Select/Special Topics in Classical Mechanics

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Module No. # 02

Lecture No. # 07

Oscillators, Resonances, Waves (i)

Greetings, welcome to unit 2 of this course on select or special topics on classical mechanics. This unit will be on oscillators, we will learn little bit about resonance and also little bit about the wave phenomena.

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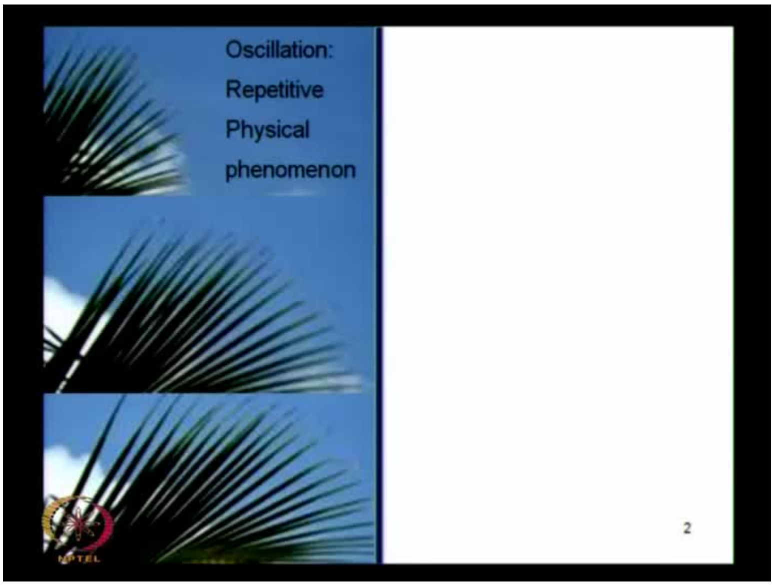

Now, oscillations are basically in the simplest form of any repetitive physical phenomenon. So, it could be just this waving of leaves, when a gentle breeze goes - past these leaves and these leaves, you know, sort of - the sorts waving, it moves a little bit, then crosses an equilibrium point and gets back. So, it could be easily set into what we call as an oscillation, so it is any repetitive phenomenon.

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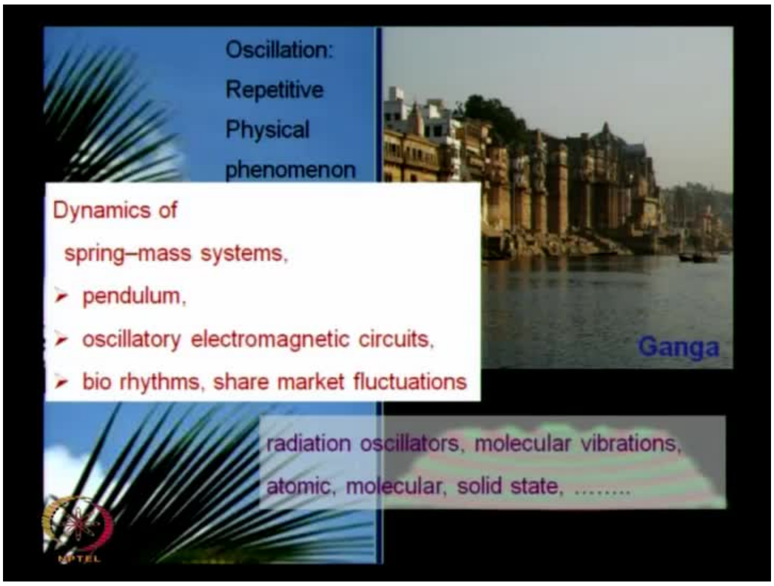

We see all the time around us in nature, in the water waves; for example, if you look at the surface of a water, then you see that the level pops up and down a little bit, then you also have this pumping of the waves, which showers along the surface and you could have water waves. You know the phenomenon of the oscillation and the wave phenomenon they really go together.

Now, this is very common phenomenon, you see it in the pendulum; for example, the light around us consist of electromagnetic waves; the electric vector and the magnetic vector, which constitutes the electromagnetic waves. They perform an oscillation about its equilibrium points and then there are bio rhythms. Then, even things like share markets, which fluctuate - I would not be talking very much about it, because I never had enough money to invest in that. But, then in other physical phenomena like molecular vibrations, in atomic physics, in molecular physics, solid state of physics and so on, oscillations too find themselves play an important role in just about every physical process in a very big way. So, it is very important to learn oscillations from the fundamental principles.

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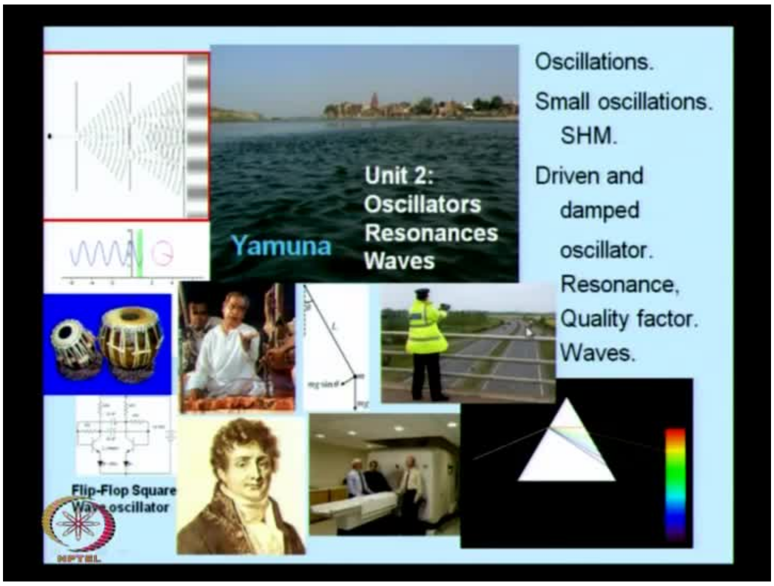

Here, you see famous experiment, which is known as Young's double slit experiment. You have got light passing through one slit over here and then the subsequent passage is through two slits, one over here and one over here. This is described often as one of the most beautiful experiments in physics (Refer Slide Time: 03:01). What you see on a screen is a fringe pattern, alternate occurrences of dark and bright fringes.

Now, to explain this, you need what is called wave theory of light. This also has its first principles; one would begin and analyze this from of the notion of oscillators. Then, in musical instruments or music of any kind - Vocal music, music coming from instruments, percussion instruments, so whatever, unfortunately, this is true also for the noise, not just for the music, but then the basic phenomenology is based on oscillation.

Light properties are described in terms of oscillation; this is the picture of a traffic police man, who is using a speed gun. What he is doing is, monitoring the speed of oncoming traffic to check if they are travelling at the speed limits or lower than the speed limits. He is always delighted when they are driving fast, so that he can charge them, but this also again is based on the principles of oscillations. Then there are quantum phenomena, in quantum mechanics, you have the phenomenon of quantum oscillators and these are again almost you ubiquitous in quantum mechanics.

Here is the picture of a nuclear magnetic resonance imaging technique, which is used to map the body tomography and this is done again using the principle of oscillation. The basic physics in electronic oscillator and so on, is what we are going to discuss in this unit (Refer Slide Time: 04:53). Here is a picture of Fourier, if you have recognized; he is one person who has contributed very rich knowledge to this field.

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Whatever our learning goals for this unit, we will first of all begin by recognizing what is a stable equilibrium, what is unstable equilibrium and what is neutral equilibrium? We will also understand what I meant by saddle points?

We will learn that if you go close enough to any point of stable equilibrium, motion can always be described by the simple harmonic oscillators. This is really amazing, because this will tell you why the physics of oscillation is so important. Because, close enough, of course, one has to define what is meant by close enough; I will discuss it as we go along this discussion.

If you go close enough to a point of stable equilibrium, motion can always be described in terms of a simple harmonic oscillator. Then, we will discover that the same physics governs a lot of phenomena, so it enables us to develop analogs between a mechanical systems and electrical systems. So, there are the same famous electromechanical analogs, then we will go on to learn the effects of damping, because damping is something which is very real, you often develop the physics for ideal systems, in which there is no damping, but then you have to deal with damping in the real physical systems. Then, we will figure out how to work with damping and then we will also learn how to deal with these oscillators when the dynamics is governed not just by damping, but also by an external driving force. So, these are some of the things that we will learn in this unit.

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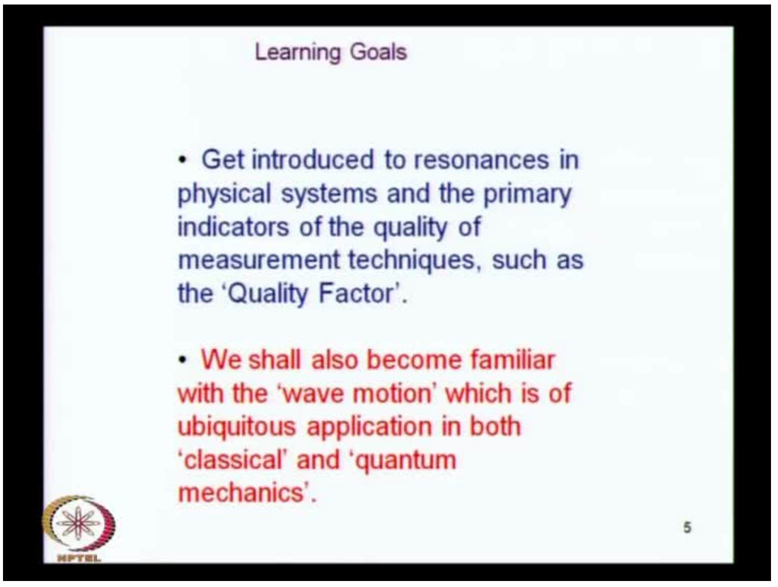

We will then introduce ourselves to the phenomenon of resonances in physical systems. Then, we will study about measurement techniques and what is known as the quality factor; these are intimately connected with the physics of resonances. Then we will also learn a little bit about wave motion, which has important consequences both in classical mechanics as well as in quantum theory.

