Fiber Optics
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Lecture - 30
Optical Fiber Components and Devices- III

In this lecture we will study the periodic refractive index modulation in the fiber core, these periodic structures are called grating. So, we are going to study fiber gratings.

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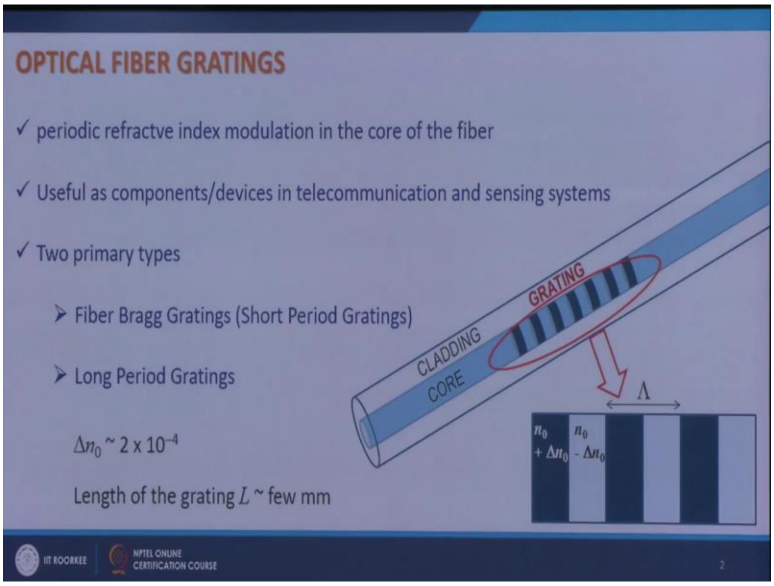

So, what we have is in a fiber if we create periodic refractive index modulation, then these kind of structures can be used as components or devices in telecommunication and sensing systems. Depending upon the periodicity of this modulation we can have short period gratings or long period gratings. Short period gratings are also referred to as fiber Bragg gratings, because the phenomenon is equivalent to Bragg reflection or Bragg diffraction in crystals. Typical refractive index modulation amplitude is $\Delta n_0 \approx 2 \times 10^{-4}$ and length of the grating is few millimeters it can be a couple of millimeters to 10 or 15 millimeter.

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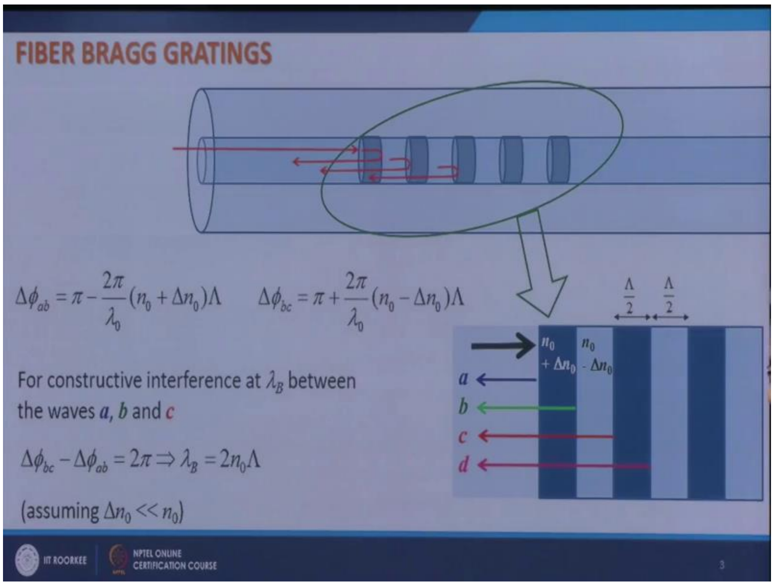

So, let us understand how does it work first we will consider fiber Bragg gratings. So, we have this refractive index modulation, we have low refractive index, high refractive index then low refractive index periodically, now when a light beam is incident here because of this index contrast a part of the beam gets reflected from this interface, part of the beam goes out and then it gets reflected from the rear interface. Similarly from all the other layers there would be reflections. What happens is that when all these reflections are add up are added up in phase then we can have a very strong reflection and this will happen at a particular wavelength.

