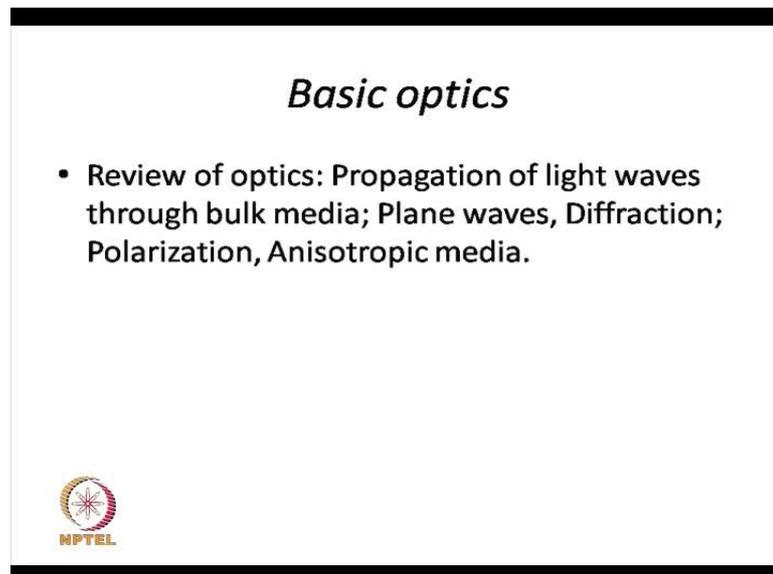


Quantum Electronics
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Module No. # 01
Brief Review of Electromagnetic Waves;
Light Propagation through Anisotropic Media.

Lecture No. # 01
Introduction

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Basic optics

- Review of optics: Propagation of light waves through bulk media; Plane waves, Diffraction; Polarization, Anisotropic media.



So, this is a course on quantum electronics; the course contents are given here, as you can see. So, we will start with some basic optics, looking at plane waves, and little bit of diffraction, polarization and discuss in bit more detail an anisotropic media.

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Nonlinear optical effects

- Nonlinear polarization, Wave equation
- *Second order nonlinear effects*: Second harmonic generation, Sum and difference frequency generation, Parametric amplification, parametric fluorescence and oscillation, Periodically poled materials and their applications in nonlinear optical devices.
- *Third order effects*: Self Phase modulation, Cross Phase modulation, Four wave mixing.



The second part actually is looking at nonlinear optical effects. You all familiar with linear optical effects; this will introduce the nonlinearity in the system. And we will primarily study things like, second harmonic generation, sum and difference frequency generation, parametric amplification, etcetera.

So, there are processes in which you can send light at a particular wavelength inside a crystal and the light coming out of the difference frequency - double the frequency for example. So, second harmonic generation, you can also come out with light with a lower frequency, which is called parametric down conversion. So, you send in blue light; you come out with red light at the output or you send in red light and come out blue light at the output.

So, these are nonlinear processes and they find a lot of applications today, and to achieve this, we need to discuss something about periodically poled materials, and so on which is given here. So, all this comes under class of second order nonlinear effects; we will also discuss briefly third order effects of self phase modulation, cross phase modulation, four wave mixing. So, till this point, we will be primarily looking at classical aspects.

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Quantum aspects of light

- Quantization of the electromagnetic field; Number states, Coherent states and Squeezed states of light and their properties; Beam splitters and interferometers; Spontaneous parametric down conversion; Concept of quantum entanglement; Application of optical parametric processes to generate squeezed states of light and entangled states.



This second part is actually looking at the quantum aspects of light. So, we will start with quantization of electromagnetic field and look at some of the concept of photons, which are arise from there. Look at states of lights, which are called coherent states, squeezed states and some other properties; we will look at some interesting devices like beam splitter; beam splitter is a very simple device, but in quantum, **it is a very very interesting property it has** and how it is used in interferometers?

There is a process called spontaneous parametric down conversion, which is a very important process; which is use today in many of the quantum applications of light. And there are states like coherent, squeezed states in which the noise in the light is much lower than what you can achieve normally with a laser.

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Nonlinear optical effects

- Nonlinear polarization, Wave equation
- *Second order nonlinear effects*: Second harmonic generation, Sum and difference frequency generation, Parametric amplification, parametric fluorescence and oscillation, Periodically poled materials and their applications in nonlinear optical devices.
- *Third order effects*: Self Phase modulation, Cross Phase modulation, Four wave mixing.



And of course, there are some very interesting entangled states of light. So, what I will assume in the course is you have a background and electromagnetism; you done Maxwell's equations, waves, etcetera. We will very briefly review the beginning and I would assume that, you have the enough mathematical background, Fourier transforms, etcetera, etcetera.

And also you have done some quantum mechanics course, because **I will not may be** I will recall some basic, **some basic**, aspects of quantum mechanics, when we start the quantum aspects, but **what** I would assume that you have done a course in quantum mechanics sometime and if not, you would have to put in some extra effort to get the background that is necessary for the course.

