Fiber Optics
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Lecture - 21
Optical Fiber Waveguide- III

In the last lecture we had done the modal analysis of optical fiber and we had obtained the transcendental equation satisfied by the propagation constants of the modes, we had also seen the cut offs of various modes.

Now in this lecture we will extend the analysis and see how the modal fields look like.

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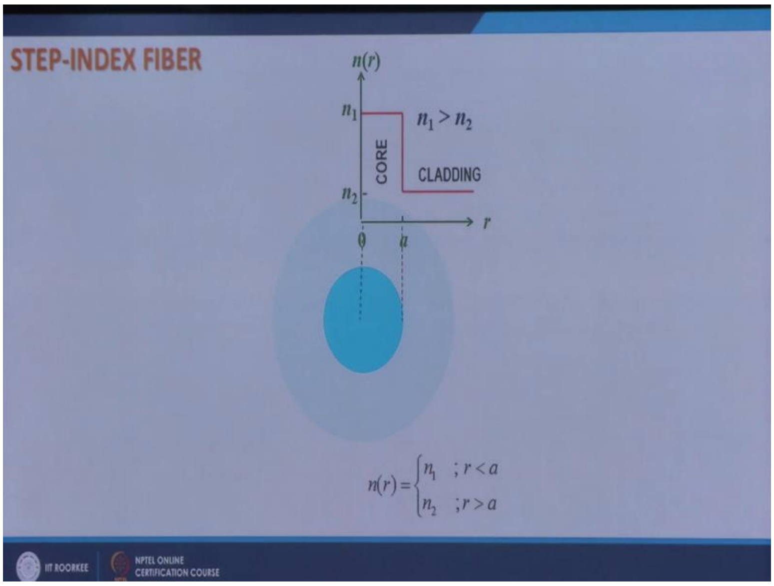

So, this is the step index fiber which we are analyzing, which has a high index core of refractive index n_1 of radius a and the cladding of refractive index n_2.

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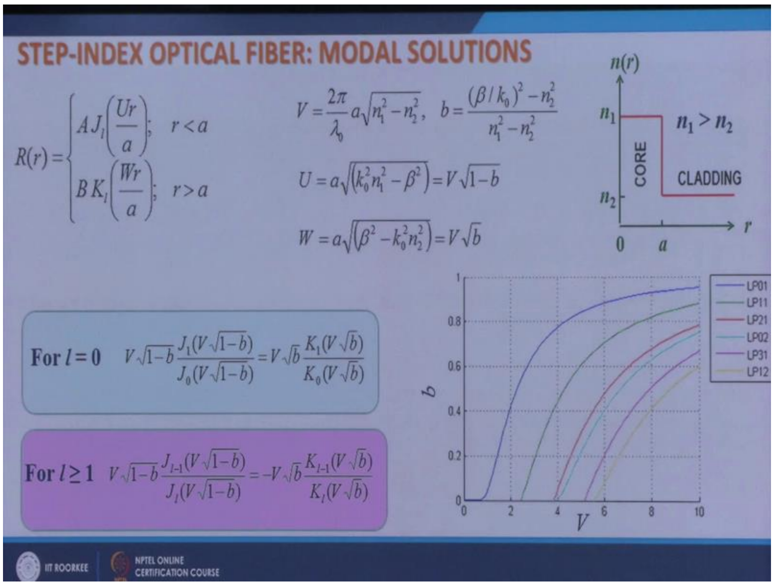

And we have done this that the radial part of the modal fields is given by $R(r)=\begin{cases} AJ_l\left(\frac{Ur}{a}\right); & r<a \\ BK_l\left(\frac{Wr}{a}\right); & r>a \end{cases}$. We had also defined the normalized frequency V normalized propagation constant $b=\frac{(\beta/k_0)^2-n_2^2}{n_1^2-n_2^2}$, $U=a\sqrt{k_0^2n_1^2-\beta^2}=V\sqrt{1-b}$, $W=a\sqrt{\beta^2-k_0^2n_2^2}$ are defined by these relations and you can also express them in terms of normalized parameters V and b. So, we had seen that the propagation constants of the linearly polarized modes of this fiber satisfy these transcendental equations or the Eigen value equations. So, after solving these equations for a given value of V, I can find out the propagation constants b of various modes of the fiber, and if I plot them as a function of V, so they look like this. So, this we had done.