So, in order to understand this let us zoom this, and consider different waves here. So, when this wave is incident here this is the incident wave, and we have the periodicity Λ. So, for convince let us divide it into $\Lambda/2$ for high refractive index region and $\Lambda/2$ low refractive index region. Let us say in high refractive index region the refractive index is $n_0 + \Delta n_0$ and in this region it is $n_0 - \Delta n_0$.

Now when this wave is incident here a wave gets reflected from this interface, let us say it is a part of the wave gets reflected from this interface let us say it is *b* then whatever portion of the wave whatever fraction of the intensity goes out here that gets reflected from other interfaces. If all these three are added up in phase if they have a phase shift of 2π or integral multiple of 2π, then we will have strong reflection.

Let us find out the condition for this strong reflection and wavelength at which this will happen. So, if I consider these 2 waves *a* and *b*, then the phase difference between *a* and *b* would be $\Delta\phi_{ab} = \pi - \frac{2\pi}{\lambda_0}(n_0 + \Delta n_0)\Lambda$, because this is the path difference from here to here; so $\Lambda/2$ and $\Lambda/2$ in the refractive index region of $n_0 + \Delta n_0$. So, this is the part difference and this pi phase shift occurs because there is a reflection from rarer to denser interphase media.

So, I have $\Delta\phi_{ab}$ the phase difference between these 2 waves, similarly I can have $\Delta\phi_{bc}$ the phase difference between these 2 waves, and if all the three waves have to have constructive interference and let us say this happens at wavelength λ_B, then if I take the total phase shift the phase shift between these and these there it should be integral multiple of 2π and for $m=1$ if I take the value of that integer as 1then it should be equal to 2π, and if I put the expressions from here the $\lambda_B = 2n_0\Lambda$.

Where I have assumed that $\Delta n_0 << n_0$, and which is the case because n_0 is typically 1.5 and $\Delta n_0 \approx 2\times10^{-4}$. So, I can neglect Δn_0 with respect to n_0.

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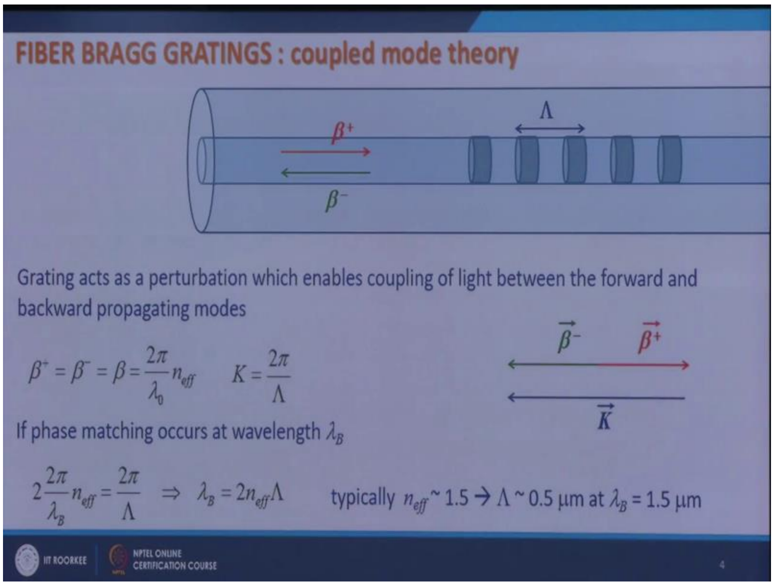

I can also understand this interaction as coupling between 2 modes. If I have a fiber let us say I have a single mode fiber. So, in a single mode fiber if I launch light from this side the mode is excited and mode propagates in this direction. If this is ideal fiber and there are no scattering centers, and if I have index matching liquid at this output end then there is no way that the light will travel back because there is no back reflection from here, there are no scattering centers. So, there is no mechanism by which the wave can travel back.