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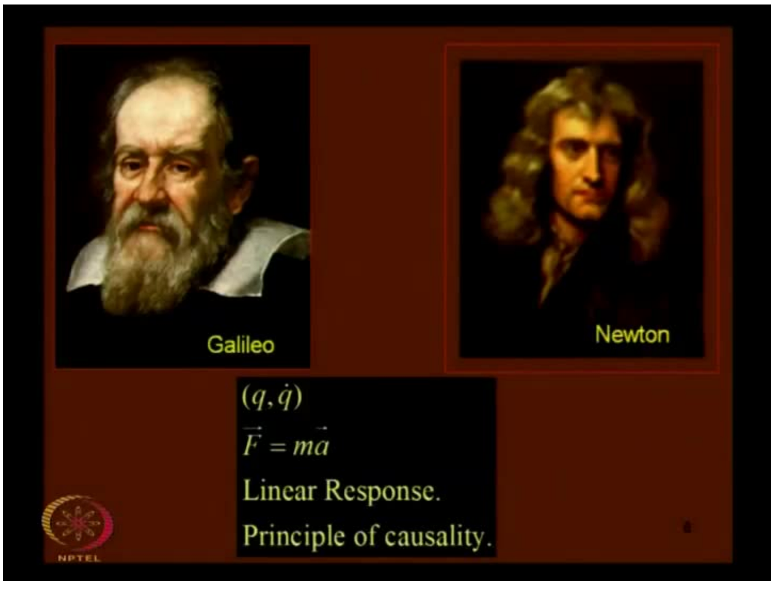

Now, in unit 1, we dealt with classical mechanics, the governing principle that we employed in unit 1 was the Newtonian formulation of classical mechanics, which is based on this linear response theory. You have got a target, how this target responds to a stimulus when the response is linearly proportional to the stimulus. You have the equation of motion namely the Newton's equation of motion, F equal to m a as we call it, it is based on this cause effect relationship. We could analyze the evolution of mechanical systems by solving this equation of motion.

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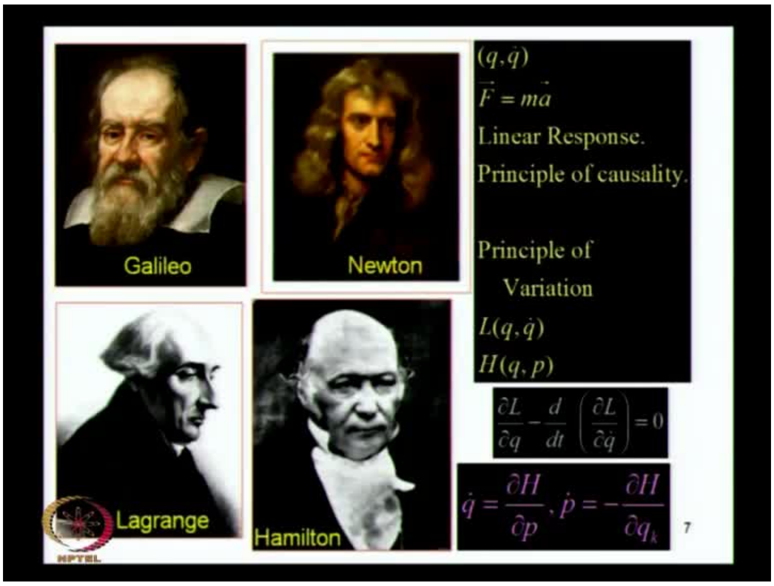

We also learned that the mechanical evolution of a classical system can also be described by using an alternative principle, not just a principle of causality, but also the principle of variation, which is based on the Hamilton's principle of extremum action. This leads to alternative differential equation, either the Lagrange's differential equation or the Hamilton's equations. These can also be used to solve the equation of motion, to observe the evolution of the mechanical system over a passage of time. With given initial conditions, you can track the development of the mechanical system. So, this is how problems are solved in mechanics. We have also learned that both of these techniques whether based on the Newtonian principle of linear response, principle of causality or in the principle of variation of Hamilton, you get essentially the same results.

Now, the objective of this course is really not so much; to develop, handle on Hamilton's principle or the Lagrange equation of motion. We introduce that to provide an alternative way of following a system evolution, but this course is not about Lagrange mechanics or Hamiltonian mechanics, because that is the very specialized area and students usually are exposed to this at somewhat latest stage of education in college.

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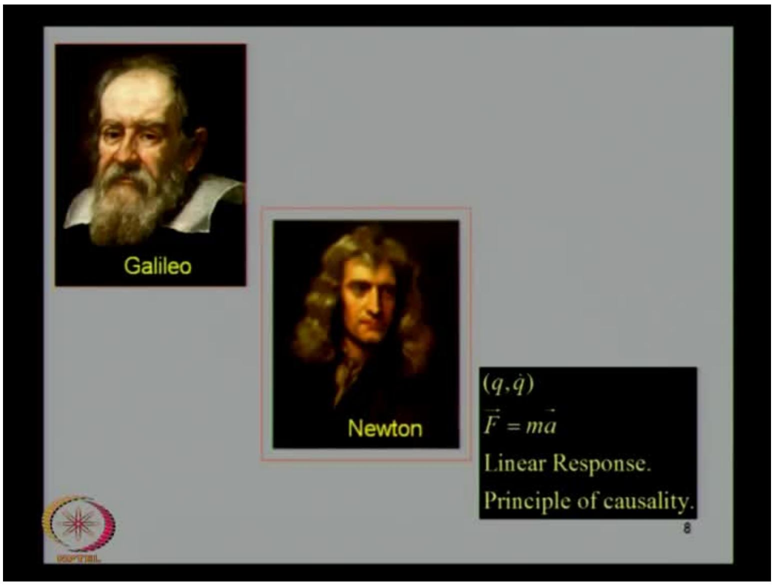

At this level, at which this course is really structured, it is only the essential concepts of the Hamiltonian principle, which were of important, so that you understand the alternative development of classical mechanics and also develops some insight in the connection between the symmetry and conservation laws, which come out so nicely in a Hamiltonian and Lagrangian formulation. What we are going to do now is, to focus on the Newtonian methods they gave us equally powerful tool to analyze the mechanical evolution of a system.

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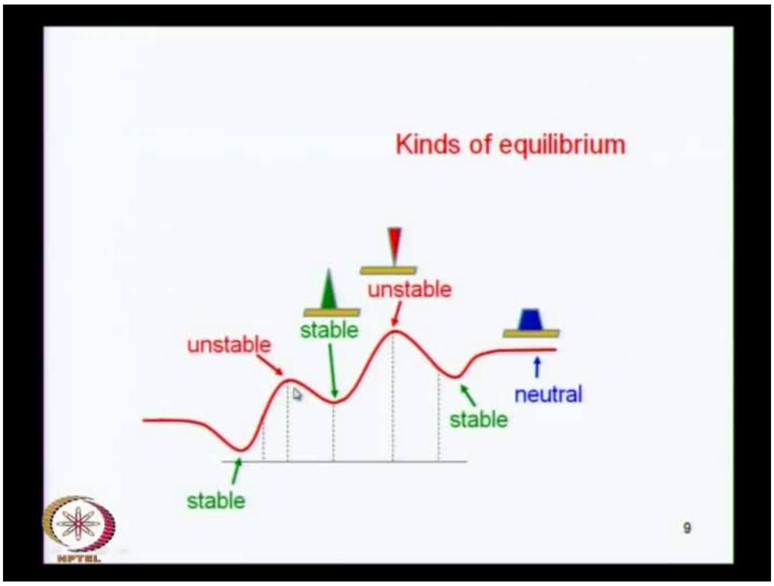

The basic idea in this is to solve the equation of motion, which is F equal to m a; then with the given initial conditions for position and velocity one can always track the evolution of the mechanical systems. So, this is our basic tool which is Newtonian mechanics and the Newton laws of motion. We will now discuss the dynamics of systems, when they are either in a state of equilibrium, in which case, they will remain in the state of equilibrium forever. If it happens to be in state of equilibrium, if there is no external force which is acting on it, then it will remains in the state of equilibrium forever, which is essentially Galileo’s law of inertia or Newton’s first law.

If it is not in a state of equilibrium, then of course, this system will respond, it will get accelerated by the stimulus, because the departing from equilibrium is indicated by the presence of interaction, which is stimulus, which will cause the system to respond in a manner so that the response is directly, linearly proportional to the stimulus. When we talk about equilibrium, there are different kinds of equilibrium that we talk about. This is simple one dimensional potential, it is flat via and then it goes to valley, then a hump, then another valley, then another hump and so on. As we know, when it is at the bottom of any of these valleys, then the system is in the state of equilibrium, just a way you see this cone placed nicely, comfortably in the state, you know that is going to stay there forever and it is very stable.

Likewise, if it is top of this hump, it is again in a state of equilibrium and it would be like inverted cone. The reason is that any slight departure from this would tends to push this system away from the point, whereas if it is close to stable equilibrium, if it is pushed slightly away - one way and another, it will tends to recover and come back to the state of equilibrium. So, this is the difference between stable equilibrium and unstable equilibrium.

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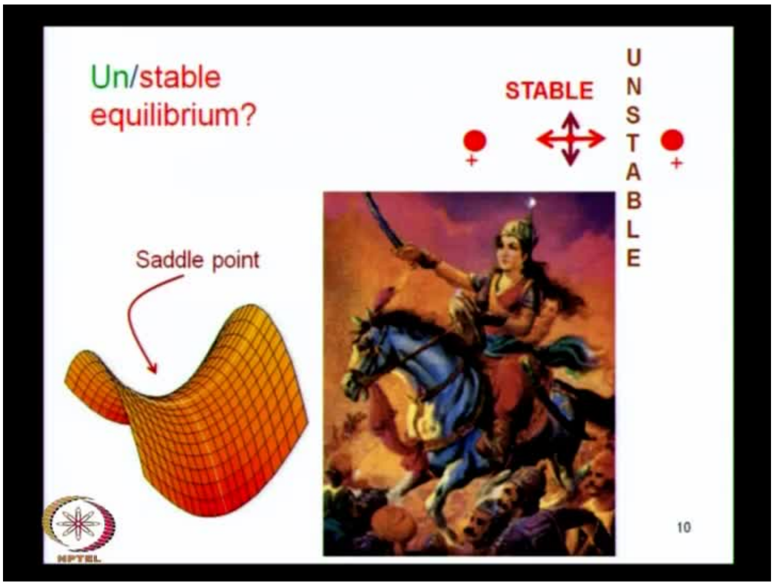

On the other hand, if it is in a region where the potential is flat, then it is in the state of what is called as a neutral equilibrium. So, these are the different kinds of equilibrium in one dimensional system. Then, let us ask a question that if you have two positive charges, you keep it test charge; a positive test charge right in the middle. Let us ask this question, is it in a state of equilibrium? Now, you have two exactly equal positive charges, a test charge is kept exactly in the middle, the question is, is it in a state of equilibrium? Obviously, yes, it is in a state of equilibrium. Now, we ask a question is it in a state of unstable equilibrium or is it in a state of stable equilibrium? That really depends on what kind of displacement we are talking about.