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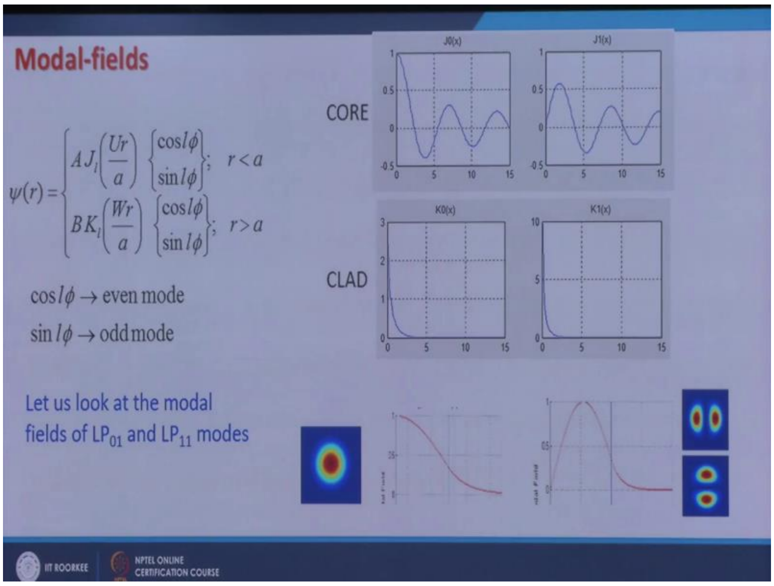

Now, let us look at how the modal fields look like. So, the total modal field would be given by the radial part and the angular part or azimuthal part. In phi direction I can have solutions $\cos l\phi$ and $\sin l\phi$ and I label these 2 solutions by 2 different names I call solution as $\cos\alpha$ even mode and $\sin\alpha$ solution as odd mode.

$$R(r)=\begin{cases} AJ_l\left(\frac{Ur}{a}\right)\begin{Bmatrix}\cos l\phi\\ \sin l\phi\end{Bmatrix}; & r<a\\ BK_l\left(\frac{Wr}{a}\right)\begin{Bmatrix}\cos l\phi\\ \sin l\phi\end{Bmatrix}; & r>a\end{cases}$$

$K_1\left(\frac{Ur}{a}\right)$ Now, let us look at the modal fields of LP_{01} and LP_{11} modes of this fiber. So, since we are looking at LP_{01} and LP_{11}, which means here $l = 0$ here $l = 1$. So, in the core I will have solution for LP_{01} mode as $J_0\left(\frac{Ur}{a}\right)$, and for LP_{11} mode $J_l\left(\frac{Ur}{a}\right)$. So, let me plot $J_0(x)$ and $J_1(x)$, they look like this. In the cladding I will have the solutions $K_0\left(\frac{Ur}{a}\right)$ and $K_0\left(\frac{Ur}{a}\right)$. So, let me plot how $K_0(x)$ and $K_1(x)$ look like. So, when I combine this k. So, in the core I will have this solution and in the cladding I will have this solution, and since these are the first modes of $l = 0$ and $l = 1$ respectively. So, I will not have any 0 in this and I will not have any 0 in this except a 0 at $r = 0$.

So, for LP_{01} mode in the core the field will go like this, and in the cladding this function will take over. For LP_{11} mode the field in the core would be given by J_1. So, it would go like this without any 0 except a_0 at r = 0 and then this K_1 function will take over. So, this is how the radial part of the modal fields would look like, what about the angular part? Angular part is simple for l = 0 you have $\cos\alpha = 1$ and there is no contribution from $\sin l\phi$ because it would be 0 everywhere. So, I will have only this kind of solution. So, you take this in radial part and then you go in angular part rotate it in phi direction and you will get this kind of variation.

However, in for LP_{11} mode, I can have $\cos\phi$ solution and $\sin\phi$ solution. So, if I take $\cos\phi$ solution that is at $\phi = 0$. I have a maximum, and then $\phi = \pi/2$. I will have 0 and so on. So, if I rotate it like this in phi direction, then I will get this kind of variation this kind of density plot or intensity pattern. If I take $\sin\phi$ solution, then $\sin\phi$ would be 0 at $\phi = 0$ and then it would be maximum at $\phi = \pi/2$ and then if I now rotate it. So, it would be 0 at $\phi = 0$ it would be maximum at $\phi = \pi/2$. So, it will give me this kind of solution. So, l = 0 mode will only be of this kind that is that is I cannot have 2-fold degeneracy for l = 0 mode as I can have for l = 1 mode here or l is equal to nonzero mode l nonzero mode. I can look at the 3 D plot of these.

