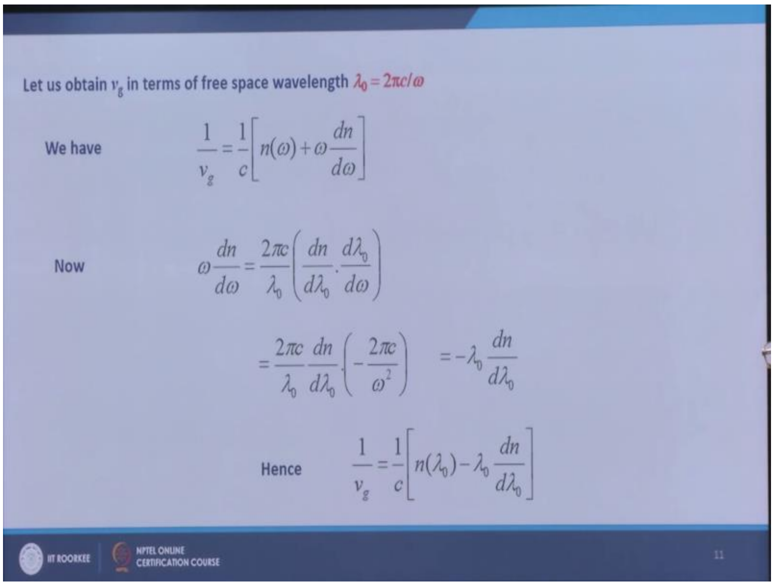

So, now let me find out this $\omega dn/d\lambda$ in terms of λ. So, $\omega \frac{dn}{d\omega} = \frac{2\pi c}{\lambda_0}(\frac{dn}{d\lambda_0}\frac{d\lambda_0}{d\omega})$ is given by this. So, it would be simply $\omega \frac{dn}{d\omega} = \frac{2\pi c}{\lambda_0}\frac{dn}{d\lambda_0}(\frac{-2\pi c}{\omega^2})$, and if I do that I get $\omega \frac{dn}{d\omega} = -\lambda_0 \frac{dn}{d\omega}$.

So, I put it back into this expression, and I get the expression of $1/v_g$ in terms of wavelength. So, $\frac{1}{v_g} = \frac{1}{c}(n(\lambda_0) - \lambda_0 \frac{dn}{d\lambda_0})$.

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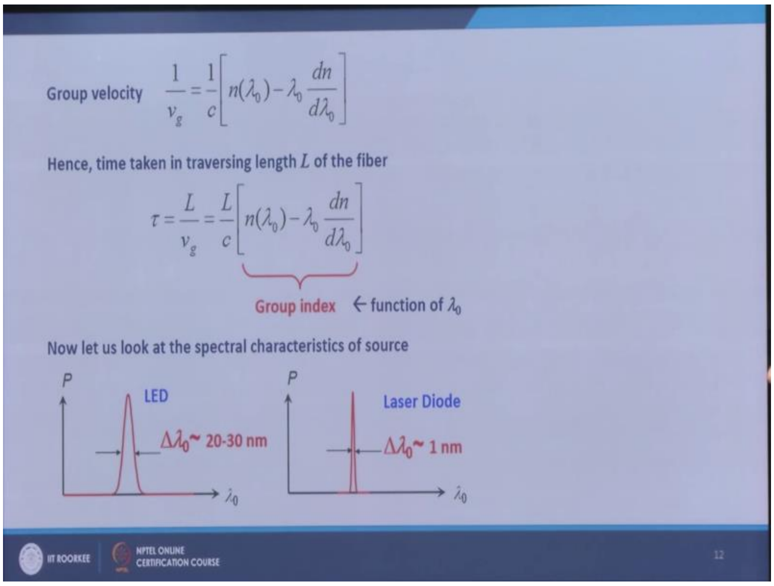

