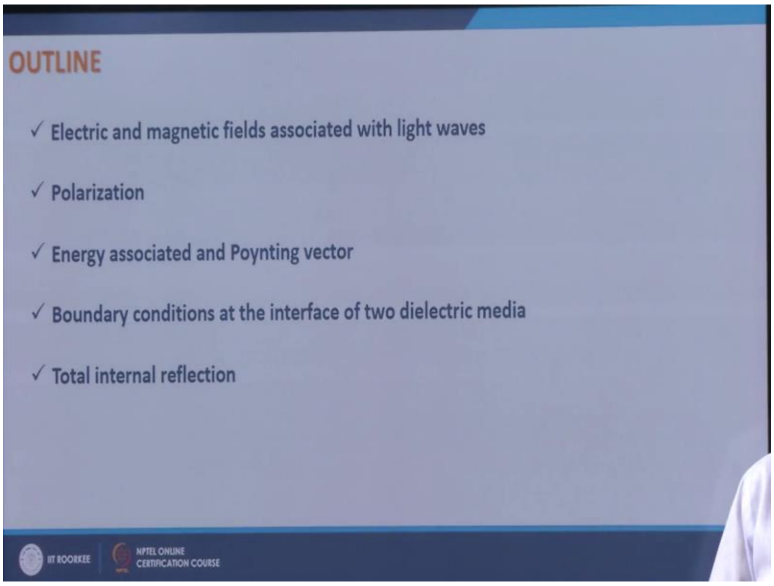
Fiber Optics
Dr. Vipul Rastogi
Department of Physics
Indian Institute of Technology, Roorkee

Lecture – 10
EM Waves in Dielectrics

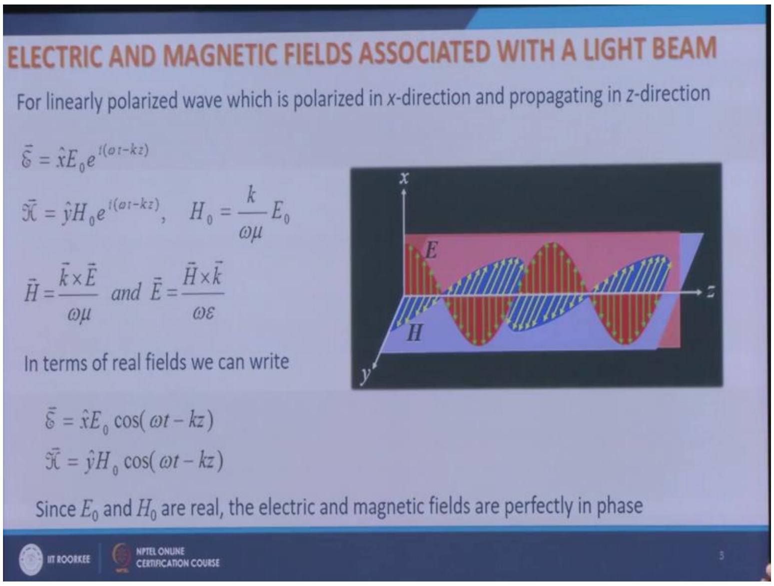
In the last lecture using Maxwell's equations we formed a wave equation and I had seen how these EM waves propagate in an infinite extended dielectric medium. In this lecture we will further looked into the propagation of EM waves in dielectrics.

(Refer Slide Time: 00:50)



The outline of the lecture is we would again look into electric and magnetic fields associated with light waves, what is polarization, what is the energy associated with these waves and we will define a vector called pointing vector for this, then if the EM wave encounters boundary if they go from one dielectric to another dielectric, then what are the conditions they should meet at the boundary of two dielectric media. And then we will look into some entry cases in total internal reflection which will become very handy when we will do wave analysis or EM wave analysis of optical waveguides.

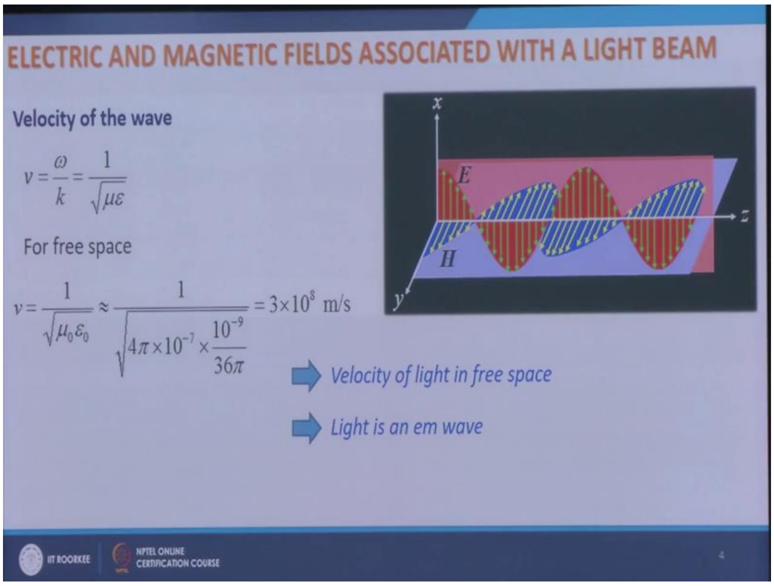
(Refer Slide Time: 01:51)



So, what are the electric and magnetic fields associated with the light beam? Well we have seen that for a linearly polarized wave which is polarized along *x* and propagating along *z*, I can write the associated electric field as $\vec{\mathcal{E}} = \hat{x}E_0 e^{i(\omega t - kz)}$, and $\vec{\mathcal{H}} = \hat{y}H_0 e^{i(\omega t - kz)}$, where the amplitude of *H* can be related to the amplitude of *E* by this expression. If I have electric field then I can find out magnetic field from this expression, and if I have magnetic field I can find out electric field by this expression. So, if I know one other can be found out because they are interrelated.