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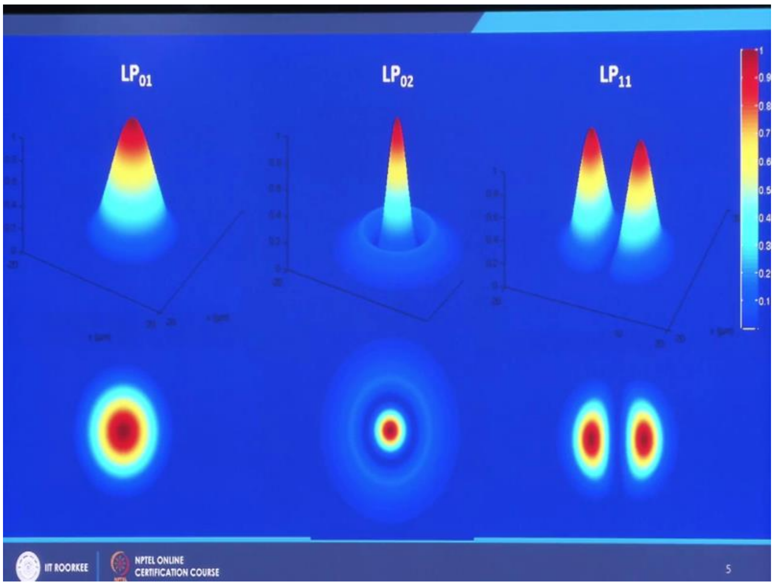

So, if I look at the various modes. So, this is how the LP_{01} mode would look like and if you view it from the top it would look like this. LP_{02} mode LP_{02} mode will admit 10 because it is the second mode in l = 0 series. So, it will admit 10 in the core. So, it will go down this is the intensity plot this is the intensity plot. So, if you look at look at it from the top it would look like this and there would be a 0 at r is equal to something here in the core itself. If you look at

LP_{11} mode it would look like this, it would not have any 0 in r direction except a 0 at $r = 0$ and these are the this is the plot of even mode even LP_{11} mode.

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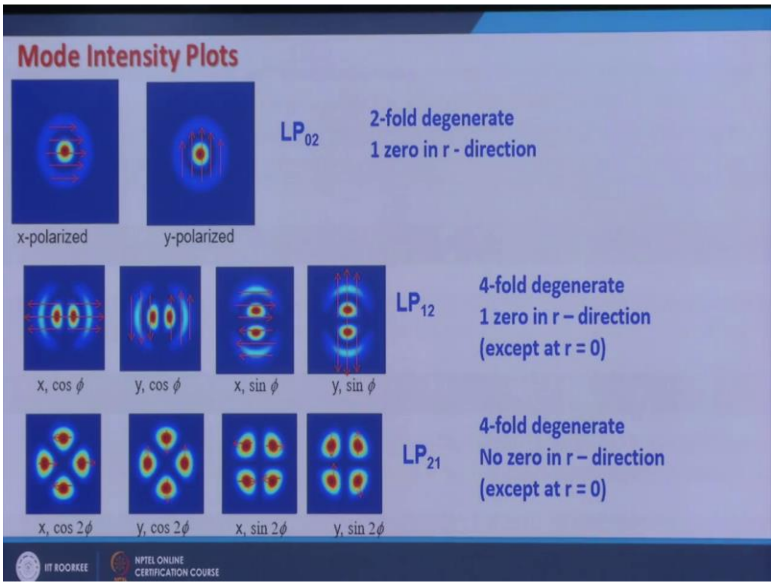

So, if I again look at these plots, now I have LP mode say or linearly polarized modes it means that I can have 2 independent or orthogonal polarization states, I can label them as *x*-polarized and *y*-polarized because that is the direction of propagation. So, I can label them as *x*-polarized and *y*-polarized. So, if I look at LP_{01} mode, then it can be *x*-polarized or *y*-polarized. So, this LP_{01} mode is twofold degenerate and it does not have any 0 in r direction. If you look at LP_{11} mode then I can have $\cos\phi$ cosine phi solution, and in cosine phi solution itself I can have 2 polarizations *y*-polarized and *x*-polarized and I can have sin phi solution. So, there also I can have *x*-polarized and *y*-polarized. So, it would be 2 it would be fourfold degenerate, and again there would be no 0 in *r*-direction except a_0 at $r = 0$. If I look at LP_{02} mode, then LP_{02} mode would again be 2-fold degenerate you will have *x*-polarized and *y* -polarized.

If you look at the second mode in l is equal to one series that is LP_{12} mode, then it will have one 0 in *r*-direction, except a 0 at $r = 0$. So, the modal fields would look like this and again this would again be fourfold degenerate. This is LP_{21} mode; now in LP_{21} mode because $l = 2$. So, $l = 2$ though. So, the 5 solutions are $\cos 2\phi$ and $\sin 2\phi$ and they will have 4 zeros in ϕ directions that you can see here and in *r*-direction there would not be any 0 at except at $r = 0$. So, so this is how the modal fields would look like for even and odd modes.