So, in general even if I do not put index matching liquid here and there are some scattering centers, then the light which comes back is very small. So, most of the light goes in forward propagating mode, this grating here acts as periodic refractive index perturbation and this perturbation can couple light from forward propagating mode to backward propagating mode. So, if I have forward propagating mode whose propagation constant is β^+, and backward propagating mode whose propagation constant β^- and of course, since it is single mode fiber. So, these modes are the same they their propagation constants magnitude are the same only their directions are different. So, this magnitude β^+ would be equal to magnitude β^- let us say it is equal to β, and if n_{eff} is the effective index then I can write it down as $\frac{2\pi}{\Lambda} n_{eff}$.

If the wave vector corresponding to this periodic refractive index modulation or the spatial frequency is $\vec{K} = \frac{2\pi}{\Lambda}$, then if this *K* is such that this length is equal to this then if a mode goes like this, and then *K* takes it here. So, the resultant would be β^-. So, in this way I can couple power from beta plus to beta minus, and the condition for this mathematically can be given by twice of this magnitude that is 2 times 2 pi over lambda *b* times *n* effective, it should be equal to the magnitude of this *K* it is 2 pi over capital lambda.

So, this gives me lambda *b* is equal to 2 times *n* effective times capital lambda. So, this particular wavelength will satisfy this condition, and all the power in this spectral component can be reflected back. Typically if I take a silica glass fiber n effective is of the order of 1.5 typical value 1.5, and if I consider the Bragg wavelength as 1.5 micrometer. So, this periodicity required is about 0.5 micrometer.

So, these are sub-micron gratings. So, this periodicity is really very small that is why these Bragg gratings are also known as short period gratings.

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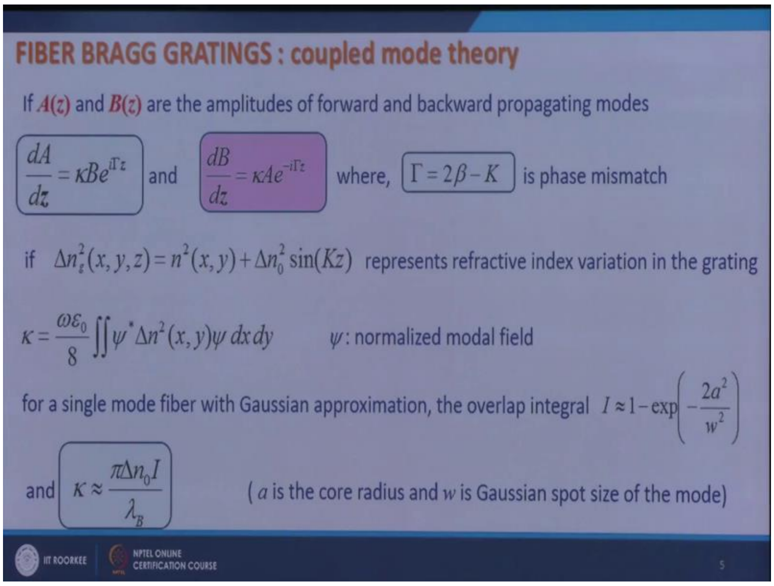

We can study the power evolution in the forward and backward propagating modes by using coupled mode theory and in coupled mode theory, the amplitude of the forward propagating mode varies with *z* according to *dA* over *dz* is equal kappa *B e* to the power *i* gamma *z* and that of backward propagating mode varies as *dB* over *dz* is equal kappa *A e* to the power minus i capital gamma *z*, where capital gamma is 2 beta minus *K* which is nothing, but phase mismatch if gamma is equal to 0 then there is phase matching between the forward propagating mode and backward propagating mode.

Intuitively, I can say that it is clear that for efficient coupling between the forward propagating mode and backward propagating modes, you should have phase matching conditions satisfied. So, gamma should be equal to 0. If I look at these 2 equations the equation of *A* contains *B* and equation of *B* contains *A*. So, these are coupled equations and this theory is known as coupled mode theory. Now this coupling between the 2 modes occurs due to periodic refractive index modulation, and in the grating region if I write down the refractive index then I can write it down as delta n g square g refers to grating *xyz* is equal to *n* square of *xy* plus delta n_0 square sign *kz*, where I have considered this periodic refractive index modulation is sinusoidal. So, this is sinusoidal refractive index modulation with special frequency *K* or wave vector *K*.

