**Fiber Optics**
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Lecture – 07
Transmission Characteristics- II

In the last lecture we had seen that when optical pulses propagate through optical fiber then they attenuate. I had also mentioned that they broaden in this lecture we are going to see how the pulses are broadened in an optical fiber and in particular in a multimode fiber.
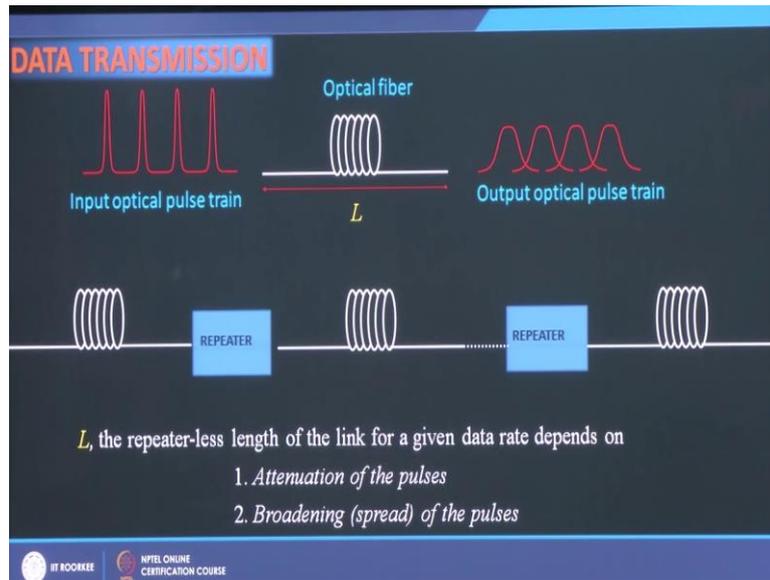
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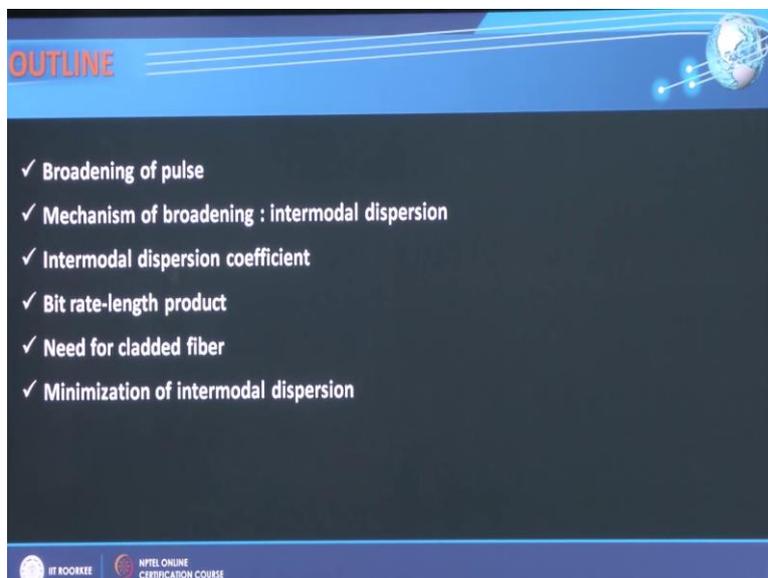
So, you are going to look at broadening of pulses; why do they broaden, mechanism of broadening, in this lecture intermodal dispersion, then what is the intermodal dispersion coefficient, then we are going to look at bit rate length product. You will see that for a given bit rate I cannot go for a very long length or if I want to keep a particular length of the fiber then bit rate I will have to compromise with.
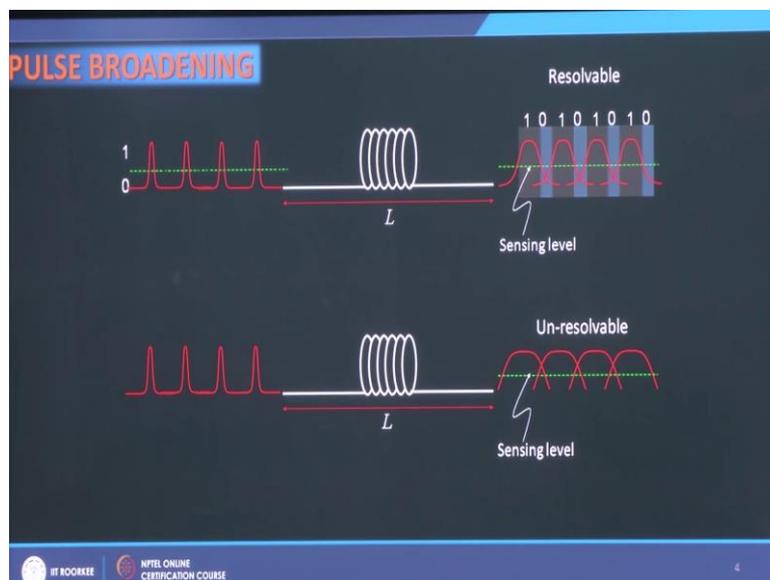
We will see in this lecture what is the need for cladded fiber, and how can I minimize intermodal dispersion.