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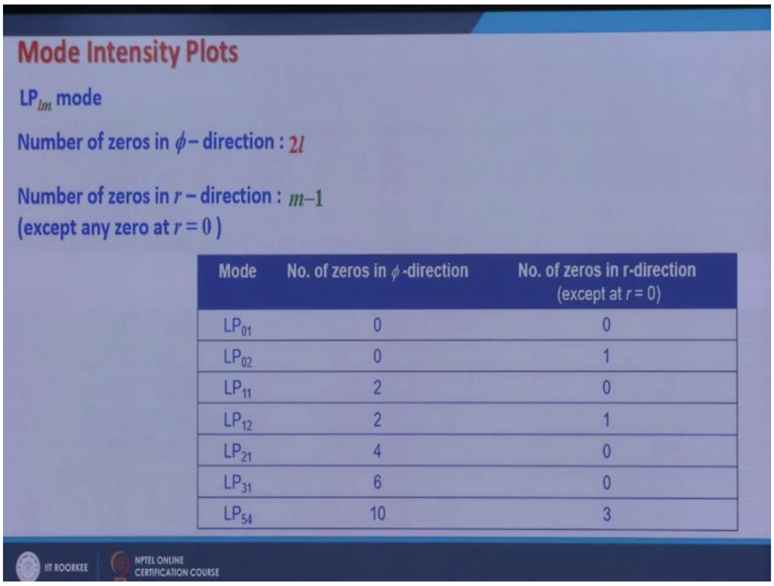

So, in general what I get for LP_{lm} mode, the number of zeros in phi direction would be 2 l and the number of zeros in r-direction would be m - 1 except any 0 at r = 0. So, if I have various mode I can immediately find out the number of zeros in r and ϕ-direction for example, in LP_{21} mode there would be 4 zeros in ϕ direction and no zero in r-direction. I always exclude a_0 at r = 0, for LP_{31} mode, I will have 6 zeros in ϕ-direction and no zero in r-direction. For LP_{54} mode I will have 10 zeros in ϕ-direction, and 3 zeros 4 – 1= 3 zeros in r-direction. So, if I know any mode if I am given any mode then I can immediately find out the number of zeros in phi and r-direction, and I can also plot the intensity patterns of those modes.

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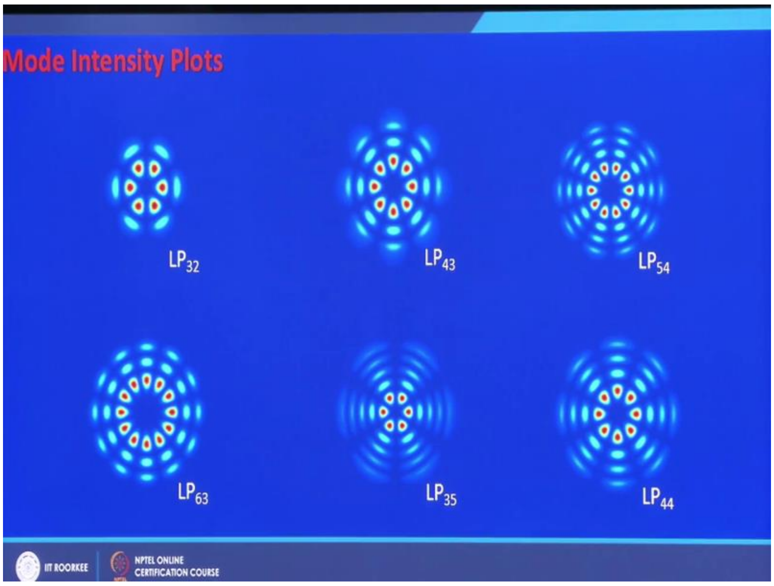

For example, these are the intensity plots of various higher order modes, this is LP_{32} mode LP_{32} mode will have 6 zeros in phi direction. So, I have 6 zeros 1, 2, 3, 4, 5, 6 zeroes in ϕ -direction, and 2 minus 1 that is one 0 in r-direction except a 0 at $r = 0$ P_{43} mode will have 8 zeros in ϕ-direction and 2 zeros in r direction. LP_{54} mode 10 zeros in ϕ-direction and 3 zeros in r direction 1 2 3. LP_{63} mode 12 zeroes in ϕ-direction 2 zeroes, 3 - 1 = 2 zeroes in r -direction and so on. So, if I am given any mode then I can immediately draw the intensity pattern corresponding to that mode.