When we work out some real quantities then instead of using phaser notation, it is useful to use the real part of these and then I can write $\vec{\mathcal{E}} = \hat{x}E_0 \cos(\omega t - kz)$, and $\vec{\mathcal{H}} = \hat{y}H_0 \cos(\omega t - kz)$. When we plot them they look like this since *E* is along *x*, then *E* goes like this the red curve and the direction of vibration of electric field vector is given by these green arrows the electric field is vibrating along *x*, and correspondingly the magnetic field is vibrating along *y*. *k* and it is represented by this blue curve. What I can notice here is that this E_0 and H_0 they are real and since they are real, because it is dielectric medium lossless medium then since they are real then the electric and magnetic fields are perfectly in phase. So, when *E* has a maximum *H* also has a maximum, and *E* goes down to 0 at the same instant *H* also goes down to 0. So, they are perfectly in phase.

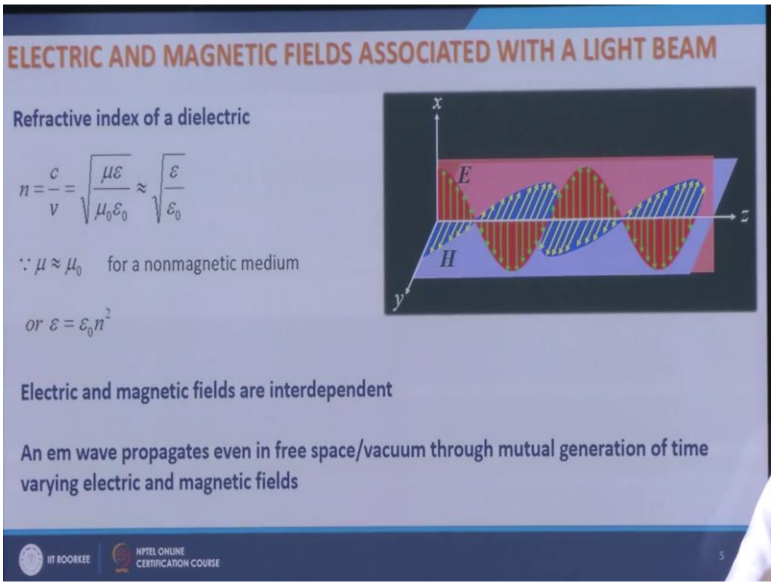
(Refer Slide Time: 04:30)



What is the velocity of this wave? In the previous lecture we had seen that ω / k is nothing, but $1/\sqrt{\mu\varepsilon}$ and this is the velocity with which the surfaces of constant phase are moving. So, these are there. So, this v is nothing, but the velocity of the wave and let me find this out for free space, for free space $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$ and I can approximate $\mu_0 = 4\pi \times 10^{-7}$ *and* $\varepsilon_0 = 10^{-9} / 36\pi$, and if I work this out it comes out to be $\mu_0 = 3\times10^8\ m/s$ which is nothing, but the velocity of light and that is how it has been established that light is an electromagnetic wave.

What is the refractive index of dielectric? When these waves move in if you go if these wave move in free space and then the wave go in dielectric medium, we find that they have different velocities. So, this difference in velocities can be associated with what is known as a certain property of the medium which is known as refractive index of the dielectric.

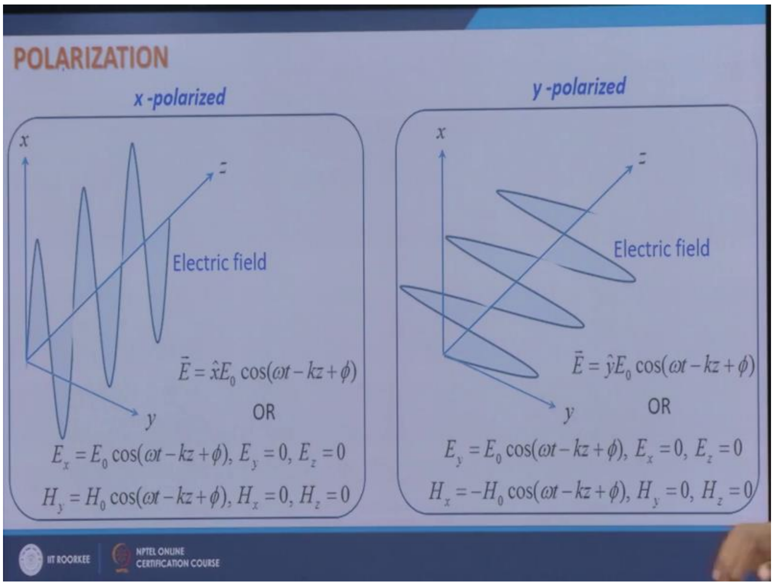
(Refer Slide Time: 06:09)



And it can be given by $n = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}}$, and since for all the dielectrics they are non-magnetic. So, I can approximate $\mu \approx \mu_0$. So, n comes out to be $n = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$ or I can write ε of a dielectric medium as $\varepsilon_0 n^2$ where ε_0 is the permittivity of free space. We see that electric and magnetic fields they are interdependent, you have time varying electric field with generates time varying magnetic field, and this time varying magnetic field intern generates time varying electric field.