If you displace it along this line, which is the transverse line between these two charges, then the charges along the horizontal line will cancel each other; the forces due to these two charges along the horizontal line will cancel each other. The one along the vertical line will tends to push this charge further and further away from the original point making it a point of unstable equilibrium, because any slide displacement along the charge pairs direction will push the charge further away and it will end up going to infinity, either to the top or to the bottom in which about direction it is displaced first. So, that is the direction of displacement, in which case, this is like unstable equilibrium.

On the other hand, if the charge meets a marginal displacement along the longitudinal axis; along the axis which connects the two charges. Then, if it is move to the left, then the force due to this will become stronger, which will post back to the equilibrium. As it does so, it will go passed this, gain a certain amount of kinetic energy, it could go passed this, then it will find itself on this side and then this charge will push it back. This kind of displacement will tend to bring the charge back to its original point all the time, so that will make it a point of stable equilibrium.

So the point - the original point itself is certainly in equilibrium, but then whether it is an unstable equilibrium or stable equilibrium depends on the direction in which the test charge is this large. So, such a point is called as a saddle point, because this is the picture of a saddle, you can see that if you place a marble at this center, which is a point of equilibrium, it will sit comfortably over there. But, if the marble receives a little bit of displacement of along the transverse at direction, which is along the width of the holes, then the marble would fly away. It would go away from the equilibrium point and that will be an unstable equilibrium.

On the other hand, if the marble is displaced along the length of the horse like this, then it will tends to come back to the original point. This is the picture of saddle point, equilibrium of this kind, which is stable with reference to the displacement in one direction, but unstable with reference to another direction, is what is called is a saddle point. So, these are the different kinds of equilibrium that one talks about, stable equilibrium, unstable equilibrium, neutral equilibrium and also a saddle point.

Now, we are going to deal with small oscillations and the terms small oscillations has a very characteristic significance in our discussion. We need to understand what is meant by small, means if an oscillation is restricted to this range is it small or it is a restricted to this range is it small or it is restricted to this range - you know, how small is small. This is the question that we need to talk about, because the smallness in the largeness is with reference to some other size parameter and we need to understand the size parameter (Refer Slide Time: 17:42).

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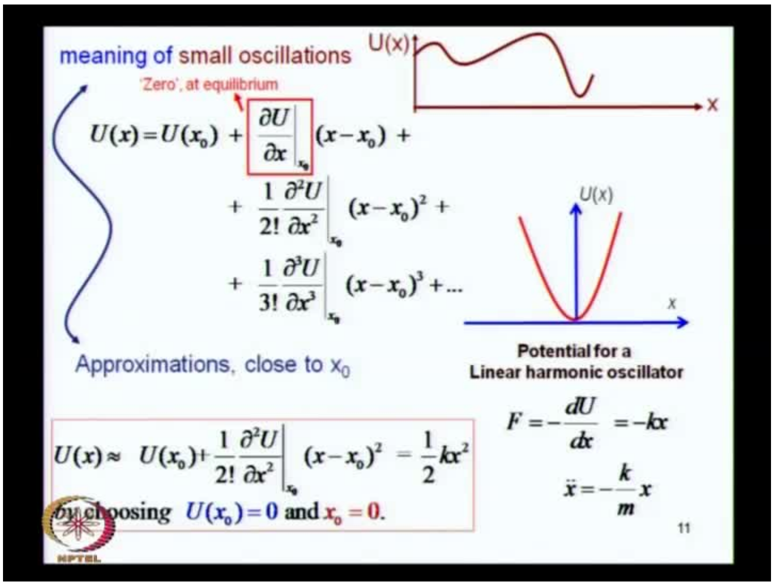

If you have a potential of this kind, you see that I have plotted a one dimensional potential, it has got humps and Valleys and so it has got position of stable equilibrium, unstable equilibrium. But, notice that this shape of the stable equilibrium over here is similar to this shape of this stable equilibrium over here, but it is not exactly the same (Refer Slide Time: 18:24).

This valley is a little bit narrower than this valley in a certain sense. This is a qualitative description, we need to quantify this. We have to develop a quantitative estimate of this, because the shape is certainly similar, but not exactly the same, there are differences. What we will do is to expand this potential about the point stable equilibrium. One can, you know, if the function is analytical, if the function is continuous, it passes its continuous derivatives, then you can have an infinite power series. So that the function at a neighboring point x will be equal to the value of the function at the equilibrium point x 0 plus the difference x minus x 0 multiplied by the derivative of the function evaluated at the point x 0. So, this will give you the first order correction, but then you really have an infinite series.

The subsequent term will be quadratic in this displacement; it will contain the second order derivative scaled by this factor of 1 over root 2. So, you are all familiar with these infinite series expansion, this is the kind of power series expansion that we will use to describe the potential at a point x in the vicinity of an equilibrium. Now, near the equilibrium notice that the tangent to this U becomes parallel to the x-axis, which means that this derivative goes to 0. If this derivative goes to 0, then this term which is the first correction first order correction to this potential vanishes. Now, this term have been vanished the potential at a neighboring point near the equilibrium is then given by the value of the potential at the equilibrium point plus this second order term and the third order term and so on.

Now, the next term will be x minus x 0 to the power 4, one after that will be x minus x zero to the power 5 and so on. The powers of x minus x 0 are increasing in successive terms, if x minus x 0 is small, then its square is smaller still; the cube is even smaller, the fourth power will be even smaller and this series will converge very rapidly.

Depending on the level of approximation you want to make, you can always find a region which is close enough to the equilibrium point, so that you can retain only this term in your approximation and ignore all successive powers of x minus x 0. If you go sufficiently close to the equilibrium point, this is the meaning of what is called as sufficiently close; sufficiently close means a region which is close enough where you can ignore the cube but not the second power of x minus x 0. That depends on the accuracy with which you want to measure these things. So, if you go sufficiently close to x minus x 0, then you can always throw off these higher order terms and this is essentially in approximation, this is not exact.

If you ignore these higher order terms, then close enough to the equilibrium point, you have got the first term, this term strikes itself off because del U by del x is 0. Then, you have got this quadratic term, if you presume that U x 0 is equals to 0, this is just choosing a gauge for the potential that you measure the potential on a scale such that this potential is equal to 0. Then, if you choose the origin of the x-axis, so that x 0 is equal to 0, then this relation is nothing but U x nearly equal to half k x square. If you plot it, if you plot u as a function of x, U essentially gets a parabolic quadratic potential (Refer Slide Time: 22:09).

So, this is the famous quadratic potential for a linear harmonic oscillator, because the restoring force, if you cause a displacement of an object in equilibrium, which is at the bottom of this valley, you push it sideways, then it will no longer remain in a state of equilibrium, it will experience force and that force is given by the negative gradient of the potential. Minus d U by dx will be the force which will act on it, if this force is proportional to the displacement, because this is a force which is always directed to the equilibrium, if you push it to the left, the force will be directed to the right, if you push it to the right, it will be directed to the left, which is why it is called as a restoring force. The restoring force is always proportional to the displacement; this gives you the equation of motion for a linear harmonic oscillator.

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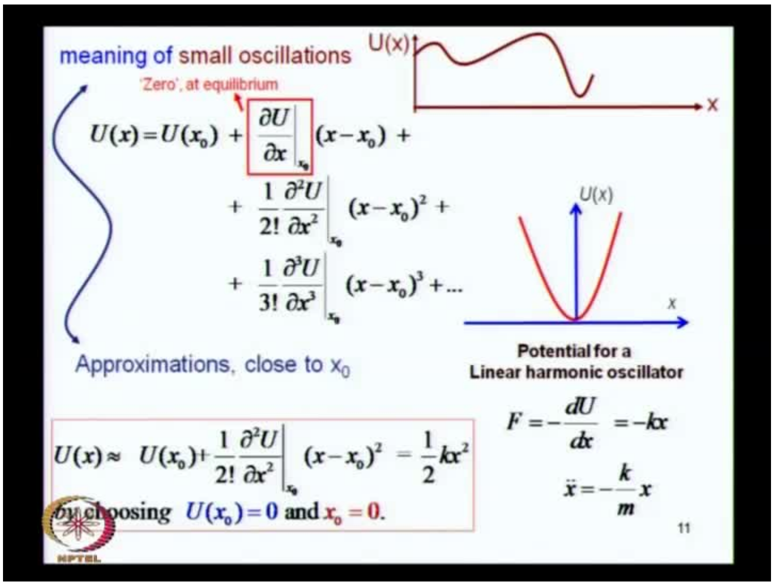

Now, why this is an oscillator will be known once we solve this differential equation? Look at the solution, because the solution will turn out to be oscillated. So, this differential equation for what is called as a linear harmonic oscillator, this is I have consider the case of an object, which is in a parabolic potential given by half k x square.

Is the harmonic oscillator only an approximation and in real sense there is anharmonicity?

Well, this is an approximation to the potential; the actual potential can always be a harmonic. Even in the case that I considered, in this potential which we have plotted, we are not pretending that the potential over here is the strictly harmonic. What we are saying is that if you go close enough to the equilibrium point, where you can through off that cube that is the pretence that pretence is what the approximation is about. Whenever you are talking about an approximation, you are saying that a certain quantity is nearly equal to the other side of an equation, is not exactly equal. When you say 2 into 2 is equal to 4, you have an exact equality, but when you say that 2 into 1.9 is nearly equal to 4, you are dealing with an approximation. Here, the left hand side gives you the exact potential at the point x, the right hand side is an infinite series.