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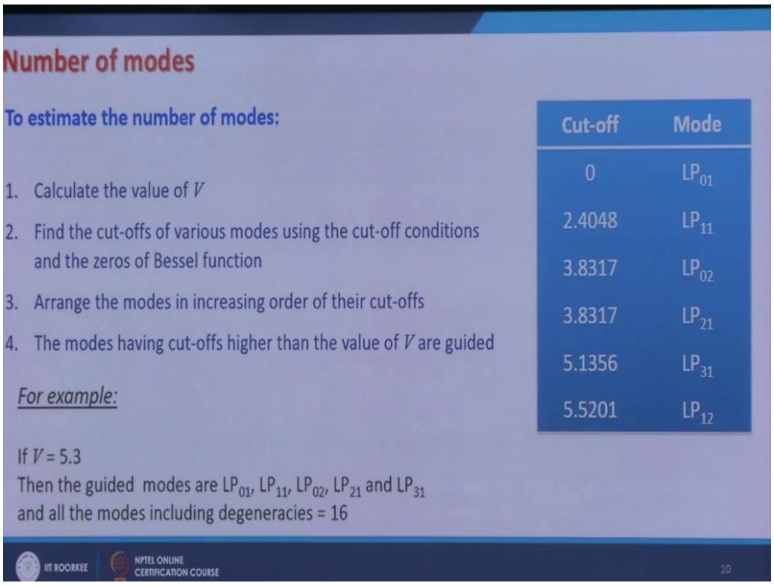

Now, the question is how do I estimate the number of modes here, in planar wave guide it was easy I just calculate the value of *V* divide that value of *V* by $\pi/2$.

And find out the closest, but greater integer and that will give me the number of modes; here the cut offs are not evenly spaced and there are several series corresponding to *l*. So, where do they fit I do not know immediately. So, what is the procedure of estimating the number of modes here? So, the first step is the same you first calculate the value of *V*. So, if you are given a fiber and the wave length, first you calculate the value of *V*, then you find out the cut offs of various modes using the cut off conditions and the zeros of Bessel functions. So, if you have the zeros of Bessel functions and cut off conditions for various modes you know for $l = 0$ mode, the cut off condition for $l = 0$ cut off conditions which are in terms of the zeros of Bessel functions.

So, you find the cut offs and then arrange the modes in the increasing order of their cut offs. So, for example, the fundamental mode LP_{01} mode has 0 cutoff, LP_{11} mode has 2.4048, 3.8317 is for LP_{02} and LP_{21}, 5.1356 is LP_{31}, 5.5201 is LP_{12} and so on. So, you arrange them arrange the modes in increasing order of their cut offs, and then locate the value of *V* here and find out how many modes are above that value of *V* and those would be the number of modes supported by the fiber. Let me work out an example if for a given fiber and given wavelength I calculate the value of *V* and it comes out to be 5.3 then I locate this 5.3 in this table which is here and I count the modes above this, 1, 2, 3, 4, 5. So, these modes LP_{01}, 11, 02, 21 and 31 these modes are guided.

I can also find out the total number of modes by including their degeneracies, I find that there are 2 modes which have $l = 0$. So, there will be only 2 fold degeneracy. So, they comprise of 4 modes, and then I have 1, 2, 3 modes which are *l* is equal to non L as *L* is equal to not 0. So, so they will have 4-fold degeneracy and they will comprise of 12 modes. So, I will have 12 plus 4. 16 total modes including the degeneracies. So, this is how I can calculate the number of modes.

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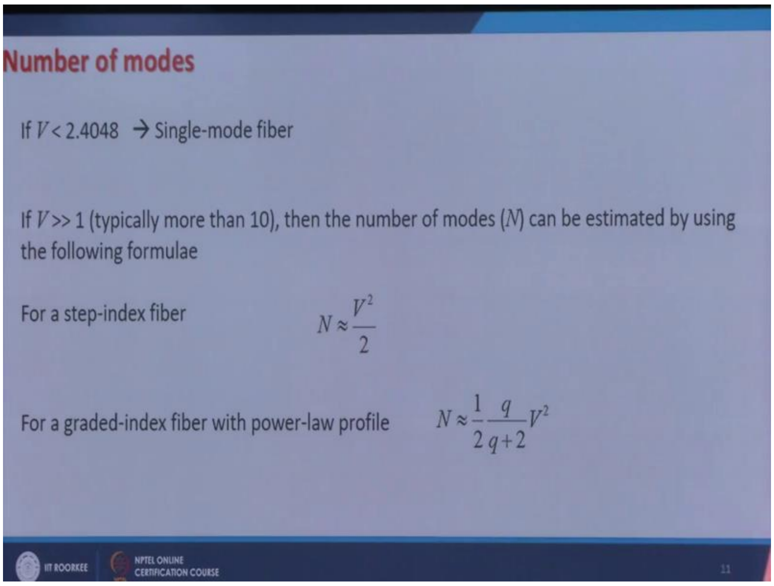