So, if I use this then I can show that this kappa which is known as coupling coefficient is given by omega epsilon naught by 8 integral over *x* and integral over *y* psi star del *n* square *x y* psi *dxdy* where psi is normalized modal field it is power normalized modal field. Now if I consider a single mode fiber and consider Gaussian approximation that is I approximate my mode, mode of the single mode fiber by a Gaussian of width w, then this integral can be written as I is equal to 1 minus exponential minus 2 a square over w square and then this kappa can be represented in terms of this integral as pi delta n_0 times I over lambda b, where a is the core radius and w is the Gaussian spot size of the mode.

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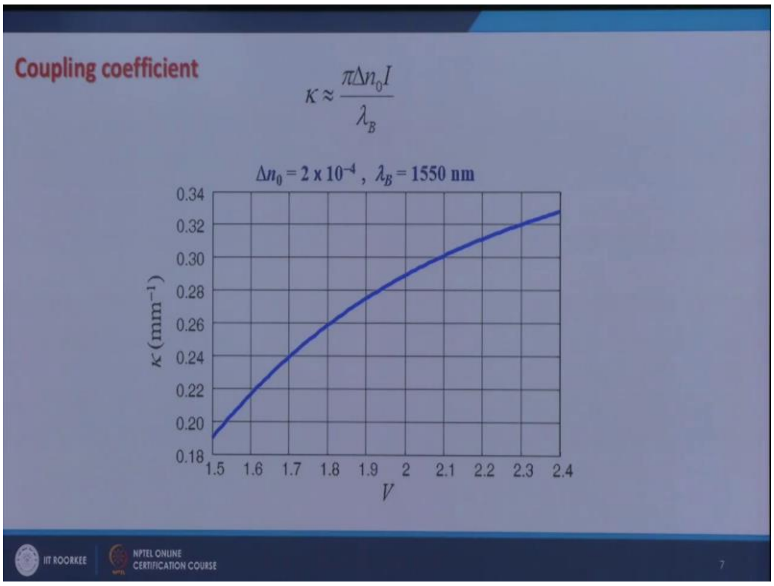

So, in order to find out kappa I need to find out overlap integral which is this, and you can see that it depends only on fiber parameters. What is the modes spot size Gaussian spot size and what is the core radius. It involves minus a square over w square, and I know that for a given fiber w over a varies with V like this, this is empirical formula given by marques. So, I can now find out how this overlap integral with vary with V it is very simple now to do this. So, I can see that as V increases, this overlap integral increases typically around between 2 and 2.4, the value of this integral is around 70 percent to 80 percent ok.

So, there is almost 78 percent to 80 percent overlap between the modes via this short period grating or fiber Bragg grating. So, this with the help of this I can find out what is the strength of interaction. So, the strength of interaction which is defined by coupling coefficient and given by pi delta n_0 I over lambda b, now I already know how I varies with V then I can find out how kappa varies with V. If I take typical value of delta n_0 as 2 into 10 to the power minus 4 and Bragg wavelength as 1550 nanometers then kappa varies with V like this.

Again I can see that that typical value of kappa for the values of *V* from 2 to 2.4 is about 0.3 millimeter inverse. So, it is around 0.3 millimeter inverse if you go to shorter values of *V* then it can be around 0.2.