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So, again I look at this slide, go back to this slide and particularly focus on the broadening of pulses. So, the input pulse train gets broaden. And let us see what happens because of this broadening.

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So, this is the input pulse train; let us say this is the sensitivity of the detector this green line represents the sensitivity of the detector. So, anything which is below this would be recorded as 0 and anything which is much above this is 1. So, when I send this pulse ~~strain~~ train through L length of optical fiber then the pulses become like this. And in this particular

scenario what I see that if this is the sensing level then this is the high pulse this is low, so high and low regions are well distinguished.

So, these pulses are resolvable. If ~~here I have 0; sorry~~ here I have 1, here I have 0 then I will receive it as 1 and 0 and my information is intact. While, if the broadening is much larger than this then the pulses will overlap and everywhere I will see 1, I will lose my 0s here and I will lose the information which I want to retrieve at the output end.

So, question is what causes the broadening of these pulses so that we can take care of this. For that let us first look at the shape of pulses what kind of pulses we are talking about and the most popular shape of the pulse is Gaussian. What is the Gaussian?

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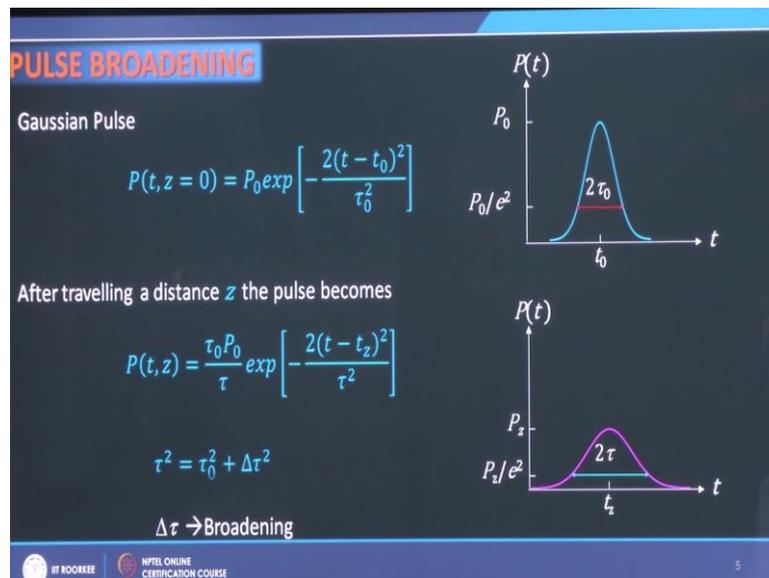
Gaussian is this particular type of variation which is expressed as $P_0 \exp\left[-\dfrac{2(t-t_0)^2}{\tau_0^2}\right]$ ~~P₀ 0 e~~

~~to the power minus 2 t minus t_0 0 square over tau not naught square~~. Where, this pulse is centered around t is equal to t_0 ~~0~~. So, this is called the position of pulse or the center of pulse and $2\tau_0$ ~~2 tau 0~~ is the width of the pulse where the power has dropped down to P_0/e^2 ~~1 over e square of its peak value~~.

So, this $2\tau_0$ ~~2 tau not naught~~ is full width and we usually call τ_0 ~~tau not naught~~ as width of the pulse. If this pulse is travelling, if I let it travel then a distance z the pulse becomes this its amplitude changes and also its width changes and position also. Now, I have a pulse which

becomes something like this, which is now centered at t is equal to t_z ~~Z~~ whose power has gone down to P_z ~~Z~~ and the width has become now τ .~~tau~~. Where τ^2 ~~tau~~ can be given by $\tau_0^2 + \Delta\tau^2$ ~~tau not naught square plus delta tau square~~, your $\Delta\tau$ ~~delta tau~~ is known as broadening of the pulse and that is what we want to find out how much is the broadening.