Of course, if $V < 2.4048$, then there would only be one mode supported of course, there would be it is twofold degenerate. So, rigorously it supports 2 modes corresponding 2 polarizations, but as a convention I call it single mode fiber because for a given polarization it has only mode. So, for $V < 2.4048$ I have a single mode fiber and this is how the cut off condition or for single mode operation is coming; because 2.4048 is the cutoff of first higher order mode which is the LP_{11} mode. If V is much much larger than one typically more than 10 or around 10, then the number of modes can be estimated using the approximate formula which are given as for a step index fiber, the approximate number of modes are $V^2/2$ and for a graded index fiber which power law profile which is characterized by profile parameter q, the number of modes can be estimated by half of $qV^2/(q + 2)$. So, if it is parabolic index fiber then $q = 2$ then the number of modes would be $V^2/4$.

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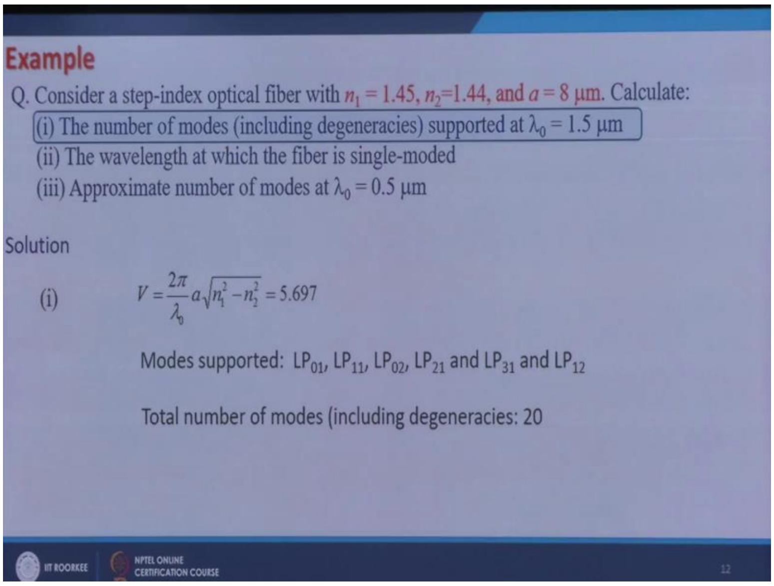

Let me work out some examples, let me consider a step index optical fiber with refractive index n_1 = 1.45, cladding refractive index n_2 = 1.44 and core radius 8 μm, and I want to find out the total number of modes including degeneracies supported by the fiber at λ_0 = 1.5 μm. So, the procedure is very simple you have to first find out the value of V, if you find out the value of V for these parameters then it comes out to be 5.697.

If you locate this value of V in the table which you have created by arranging the modes in increasing order of their cut offs, then you find that these modes are supported LP_{01}, 11, 02, 21, 31 and 12. Now if you include the degeneracies of all these modes and calculate the total number of modes then they come out to be 20. So, with this value of V the number of modes would be around 20, you can also see that if you calculate using $V^2/2$, $V^2/2$would roughly be because it is 6 it is close to 6. So, 6 squares 36, 36 divided by 2 is approximately 18. So, little more than 18 and here you are getting 20. So, they are close.

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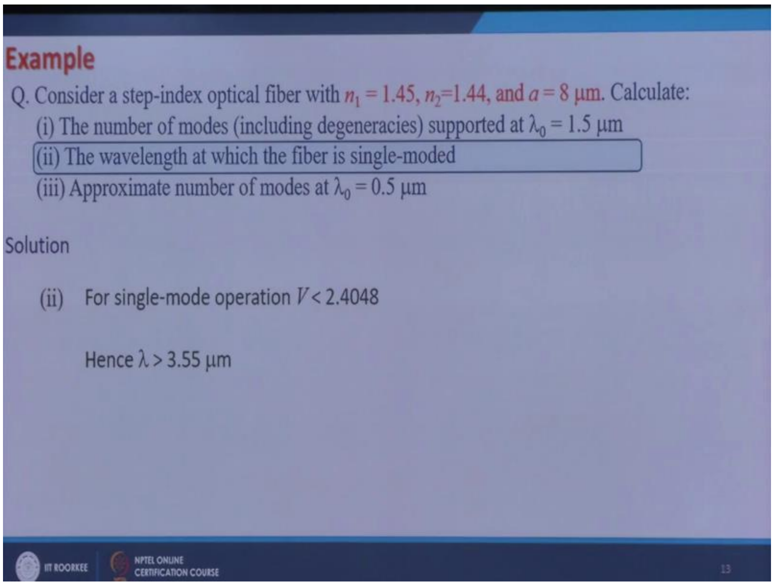

Second is the wave length at which the fiber is single moded, if I want to find out the wave length at which the fiber is single moded or the wavelength range in which the fiber is single moded then I know the single mode condition for a fiber is V should be less than 2.4048 and I have the expression for V.