If you consider all the infinite series, you have equality, but if you say that x minus x 0 are small enough that I can ignore higher order terms. So, in real potentials, these higher order terms will always be present, because a potential does require this infinite power series to be used. So, it is an approximation, it is a good enough approximation in a large number of physical situations, which is why it is powerful.

Now, the reason is it is useful in such a large number of situations, because in every situation you can always go close enough to the equilibrium point. If you go sufficiently close, either over here or over here, you can always find a region, where you can find that you are comfortable ignoring this term compare to the quadratic term (Refer Slide Time: 26:35). If you go close enough, then you have a quadratic approximation that is what you are approximately in a harmonic oscillator, otherwise harmonicity is always present.

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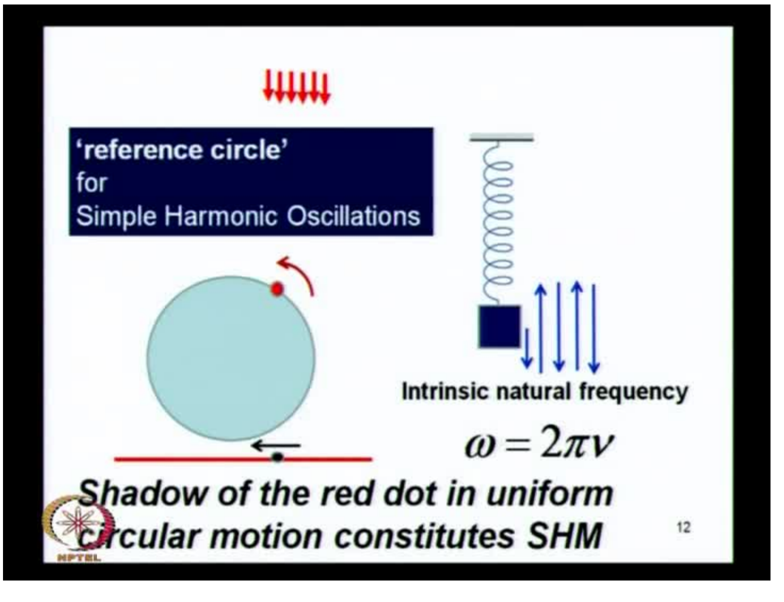

This kind of situation happens for a mastering oscillator; for example, if you have a mass suspended with a spring, you push it a little bit, then let go, then the spring has got a restoring force, because it has got the elastic properties. So, there is a tendency of the material to recover from the strange, then it tries to come back to it is original configuration. So, it will come back, then overshoot the position of equilibrium, because it would have required a little bit of kinetic energy, there will be these continuous transformations between kinetic energy and potential energy, the mass will be set into oscillation. So, this will be a periodic kind of behavior, this will constitute an oscillator.

Now, this oscillator will go through one cycle at a certain rate, it will go through a certain number of oscillations in a given unit of time, so that is your frequency nu, twice by nu is what is called as a circular frequency. Why it is called as a circular frequency will also become clear, as we discuss it in the next few minutes. The reason it is called as a circular frequency is because of it is connection with what is called as uniform circular motion, because the equation of motion which governs this.

It is the same that would govern the motion of the shadow of an object which is a circular motion, so if you have this red piece of ball, which is a uniform circular motion in this vertical plane, you have got light shining on it from the top. If you look at the shadow of this red circle, which is following over here, then as this red ball is going along this arc, this shadow will be moving from right to the left. When it comes over here by the time it goes to the extreme point over here, then it continues its motion over here, the shadow will move back and start moving from left to the right, so the shadow will exercise an oscillatory motion (Refer Slide Time: 29:00). So, there is a connection between circular motion and a periodic motion, which is why the corresponding frequency is called a circular frequency, this actually is that the shadow itself will constitute a simple harmonic motion.

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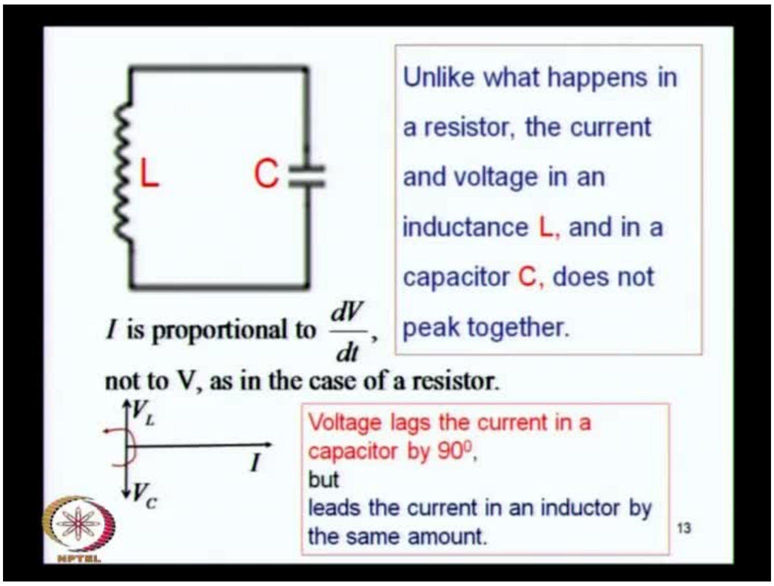

Now, this is called as a reference circle for the simple harmonic oscillation. Now, this kind of phenomenon takes place in completely different kinds of systems, so we are no more talking about a mechanical system. Suppose you just have a set electrical circuit, which is made up of an inductance L and a capacitor C. Let us say, you have a choice capacitor, then you connect it up and let go. Now, what is going to happen is that the current and voltage in the inductance L and in the capacitor, they do not go to the peak together that is kind of thing happens only in resistor circuit, but if you have non-resistive components, if you have a inductive component - an inductance or if you have a capacitance, then the current and voltage do not go in same step.

What happens is that the current is actually proportional to dV by dt rather than to the voltage. When it is proportional to the voltage itself, you have an ohmic circuit, so that is the dynamics, is governed by the Ohm’s law that is the resistive circuit. But, in a circuit, in which, you have got an inductance and the capacitance then the current is proportional to the rate change of the voltage rather than the voltage itself.

In such circuits, what happens is that the voltage and the current do not peak together, the voltage actually lacks the current in a capacitor by 90 degrees and in an inductance, it leads the current; so how would you show it in a diagram? If you pick a direction for the current, then the voltage lacks the current in the capacitor by 90 degrees, so that is shown by this arrow by here and it leads the current by 90 degree, as it is shown by this arrow over here. So, this diagram which keeps track of the phase, this is called as the phasor diagram (Refer Slide Time: 31:52). We find that these systems - system of this kind, it is an electrical system, but it is described by essentially the same kind of equation as is involved in the description of a mass spring simple harmonic oscillator.

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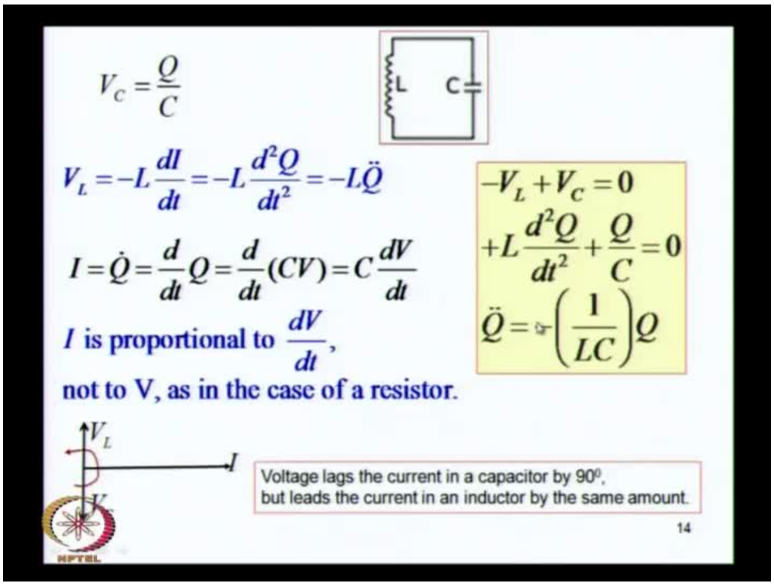

Let us see, how this equation of the motion is set up? You have got the voltage across the capacitance, which is given by this ratio of charge over capacitance. The voltage across the inductance is given by - this comes from the Faraday length kinds of consideration, you have the L d I by dt factor, the current itself is d Q by dt, so you get the second time derivative of the charge; the second time derivative is indicated by these two dots. So, the dot in our notation, let me remind you, as we did in the unit 1, as well represent the derivative with respect to time, two dots represent the second order derivative with respect to time.

Now, it is very simple, because we have got the current which is the rate of change, how it flows, how the charge flows? It will be given by d Q by dt; Q is surface C V, so this goes as d C times dV by dt, because the capacitance C is a constant. We see that the current is actually proportional to dV by dt; this is the point I was making earlier that in the resistor circuit, the current is directly to the potential V, like in Ohmic circuits and purely resistive circuits, but not in L C circuits.

Now, if you just set up this basic relationship minus V L plus V C is equal to 0, as it will have to be. Then, you just substitute these two quantities; you get an equation of motion for the charge.

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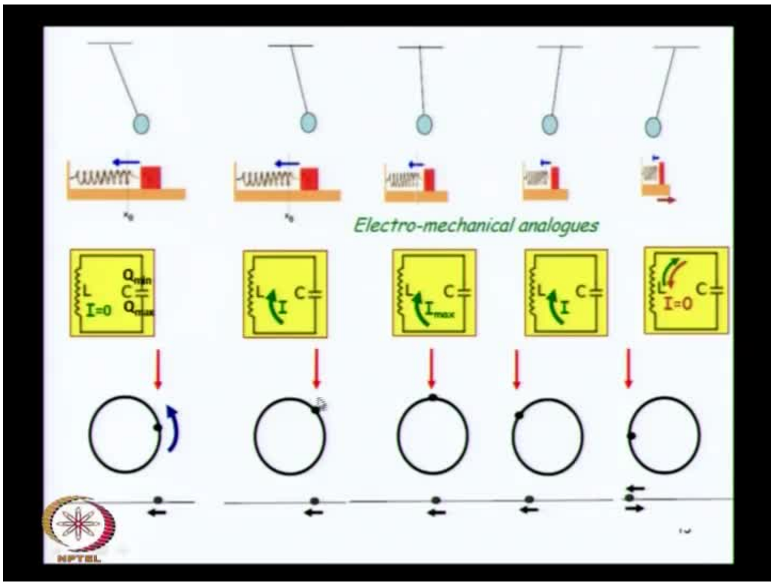

The second time derivative of the charge is directly proportional to the charge, this is the proportionality 1 over LC and there is a minus sign. Now, you will immediately recognize that this is very similar to the equation of motion for a simple harmonic oscillator.