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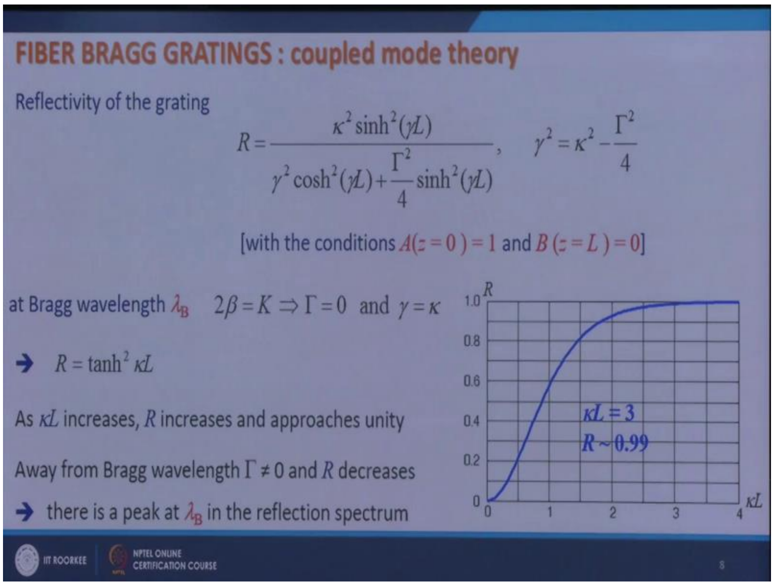

Now, with the help of this since I know how, *a* which is the amplitude of forward propagating mode, and *b* which is the amplitude of backward propagating mode. So, the equations describing the variations of *a* and *b* with *z,* I already know. So, if I solve these equations for given kappa for given fiber for giving grating, then I can find out how much would be the reflectivity. Because from these amplitudes I can find out the powers and if I find out the ratio between the reflected power at the input end of the fiber, and the incident power at the input end of the fiber, then I can know what is the reflectivity.

If I work that out then reflectivity comes out to be kappa square, sin hyperbolic square gamma *z*, divided by gamma square cosine hyperbolic square gamma *L* plus capital gamma square by 4 sin hyperbolic square gamma *L*, where gamma square small gamma square is kappa square minus capital gamma square by 4. And this expression has been obtained using the conditions that at the input end of the fiber that is at *z* is equal to 0 *A* is equal to 1 that is all the power is in forward propagating mode, and *B* at *z* s equal to l is 0. So, when light reaches to the end of the grating then at that value of *z* the power in reflected mode or the backward mode is 0, which means that all the power gets reflected at that wavelength lambda

B phase matching condition is satisfied. So, 2 beta is equal to kappa which means capital gamma is equal to 0, if capital gamma is equal to 0 then small gamma is equal to kappa.

If I use this in this expression, then the reflectivity of the grating comes out to be ten hyperbolic square kappa *L*. If I plot this reflectivity as a function of kappa L, then what I see that as I increase kappa *L* the reflectivity of the grating increases, and eventually for large values of kappa *L* it will eventually reach the value unity, there would be hundred percent reflection, but that will occur at the value of kappa *L* is equal to infinite. But what I can have is that at kappa *L* is equal to three if I work this out here, read this out then at kappa *L* is equal to 3 reflectivity is almost 99 percent.

So, this value of kappa *L* is sufficient for me, this happens when I am at Bragg wavelength. So, if I choose let us say kappa l is equal to 3 then at Bragg wavelength the reflectivity is 99 percent, now if I slightly move away from Bragg wavelength on either side then phase matching condition is not satisfied and the reflectivity will drop down. So, what I anticipate that at lambda is equal to lambda *b* there should be a peak in the reflection spectrum, and as I move away from this lambda *b* the reflectivity would fall down.

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Here, I show how the power in the transmitted and reflected modes vary as a function of *z*, what is the evolution of power in the forward and backward propagating modes for different values of kappa. For a given grating length *L* is equal to let us say 5 millimeters I can see that when kappa is the small value say 0.1 milliliter inverse, then ~~then~~ even at l is equal to 5. I have the transmitted power, transmitted power drops down only by 20 percent and it will almost saturate, and the reflected power is maximum it is around 20 percent, it will take very long length for this power to go beyond this.

However, if I increase kappa then I can increase this power, the transmitted power drops down and the power in reflection mode increases and when I have the value of kappa around 0.4, then I can see that the power in reflected mode is or backward propagating mode is more than 90 percent. And if I have the combination of kappa and *L* is equal to three then of course, the reflected power would be 99 percent here kappa times l is only 2.4 times 5.