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Assessment scheme	
• Minor I	20
• Minor II	20
• Quizzes & assignments	20
• Major Test	40
• Total	100



(Refer Slide Time: 04:17)

Suggested texts and reference material
• <i>Quantum Electronics</i> , A Yariv, John Wiley, NY, 1989
• <i>Optical Electronics</i> , A Ghatak and K Thyagarajan, Cambridge Univ Press, UK, 1989
• <i>Introductory quantum optics</i> , C. Gerry and P. Knight, Cambridge Univ. Press, UK, 2006
• <i>Quantum Optics: An Introduction</i> , M.Fox, Oxford Univ. Press, UK, 2006

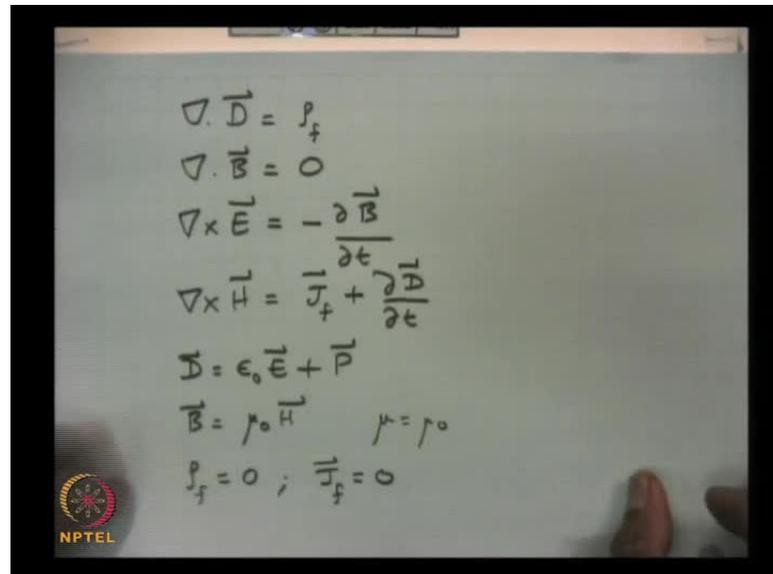


So, as I said here, there will be quiz every week and may be at the end of the semester we will have some assignments, term papers or assignments as we go through the semester and so **as I am written here**, if you have any difficulty at any time, you most welcome to contact me.

So, today let us start with some basic optics; what is the plane wave, what is the meaning of a plane wave? – Anyone - What is plane in a plane wave? The wave front is a plane.

So, first, of let me start with writing the Maxwell's equations. So, can one of you tell me the Maxwell's equations in media, what is the first equation?

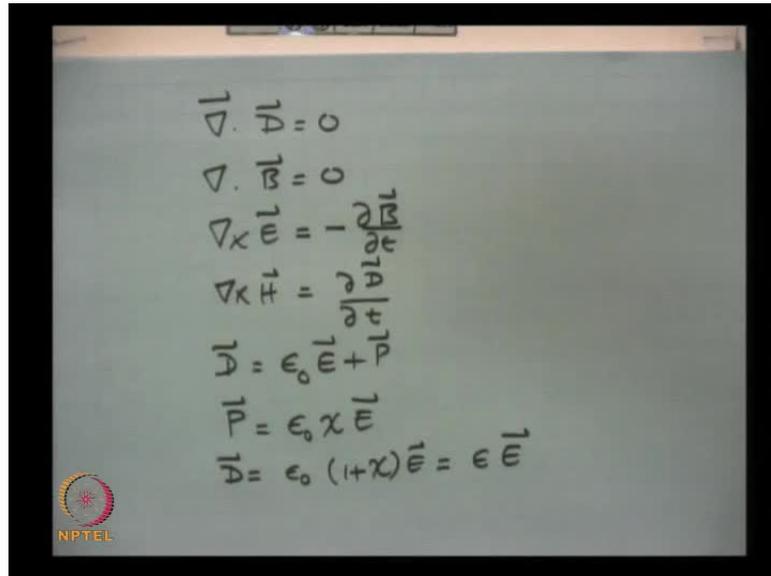
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$$\begin{aligned}\nabla \cdot \vec{D} &= \rho_f \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \\ \vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \vec{B} &= \mu_0 \vec{H} \quad \mu = \mu_0 \\ \rho_f &= 0 ; \vec{J}_f = 0\end{aligned}$$

Divergence D is equal to rho f, where rho f is the free charge density. Divergence B is equal to 0, curl E is equal to minus del B by del t and curl H is equal to J f plus del D by del t and how do you define D? D is defined as epsilon 0 E plus p and we will in towards the course, we will assume the media to be non-magnetic. So, we will assume B is equal to mu naught H .

So, we will assume mu is equal to mu naught, for all media. Now, rho f is the free charge density. So, again in the course, we will not have any free charges anywhere. So, rho f will be assumed to be 0 and no free currents J f is also equal to 0.

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$$\begin{aligned}\nabla \cdot \vec{D} &= 0 \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} \\ \vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \vec{P} &= \epsilon_0 \chi \vec{E} \\ \vec{D} &= \epsilon_0 (1 + \chi) \vec{E} = \epsilon \vec{E}\end{aligned}$$

So, primarily we will have the four equations as follows: so, divergence D is equal to 0 divergence B is equal to 0 curl E is minus del B by del t and curl H is equal to del D by del t with D is equal to epsilon 0 E plus P.

So, normally you write this P in terms of E as epsilon 0 chi E and so you write D as epsilon 0 into 1 plus chi into E, which is written as epsilon E chi is the electric susceptibility and epsilon is the permittivity of the medium epsilon 0 is the permittivity of free space.

Now, if chi or epsilon is independent of position that means it does not depend on x y z the medium is called homogeneous, if this when I write this equation P is equal to epsilon 0 chi E, I am assuming that the polarization is proportional to electric field.