In fact, as you see, if you have a simple pendulum, which is stretch out to it is one extreme position and you let go; this is the same kind of situation is that of a mass spring oscillator, we consider those cases, in which, we presume that this is writing on a frictionless surface just like this pendulum. In the first approximation, we do not deal with damping, so we will presume that it is not meeting any resistance of the air medium nor at the point of fulcrum or at the point of support or anywhere. Then situation is very similar to that in a LC circuit, in which, you have got the capacitance charge maximum, there is no current, you just connect the whole circuit with reference to the shadow of this ball, which is going on an uniform circular motion. We see the shadow of this ball in the light, which is following from the top, then this is the shadow - then this shadow will start moving from this end to the left (Refer Slide Time: 35:19).

If you look at it just a moment later, then this angle would have decreased, this mass would have moved further here, the current would have started flowing over here and this shadow will be in a state of motion towards the equilibrium point. If you see it a few moments later, you will see that you can actually find the exact one to one correspondence between the motion of this pendulum, the mass spring oscillator, the LC circuit, or the uniform circular motion and it is shadow, they all describe essentially the same phenomenology.

At the extreme position on the left, when this pendulum has come all the way to its leftmost orientation, then the current will come to 0 and then it will start flowing backward. This will set the whole system, whatever the system is, whether it is a pendulum or a mass spring oscillator or an L C circuit or the shadow, what you are going to see is an oscillator.

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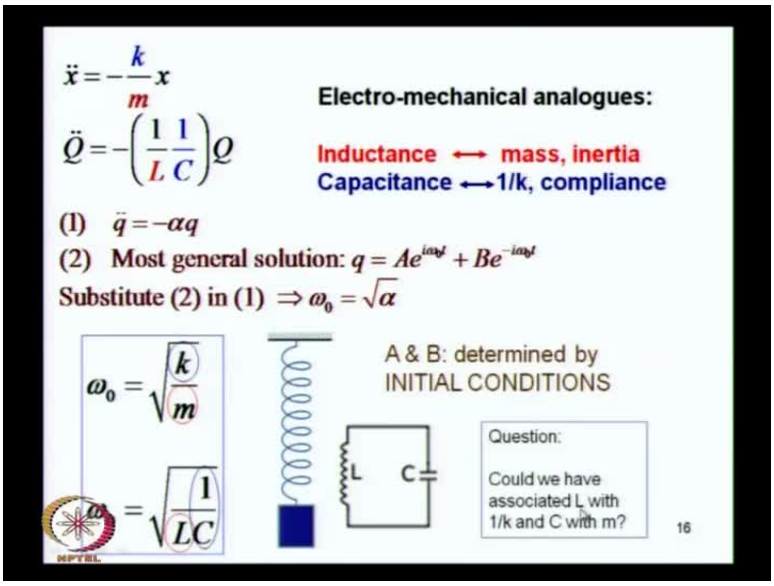

These are the electromechanical analogues, you see it in the equation of motion that in the mass spring oscillator the equation of motion is given by the acceleration, given by minus k over m times x. In the L C circuit, the second derivative of the charge; this is the second derivative of position - second time derivative of position, this is the second time derivative of charge, these are proportional to the corresponding quantities x over here, Q over here. Then, if you look at the proportionality, which is k over m, 1 over L, C over here, both having the same minus sign over here, then you can immediately see a correspondence between the quantities k m and L C. You see that the inductance place the same role in electric circuit as mass or inertia does in mechanical systems, a capacitance place the same role as the inverse spring constant, which is sometimes called as compliance.

This is the general equation of motion, now we will not worry about calling it is x for position displacement or charge for the corresponding analog in the electrical circuits. We will write this for some degree of freedom which is q; so q is our typical prototype of the displacement. It could be a position displacement, it could be angle, no matter what (Refer Slide Time: 37:53).

The equation of motion that we looking at is that the second time derivative of q, is proportional to the q itself, it is always directed to the word q equal to 0, so there is a minus sign, there is a proportionality alpha, this is the second order differential equation. Its most general solution will contain two unknown constants A and B, which will have to be determined; this is the most general solution. If you just put this solution in the differential equation, then it is matter of simple algebra to find that this omega 0 is nothing but the square root of alpha.

So, A and B are to be determined, these will be determine by the initial condition; you need two initial conditions for this: one is what is the value of q at t equal to 0 and what is the value of q dot at t equal to 0. So, these are the initial position and initial velocity for a mechanical system.

We can immediately see that if you use this general differential equation for an arbitrary degree of freedom q, then whether it is a mechanical system or an electrical circuit, you can get the solutions by solving the same differential equation. You do not have to do this physics again and again for different systems. You just set up one equation of motion and then it will be applicable to every situation, where you can use the quadratic potential approximation.

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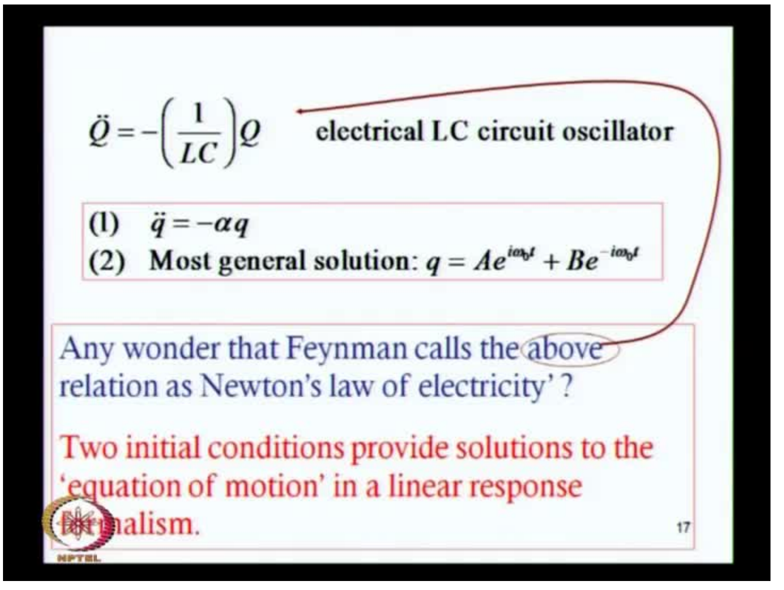

Now, I will leave you with this question, if we could have associated L with 1 over k and C with m - but I let you worry about it. So, this is the differential equation for the charge, Feynman in fact calls this in his famous lectures as Newton's law of electricity, means of course, Newton's law of motion is was meant for mechanical systems, at the time of Newton electrical circuits did not exist. Newton did not constitute this for any law, any electrical circuit, but this equation of motion is what Feynman calls as Newton's law of electricity, obviously for the reason that you have two initial conditions, which provide the solution for the equation of motion. Essentially, in the same formalism, in which, Newtonian mechanics is developed in the linear response formalism, because it is the same kind of situation which governs the dynamics.

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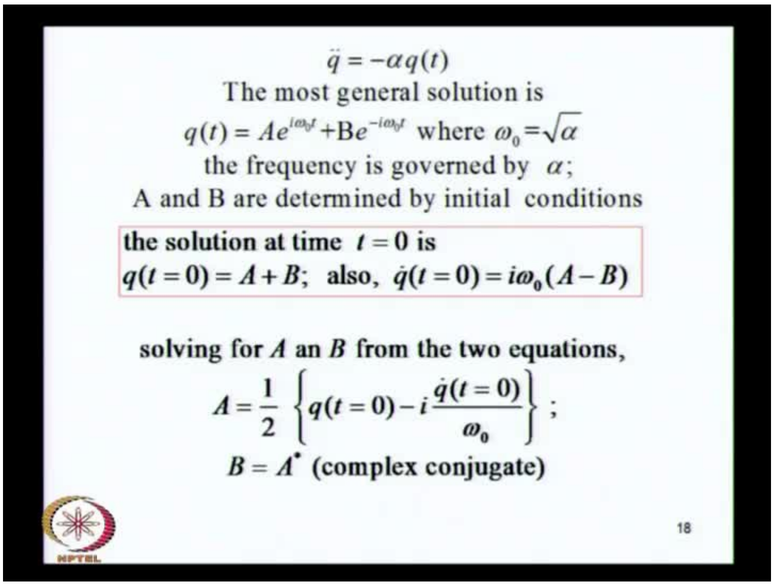

So, this is our story here, we have to determine A and B by initial conditions. The frequency nu or twice by nu which is omega 0, is determined by whatever this proportionality is between acceleration and the position, this is what determines the frequency. Time t equal to 0, if we put t equal to 0, e to the power 0 will give you one; same here, q at t equal to 0 will give you A plus B. So, q at t equal to 0 is A plus B, q dot will give you i omega 0 times A minus B, because you know the exponential function is the easiest one to differentiate, it is the derivative of e to i x or e to the x is the same as e to the x and then you can take one derivatives. So, this is the very easy function to deal with in element calculus.

You can then determine A and B, you have got two equations for these two unknowns A and B; one is this q at t equal to 0 must be the sum of A and B, the difference of A and B multiplied by i omega 0 is equal to the initial velocity. From the initial position then the initial velocity, you can get the values of A and B and that really set of the problem for you. Once you have it, you can predict what q will be at any time t, your problem is solved, because that is exactly what the mechanical problem is a marked. That if you can trace the evaluation of the mechanical system and predict what the position will be at a later time or what the velocity will be a later time, given what the initial position and velocity is, then you have solved the mechanical problem, so that is the complete solution.