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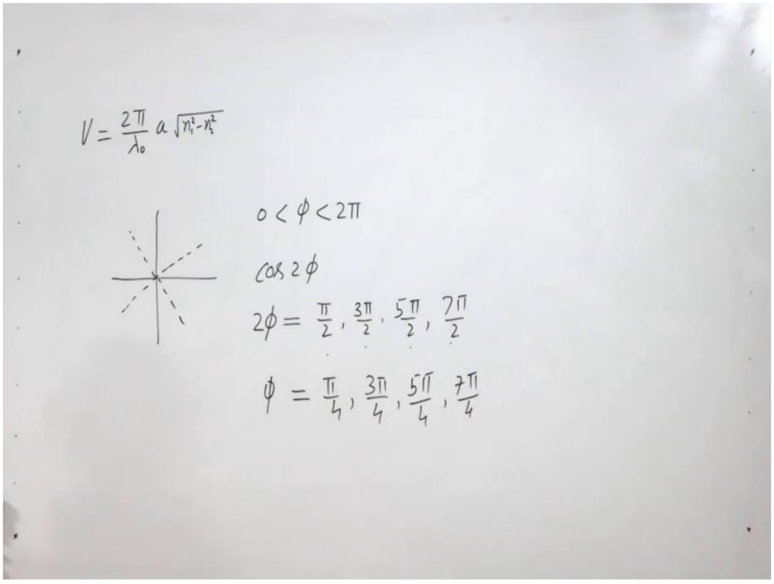

Which goes as $V = \frac{2\pi}{\lambda_0} a\sqrt{n_1^2 - n_2^2}$. So, if I find lambda naught from here, then I will see that for lambda naught greater than 3.55 μm, V would be less than this and therefore, in this range of wave length the fiber would be single moded.

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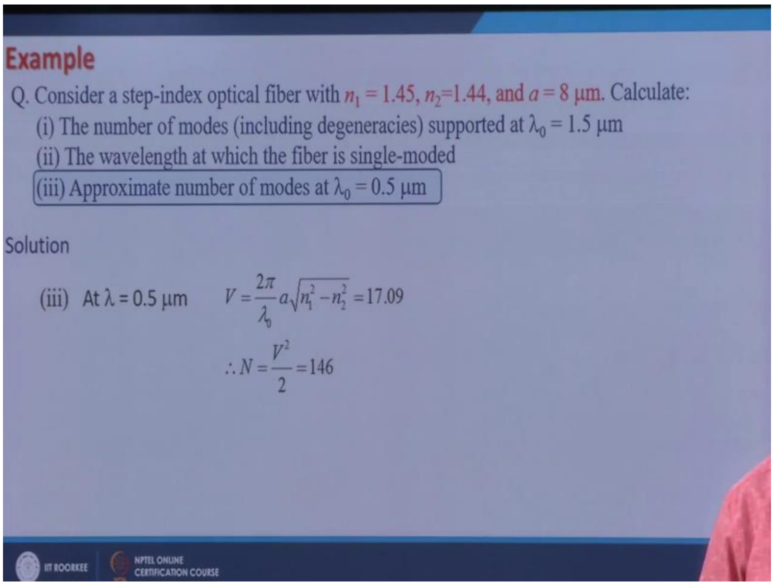

Third part is approximate number of modes at lambda naught is equal to 0.5 μm. So, I again calculate the value of V at 0.5 μm, and it comes out to be about 17 which is much larger than one, then I can find out the approximate number of modes by $V^2/2$ and it comes out to be about 146. So, this fiber will support 146 modes.

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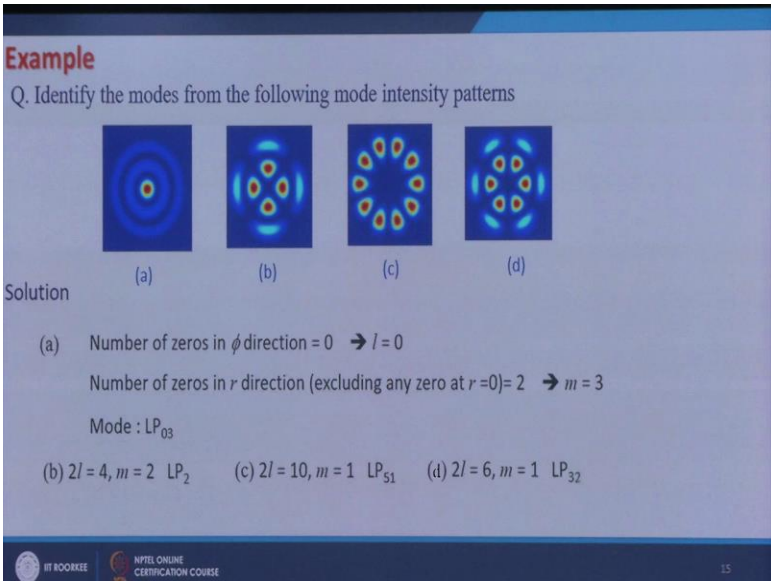