You may notice here that in case of Gaussian pulse after travelling the Gaussian pulse remains Gaussian. However, its amplitude and width changes, but the function remains the same.

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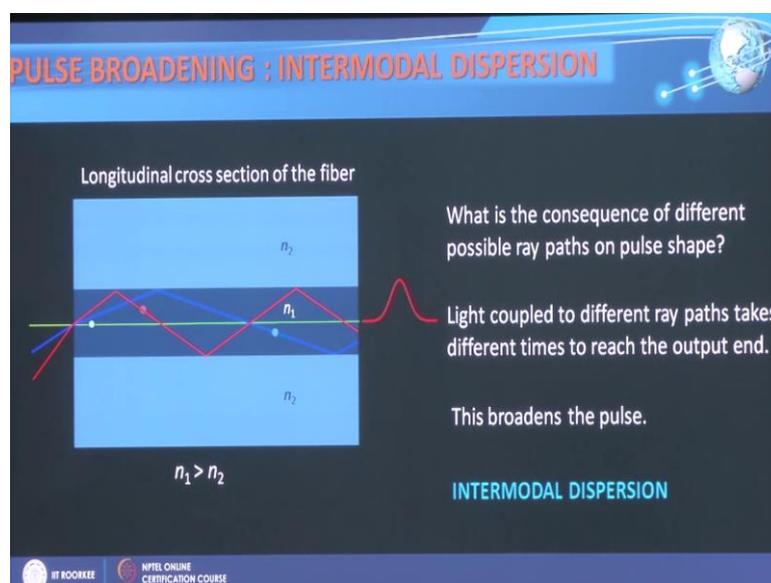
So, why it is broadened? What I see that if I look at a multimode fiber then in a multimode fiber I can have various ray paths: if I launch a pulse of light into this fiber then some of the energy can go into this ray path ~~past~~, some of the energy can go into this blue one, some of the energy can go into red one. And what happens is I can immediately see that if the energy is going into this path then it travels the least distance to traverse a certain length of the fiber.

However this red one it takes longer path. So, the light is coupled into all these ray paths then these ray past take different times to reach the output end and because of this there is broadening. So, I ~~sank~~ send the pulse like this and at ~~add~~ output and I receive the pulse like this. And this kind of broadening is known as intermodal dispersion, because different ray paths can be called different modes of propagation of the system. So, this is known as intermodal dispersion.

Let us now find out how much ~~gardening~~ broadening happens due to this intermodal dispersion.

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So, I have again three rays here white one which is axial ray ~~accelerate~~, then there is the green one and the red one. So, there can be several rays various rays like this. In order to find out the expression for broadening let me consider a ray which makes an angle theta from the fiber axis and correspondingly it makes an angle phi from the normal to the core cladding interface and this ray goes like this. When it reaches path B when it reaches point B it repeats itself after this. So, I can see these two points are kind of identical.

So, if I can find out the time taken by this ray in traversing this part AB then I can find out what would be the time taken by traversing L length of the fiber, because that L length can be comprising these several AB's.

So, what would be the time taken in reaching from A to B? If it follows this path red path well it is very simple: AC plus CB divided by the velocity of light in the refractive index region of n_1, which is c/n_1 ~~C by n₁ 1~~. So, $t = \dfrac{AC + CB}{c/n_1}$ ~~t is equal to AC plus CB divided by c over n₁ 1~~. Now from here I can see $AC\cos\theta = AB/2$ ~~AC cos theta is equal to AB by 2~~ and similarly $CB\cos\theta = AB/2$ ~~CB cos theta is equal to AB by 2~~. I can immediately see that this angle θ ~~theta~~ would be equal to this angle θ ~~theta~~, because this θ ~~theta~~ is equal to this θ ~~theta~~ and then it should be equal to this.

Now, if I find out $AC + CB$ then $AC + CB$ from here is $AB/\cos\theta$ ~~AB over cos theta~~. And in this way I will have $t = n_1 L / c\cos\theta$ ~~t is equal to n₁ 1 AB over c cos theta~~. And therefore, for *L* length of the fiber it would be $n_1 L / c\cos\theta$ ~~n₁ 1 L over c cos theta~~. So, it should come somewhere here. Since time taken for traversing AB length of the fiber is $n_1 L / c\cos\theta$ ~~n₁ 1 AB over c cos theta~~. So, the time taken in traversing *L* length of the fiber would be $n_1 L / c\cos\theta$. ~~n₁ 1 L over c cos theta~~.