Now look at it, if I have a single wave having wavelengths λ_0 and there is there are no other wavelengths components, then this $\frac{dn}{d\lambda_0}$ is 0 and I simply get $1/v_g = n/c$ *or* $v_g = c/n$ which is nothing, but the phase velocity. But when I have a group of wave then this extra term comes into picture. From here you might think that in this way the refractive index which is felt by the group of waves has decreased. You can see that now the refractive index from individual wave to group of waves, now changes as $n(\lambda_0) - \lambda_0 \frac{dn}{d\lambda_0}$. So, the group of waves will feel this refractive index of the medium, has it decreased? If I compare it with the individual wave no because $\frac{dn}{d\lambda_0}$ is negative. So, the refractive index which is also refractive index of the group of waves which is also known as group index has basically increased. So, group of index has increased and this is how the group velocity has decreased now. So, individual waves they travel with faster velocity, but the group moved with a slower velocity group moves bit slower much lower, others pull others pull them down. One individual wave tries to move fast, but others is no you cannot go that fast and they pull, they pull them down. So, group index is always larger than the index of the individual wave.

Now, let us find out what is the time taken in traversing length L of the fiber. So, this would be given by L/v_g, and simply L/c times group index and this group index is of course, a function of lambda. Now let us look at the spectral characteristics of the source, what wavelength what values of lambda naught I have. See if I take the LED then this is the line width, and if I take a laser diode this is the line width, and all the wavelengths components which fall into these line width would now contribute here they have different transit times. What would be the implication of this on the broadening how much would be the broadening. So, it is very simple, that if the source has the spectral width $\Delta\lambda_0$, they the temporal broadening of the pulse would be simply given by $\Delta\tau = \frac{d\tau}{d\lambda_0}\Delta\lambda_0$ while considering only first order term.

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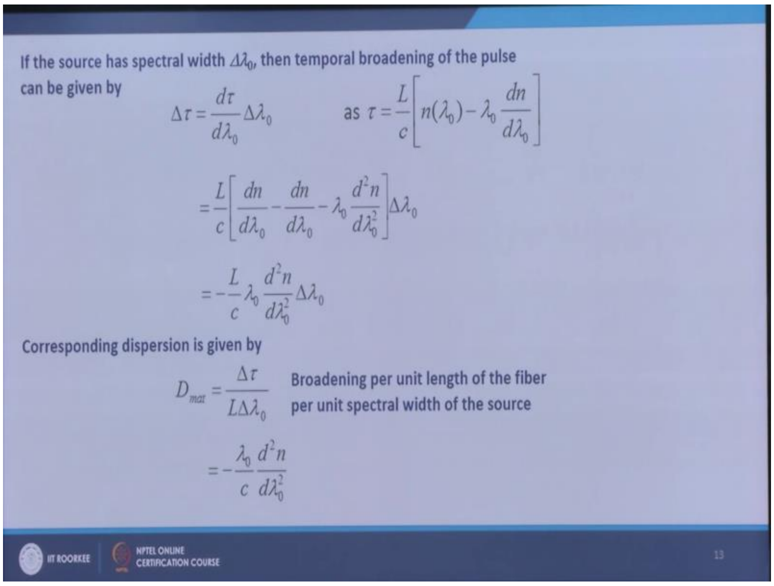

And since $\tau = \frac{L}{c}[n(\lambda_0) - \lambda_0 \frac{dn}{d\lambda_0}]$ then this would be simply now I just take $\frac{d\tau}{d\lambda_0}$ from here. So, $\frac{L}{c}[\frac{dn}{d\lambda_0} - \frac{dn}{d\lambda_0} - \frac{d^2n}{d\lambda_0^2}]$ and this would simply be $\frac{L}{c}[\frac{dn}{d\lambda_0} - \frac{dn}{d\lambda_0} - \frac{d^2n}{d\lambda_0^2}]$. And usually what we do we define this broadening in terms of dispersion coefficient, and here it would be called material dispersion coefficient.