So, that means it is a linear medium. The relationship between P and E is linear. So, it is called a linear medium and if I assume chi to be a scalar quantity, this equation tells me P and E are parallel to each other and so, D and E are also parallel to each other and that is an isotropic medium.

So, this equation, when I write this equation, I assume the medium to be linear, if chi is independent of x y z, it is homogeneous and if chi is a scalar, it is an isotropic medium. So, this equation here when you normally write, D is equal to epsilon E with epsilon

independent of position and epsilon is a scalar it corresponds to linear homogeneous isotropic medium.

In the course, that we do here we will assume homogeneity, we will not look at non inhomogeneous media, but we will come out of this restriction on linearity and isotropic. So, we will first look at anisotropic media in which D and E may not be parallel.

And once we understand those properties, we will go into a system, where this equation have to be modified P is no more proportional to E, you can have higher order contributions coming from higher expressions in E vector.

(Refer Slide Time: 10:00)

The image shows a chalkboard with the following handwritten equations:

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{k} \cdot \vec{r} = \text{Constant}$$

$$k_x x + k_y y + k_z z = \text{Constant}$$

$$\nabla \cdot \vec{D} = 0$$

$$\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0$$

$$\nabla \cdot [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] = 0$$

$$\vec{k} \cdot \vec{E}_0 = 0$$

In the bottom left corner of the chalkboard, there is a small circular logo with the text "NPTEL" below it.

So, before we move on to that let me just, very close quickly recall, basic electromagnetism in terms of plane waves and what are the properties of plane waves? So, first is let me take an electric vector of this form, E is equal to E 0 exponential i k dot r minus omega t that is the electric vector. This is the propagating wave because it has expressional form k dot r minus omega t .

How do I check whether this is a plane wave? This component represents the phase of the wave, so surface of constant phase at any time will be k dot r is equal to constant. This means k x x plus k y y plus k z z is equal to constant. And that is an equation of a plane a x plus b y plus c z is equal to constant, is an equation of a plane and this plane that the orientation of plane is perpendicular to the vector defined by k x k y k z.

If you go back and look at geometry, you will see that this is an equation of a plane and that plane is perpendicular to the vector, whose components are k_x , k_y , k_z . So, this is a plane wave, because the wavefront or the surface of constant phase is the plane and this surface of constant phase is perpendicular to the k vector. k vector represents the direction of propagation of the wave; the direction of propagation of the phasefront of the wave. So, the phasefront may be like this; so plane wavefront is going like this.

So, this direction a k vector direction is like this Refer slide time 12:08 and that is a phasefront. So, k vector is like this and the corresponding electric field is given by this equation, which I have written, here also we have by writing exponential minus $i\omega t$, I am saying that the wave is a monochromatic wave, because this wave have this form for all times. So, this is only one frequency present.

If I take time dependence in this E_0 vector or if I take a phase, which is time dependent inside the exponential. The wave will be no more monochromatic or if I say that this is electric field from t is equal to 0 to capital T . I restricted the length of the wave train; the wave is no more monochromatic.

So, this expression represents the monochromatic wave. A plane wave propagating along the k vector direction, can I have an arbitrary E_0 vector direction, please note that this electric field has to satisfy all the Maxwell's equations.

So, let me look at the first equation. Now, so Divergence D is equal to 0. Now, we are assuming a linear homogeneous isotropic medium. So, D is equal to ϵE where ϵ is a constant. So, this implies that divergence E is equal to 0.

So, ϵ is a scalar constant. So, when you take a divergence of D vector, you can divergence E is equal to 0. So, this means that $\text{div} \cdot E_0 \text{ vector } e^{-i(k_x x + k_y y + k_z z - \omega t)}$ is equal to 0.

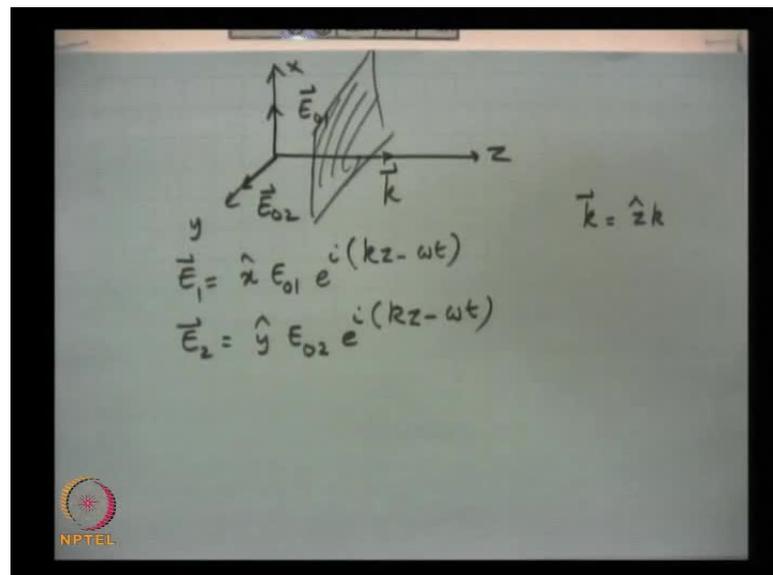
Now, I will leave to you would expand this E_0 is a constant but there is a x dependence in the exponential factors. So, if you expand this, you can show that this equation implies; so this divergence has $i \text{cap} \text{del} \text{ by } \text{del} x + J \text{cap} \text{del} \text{ by } \text{del} y + K \text{cap} \text{del} \text{ by } \text{del} z$. So, you expand this inside. So, there are x, y, z components of this vector, here you take the divergence and you can show that this implies $k \cdot E_0$ is equal to 0.