So, this is what I have.

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Now, what is the range of theta for light guidance by total internal reflection? You can have an axial ray which corresponds to theta is equal to 0 and then you can have a ray which corresponds to $\theta = \theta_0$ ~~theta is equal to theta C~~, which in turn corresponds to the critical angle. So, your minimum time would be for axial ray and it is corresponding to θ ~~theta~~ is equal to 0 given by $n_1 L / c$ ~~n 1 L over C~~. While the maximum time would be taken by the ray which makes an angle θ_c ~~theta C~~ from the axis, so, $t_{max} = n_1 L / c\cos\theta_c$ ~~t max is equal to n₁ 1 L over c cos theta C~~. And because of the time difference between this and this I will have broadening. From here I can immediately have $\cos\theta_c = \sin\phi_c = \dfrac{n_2}{n_1}$ ~~cos theta C is equal to sine phi C which~~

~~is n₂ 2 over n₁ 1~~. So, $t_{max} = \dfrac{n_1^2 L}{cn_2}$ ~~t max is n₁ 1 square L over c n₂ 2~~, ok.

Now, if all the rays are excited simultaneously at the input end then the time interval occupied by the rays at the output end now would be $\Delta\tau = t_{max} - t_{min} = \frac{n_1 L}{c}(\frac{n_1}{n_2} - 1)$ ~~t~~_{~~max~~} ~~max~~ ~~minus t~~_{~~min~~} ~~min which are represent by delta tau broadening, and it would be simply n₁ 1 L by c times n₁ 1 over n₂ 2 minus 1.~~

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Let us work out an example: how much broadening I am going to have for a typical fiber. If I have a typical multimode fiber and I send a very narrow pulse or impulse through that fiber then what would be the broadening. For a typical multimode fiber n_1 ~~1~~ is equal to 1.5 and relative index difference between the core and the cladding is typically 0.01 or 1 percent.

So, now for these values if I calculate $\Delta\tau$ ~~delta tau~~ which is given by $\Delta\tau = \frac{n_1 L}{c}(\frac{n_1}{n_2} - 1)$ ~~n₁ 1~~ ~~L by c n₁ 1 by n₂ 2 minus 1~~ and let me do it for 1 kilometer length of the fiber L is equal to 1 kilometer then I find out the $\Delta\tau$ ~~delta tau~~ comes out to be 50 nanoseconds: 50 nanoseconds for 1 kilometer; which means that if I send an impulse then it will become a pulse of 50 nanoseconds duration after traveling through 1 kilometer length of the fiber. So, let me send this very narrow pulse through this optical fiber and as it propagates through optical fiber that it broadens. So, you can see that it broadens.

So, this pulse at the input end will come out to be like this at output and after 1 kilometer length of the fiber. Let me see what happens in the case of pulse strain.
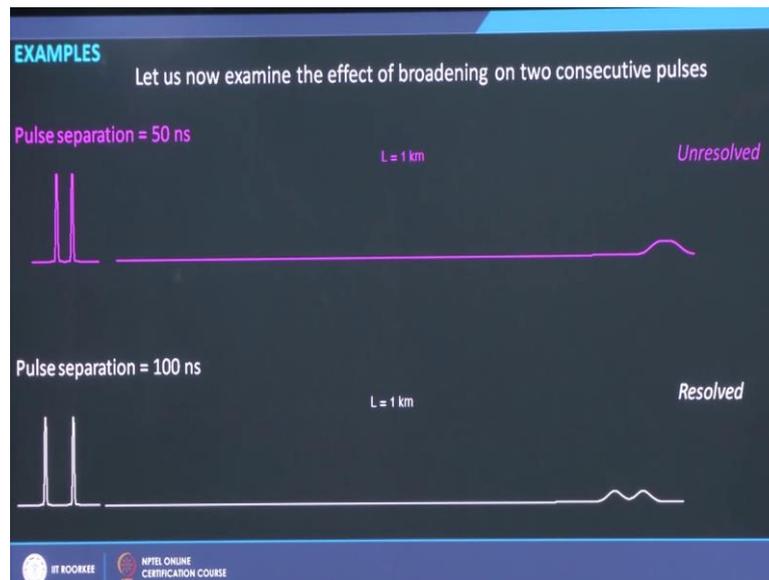
If I now send two pulses like this which are separated in time by 50 nanoseconds and let me send it to 1 kilometer length of an optical fiber, then what happens is that as the pulses propagate through fiber they broaden, you can see the broadening. Now you see they have started overlapping. And around this point you can see that if the sensitivity of detector is somewhere here then you cannot make out where is your 0 where is your 1 they have completely overlapped, you cannot distinguish between the two pulses and here at 1 kilometer length you cannot say that there were two pulses initially, ok. So, they are unresolved you cannot resolve these pulses.