Since the fiber length is measured in kilometers and the line width of the source is measured in nanometers. So, I find the broadening of the pulse per kilo meter length of the fiber, and per nanometer spectral width of the source and. So, the dispersion coefficient is given as delta tau divided by per kilometer length of the fiber per nanometer spectral width of the source, and we usually represent this delta tau in picoseconds per kilometer nanometers. So, this would now be simply $-\frac{\lambda_0}{c}\frac{d^2n}{d\lambda_0^2}$.

Let us work out with the dimensions here. So, if you go back it is lambda naught it is $\frac{d^2n}{d\lambda_0^2}$. So, what I do now I multiply numerator and denominator by λ_0.

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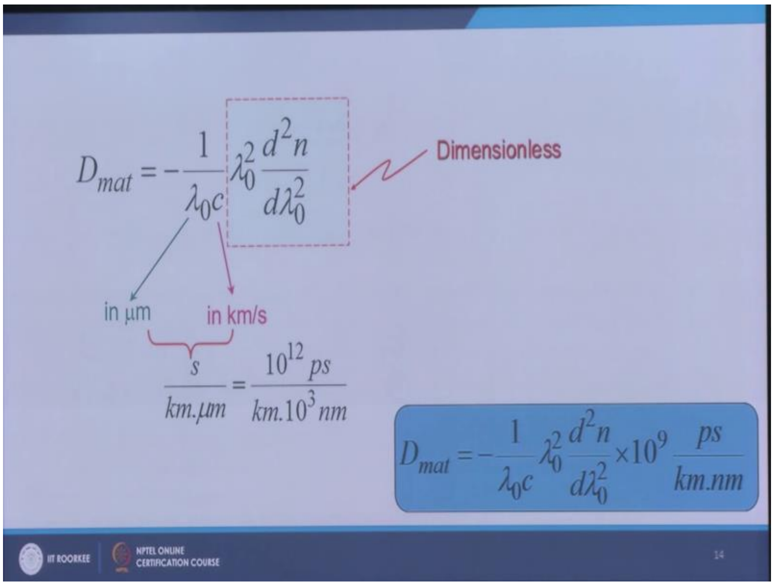

So, I can write this down like this. The motivation for writing this down is that now I can get this as a dimensionless quantity. So, now, the dimensions are coming from here. So, let me express λ_0 in micrometer and c in kilometers per second, then the dimensions would be seconds per kilometer times micrometer, and let me convert it into picoseconds per kilometer nanometers. So, this would be 10 to the power 12 picoseconds per kilo meters times thousand nanometers. So, it would simply be this much times 10 to the power 9 picoseconds per kilometer nanometer.

So, you can use this expression to calculate material dispersion coefficient in picoseconds per kilometer nanometer, provided that you put the velocity of light c in kilometers per second and wavelength of light in micrometer.

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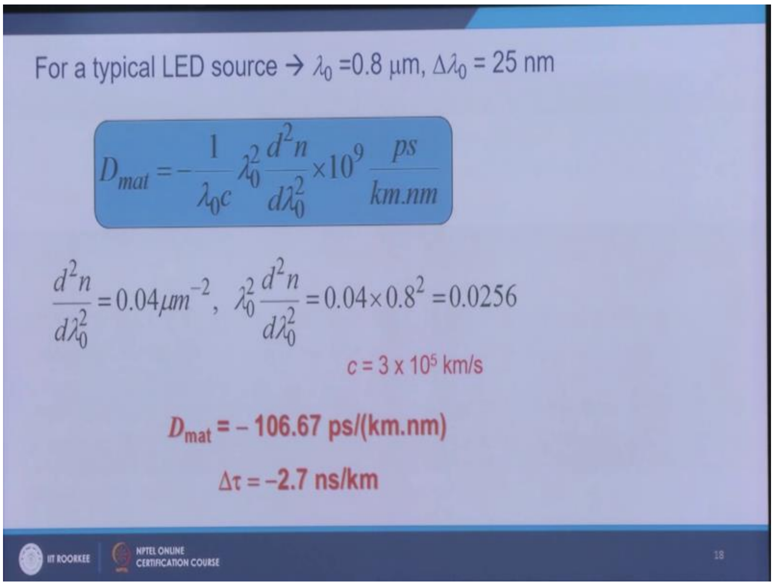

So, that is what I have written here. So, now, let us look. So, if I again go back and see that this material dispersion coefficient depends upon how the refractive index of the material changes with wavelength.