So, this is nothing but saying that electromagnetic waves are transverse, because \vec{k} vector direction represent, the direction of propagation of the wave and \vec{E}_0 vector has to be perpendicular to \vec{k} vector.

So, this implies that the electric vector is always perpendicular to the \vec{k} vector direction in a plane wave. Now, there are two independent directions for any given direction of propagation, if the \vec{k} vector is like this. So, the electric vector \vec{E}_0 vector has to be in this plane, so I can have two independent orientations.

So, if this orientation is like this or like this. There are two perpendicular orientation, which are independent to each other. So, I will define two independent polarization shades of light, for every propagation direction defined by \vec{k} vector that can be two different directions of \vec{E}_0 vectors, which are independent and both perpendicular to \vec{k} vector.

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For example, if I choose a propagation along z direction, suppose \vec{k} vector is like this, this is x y, I can have a electric vector. So, \vec{E}_0 vector could be like this or \vec{E}_0 could be like this. So, this is E_{01} , E_{02} and these are two independent directions of \vec{E}_0 vector both corresponding to wave propagating along a direction which I have called as z.

So, if I have for example, if I write E is equal to $\hat{x} E_{01} e^{i(kz - \omega t)}$. Now, what happens to $\vec{k} \cdot \vec{r} - \omega t$. So, remember \vec{k} vector becomes $\hat{z} k$ because \vec{k}

vector is along this z direction my choice of z direction is along the k vector direction. So, $\vec{k} \cdot \vec{r}$ becomes simply $k_z z$ so this is $k_z z - \omega t$.

So, what is this? These are planes. Now, $k_z z = \text{constant}$ is a plane parallel to the x y plane. So, these are plane waves. So, the planes of wave wavefront are like this; so it is propagating like this in the z direction. The phasefront is perpendicular to the z direction.

I can have another one. So, \vec{E}_2 vector - for example, is $\hat{y} E_0 e^{i(k_z z - \omega t)}$ and these are two independent polarization states. These are two independent polarization states - one along the x cap direction; one along the y cap direction and you give me any polarization state **which is** which has this electric vector in the x y plane, I can write it as a superposition of x cap and y cap components.

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$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{k} \cdot \vec{r} = \text{constant}$$

$$k_x x + k_y y + k_z z = \text{constant}$$

$$\nabla \cdot \vec{D} = 0$$

$$\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0$$

$$\nabla \cdot [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] = 0$$

$$\vec{k} \cdot \vec{E}_0 = 0$$

So, this is one basis set, I can use these two linearly polarized states of light as a base is to describe any state of polarization of a light wave propagating along this z direction now what should be the direction of the magnetic field of this electric field. So, if I take this electric vector is \vec{E} is equal to $\vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$.

(Refer Slide Time: 18:45)

The whiteboard contains the following handwritten equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{B} = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$
$$i \vec{k} \times \vec{E} = i \omega \vec{B}$$
$$\boxed{\vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E}}$$

The NPTEL logo is visible in the bottom left corner of the whiteboard image.

I can use another Maxwell's equation and find out the direction of the magnetic field. So, for example, if I take this equation del cross E is equal to minus del B by del t. Now, the magnetic field can also be written as B 0 vector exponential i k dot r minus omega t.

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The whiteboard contains the following handwritten equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{B} = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$
$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$
$$\vec{k} \cdot \vec{r} = \text{Constant}$$
$$k_x x + k_y y + k_z z = \text{Constant}$$
$$\nabla \cdot \vec{D} = 0$$
$$\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0$$
$$\nabla \cdot (\vec{r} e^{i(k_x x + k_y y + k_z z - \omega t)}) = 0$$

The NPTEL logo is visible in the bottom left corner of the whiteboard image.

It has the same phase distribution, as the electric field and its magnitude is B 0 vector magnitude and direction are defined by the B 0 vector. So, if I substitute into this equation, again you can use this electric field; substitute into this curl on the left hand side and this magnetic field on the right hand side; what you can show is... so ok now.

Let me tell you for this kind of an expression, just like I showed you divergence means simply $\mathbf{k} \cdot \mathbf{curl}$ would imply $\mathbf{k} \times$. So, I leave this to you again, substitute the electric field vector calculate the curl of the vector and you can show this implies $\mathbf{k} \times \mathbf{E}$ is equal to $i \omega$ times \mathbf{B} or \mathbf{B} is equal to $1/\omega \mathbf{k} \times \mathbf{E}$.

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Handwritten mathematical derivation on a whiteboard:

$$\vec{k} \cdot \vec{r} = \text{Constant}$$

$$k_x x + k_y y + k_z z = \text{Constant}$$

$$\nabla \cdot \vec{D} = 0$$

$$\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0$$

$$\nabla \cdot [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] = 0$$

$$\vec{k} \cdot \vec{E}_0 = 0$$

$$\boxed{\vec{k} \cdot \vec{E} = 0}$$

The NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, this is just review of the electromagnetics that you must have done some time back. So, that is the relationship between the magnetic field and the electric field. So, this equation told me this is actually $\mathbf{E} = 0$ vectors is \mathbf{E} vector. So, this also tells me $\mathbf{k} \cdot \mathbf{E}$ is equal to 0.