So, what you will have to do? If you want to keep the length of the fiber as 1 kilometer and you still want to resolve them at output end then you will have to increase the separation between the pulses. So, that they are resolvable at the output end. So, let me now increase the separation between the pulses and let me make it 100 nanosecond.

Now if I propagate these through this fiber. Now, I can see the pulses are broadened, but they are resolvable, they are still resolvable ok. So, for a given broadening this was the case for delta tau corresponding to 50 nanoseconds. So, for a given broadening or for a given fiber if I want to keep the length constant 1 kilometer then in order to retrieve information at output end I will have to have large separation between the pulses; large separation means that you are now able to send less number of pulses per second, because separation between the pulses has been increased, which means that your data rate is now smaller, ok.

In fact, we can relate this delta tau to this data rate or the information carrying capacity the data rate or bit rate I can relate this delta tau to this bit rate B, ok.

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A precise relation between B and τ ~~tau~~ depends upon several things; like pulse shape or coding scheme. But intuitively it is clear that this $\Delta\tau$ ~~delta tau~~ should be less than ~~t~~ T_B; what is ~~t~~ T_B, ~~t~~ T_B is the allocated bit slot which is nothing but 1 over B if B is the bit rate then ~~t~~ T_B is equal to 1 over B is the allocated bit slot; which means that this B times delta tau should be less than 1. And $\Delta\tau$ ~~delta tau~~ I have from previous slide $\Delta\tau = \dfrac{n_1 L}{c}(\dfrac{n_1}{n_2}-1)$ ~~n₁ 1 L over c n₁ 1~~

~~minus n₂ 2 minus 1.~~ So, $BL < \dfrac{n_2 c}{n_1(n_1 - n_2)}$ ~~B times this should be less than 1. Or B times L should be less than n to C over n₁ 1 times n₁ 1 minus n₂ 2.~~

So, for a given fiber where n_1 ~~1~~ and n_2 ~~2~~ are fixed your B times L is constant. So, for a given fiber B times L is constant that is bit rate length product is always constant. What is the implication of this?

Let us consider two cases. Case one: it is of un-cladded fiber, where I have core of glass of refractive index 1.5 and (Refer Time: 18:12) works as cladding. So, n_1 ~~2~~ is equal to 1. In this case let me find out this B times L, and B times L if I plug in these values here a small c is the speed of light 3 into 10 to the power 8 meters per second. Then it comes out to be 0.4 Gbps times meter, which means that you can send pulses at the rate of 0.4 GB per second or 100 MB per second only over 1 meter length of the fiber.

So, if you send pulses at this rate the information can be retrieved just within 1 meter, otherwise you will lose information. of course, you would not like to send information only for a meter. If I talk about kilometer then 1 kilometer length of the fiber can sustain a bit rate only up to 0.4 megabits per second. And if you remember that if I am trying to send a video then video requires a data rate of about 100 mbps. So, you cannot use it.

Now like we take another case, we are which is of cladded fiber where n_1 ~~1~~ is equal to 1.5 and delta the relative index difference is 1 percent then what is B times L. Then B times L comes out to be 20 Gbps over 1 meter or 1 mbps over 20 kilometers So, you can send data at the rate of 1 megabits per second over 20 kilometer length of the fiber or if you want to keep it within campus typically 2 kilometer length then the data rate can be 10 mbps.

So, if still it is not sufficient; 10 Mbps is nothing actually. So, we will have to take care of this. So, what I see here that is smaller is the index difference; if smaller is the index difference between the core and cladding higher would be the data. That is why I cannot use

unclad fiber, I cannot use unclad fiber and I need a cladding to have a telecom fiber. So, this clearly shows the importance of the cladding in the fiber.