Now, let me work out some examples on identification of modes from they are given intensity patterns, how do I identify various modes from their given intensity patterns. So, let us look at this, this one is easy. If I go in phi direction, I take any value of r and go in phi direction I do not encounter any 0. So, the number of zeros in phi direction is 0 which means 12 is 0, which means l = 0. Second thing I do is I count the number of zeros in r direction. So, I take any value of phi and for that value of phi I move in r direction and an end I encounter 1 and 2 zeros, the number of zeros in r direction is 2. So, m = 3 because m - 1 = 2.

So, this mode is LP_{03} mode, similarly for this mode the number of zeros in phi direction the number of zeros in ϕ-direction is 4. So, 2 l is equal to 4l - 2 and number of zeros in r direction you exclude 0 here. So, it is only one. So, m minus 1 is equal to 1 which means m is equal to 2. So, this is LP_{22} mode. Similarly, here it is LP_{51} mode and this is LP_{32} mode. So, in this way I can identify any mode.

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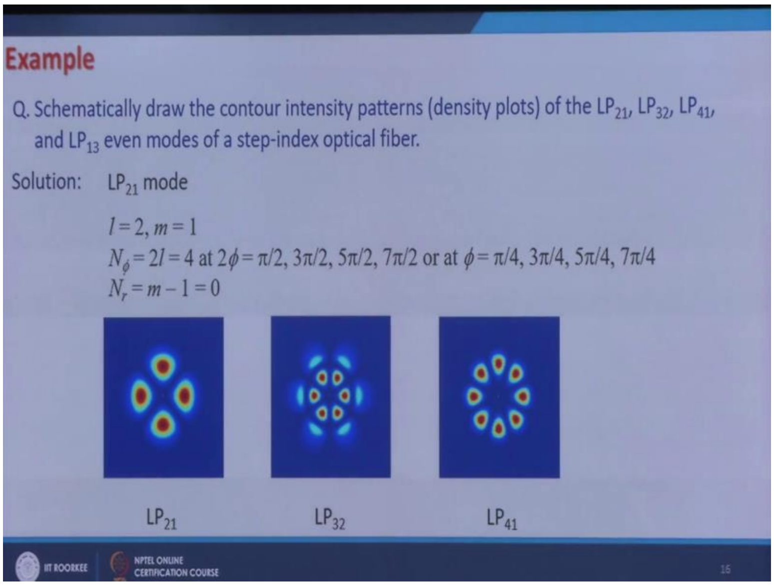

Next example is schematically draw the contour intensity plots or density plots of these modes, and I just want to draw the patterns of even mode. So, cosine α solution. So, let me do it for LP_{21} mode. So, for LP_{21} mode what I have l = 0. So, number of zeros in μm direction are 2l. So, they are 4. So, now, where these 4 zeros are located? I know that if you go in phi direction then ϕ ranges from 0 to 2π, now you have solution which has $\cos 2\pi$. So, you now find out the zeros of this cosine 2π they will occur at $\pi/2$ then $3\pi/2$ and where do you stop you stop because phi goals phi goes up to 2π then 2ϕ will go up to 4π. So, until you cross or reach 4π you do not stop. So, $3\pi/2$, $5\pi/2$ and then $7\pi/2$, after that it would be $9\pi/2$ which is more than 2π. So, you stop here. So, here these 4 zeros would be located which means phi is equal to $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$.

So, here you have 0 this is $\pi / 4$, this is $2\pi / 4$, this is $3\pi / 4$, $4\pi / 4$, $5\pi / 4$, $6\pi / 4$ and this is $7\pi / 4$. So, here you have 0 and in between you will have the intensity and m = 1. So, there would not be any 0 in r-direction except at r = 0. So, you will have intensity something like this. So, this is how you will plot the intensity. So, this is how it would look like similarly for LP_{32} mode you will have 6 zeros in ϕ-direction and you can in the same way you can calculate the values of phi where the zeros will occur and since m = 2. So, there would be 1 0 in r-direction, in the same way you can plot for LP_{41} mode and LP_{13} mode.

So, today we have learned about the mode modal fields, in the next class we would look in to what fraction of power of these modes is confined in the core and of and about various parameters of a single mode fiber.

Thank you.