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$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{B} = B_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$
$$i \vec{k} \times \vec{E} = i \omega \vec{B}$$
$$\vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E}$$
$$\vec{S} = \vec{E} \times \vec{H}$$

So, the electric field of this wave is perpendicular to k vector direction. The magnetic field is perpendicular to k vector and E vector $k \times E$ is perpendicular to k and E and so, B is perpendicular to k as well as E .

So, for example if I choose my k vector like this E is here, B will be here, E , B and k form a right handed coordinate system. Both E and B are perpendicular to k vector E and B are perpendicular to each other with E , B and k forming a right handed coordinate system.

So, this in plane electromagnetic waves, k vector is always perpendicular to E vector and B vector. Please note that we are discussing right now, isotropic media in anisotropic media things **were** will change. So, in media like glass or air or water, the direction of electric vector and magnetic vector are both perpendicular to k vector; and E and B are perpendicular to each other.

Now, what define the direction of propagation of energy? What is vector, the pointing vector. So, pointing vector, is given by $E \times H$ and in this case, because we are assuming μ is equal to μ_0 and scalar B and H , have the same direction. So, this is also the direction of H and this is also the direction of E , D vector, because D is equal to ϵE and ϵ is a scalar.

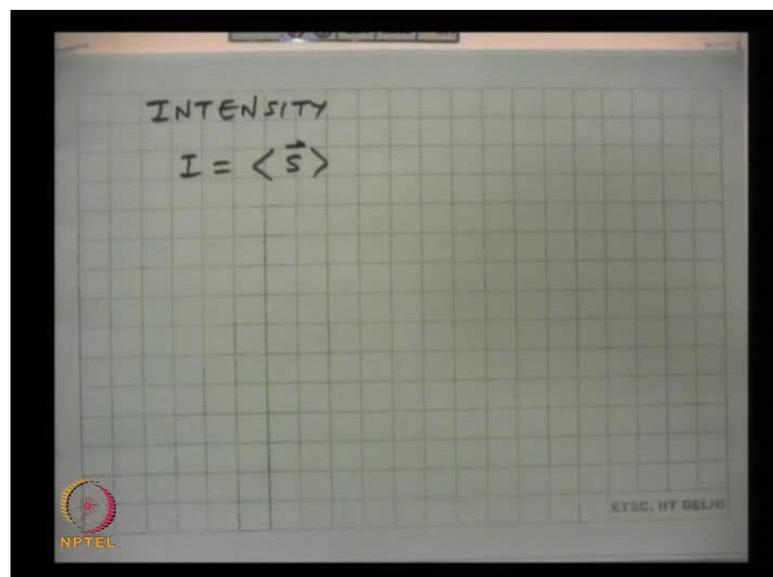
So, D and E are parallel, B and H are parallel, E B and K form a right handed coordinate system and so, S vector is also like this, E cross H is also in the same direction as k vector. So, the wavefront propagates like this, the energy also propagates like this.

Why I am telling you this is because in anisotropic media, this does not happen. I will show you that k and S are not parallel to each other. So, wavefront is propagating in one direction, but the energy is propagating in a slightly different direction.

Now, E D and k here are forming a right handed coordinate system. So, this media called right handed media. There is a lot of work going on these days, in a medium called metamaterial or a negative refractive index medium or left handed medium in which E B and k form a left handed coordinate system.

The properties of that medium must be different from the properties of this media. So, we will not touch upon that in the course, but this is just to tell you that in normal media that we are looking at E B and k are at right angles to each other and form a right handed coordinate system.

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Now, I can actually calculate, what is called as the intensity of the wave? How much in energy that the wave is carrying per unit time; so, this is defined as the time average of the pointing vector, so intensity is defined as the time average of the pointing vector.

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$$\begin{aligned}\vec{E} &= \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \\ \vec{k} \cdot \vec{r} &= \text{Constant} \\ k_x x + k_y y + k_z z &= \text{Constant} \\ \nabla \cdot \vec{D} &= 0 \\ \vec{D} &= \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0 \\ \nabla \cdot [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] &= 0 \\ \vec{k} \cdot \vec{E}_0 &= 0 \\ \boxed{\vec{k} \cdot \vec{E} = 0}\end{aligned}$$

Now, let me recall, please note here that - let me come back - to this equation and emphasize; one more point we are writing a complex equation, but electric fields are always real; magnetic fields are always real.

Why am I using a complex equation? This is mathematically much easier to work with, but then finally, I must remember that all quantities are real. So, I have taken the real part of electric vector, real part of magnetic field vector, etcetera.

But remember, that the sum of the real parts of two numbers is equal to the real part of the sum of the two complex numbers. But the real part of the product of two complex numbers is not equal to the product of the real parts of complex numbers.

So, whenever operations like product between complex numbers or squaring. These are called nonlinear operations; a plus b whole square is not equal to a square plus b square; this squaring is not a linear operation.

So, whenever you have a nonlinear operation, I would be careful, I should not use this equation like this, I have to use always **use** real fields. So, either I use like plus this complex conjugate, which is what we will do to make the quantity always real or I must use cosine or sine functions.