So, there is mutual generation of electric and magnetic fields, and it is due to this mutual generation of electric and magnetic fields, EM wave propagates even in free space or in vacuum they do not require any medium to propagate.

(Refer Slide Time: 07:31)



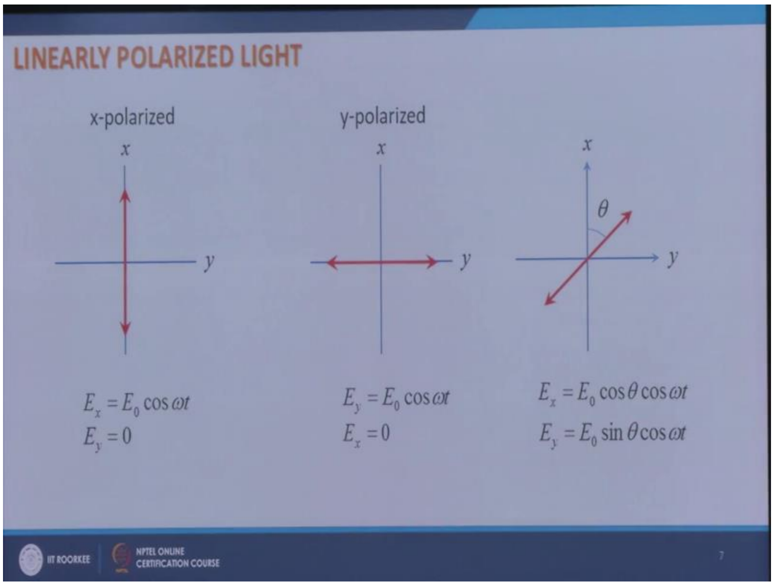
Now, let us look at what is the meaning of polarization, if the electric field vector associated with an EM wave vibrates in *x* direction the oscillations are in *x* direction, and there is a propagation in *z* direction, it is something like this if I take a string I attach one end of by string on the front wall, and then one and I shake in *x* direction. So, I am vibrating the listing in *x* direction and generating the wave in *z* direction. So, this *x* which is the direction of vibration of my hand or this end of the string or here it is the electric field, it is the direction of polarization.

And the electric field in general now can be given by of *x* polarized wave as $\vec{E} = \hat{x}E_0 \cos(\omega t - kz + \phi)$, where ϕ is an arbitrary phase it decides where you start your clock. So, I can also write this down as in terms of *x, y, z* components of electric field, as $\vec{E}_x = E_0 \cos(\omega t - kz + \phi),\ E_y = 0,\ E_z = 0$ because they are transverse waves.

Correspondingly I can get the components of magnetic fields, now $\vec{H}_x = H_0 \cos(\omega t - kz + \phi),\ H_x = 0,\ H_z = 0$. So, this is *x* polarized wave if I can also shake by string in *y* direction, instead of *x* direction I can shake that string in *y* direction and then the wave goes something like this. It again travels in *z* direction, but now the displacement is in *y* direction. So, here $\vec{E} = \hat{y}E_0 \cos(\omega t - kz + \phi)$ or you can again write them down in terms of the components of electric and magnetic field and these are the components you will get corresponding to this *y* polarized wave.

Now, if I take a transverse plane, here it is *x*-*y* plane and notice the vibration of electric field vector then in *x* polarized.

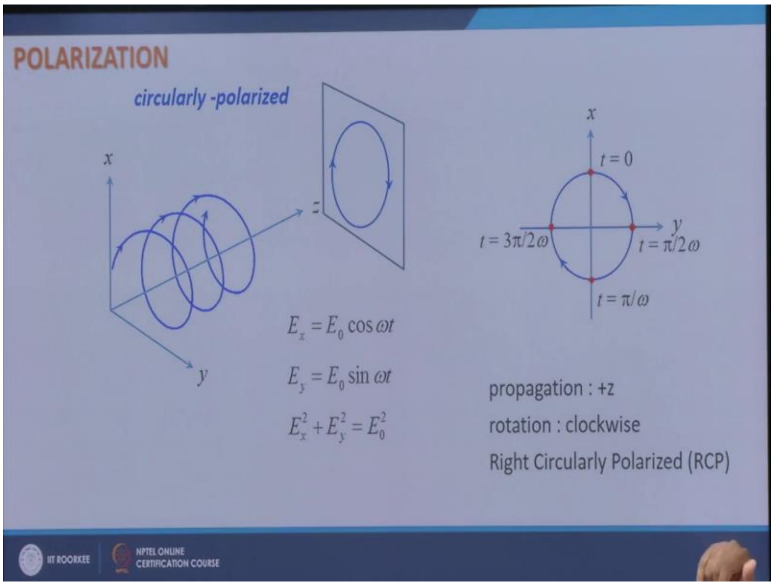
(Refer Slide Time: 10:15)



I see that the vibration is like this for *y* polarized the vibration is like this, but I can also shake the string in this direction this is *x* this is *y*, but I can shake it in this direction also. Then I will create a wave which goes like this, the direction of vibration is now slanted, it make certain angle say θ from *x* axis then this is this I called a linearly polarized wave which has both the components *x* and *y*. In this case you have only *x* component in this case you have only *y* component, but you can have *x* and *y* both components. And the amplitude of these components we will depend upon what is the direction of vibration of electric field vector.