Question is how can I minimize this intermodal dispersion. So, for that let me look at the origin of this intermodal dispersion. And the origin is there different rays have different transit times through particular length of the fiber. So, if somehow I can minimize the difference in transit times corresponding to different ray paths then I can minimize intermodal dispersion.

For that I can use a graded index fiber. What happens in a graded index fiber? For an axial rate the path is like this.

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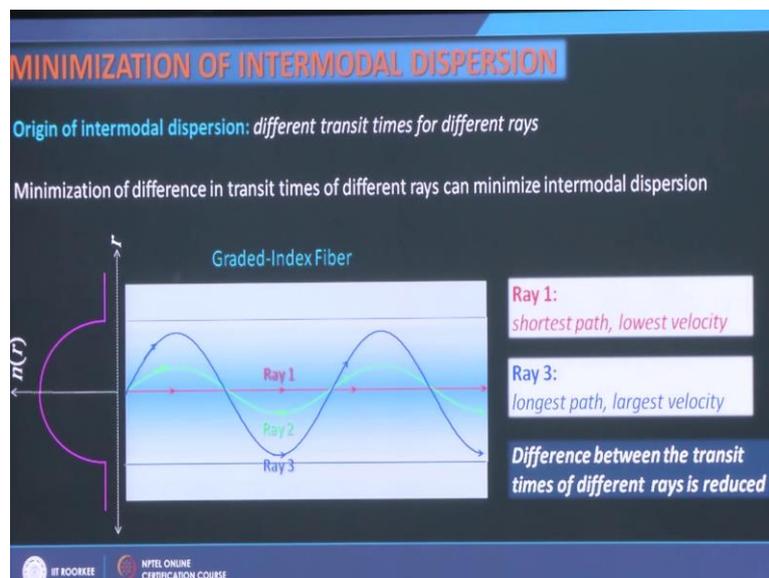
So, the path is shortest, but here I have highest refractive index. So, the velocity of light is the smallest. So, ray 1 has shortest path, but lowest velocity also, it is slowest. While this ray it has longer path, but it traverses in medium from here to here the refractive index changes and in fact refractive index decreases. So, the velocity increases. And for this ray this ray spends most of its time in a smaller refractive index region. So, it will have largest velocity.

So, in this way the difference between the transit times of different rays can be reduced. If I look at typical graded index fiber defined by power low profile then this is the refractive index variation in the core.

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And if I find out the value of delta tau the expression for delta tau for a fiber which corresponds to q is equal to 2 and called parabolic index fiber then delta tau comes out to be like this.

Now, if I take a fiber with n_2 ~~2~~ is equal to 1.45 and delta is equal to 1 percent then delta tau comes out to be about 0.25 nano seconds per kilometer. We call that for step index fiber; for a step index fiber delta tau was something like 50 nanoseconds per kilometer. So, I can bring it down from 50 nanoseconds per kilometer to 0.25 nano seconds per kilometer.

I can also find out what is the optimum profile, what is the optimal value of q for which my intermodal dispersion would be minimum. And it can be shown that this optimum profile of course, depends upon the value of n_1 ~~1~~ and n_2 ~~2~~ and it can be $q = 2\dfrac{n_2}{n_1} = 2\sqrt{1 - 2\Delta} \approx 2 - 2\Delta$

~~given y₂ 2 times n₂ 2 over n₁ 1 or approximated by 2 minus 2 delta~~.

So, now if I take n_1 ~~1~~ is equal to 1.46 and delta is equal to 1 percent then for step index fiber which corresponds to q is equal to infinity the intermodal dispersion is 50 nanoseconds per kilometer, for parabolic index fiber which corresponds to q is equal to 2 this dispersion is 0.25 nano seconds per kilometer. And the optimum profile across at around 1.98, q is equal to 1.98. And intermodal dispersion can be brought down to 0.0625 nano seconds per kilometer.

So, you can tremendously bring down the intermodal dispersion if you are using a graded index fiber. Of course, if you want to completely get rid of intermodal dispersion then you should use a single mode fiber. The question comes: why cannot we completely get rid of intermodal dispersion and you single mode fiber instead of doing all this. The answer is that this fiber has core diameter of about 50 micro meter, and single mode fiber has a core diameter of about 10 micrometer. So, if you go from multimode to single mode then it becomes very expensive, because your components will become more precise they have to be compactified. So, cost increases.