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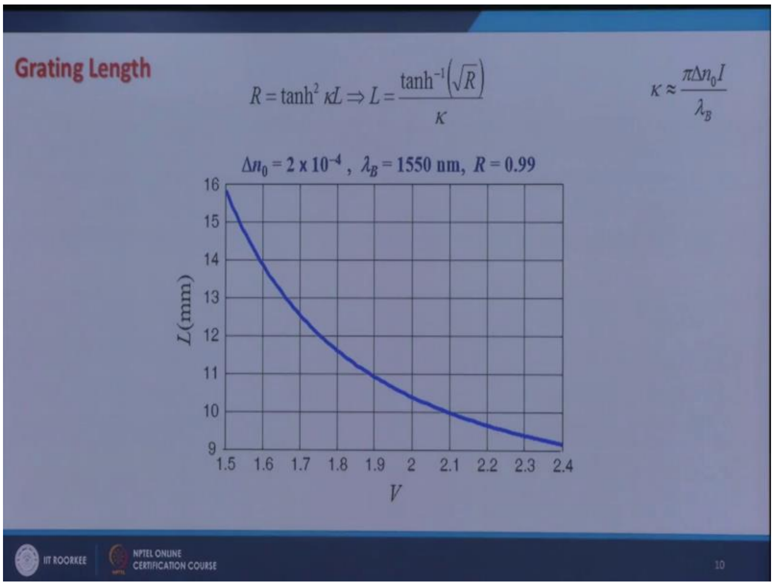

Let us find out what is the optimum grating length, if I want to have reflectivity let us say 99 percent. I can always find out the grating length required for a desired value of reflectivity, because I know that the reflectivity varies at tan hyperbolic square kappa *L*. So, *L* would be tan hyperbolic inverse square root of *R* over kappa. This plot is for delta *n* is 0 is equal to 2 into 10 to the power minus 4 lambda *B* is equal 1550 nanometers and for the value of *R* 0.99. So, in order to have 99 percent reflection.

So, if I vary my fiber parameter that is I vary the value of *V* then this is how the length of the grating has to be tuned to achieve 99 percent reflectivity. I can see that if the value of *V* is large, then the grating length required is the smaller, and for smaller values of *V*. I require much larger grating length. It is obvious because if I increase the value of *V* my mode is more and more confined in the core and the overlap integral increases. While if I go to shorter wavelength then the field spreads out from the core and the overlap integral mind it that this overlap integral since grating is only in the core region.

So, it is the power in the ~~in the~~ core region that matters, so, if power in the core region is small then the overlap integral would be smaller, and hence you will require much longer grating to reflect back 99 percent power.

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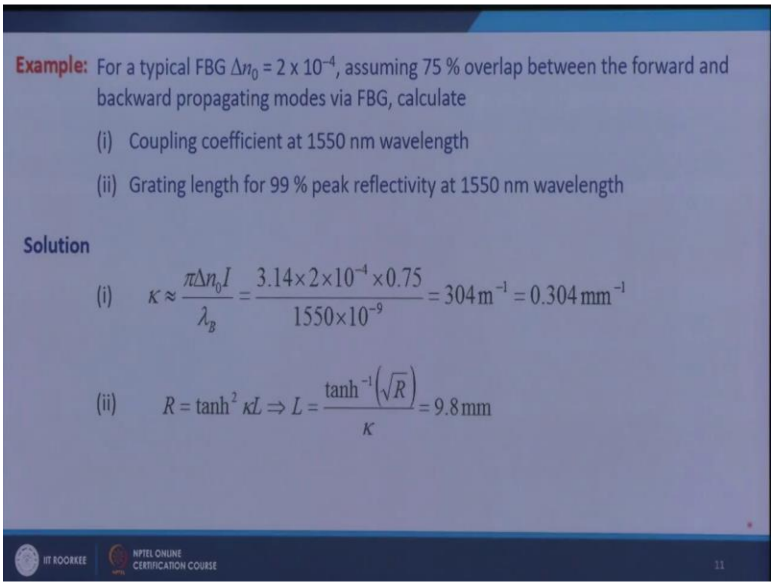