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INTENSITY

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{k} \cdot \vec{r} = \text{Constant}$$
$$k_x x + k_y y + k_z z = \text{Constant}$$
$$\nabla \cdot \vec{D} = 0$$
$$\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0$$
$$\nabla \cdot (\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}) = 0$$
$$\vec{k} \cdot \vec{E}_0 = 0$$

$\vec{k} \cdot \vec{E} = 0$

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So, **when you...**, if you recall your electromagnetics, if I actually start with cosine forms of electric vector, then I can have to calculate \vec{E} vector real fields, \vec{H} vector real fields. Calculate $\vec{E} \times \vec{H}$. The pointing vector, which will be time dependent, because \vec{E} will be of the form of $\cos k_x x + k_y y + k_z z - \omega t$ \vec{H} will be $\cos k_x x + k_y y + k_z z - \omega t$. So, $\vec{E} \times \vec{H}$ will be $\cos^2 k_x x + k_y y + k_z z - \omega t$.

But intensity is the time average. The energy crossing per unit area, per unit time perpendicular to the propagation direction, so when I take a time average, the \cos^2 factor will give you a factor of half.

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$$\begin{aligned}
 \text{INTENSITY} \\
 \mathbf{I} = \langle \vec{S} \rangle &= \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*] \\
 &= \frac{1}{2} \text{Re} [\vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \times \frac{1}{\omega \mu_0} \vec{k} \times \vec{E}^*] \\
 &= \frac{1}{2} \text{Re} [\vec{E} \times (\vec{k} \times \vec{E}^*)] \frac{1}{\omega \mu_0} \\
 &= \frac{1}{2\omega \mu_0} \text{Re} [|\vec{E}|^2 \vec{k} - 0] \\
 &= \frac{k}{2\omega \mu_0} |\vec{E}_0|^2 = \frac{k}{2\omega \mu_0} |\vec{E}_0|^2 \hat{k}
 \end{aligned}$$

Now, **in** when I start to use complex notation for electric vector, I have to **I what I** get equivalent, that this will be half of real part of E cross H star taking time average, with real electric fields and real magnetic fields is the same as taking the real part of E cross H star with the factor of half outside.

So, may be you have done in the course, or if you have not done please take a cosine form of this calculate. Assuming real fields E cross H take a time average. Assume complex fields, calculate H, then calculate this and you will find the same thing.

So, let me calculate this for the electric and magnetic fields, we have got; so this is equal **to...**. So, this is a vector E cross H star gives me a vector, which is the direction of propagation, which we know its k vector direction.

Because E, H and k form a right handed system. So, E cross H will be along the k vector direction, so that is the direction of energy flow. So, let me substitute this here; so half of real part of... So, E is E naught e to the power i k dot r minus omega t cross H star. Now H, we have calculated B here.

So, this is 1 by omega mu 0 k cross. Let me keep this E. So, let me, **let me**, write this as half of E star; this is half of real part of E cross k cross E star and there is 1 by omega mu 0 comes out.

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$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{B} = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$
$$i \vec{k} \times \vec{E} = i \omega \vec{B}$$
$$\vec{B} = \frac{1}{\omega} \vec{k} \times \vec{E}$$
$$\vec{S} = \vec{E} \times \vec{H}$$

\vec{D}, \vec{E}

\vec{k}

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Remember, this is H star; so if I use this equation, this B star becomes $\frac{1}{\omega} \vec{k} \times \vec{E}$ and so what is this quantity here? $\vec{E} \times \vec{k} \times \vec{E}$ a cross \vec{B} cross c , can you expand this and tell me.

So, this is E or E star. So, this is real part of modulus E square into k minus 0 and modulus E square is anyway real. So, this is simply k vector by $2 \omega \mu_0$ into modulus E square. This is actually modulus E square modulus E_0 square and E_0 is **the electric** the amplitude of electric field here. So, actually this intensity has this vector here; simply because I am using k cross H star here. So, this I can write, this as k by $2 \omega \mu_0$ modulus E_0 square into k unit vector.

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The image shows a handwritten derivation on a grid background. At the top, the equation is $I = \frac{k}{2\omega\mu_0} |E_0|^2$. Below it, $k = \frac{\omega \cdot n}{c}$ is written. A box contains the equation $I = \frac{n}{2c\mu_0} |E_0|^2$. To the right of the box, the relationship $c^2 = \frac{1}{\epsilon_0\mu_0}$ is written. Below the boxed equation, it is simplified to $I = \frac{nc\epsilon_0}{2} |E_0|^2$. In the bottom left corner, there is an NPTEL logo. In the bottom right corner, the text 'IIT DELHI' is visible.

This vector simply implies that the energy is flowing along the direction of k vector and the magnitude is simply k by $2\omega\mu_0$ mod E square; many times you will write this in a slightly different form. So, let me forget about the vector here. So, this is k by $2\omega\mu_0$ mod E naught square. Now, how is k related to ω ? k is equal to ω by the velocity of the wave. So, if there is the medium. There is a refractive index; so, this intensity becomes, simply n by $2c\mu_0$ mod E naught square.

Now, you know that c square is 1 by $\epsilon_0\mu_0$. So, you can also write this as $nc\epsilon_0$ by 2 mod E naught square, we just write $c\epsilon_0$ is 1 by μ_0 , then you get $n\epsilon_0$ by 2 mod E naught square. So, these are all interchangeable forms of the intensity, it depends on the refractive index in the medium, because of velocity of light, velocity of the electromagnetic wave depends on refractive index, and it depends on the peak electric field in turn.

Starts energy crossing per unit time, per unit area and this is an expression, which you will frequently use. Later on, when we look at all the nonlinear effects, because we need to calculate how much of light which I shine gets converted to the new frequency?