That is why if you do not want to have very high data rate you want to use it within a campus; in short distance communication, in local area networks, it is referable that you use a multimode fiber with optimum profile.
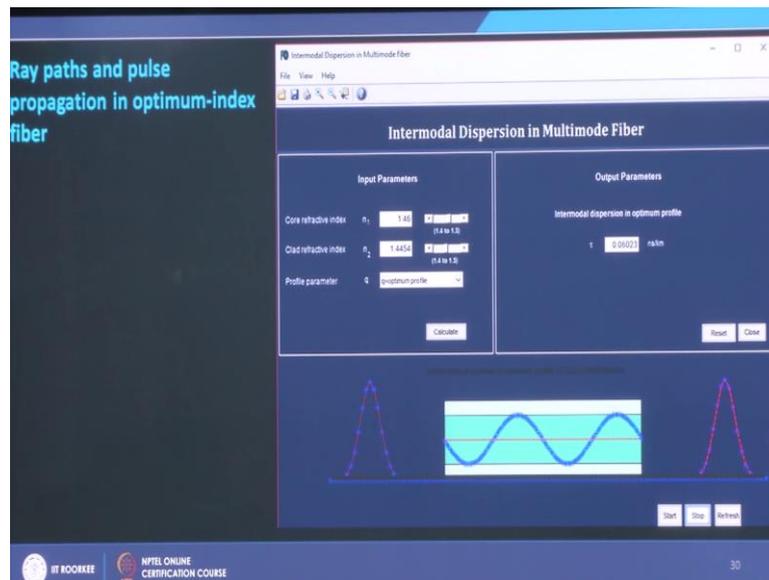
So, let us now look at ray path and pulse propagation in step index fiber. So, what we have done here? We have launched these pulses, I have launched this pulse and the red one corresponds to the energy coupled into the axial ray and blue one is the energy coupled into the ray which is launched at an angle. And let us see when the light propagates along these two paths what happens to this.

So, you can see that in the time when this axial ray reaches here this comes somewhere here. And you can see that the axial rate reaches faster while this ray at an angle reaches slower, it reaches later. So, by the time this blue one reaches here the red one will go much ahead. So, there would be broadening because of this.

If I now see the same thing for optimum index fiber then by the time the red one reaches here the blue one reaches here. So, they traverse the same length of the fiber; it reaches red one reaches here blue also reaches here. So, they go along and at the output end both of them will reach almost at the same time. So, the intermodal dispersion is minimized.

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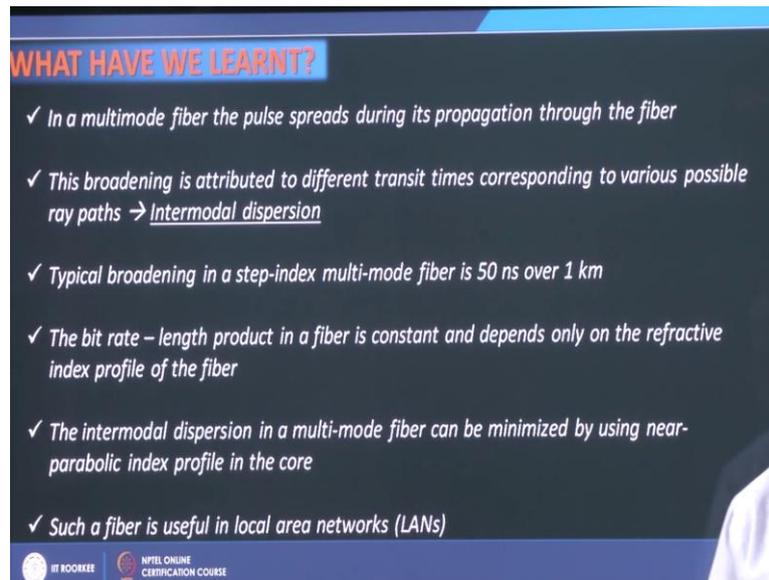
So, what we have learnt in this lecture. In a multimode fiber the pulse spreads during propagation through the fiber and the broadening of this pulse is attributed to different transit times corresponding to various possible ray paths. And this is known as intermodal dispersion. Typical broadening in my step index multimode fiber is 50 nanoseconds over 1 kilometer. The bit rate length product in a fiber is a constant and it depends only on the refractive index profile of the fiber. In a multimode fiber the intermodal dispersion can be minimized by using a near parabolic profile in the core. And such fibers are useful in local area networks.

Thank you.