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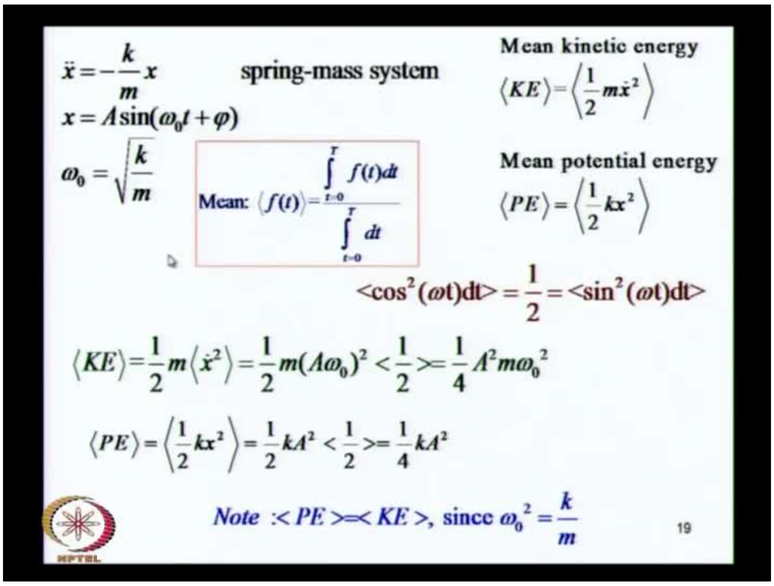

Now, what happens in ideal oscillator - I am presuming this is an ideal oscillator, by which, I mean that I am having faith in the approximation we have made. You know, we are pretending that the other terms, which we have ignored, the harmonic terms that we talked about little while ago, they are of no importance.

If it is an ideal system, in which this quadratic potential approximation gives us the true mechanical evolution of the system, then the average kinetic energy - what is meant by average? Average is you can see that the kinetic energy at different points of the oscillation will not be the same, but if you take the average, if you add up the kinetic energy at every instant of time, over the entire cycle, then you get divided by the total time period for that cycle, then you get the average kinetic energy. It is this average which I am talking about, it is indicated by writing kinetic energy in these angular brackets, so this angular bracket is my notation to represent the mean kinetic energy or the average kinetic energy.

Likewise, we have the mean potential energy, which is the average of this half k x square. The average will be determined by taking this time integral of any function over one period and dividing it by the total time period itself; that is how you will determine average. When you have to determine the average kinetic energy, you will need the average of x dot square. x dot will come from the time derivative of this, so you will get from the derivative of the sin function, you will get the cosine function. You will need the cosine square, whereas when you take the average of the potential energy, you will need to take the average of x square, which will be the average of the sin square function. In one case, you need the average of the sin square function, in the other, you need the average of the cos square function. Over the entire angle of 2 pi, this average is exactly equal to half or both, the sin square as well as the cosine square.

This enable us to determine what is the average kinetic energy by taking half m times the average of x dot square, this will turn out to be - this is elementary algebra, let you work out the details, so this is one fourth a square m omega 0 square. The reason is this ratio k over m is the square of omega 0.

As a result of that you get one fourth a square m omega 0 square, average of the potential energy also has got a factor of one fourth a square, here you have got k, but these two are exactly equal to each other, because omega 0 square is nothing but k over m. So, the average kinetic energy and the average potential energy are exactly equal. Now, this is quite appropriate for an ideal oscillator.

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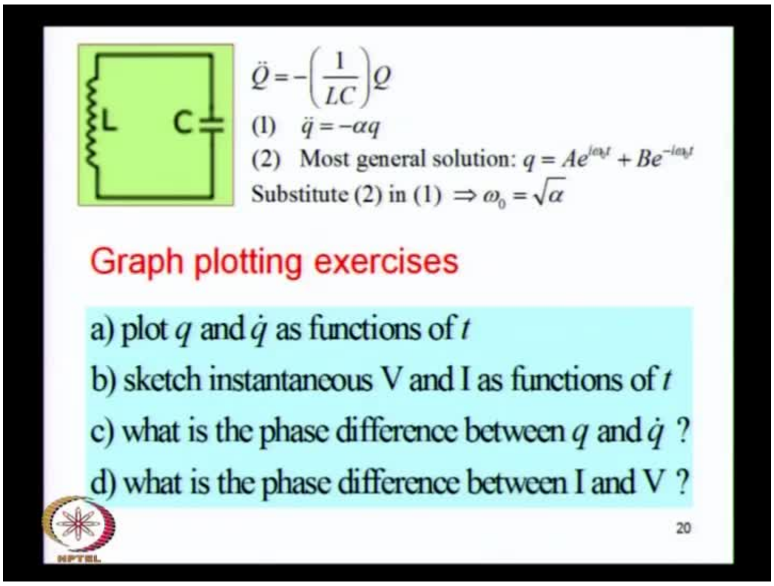

Now, I will set some exercises over here. We have already discussed this electrical circuit. We have already got the general solution; I suggest that you actually plot on a piece of graph paper the position and velocity as functions of time, also plot instantaneous voltage and instantaneous current as function of time. Find out what is the phase difference between position and velocity, likewise what is the phase difference between current and voltage? These are nice graphs to be plotted; it will help you to develop some insight into the physics of these systems.

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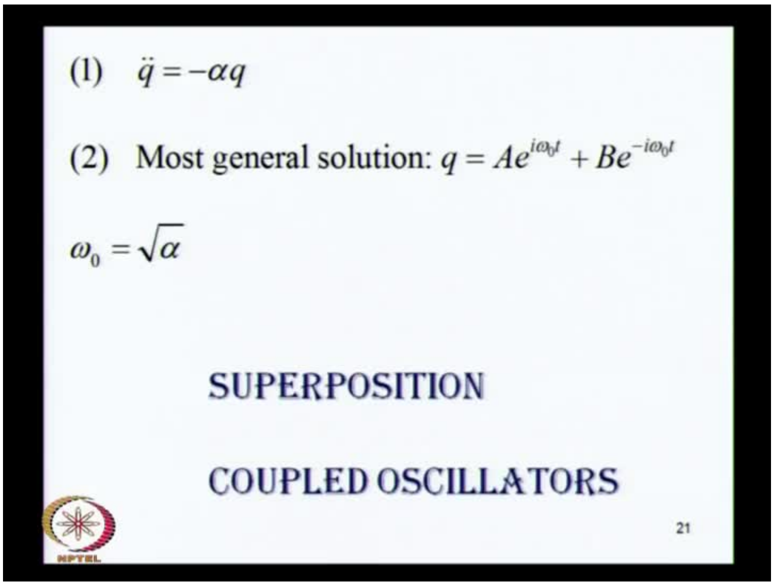

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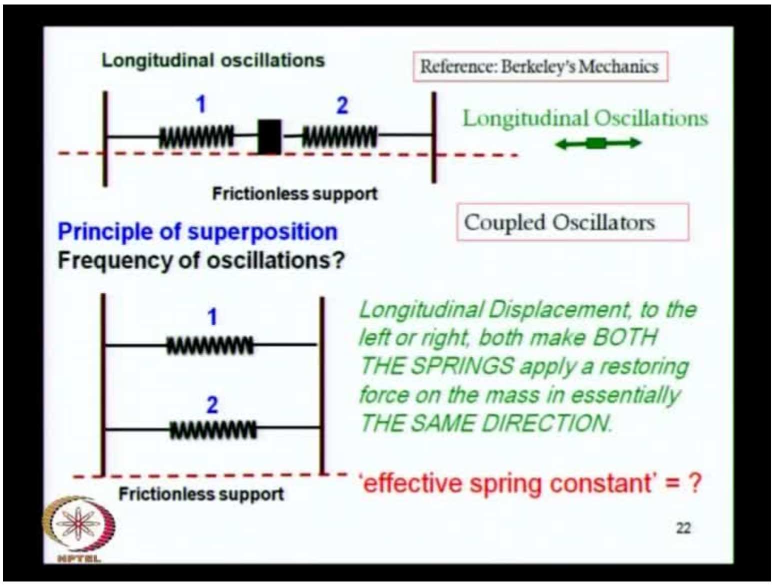

Now, we will talk about coupled oscillators, because one of the very fundamental principles in physics, which holds in a large number of physical phenomena, not that it holds in every case, but the principle of superposition, is a very strong principle in physics. A large number of physical systems are governed by the principle of superposition; we will now discuss it in the context of coupled oscillators.

If you have two oscillators, they are coupled and then what will be the net motion? So, this will be described by the superposition of two oscillators. The very nice example, which I have drawn from Berkeley's physics course; the volume one is on mechanics, there are simple examples of this kind that you have got a mass, which is attached to two springs 1 in this side and 1 on the other or all the two springs which are held together, they have this kind of the configuration as shown in this, they can carry out oscillation on a frictionless support.

In both these pictures, there is one thing which is common that if you displace this to the left, this spring, when spring 1 will push this mass back to the right and spring 2 will also pull it towards the right. So, the forces of the two springs will be in the same direction, although in one case it is pushed, in the other case it is pulled, but it will be in the same direction; it is also the same in this configuration. The effect of the two springs will be supportive of each other. So, if they are springs of the same spring constant, you can easily determine what the effective spring constant of this combination of this spring will be like. Because, you know that they are acting similarly, they are acting in the same direction; they are supporting each other, so they will enhance each other's elastic properties.