So, these are some examples of linearly polarized light, I have another possibility I can shake the string like this. Again the movement of my hand is confined to *x*-*y* plane and I am shaking the string like this and I generate a wave like this, then it is known as circularly polarized light.

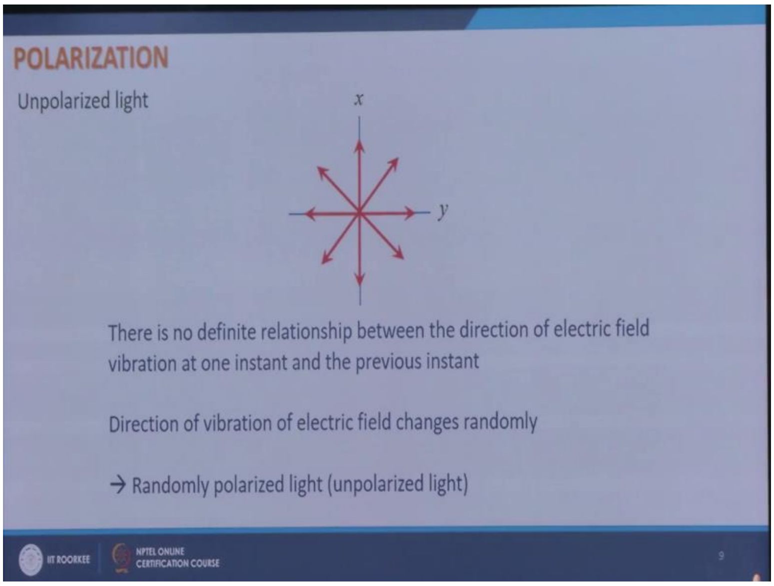
(Refer Slide Time: 11:45)



So, $E_x^2 + E_y^2 = E_0^2$ if I trace the variation of the tip of electric field vector on a transverse plane, it will go like this. So, how can I represent it are since it is a circle. So, institutively I can say that if $E_x = E_0 \cos \omega t$, then $E_y = E_0 \sin \omega t$. So, that, which is the equation of a circle. If I now find out how the tip of electric field vector goes in *x*-*y* plane, I plot this *t* is equal to 0 to *t*. I will have $E_x = E_0$, $E_y = 0$. So, I will be somewhere here. At $t = \pi / 2\omega$ I will come here here, $t = 3\pi / 2\omega$ here.

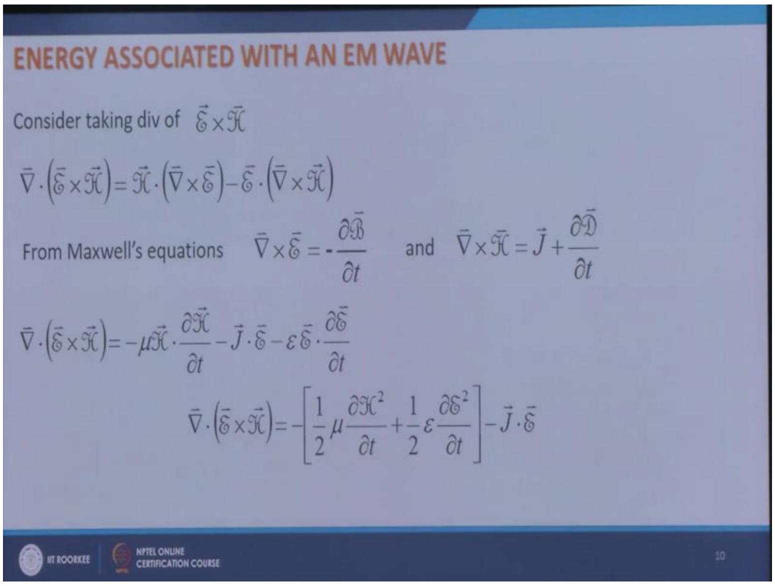
So, as time passes it moves in this direction this is clockwise. So, the direction if the direction of propagation I have fixed as plus *z* and rotation is clockwise, then I can call this as circular right circularly polarized light or RCP it is a convention whether you call it right circularly or left circularly. So, if you call this as right circularly then that one which is the anticlockwise and direction of propagation is plus *z* always then it would be left circularly. So, I call it RCP.

(Refer Slide Time: 13:59)



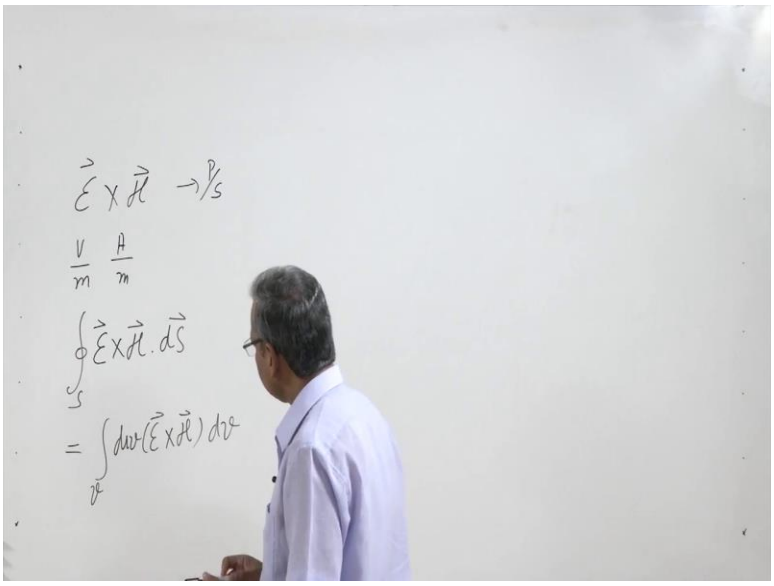
Then we often here term un polarized light, what is un polarized light; well if it one instant I look at the direction of electric field vector if it is *x* and the next incident it becomes like this, and another instant it becomes like this, and then like this then I do not have any correlation between the direction of electric fields at different instances, they change their direction randomly then such kind of light is known as randomly polarized light or un polarized light people call it un polarized light, but I prefer to use randomly polarized light.