Now **(())** the intensity is simply depends on the electric field. The question is, if I launch an electric field with a certain frequency, the frequency I get new waves at new frequencies, which will have their own electric fields. So, I can calculate the intensity of

that wave, of the new frequency using the same expression for the electric field of that wave.

So, if I have a medium and if I launch omega frequency, it has certain intensity. So, at the wave propagates, it will generate two omega frequency, which means it will generate electric fields at 2 omega. So that electric field will define what is the intensity of the wave at two omega frequency.

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$$\begin{aligned}\nabla \cdot \vec{D} &= 0 \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} &= \frac{\partial \vec{D}}{\partial t} \\ \vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \vec{P} &= \epsilon_0 \chi \vec{E} \\ \vec{D} &= \epsilon_0 (1 + \chi) \vec{E} = \epsilon \vec{E}\end{aligned}$$

All these remains the same, the electric field because these are essentially, the change that will happen is how is P related to E. This equation which we wrote earlier will get modified. This equation and this equation will get completely modified because P is no more proportional to E.

So, if I launch an electric field, a certain wave at a certain electric field. What is polarization? Polarization is the dipole moment per unit volume, so when I launch a wave with a certain electric field, the polarization is not proportional to e vector and as I would show you there will be components of polarization at new frequencies.

And because polarization represents dipole moment per unit volume that means the dipoles are oscillating at new frequencies which means they will radiate new frequencies and I will see those frequencies coming out if I satisfies certain conditions.

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$$I = \frac{k}{2\omega\mu_0} |E_0|^2$$
$$k = \frac{\omega \cdot n}{c}$$
$$I = \frac{n}{2c\mu_0} |E_0|^2$$
$$= \frac{nc\epsilon_0}{2} |E_0|^2$$
$$c^2 = \frac{1}{\epsilon_0\mu_0}$$

So, that will become clear; so this equation is going to change in nonlinear media but the equation for intensity the pointing vector everything remains the same because actually nonlinear medium is the light, which enters interacts with the medium and generates new electric fields. Normally, in linear media the light interacts with the medium and generates the same frequencies it does not generate new frequencies.

So, whatever light you have put in frequency that is a same frequency, which comes out gets scattered in different directions. For examples or gets attenuated, gets amplified, etcetera, etcetera but in nonlinear effects you will generate new frequencies, with new electric fields and new magnetic fields.

(Refer Slide Time: 35:03)

Handwritten mathematical derivation on a chalkboard:

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{k} \cdot \vec{r} = \text{Constant}$$

$$k_x x + k_y y + k_z z = \text{Constant}$$

$$\nabla \cdot \vec{D} = 0$$

$$\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0$$

$$\nabla \cdot [\vec{E}_0 e^{i(k_x x + k_y y + k_z z - \omega t)}] = 0$$

$$\vec{k} \cdot \vec{E}_0 = 0$$

$$\boxed{\vec{k} \cdot \vec{E} = 0}$$

The NPTEL logo is visible in the bottom left corner of the slide.

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Handwritten mathematical derivation and diagram on a chalkboard:

$$\vec{E}_0 = (\hat{x} + i\hat{y}) E_0$$

$$\vec{E} = E_0 (\hat{x} + i\hat{y}) e^{i(kz - \omega t)}$$

$$\vec{E} = E_0 \hat{x} \cos(kz - \omega t) + E_0 \hat{y} \sin(kz - \omega t)$$

At $z=0$

$$\vec{E}(t) = E_0 \hat{x} \cos \omega t + E_0 \hat{y} \sin \omega t$$

The diagram shows a 2D coordinate system with x and y axes. A vector \vec{E} is shown at an angle in the first quadrant. The angle is labeled $t = \Delta t$ and the initial position is labeled $t = 0$. The text "RCP" (Right Circular Polarization) is written to the right of the diagram. The NPTEL logo is visible in the bottom left corner of the slide.

Now, let me look at this equation again, this equation I have assumed this vector and E_0 need not necessarily be real. For example, let me look at a wave like this, so let me assume my E_0 vector is \hat{x} cap plus i \hat{y} cap into some E_0 and because I am choosing my E_0 vector in the x y plane, I choose the k vector along this the z direction.

So, let me write this electric field E is equal to E_0 times \hat{x} cap plus i \hat{y} cap $e^{i(kz - \omega t)}$ propagation direction is that and the electric vector has x and y components. what a wave, what is this wave? So, let me try to analyze this wave. This

is be circularly polarized wave. So, how do I get this? So, let me look at the real part of this, so real part of this vector will be $E_0 x \text{ cap into cosine } k z \text{ minus } \omega t$ and what will be the real part of the second term minus sine $k z \text{ minus } \omega t$.

Because this will be cosine $k z \text{ minus } \omega t$ plus i sine $k z \text{ minus } \omega t$, so, with this i get a minus sign here. So, let me look at a point, which I call z is equal to 0. So, z is equal to 0 the E vector; where is the time as $E_0 x \text{ cap cos } \omega t$ plus $E_0 y \text{ cap sin } \omega t$ I choose a plane, which I call z is equal to 0 and I monitor the electric field as the function of time on that plane.

The electric field of the wave on that plane will vary, as this expression shows. So, let me write let me try do draw figure and see what this implies this is x this is y so z is towards me so t is equal to 0 what is the directional electric field $y \text{ cap component}$ is 0 is $x \text{ cap}$ so it like this this is a t is equal to 0 at a slightly related time this will decrease from 1 so $x \text{ cap component}$ will decrease and $y \text{ cap component}$ will increase and will be positive.