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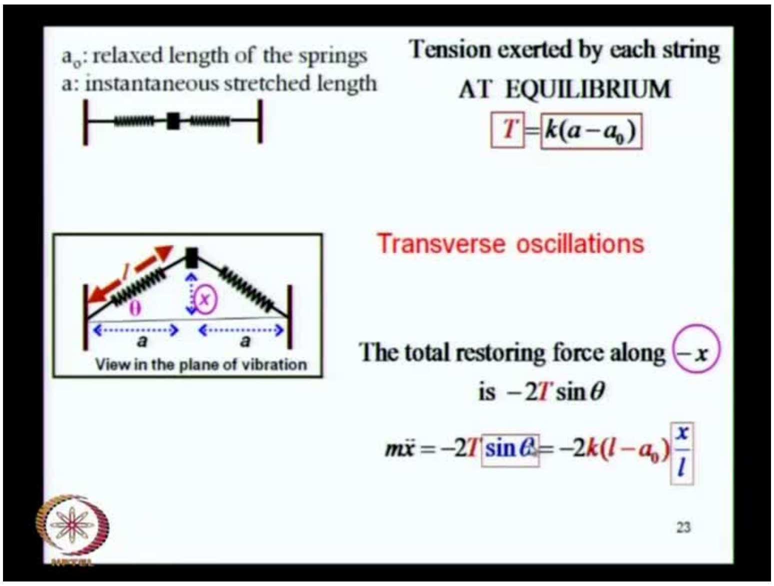

Now, what happens if you provide it with the transverse displacement? So, if you have a relax length of the spring which is a 0, if the instantaneous stretched length is a, then the stretch is through this difference a minus a 0, so the tension will be proportional to the stretch and the proportionality will be the spring constants; so that is the tension. If you have a displacement in the transverse direction, then this stretch will then sort of induce this object to be brought back towards the equilibrium. The total restoring force will be minus 2T sin theta, because it is a component of this tension along this direction which will come in. So, there will be a sin theta factor, sin theta as you can see from this figure is nothing but the ratio x over l, so you have 2k l minus a 0 x over l.

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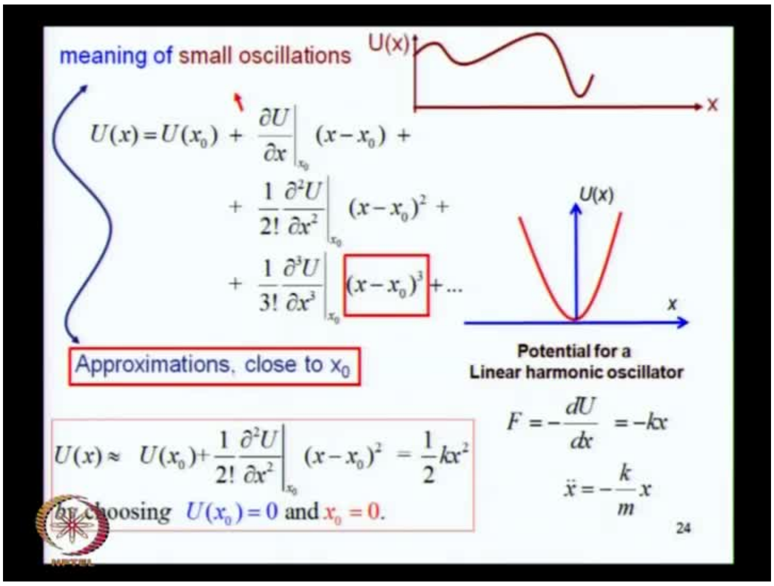

So, this is the kind of transverse oscillation that you can expect when you have a mass which is sandwiched between two springs, but set into oscillations not along the length of these springs, but along the transverse direction; so this is again an example of two springs acting together.

Again, we have already talked about small oscillations. I mention that you need to go close enough to the equilibrium point; this is what develops the notion of small oscillations, this gives - generates in our minds a feeling that the small oscillations, because in totally we have some idea of what is small oscillation is. So, if I say that an oscillation is like this, you will agree that it is small, but if I say that the oscillation is like this, you might tends to worry that I am no longer talking about small oscillations.

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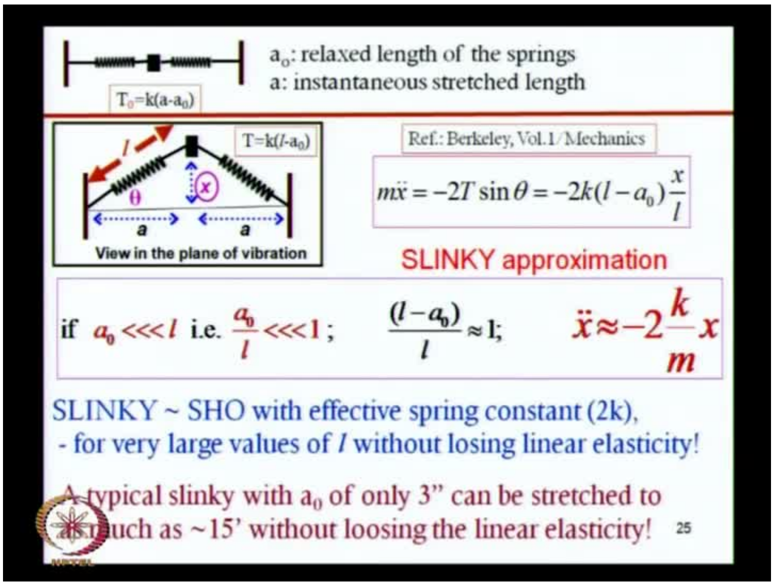

Now, one has to understand that all this at the back of the analysis has a certain approximations came. This smallness is always in the context of that approximations scheme. I am going to show you an example of what is called as a slinky. That you have these transverse oscillations, we have already written this equation of motion for the acceleration of an object. Here, minus 2T sin theta, we have written T is k l minus a 0, sin theta is x over l, now we can develop an approximation, which is known as the slinky approximation.

Now, this approximation can be made when a 0 is much less than l, because what happens when a 0 is much less than l, a 0 or l this ratio becomes much less than one, this factor which is l minus a 0 over l, this factor becomes nearly equal to 1. So, you can actually cancel l minus a 0 over l, this numerator l minus a 0 becomes nearly equal to l in this approximation and then resultant approximation is this x double dot. So, if you take m to the other side of the equation, it will come in the denominator, so this will become minus 2 k over m times x in the approximation that l minus a 0 over l is nearly equal to 1.

Now, this is obviously not exact, nevertheless it is a good approximation provided a 0 is much less than l, what we talking about? If the relax length of the spring is much less than the instantaneous stretch length, in that approximation, you have an equation of motion, which is exactly equal to that of the simple harmonic oscillator that the acceleration is proportional to the displacement. It is dictated by this minus sign, so it is always in the direction towards the equilibrium. The proportionality, which we call as the spring constant is scaled up by a factor of 2 in this case. So, this is what is called as the slinky approximation.

In this approximation, even if l itself is quite large, you can still have simple harmonic oscillation and then siddharth is going to show us the slinky. Siddharth would do you just come up. Then the cameras can be directed at this slinky, Siddharth likes to play with this, he has got what is called as a slinky. You can just hold it and show you, he will show you how this works. This is fairly large kind of oscillation; this is not something that - in the intuitive sense you will call as a small oscillation. This is of course not what you might call as a small oscillation, but in the context in which we are discussing it, in the approximation which works of; thank you Siddharth, could you get the image? You got it, ok.

So, in the context in which we are talking about, this is an excellent approximation and this is called as a slinky approximation. For this slinky, you have large values of a l, this slinky works as a simple harmonic oscillator over fairly large size oscillation without losing what we call as a linear elasticity; something with in the limit of the Hook's law, if you might want to call it that way. Typical slinky of a 0, which is about 3 inches; I have believed that about the size of the slinky that you heard about 3 inches more or less. Then, if you stretch it, it would go as much as you know 10 feet, maybe 12 to 15 feet and that is the typical slinky. If you really stretch it without losing the linear elasticity, it can still exhibit simple harmonic oscillations.

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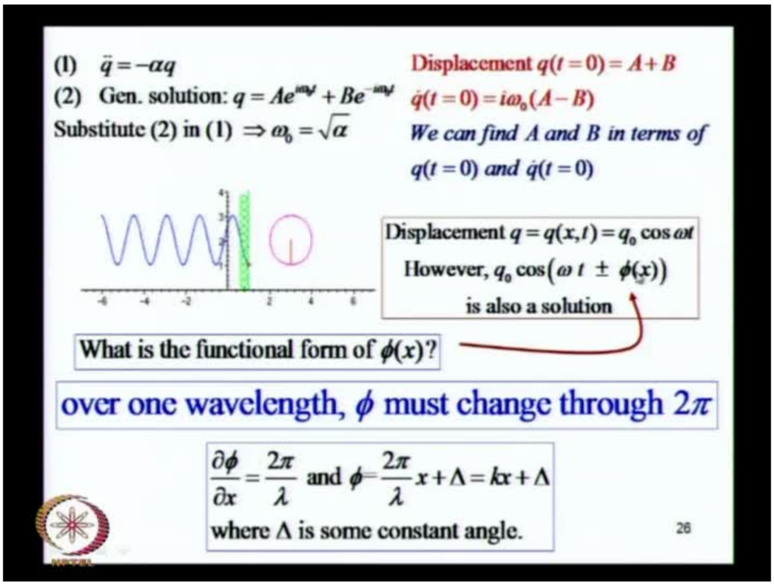

This phenomenon is very widespread, it describes a large number of situation as you can see from this figure, you are governed - the dynamics is governed by the simple equation of motion. We find that if q 0 cosine omega t is a function which represents the solution to the differential equation, then if you phase shift to the argument of the cosine function by an angle phi which must be a constant - but, constant with respect to what? Constant with respect to time, it can certainly depend on some other variable, so it can depend on x.

If you have a solution in which the argument of the cosine function is phase shifted through an angle which is independent of time, it must be independent of time, but it can depend on x, then this is also a solution of the same differential equation. You can have the solution of different kinds, they will all be phase shifted from each other determined by the initial conditions. We ask the general question, what should be the functional form of phi?

What should be the dependence on x of the function phi, what can it be? Because, phi some arbitrary function of the x in the main, in which we represented over here, now we are asking what should be the exact dependence on x of this function phi? Now, what we do know is that over one wave length phi must change by 2 pi, so the rate of change of phi with respect to x which is d phi by dx, must be 2 pi by lambda.

Because, over one wave length, the angle phi should change by 2 pi, so the rate of the change must be equal to 2 pi by lambda. Now, you got a simple differential equation, just integrated, the solution is obvious phi must be 2 pi by lambda times x plus some constant of integration, which you cannot rule out. So, this is the dependence of phi on x phi, must depend linearly on x, so phi is equal to k x plus delta is your solution and you get the exact dependence on the argument x of phi.