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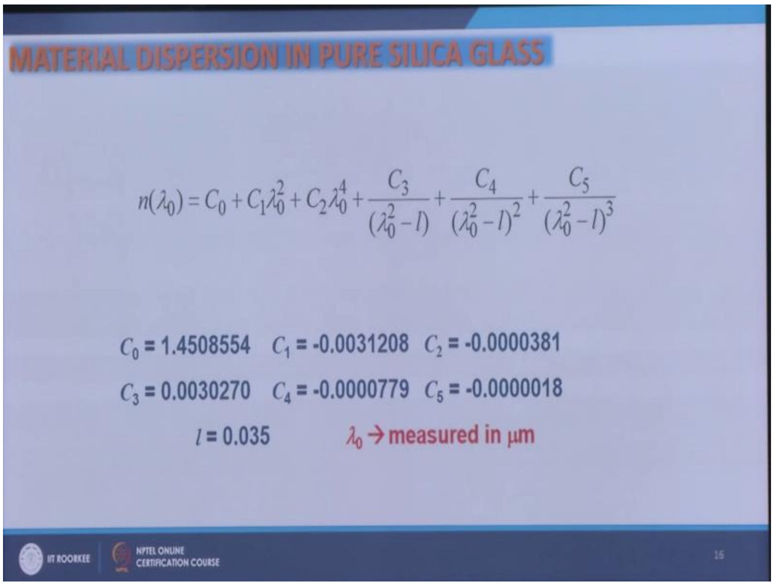

So, I need to know what is the second derivative of n with respect to λ, and this is the characteristic of a material for different materials it would be different. So, different materials will exhibit different material dispersion characteristics.

Let me do this for pure silica glass, it is fused silica and the variation of effective index with wavelength or pure silica glass is given by this, where various constants have these values and lambda naught is measured in micrometer. What is done is basically you measure the refractive index of the material at different wavelengths, and then fit this kind of relationship to find out the values of these constants. So, these values are also experiment dependent, but these are quite accepted values in the literature.

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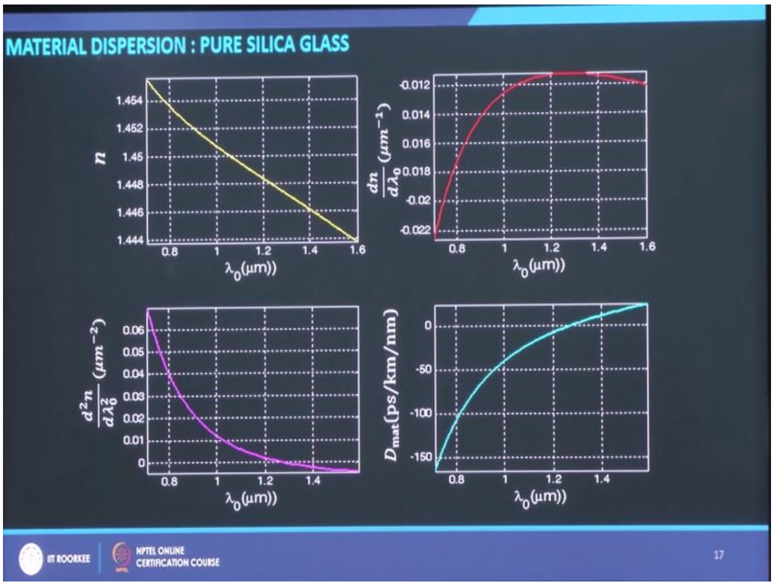