Let us work out few examples, if I have a typical fiber Bragg grading with delta n_0 is equal to 2 into 10 to the power minus 4, and I assume 75 percent overlap between the forward and backward propagating modes via fiber Bragg grating, then let us calculate coupling coefficient at 1550 nanometer wavelength and the required grating length for 99 percent peak reflectivity at 1550 nanometer wavelength. I know the coupling coefficient is given by pi delta n_0 times I over lambda B and delta n_0 is 2 into 10 to the power minus 4, I is 0.75 and lambda B is 1550 nanometer. So, this gives me the value of kappa about 0.3 millimeter inwards.

What is the required grating length for 99 percent reflect peak reflectivity. So, I know the reflectivity is given by tan hyperbolic square kappa L. So, l would be tan hyperbolic inverse square root of R over kappa ~~r~~ is 0.99 to have peak reflectivity 0.99 percent, and the value of kappa I have obtained at 1550 nanometer wavelength as 0.0 0.3040. So, if I put these values here, I will get the grating length about ten millimeters or 1 centimeter.

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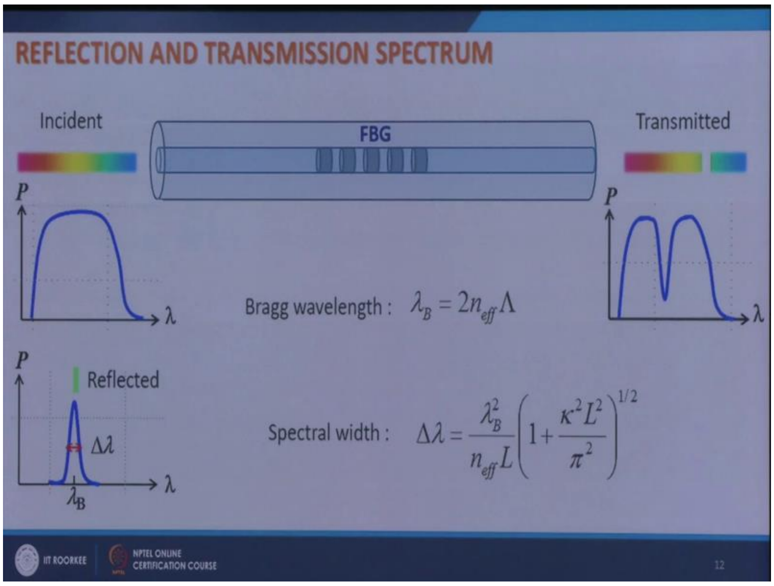

Let us now look at reflection and transmission spectrum of the grating. if I incident this spectrum which is broadband spectrum, then out of all these wavelengths only one wavelength which satisfies the condition 2 *n* effective capital lambda is equal to lambda *B*, that wavelength will be reflected and all the other wavelengths will pass through. So, in the transmitted spectrum this wavelength lambda *B* would be missing and in the reflection spectrum this wavelength will appear.

Then what is the spectral width of this? This spectral width of this reflection spectrum is given by delta lambda is equal to lambda *B* square divided by *n* effective times *L*, times one plus kappa square l square over pi square to the power half. So, if I know all these values *n* effective kappa *L* is given for given reflectivity, and then it will depend upon what is the value of *n* effective what is the value of *L*.