(Refer Slide Time: 14:46)



Then what is the energy associated with an EM wave. For that let me considered taking $\vec{\nabla}.(\vec{\mathscr{E}} \times \vec{\mathscr{H}})$, you may think why suddenly I am taking the $\vec{\nabla}.(\vec{\mathscr{E}} \times \vec{\mathscr{H}})$; well I want to find out energy associated with an EM wave, and right now I am considering medium which is isotropic medium and in an isotropic medium what I have the energy flows in the direction of wave. Wave is propagating like this and energy also flows like this, and I know that $\vec{\mathscr{E}} \times \vec{\mathscr{H}}$ represents the direction of flow direction of propagation of wave; it is in the same direction as the wave vector.

(Refer Slide Time: 15:55)



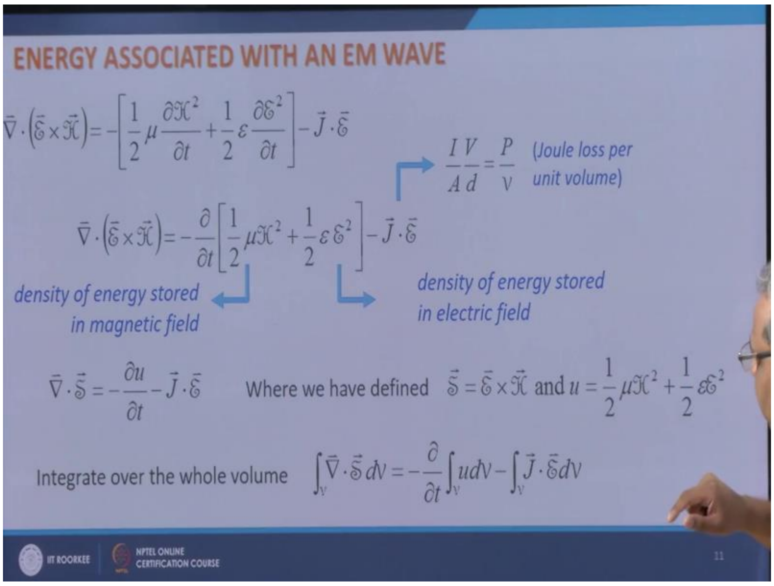
And I also notice that $\vec{\mathscr{E}}$ is I take $\vec{\mathscr{E}} \times \vec{\mathscr{H}}$, $\vec{\mathscr{E}}$ is volts/meter and $\vec{\mathscr{H}}$ is amperes per meter. So, *v* times a volts times ampere will give you power. So, it is power per meter square. So, it is kind of intensity. So, it has got the units of power per area power per unit area.

Let us say *S*, and if I want to find out power then I should multiply it by *S*. So, what I should do if I take the entire cross sectional area. So, I do this and integrated over the whole area. This $\oint_{S} d\vec{S}\, \vec{\mathscr{E}} \times \vec{\mathscr{H}}$ from divergence theorem, I know that it is equal to divergence over integral over whole volume v, which is enclosed by this surface times divergence of $\oint_{V} dv v \vec{\mathscr{E}} \times \vec{\mathscr{H}}$, *v* is the volume.

So, this basically will give me the power flowing out, that is why I consider taking $\nabla.(\vec{\mathcal{E}}\times\vec{\mathcal{H}})$ and see by using now Maxwell's equations, whether it really represents the power or not. So, I take $\vec{\nabla}.(\vec{\mathcal{E}}\times\vec{\mathcal{H}})$ which I can expand know as $\vec{\mathcal{H}}.\vec{\nabla}\times\vec{\mathcal{E}} - \vec{\mathcal{E}}.\vec{\nabla}\times\vec{\mathcal{H}}$, and from Maxwell's equations I know $\vec{\nabla}.(\vec{\mathcal{E}}\times\vec{\mathcal{H}}) = -[\frac{1}{2}\mu\frac{\partial\mathcal{H}^2}{\partial t}+\frac{1}{2}\varepsilon\frac{\partial\mathcal{E}^2}{\partial t}]-\vec{J}.\vec{\mathcal{E}}$. So, this $\vec{\nabla}.(\vec{\mathcal{E}}\times\vec{\mathcal{H}}) = -\frac{\partial}{\partial t}[\frac{1}{2}\mu\mathcal{H}^2+\frac{1}{2}\varepsilon\mathcal{E}^2]-\vec{J}.\vec{\mathcal{E}}$. So, $-\mu\mathcal{H}$ comes here. So, it becomes $\vec{\mathcal{H}}.\frac{\partial\mathcal{H}}{\partial t}$ and here you have *J* here you have e. So, $\vec{\mathcal{E}}.\vec{J}\ or\ -\vec{J}.\vec{\mathcal{E}} - \frac{1}{2}\varepsilon\vec{\mathcal{E}}.\frac{\partial\vec{\mathcal{E}}}{\partial t}$ So, this I can also write as $\vec{\mathcal{H}}.\frac{\partial\vec{\mathcal{H}}}{\partial t}=\frac{1}{2}\frac{\partial\vec{\mathcal{H}}^2}{\partial t}$.