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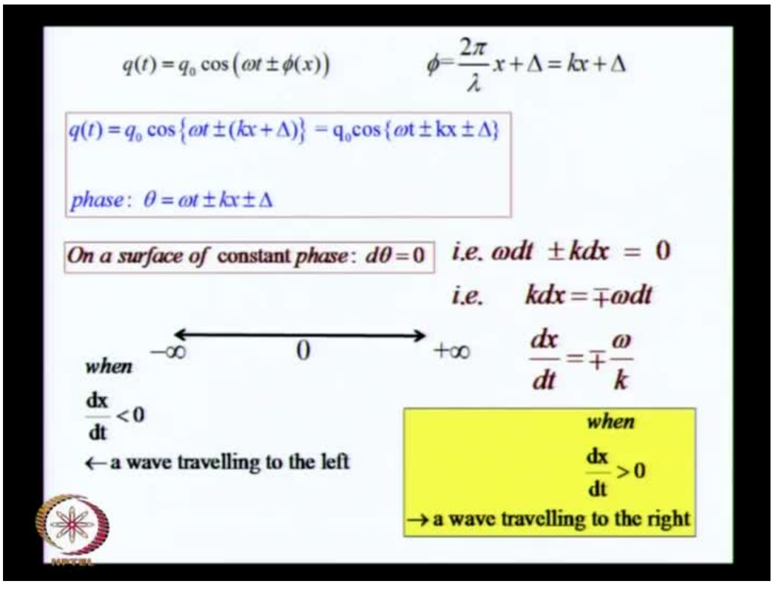

Here, you have phi equal to k x plus delta and the total phase of the cosine function is this angle here, which has got a time parameter, it has got a phase parameter. The time parameter is scaled by the circular frequency omega and this phase parameter is scalded by the spring constant k. Now, this spring constant is some appropriate factor in your linear harmonic oscillators, so you have to invoke the electro mechanical analogs if you are talking about the electrical circuit.

In the electrical circuit, k is not the spring constant, but you have already met the electromechanical analog, so you can replace mass by the inductance and the capacitance or with the inverse spring constant by the compliance. You have these oscillations, if you have got an x axis with an origin over here, then this is the phase of the cosine angle, if this is in an extended medium, so you do not have just a point which is an oscillation, but you got a whole medium (Refer Slide Time: 59:18). In this medium, this entire medium is having a set into some kind of oscillation; in this entire medium, what is the connection between oscillations at one point of the medium with that to another point of the medium. Are these oscillations in step with each other? Are they in step or are they out of step? Are they in phase or are they out of phase? Now, at some of the points, in this medium, they will be in step and some of the points, they will be out of step.

You can ask if you connect all the points in this medium at which the oscillation is in step, then you get a surface in a medium, it could be a flat surface, it could be a spherical surface, it could be a cylindrical surface or it could be a warp surface; it could have any kind of shape. But, a surface of constant phase will be described by all those points at which this theta is the same, the phase angle is the same. If this angle is the same, it has got a certain value which is a constant for all the point on the surface, then for all the point in the surface, the angle theta being the same, the variation in this angle is 0.

That is the equivalence statement of surface of a constant phase, because all the phase angles for all the points on the surface is essentially the same, so there is no change in the phase angle from one point to the other point. So, this gives us the definition of a surface of constant phase. A surface of constant phase is then such a surface in the medium at which all the points are in a state of oscillation, which are in step, in phase with each other, or exactly in the same step, they are described by the condition d theta is equal to zero. Now, theta is equal to this omega t plus or minus k x plus or minus delta, if the differential in this is equal to 0, then the differential on the right hand side is equal to 0 of which omega is a constant, k is a constant and delta is a constant.

The differential in omega is 0, the only parameter which you are changing in the right hand side is t and x, so dt theta equal to 0 gives us this relation omega dt plus or minus k dx equal to 0, because this arbitrary angle delta is a constant, so omega dt plus or minus k dx is equal to 0, which essentially means that if you take dx by dt, it will be either minus or plus the ratio omega over k. Now, omega over k is an intrinsically positive constant quantity; omega is a frequency, it is an intrinsically positive quantity, k is an intrinsically positive quantity, it is a spring constant, so dx by dt can be either negative or positive, both the possibilities are open. We therefore ask what is the nature of motion when dx by dt is negative and what is the nature of motion, when dx by dt is positive?

When dx by dt is negative, then x changes with time, but x decreases with time, because a decrease in x as time increases, this is the ratio of delta x over delta t, so as delta t increases, as the denominator increases if the numerator decreases, then the ratio will be negative. So, dx by dt will be negative, this will obviously represent x decreasing with time, the only way x could decrease as time increases is if the surface of constant wave is moving from the right to the left. So, this is a wave which is travelling to the left.

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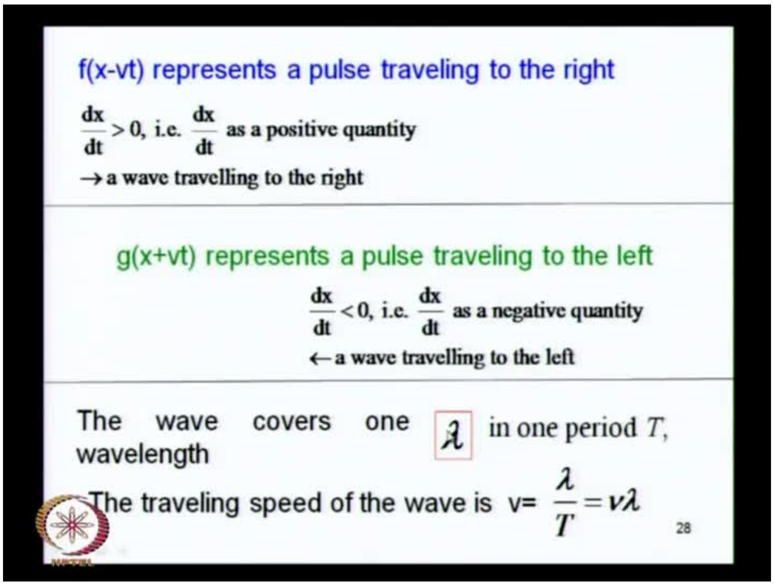

On the other hand, if dx by dt is positive, then x increases as time increases. x increases as time increases if and only if, the surface of constant phase moves from the left to the right. So, the dx by dt is what tells us the direction in which the surface of constant wave is travelling and this is how you recognize whether a wave motion is travelling either to the left or to the right.

This is true for any function, which is not just a cosine function; it could be some arbitrary function. If the functional form is generally given by x minus v t, which is some description of a pulse, then if dx by dt is positive, then you have the pulse which is travelling to the right. If you have a function where this sign is plus, then dx by dt will be less than 0, you will have pulse which is travelling to the left. So, a function x minus v t will be a pulse travelling to the right, whereas a function g x plus v t will be pulse which travels to the left.

Now, the travelling speed is automatically determined, because a wave covers one wave length in one periodic time, so the ratio of this distance over time is what gives you the velocity. This 1 over t is nothing but the frequency, so v equal to nu lambda is a very famous relationship that you would have used.

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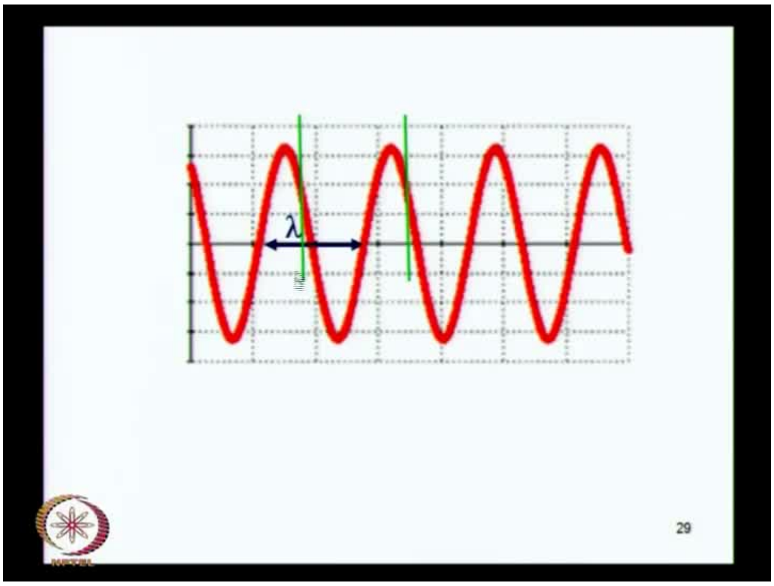

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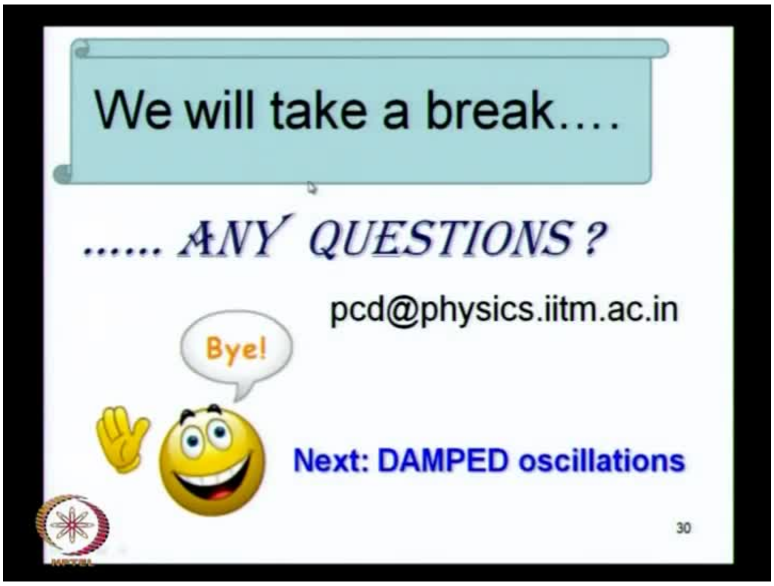

You can see the picture that if you have emotion of this kind, you have got one wavelength which is described between these two points, if you take any point over here and see how this propagates in one direction, and then you know you have the corresponding propagation of wave. We will take a little break over here, if there are any questions I will be happy to answer; otherwise we will take a break, and then we will be discussing how motion of this oscillator gets affected when damping is introduced.