So, now what I do? I find out the variation of n with respect to λ. I plot the variation of n with respect to λ or fused silica glass, and it looks like this. Then $\frac{dn}{d\lambda_0}$ in the units of micro meter inverse it goes like this, and what I find that it changes its slope from here to here, and it has got a maximum somewhere here and it tells me that and remember that in dispersion coefficient I need to have $\frac{d^2n}{d\lambda_0^2}$, which means that it should cross 0 somewhere here. So, now, I plot $\frac{d^2n}{d\lambda_0^2}$ in micrometer to the power minus 2, and I see the variation looks like this, and it crosses 0 somewhere here. And if I find out $\frac{d^2n}{d\lambda_0^2}$ at certain wavelengths say 800 nanometer or 0.8 micrometer, which was the wavelength, used earlier for optical fiber communication around 800 or 850 nanometer.

So, I find that is $\frac{d^2n}{d\lambda_0^2}$ is about 0.04 micrometer to the power minus 2, and here if I plot this material dispersion coefficient using this in picoseconds per kilometer nanometer, then corresponding D_{mat} goes something like this and I find that since there is zero crossing around this wavelength. So, material dispersion is zero at this wavelength, and this wavelength is around 1.27 micrometer. Now let me work out some numbers for a typical led source, if I take $\lambda_0 = 0.8\,\mu m$, $\Delta\lambda_0 = 25\,nm$ and from here if I find out D_{mat} I have already seen that $\frac{d^2n}{d\lambda_0^2}$ is 0.04 micrometer to the power minus 2.

So, if I put these numbers here I find that material dispersion coefficient at this wavelength comes out to be about 106 picoseconds per kilo meter nanometer. Corresponding broadening will come out to be minus 2.7 nano seconds per kilometer well broadening would be always I have to take the magnitude of that. This dispersion coefficient is negative, what is the meaning of negative and positive we will learn as we go along.

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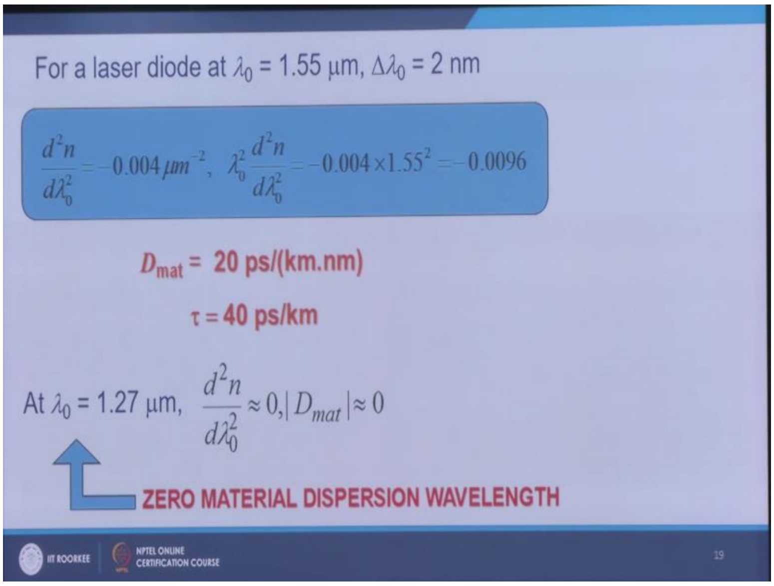

But there would be a broadening of about p nanosecond every kilometer. If I take a laser diode at 1.55 micrometer and delta lambda naught about 2 micrometer sorry 2 nanometers, at this wavelength $\frac{d^2n}{d\lambda_0^2}$ comes out to be minus 0.004 micrometer to the power minus 2, and it gives me material dispersion coefficient as 20 picoseconds per kilometer nanometer and now the broadening is only 40 picoseconds.