So, this $\vec{\nabla}.(\vec{\mathcal{E}}\times\vec{\mathcal{H}})$ can now be written something like this.

(Refer Slide Time: 19:00)

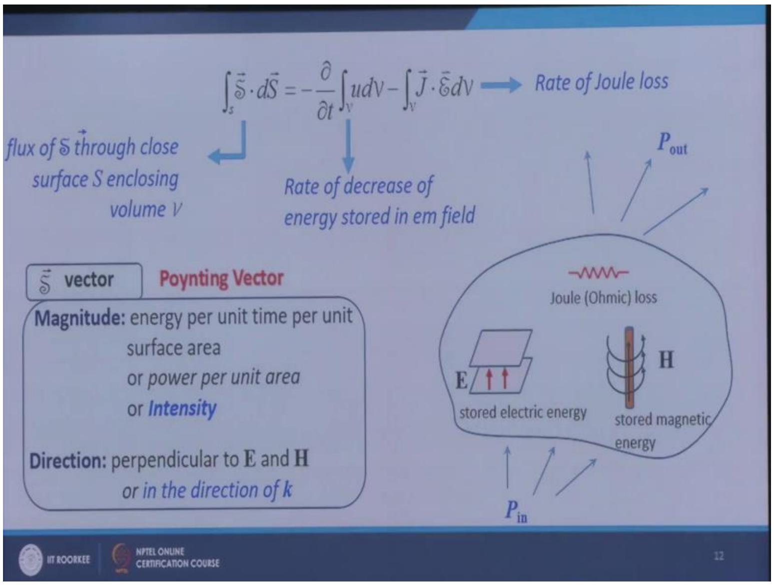


Now let me take this $\partial/\partial t$ outside, then it is $\frac{1}{2}\mu\frac{\partial\mathcal{H}^2}{\partial t}+\frac{1}{2}\varepsilon\frac{\partial\mathcal{E}^2}{\partial t}-\vec{J}.\vec{\mathcal{E}}$. What is this we know that $\frac{1}{2}\mu\frac{\partial\mathcal{H}^2}{\partial t}$ is nothing, but the density of energy stored in magnetic field and this is nothing, but the density of energy stored in electric field, what is $\vec{J}.\vec{\mathcal{E}}$? *J* is current per unit area this is current density and $\vec{\mathcal{E}}$ is volts/meter potential divided by the distance. So, this is nothing, but power *v* times I is power and a times d is volume. So, this is nothing, but joule loss per unit volume. So, if you have a resistor. So, P would be the power dissipated in register per unit volume. So, this is ohmic loss or joule loss per unit volume.

Now, let me represent this $\vec{\mathcal{E}} \times \vec{\mathcal{H}} = \vec{\mathbb{S}}$ some vector $\vec{\mathbb{S}}$, and this energy density as u which is the energy density of a stored energy in electromagnetic field, then I have this and let me integrated over the whole volume then I get this is $\oint_S \vec{\mathbb{S}}.d\vec{S} = -\frac{\partial}{\partial t}\oint_V udV - \oint_V dV\vec{J}.\vec{\mathcal{E}}$

So, this is what it is again here this is nothing, but the rate of decrease of energy stored in EM field, u times dv will give you energy and this is nothing, but the rate of Joule loss.

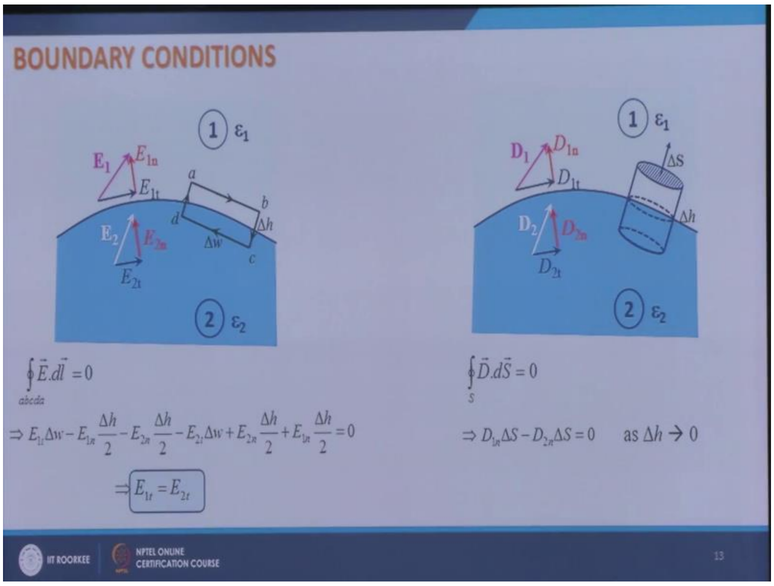
(Refer Slide Time: 21:01)



So, this SD what is $\vec{\mathbb{S}}$ now? I can now interpret $\vec{\mathbb{S}}$. So, this $\vec{\mathbb{S}}$ is nothing, but here if I look in this form $\vec{\mathbb{S}}.d\vec{S}$ it is nothing, but the flux of this vector $\vec{\mathbb{S}}$ through closed surface $\vec{\mathbb{S}}$ which encloses this volume V; if I look at this. So, if this is the volume and this is the surface area $\vec{\mathbb{S}}$ and I have some power P_{in} some power, P_{out} out then this is the net output flux through this volume and what is happening here? I have a stored electric energy is stored magnetic energy and joule loss. So, this $\vec{\mathbb{S}}$ can now be interpreted as the energy per unit time per unit area which flows out.