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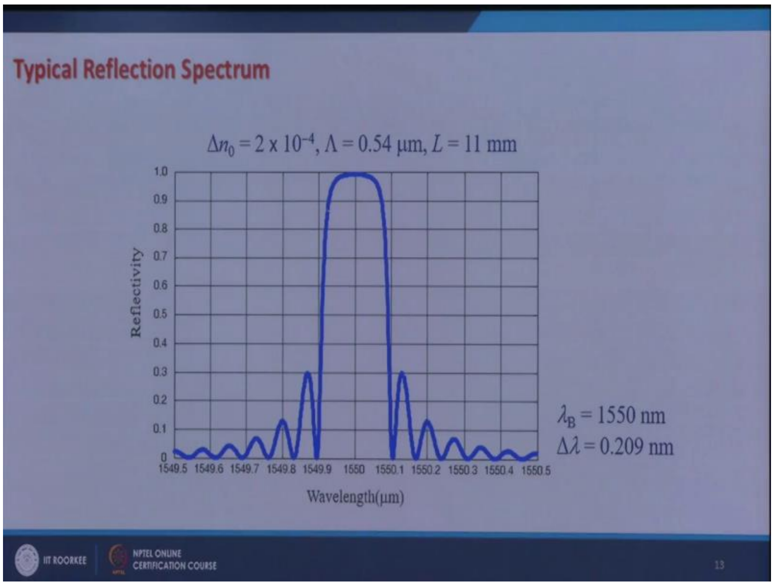

This is typical reflection spectrum for delta n_0 is equal to 2 into 10 to the power minus 4,grating period about 0.54 micro meter and L is equal to 11 millimeter for a typical single mode fiber.

So, this grating period with for a given for the given single mode fiber gives me lambda B is equal to 1550 nanometers. So, I get the peak reflectivity at 1550 nanometers and look at the value of delta lambda. Delta lambda is about 0.2 nanometers, delta lambda is the width at which the power falls down to 50 percent of its peak value. So, it is about this width. So, delta lambda here ~~in~~ in reflection spectrum of fiber Bragg grating is really very small, it is 0.2 nanometer or even less depending upon the length of the grating depending upon other grating parameters and fiber parameters.

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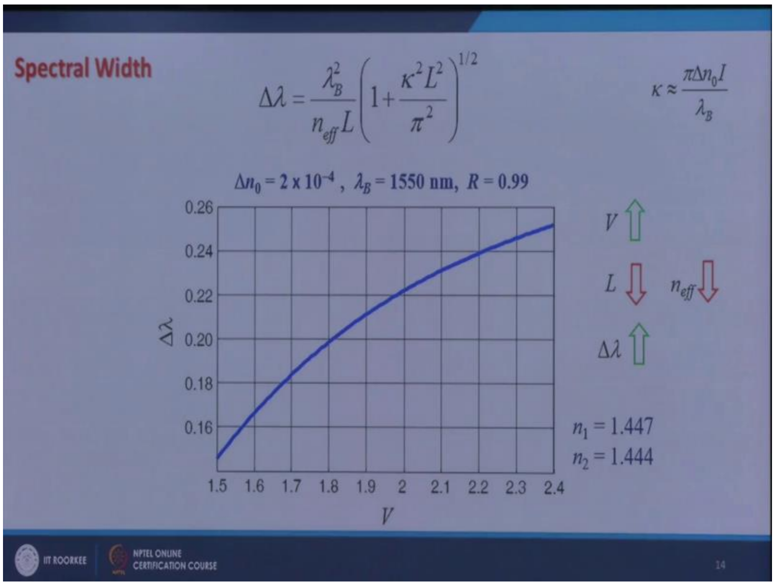

If I look at the spectral width for a typical fiber given by n_1 is equal to 1.447 and n_2 is equal to 1.444 then if I change the value of V, then delta lambda changes according to this. For reflectivity 99 percent I know for reflectivity 99 percent kappa L is equal to 3. So, all the values here in this bracket is fixed now it will depend only on the grating length, and n effective ~~n effective~~ means the fiber and the Bragg wavelength which also depends upon fiber parameters.

So, what I see that that as I vary V that is I change my fiber, then delta lambda varies between let us say 0.15 to 0.25 nanometers, it is in nanometers. Why do I obtain this kind of variation how can I understand this kind of variation? If I increase V then L decreases this I have already seen that if I increase V the length required for 99 percent reflection decreases, if I increase V. n effective also decreases. So, if L decreases an n effective decreases then delta lambda would increase. So, this is how, I can understand this curve.

In the next lecture I will work out some more examples, and look into some applications of fiber Bragg grating.

Thank you.