So, you can see when I go from an LED at 800 nanometer wavelength to a laser diode at 1550 nanometer wavelength, the material dispersion comes down drastically from 3 nano seconds per kilometer if you go back, 3 nano seconds per kilometer to 40 picoseconds per kilometer and if I use a wavelength around 1.27 micrometer then it is almost 0. So, this wavelength is also known as 0 material dispersion wavelength and that is why the earlier fibers the fiber which is already laid on seabed, most of that fiber is optimized around 1.27 or 1.3 micrometer wavelength.

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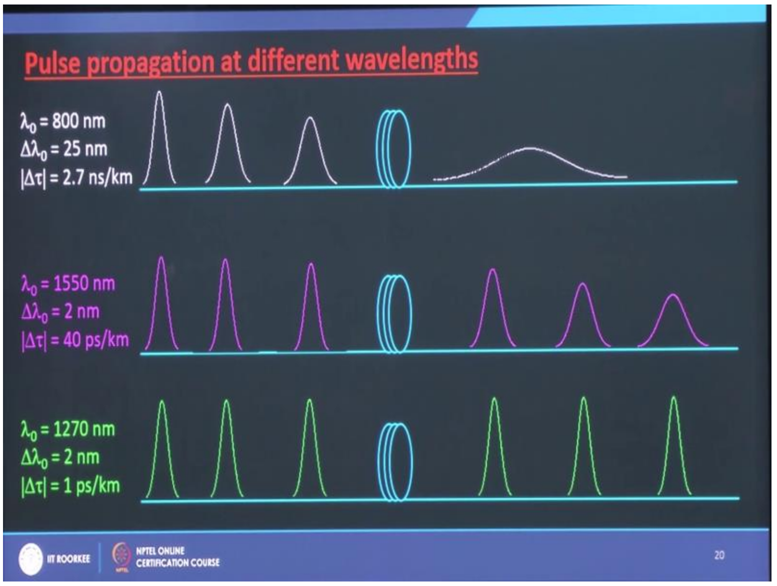

Now, let us look at pulse propagation at different wavelengths this is the LED at 800 nanometer with line width 25 nanometer and broadening of 2.7 nano seconds per kilometer. So, if I now send this pulse I see that it broadens very quickly, when I use a laser diode at 1550 nanometer wavelength which gives me a broadening of 40 picoseconds per kilometer nanometer, then it also broadens the pulse there, but the broadening is not that much. But if I use 1270 nanometer laser diode then the broadening is very small around one picoseconds per kilometer length of the fiber and at this wavelength the pulses is retain their shape over very long distances.

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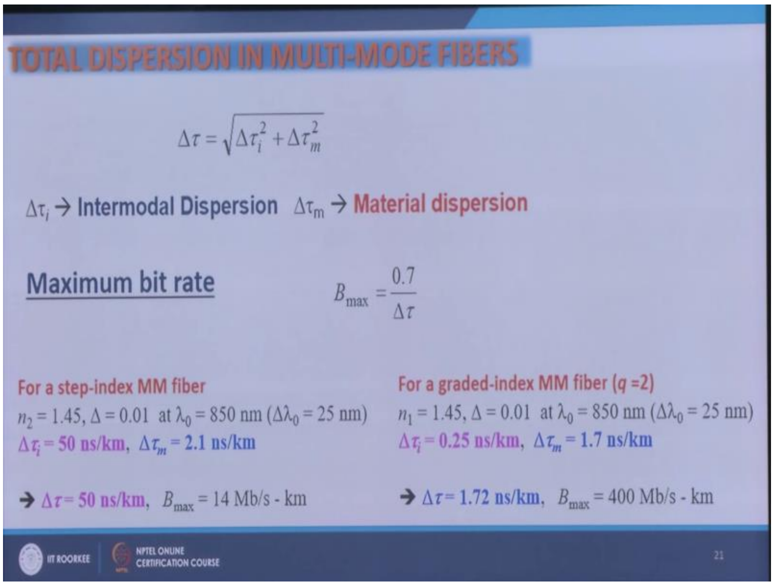