So, this is this vector has a magnitude which shows you energy per unit time per unit surface area or power per unit area or intensity as I have seen here and its direction is perpendicular to E and H, which is in the direction of k and this vector is known as pointing vector, and represents intensity associated with an electromagnetic wave. So, this would be useful when we will calculate the energy associated with modes. Another useful thing would be which we will use in our further analysis of optical waveguides and fiber or boundary conditions, what are the boundary conditions.

(Refer Slide Time: 23:00)



If I have two media medium 1 and medium 2, these are dielectric media medium 1 has dielectric permittivity ε_1 medium 2 has dielectric permittivity ε_2, and if there is an electromagnetic field here then if I know the field here I can find out the field here or I can relate the field in region 1 to field in region 2 using some condition ok.

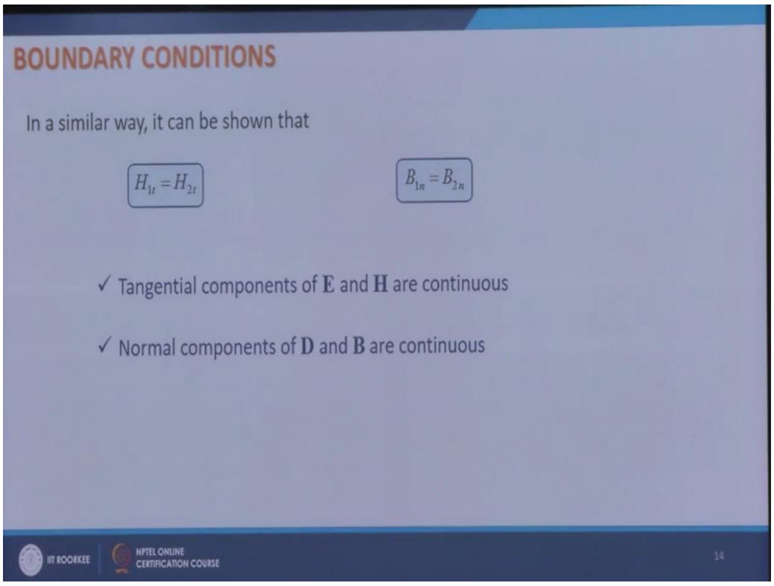
With what condition I can relate the fields here with the fields here. So, if I consider that the electric field here is E_1 let us in this direction, and electric field is E_2 let us say in this direction. Then these fields I can always break into two components, one is tangential to this surface interface and one is normal to this. So, for E_1 as well as E_2 and then I can try to find out the relationships between these tangential and normal components for that what I do? I consider a closed loop something like this abcda and I know that E dot dl over this closed loop should be equal to 0. If I apply this I go from a to b, I need to take tangential component to the surface. So, it is $E_{1t}\Delta w$, Δw is the path this the very small loop. So, that it can represent a point when Δw and Δh tend to 0 and a surface and interface when Δh tends to 0, then I have from here to here let us say it is $\Delta h/2$ here $\Delta h/2$ here.

So, $E_{1t}\Delta w - E_{1n}\dfrac{\Delta h}{2} - E_{2n}\dfrac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\dfrac{\Delta h}{2} + E_{1n}\dfrac{\Delta h}{2} = 0$. So, if I do this I find that E_{1t} should be equal to E_{2t}. So, the tangential component of the field here and the field here they should be continuous, the tangential component of electric field should be continuous what about normal component? For normal component I consider a pill box like this very small pillbox let me take initially the height Δh and it has got some surface area Δs. And I consider D_1 and D_2 vectors here, they are dielectric displacement vectors I again resolve them into the components which are parallel to the surface and perpendicular to the surface.

Now, in this a small pill box I apply Gauss's law, which gives me $\vec{D}.d\vec{s} = 0$ should be equal to 0. When I do this then I find is they it would be D_{1n} times delta s, this is D_{2n} times delta s with the negative sign and what is the flux through this curved surface? This flux through this curved surface would be 0 as $\Delta h \to 0$ to obtain this boundary to close down to this interface. So, this gives me $D_{1n} = D_{2n}$, which means that normal component of D should be continuous. So, I have got tangential component of E should be continuous and normal component of D should be continuous at the interface of two dielectric media provided that there are no free charges at the interface and this is true in case of dielectrics there are no free charges no free currents.

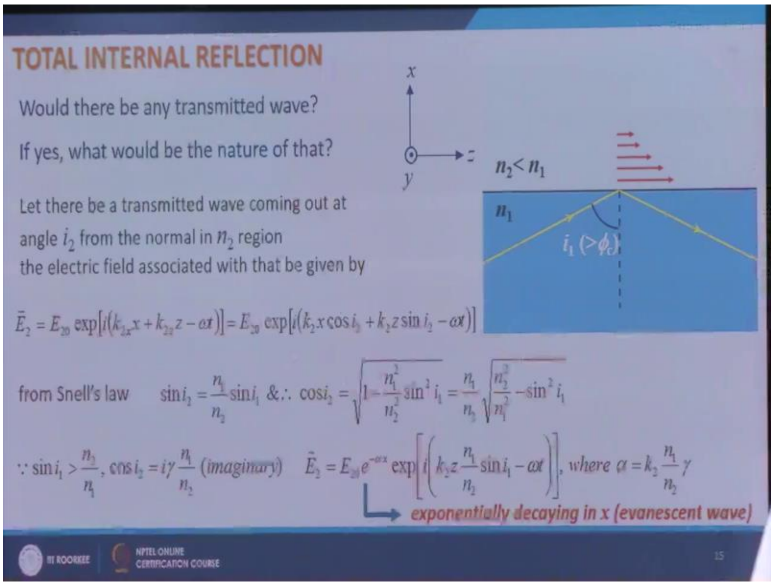
In a similar way I can show that for magnetic fields the tangential component of H and normal component of B should be continuous.