So this will be at t is equal to Δt so electric vector have moved like this and you can analyze this this will keep rotating as a function of time and the direction of rotation corresponds to right handed screw because z direction is like this if I rotate the screw in this screw direction it will unscrew towards me

So for a rotation like this and movement like this corresponds to right handed screw direction and this is called a right circularly polarized because circularly polarized because the length of the electric vector will always be E_0 square

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$$\vec{E}_0 = (\hat{x} - i\hat{y}) E_0 \quad \text{LCP}$$

$$\vec{E}_0 = (\hat{x} + i\hat{y}) E_0$$

$$\vec{E} = E_0 (\hat{x} + i\hat{y}) e^{i(kz - \omega t)}$$

$$\vec{E} = E_0 \hat{x} \cos(kz - \omega t) + E_0 \hat{y} \sin(kz - \omega t)$$
 At $z=0$

$$\vec{E}(t) = E_0 \hat{x} \cos \omega t + E_0 \hat{y} \sin \omega t$$

RCP

It not change with time and this raised circular because the direction of rotation of electric field electric vector corresponds to a right handed screw so you can write for example if you take an expression in which E_0 vector is x cap minus i y cap E_0 this will correspond to a left circularly polarized wave

So what is making it circularly polarized is simply because the two components x cap and y cap components are being added with the π by 2 phase difference this i corresponds to a phase change of π by 2

E to the power i π by 2 is i so i means exponential i π by 2 which means a π by 2 phase difference between x cap and y cap components which you can see here y is $\cos \omega t$ the other is $\sin \omega t$

So in this you can actually show that these are two orthogonally polarized states of polarization a right circular polarization and a left circular polarization are orthogonal in the sense that they are independent polarization states and you can write any polarization state as a linear combination of a right circular and a left circular

Just like you could write any polarization as a linear combination of an x cap component and a y cap component which means you can write any polarization as a superposition of two linearly polarized light waves you can write any polarization also as a superposition

of any two circularly polarized light waves one right circular one left circular these are like basis sets

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$$\vec{E}_0 = (\hat{x} - i\hat{y}) E_0 \quad \text{LCP}$$

$$\vec{E}_0 = (\hat{x} + i\hat{y}) E_0$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

I can use vertically at horizontally polarized as a basis set I can use these two as basis sets I can use these two as basis sets I can use any basis set I can use this and this as basis set in fact I can generalize this even further and I can write E 0 vector as say x cap plus i beta y cap

The components may not be equal I can add with unequal amplitude x cap and y cap components and I will get elliptically polarized light you can show that if you look at the same diagram again you can see that the electric vector is rotating but the length of electric vector also changes with time that is called an electrically polarized light and that is the most general polarization state for light a linear polarization circular polarization or special cases of elliptically polarized light

So in fact you can use a right elliptical and a left elliptical as two basis for writing any polarization state so in fact for example if you want to say like this suppose I make a column matrix so E x and E y components

So the linear bases corresponds to the two independent factors 0 1 1 0 and 0 1 the other one corresponds to 1 i and 1 minus i with of course you have to normalize this I have to

put 1 by root 2 here its writing a column vector with two elements as a linear combination of these two or these two in fact infinite number of basis sets

So this is nothing but stating that I can use linearly polarized states as basis sets or circularly polarized states as basis sets and which basis sets will I use depends on the problem we will find an anisotropic media the basis set that we will use is the linear polarization states because what you will find is what are called as the eigenmodes of propagation or linearly polarized states

You have effects where you should use circularly polarized basis set I do not know whether you heard a Faraday effect if you take a material and apply a magnetic field suppose the magnetic field is applied in this direction and you launch a light wave polarized like this as it propagates it rotates its polarization state it is like an optical rotation

So light polarized vertically as its propagates through the rotates in the presence of the magnetic field and this is called Faraday effect Michael Faraday discovered this and to analyze this problem is better to use this basis sets right circular and left circular basis sets

So the basis sets will depend on the problem at hand and in the anisotropic media we will discuss we will all the using this vertically and horizontally polarized or linearly polarized states as the basis sets for the course

So I think that we finished today's course so what I will start in the next class is essentially will recall a bit of diffraction I will not going to much detail and then we will start anisotropic media we will discuss anisotropic media and I do not know have you studied anisotropic media in any course before

Studied (())

In detail

Studied (())

It invoke tenses but does not I mean it is application of tenses to studying life propagation any questions

(Refer Slide Time: 40:59)

$$\vec{E}_0 = (\hat{x} - i\hat{y}) E_0 \quad \text{LCP}$$
$$\vec{E}_0 = (\hat{x} + i\hat{y}) E_0$$
$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \begin{pmatrix} 1 \\ -i \end{pmatrix} \frac{1}{\sqrt{2}}$$

Parade politics medium not or inNo in vacuum the polarization state does not change at all you can launch any polarization state it propagates without any change

(())

Which property? (())

What will happen is in there are media in which epsilon and mu are both negative metals have a negative epsilon so there are people who now make media which are microstructure media in which both epsilon and mu are negative which means that D and E are antiparallel and B and H are also anti-parallel and in this case you can that S vector and k vector are opposite to each other so the wavefront is going like this but energy is going backwards.

So, this is that our backward propagating waves and these exists in wave guides and many other phenomena but this is now being analyze for bulk media in which you can make an artificial medium which has negative epsilon negative mu at a certain frequency.