Then there is total dispersion in multimode fiber, total dispersion will comprise both the intermodal and material. So, I can then find out the total dispersion by using the information of intermodal as well as material dispersion. Then the maximum bit rate I can find out if I know the total dispersion by 0.7 over delta tau, and I know that beam max times delta tau should be less than one, but I have taken this vector 0.7 here it is corresponding to non return to zero coding. Now I take 2 examples for a step index, multimode fiber where n_2 is equal to 1.45 delta is equal to 0.01 and I have wavelengths which is 850 nanometer please LED of 25 nanometer line width.

So, here delta tau m material dispersion is about 290 seconds per kilometer, while intermodal dispersion is 50 nanoseconds per kilometer. So, total dispersion comes out to be 50 nanoseconds per kilometer this dominates, and if I find out B_{max} from here it is about 40 Mbps over a kilometer. If I take a graded index fiber graded index multimode fiber with q is equal to 2 which is which has got parabolic index variation, then I find that intermodal dispersion comes down to 0.25 nano seconds per kilometer, while material dispersion is 1.7 nano seconds per kilometer. So, this dominates total dispersion is about 1.72 and B_{max} can be 400 Mbps over a kilometer 400 Mbps over a kilometer. So, it increases from 14 Mbps to 400 mbps.

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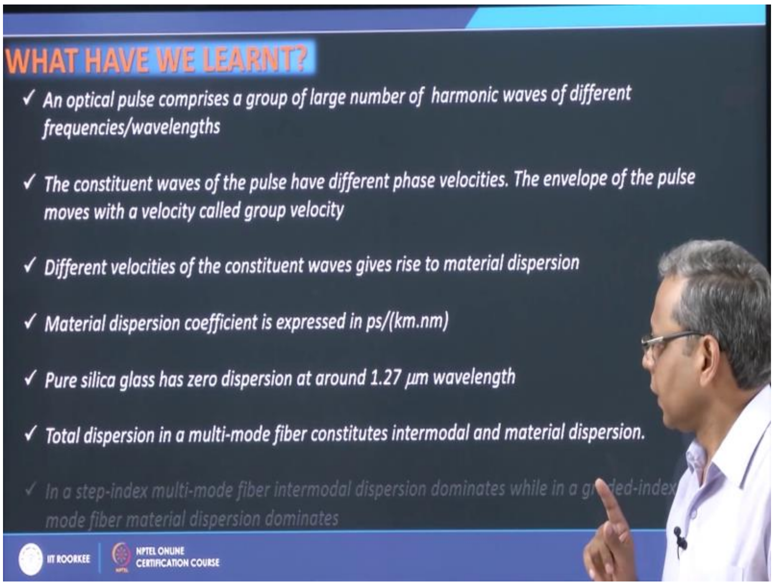

So, what we have learnt in this lecture that, an optical pulse comprises a group of large number of harmonic waves of different frequencies or wavelengths, these constituent waves of pulse have different phase velocities, the envelope of the pulse moves with a velocity called group velocity, different velocities of constituent waves give rise to what is known as material dispersion and material dispersion coefficient we usually represent in terms of picoseconds per kilometer nanometer. Pure silica glass has 0 dispersion around 1.27 micrometer wavelength, and total dispersion will contain now both intermodal as well as material dispersion, in a multi mode step index fiber the intermodal dispersion will dominate while in a graded index multimode fiber material dispersion dominates.

So, this is all about some basic transmission characteristics of an optical fiber. In the next few modules we will do some rigorous analysis of light propagation in optical fibers, before that we will understand how light propagates in infinitely extended medium, then a very simple planar waveguide. So, we will do in the next few lectures now.

Thank you.