(Refer Slide Time: 27:50)



So, these are the boundary conditions that I will use whenever I will encounter an interface between two dielectric media. Now lastly let me revisit total internal reflection and understand some intricacies. You remember that when I did the ray theory, ray theory simply tells me that a ray goes like this and it comes back there is no energy which flows out into this medium.

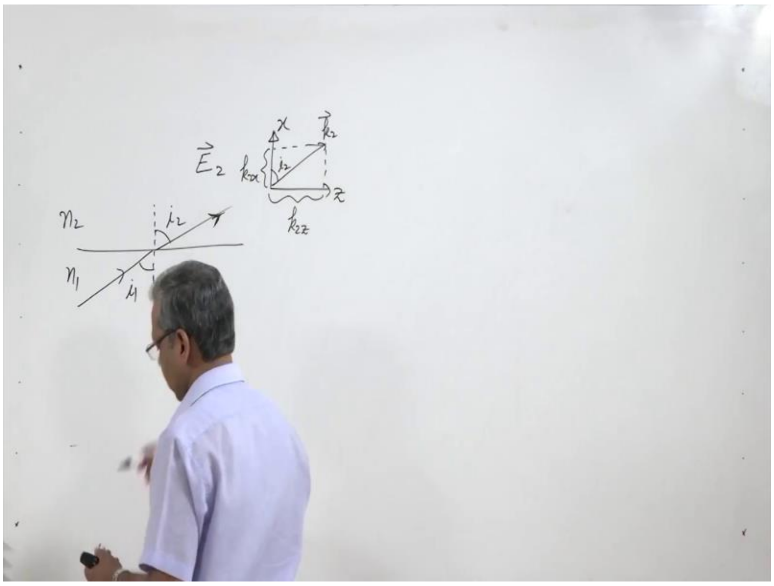
(Refer Slide Time: 28:29)



If this is the medium of refractive index n_1 this is of n_2, where n_2 is a smaller than n_1 then ray theory tells me if I launch array which makes an angle larger than the critical angle, then it this ray will come back into the same medium and there would be no transmitted ray in this. But if I talk about waves and with waves, these are EM waves associated electric and magnetic fields.

Let me assume that there is some translated wave also, there is some transmitted wave and then find out what would be the nature of that. So, first of all I would ask would there be any transmitted wave at if yes what would be the nature of that. So, what I do? I like that there be a transmitted wave which is coming out at an angle i_2.

(Refer Slide Time: 29:46)



So, if this is n_1 this is n_2, this is the interface and this is the normal, this is the incident one at i_1 and let me say here I have at i_2 this is the transmitted one. So, the field associated with this would be given by E_2 let us say E_2 is the field here, E_2 is the field associated with this wave these are waves not raise would be now E20, E to the power i know since it is in x-z plane, this is z this is x. So, this is an x-z plane.

So, I will have a component along x of wave vector, which is k_{2x}, k_2 is like this. So, here you will have k_{2x} and here you will have k_{2z}. So, I will have $e^{(ik_{2x}x+ik_{2z}z-\omega t)}$. What is k_2x? $k_{2x}=k_2\cos i$, this is i_2. So, that is what I have written k_2 cosign i_2 times x and similar this is k_2 sign i_2 time z minus omega t. If I apply a Snell's law here then I know that sin i_2 would be $n_1/n_2\sin i_1$. So, I will get $\sin i_2$ and correspondingly $\cos i_2=1-\frac{n_1^2}{n_2^2}\sin^2 i_1$ and from here if I take this n_1/n_2 out. So, this would be $\cos i_2=\frac{n_1^2}{n_2^2}(\frac{n_2^2}{n_1^2}-\sin^2 i_1)$, and since this i_1 is greater than phi c for total internal reflection.

So, $\sin i_1>\frac{n_2}{n_1}$, if $\sin i_1>\frac{n_2}{n_1}$ then this quantity would be imaginary. So, I will have this $\cos i_2=i\gamma\frac{n_1}{n_2}$, let me represent this as i gamma which is imaginary now. So, now, if I put these values of $\sin i_2\ and \cos i_2$ back into this, what I get that all right corresponding to z it is find k_2z, $n_1 / n_2 \sin(i_1 - \omega t)$, but corresponding to this I have *i* gamma vector and this *i* gamma when it is multiplied by this *i*, it becomes exponentially decaying.

So, I will get some $E_{20}e^{-\alpha x}$ times this where α is this. So, what I have got? I have a transmitted wave whose amplitude decreases with *x* exponentially. So, I have a transmitted wave whose amplitude decreases exponentially in *x* direction, this is known as evanescent wave. So, with total internal reflection there is an associated transmitted wave in lower index medium, whose amplitude decreases exponentially and this wave is known as evanescent wave. This is going to be useful when we will do the analysis of optical waveguides and optical fibers.

Thank you.