Strength and Vibration of Marine Structures
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Lecture - 8
Statically Indeterminate Structures – II

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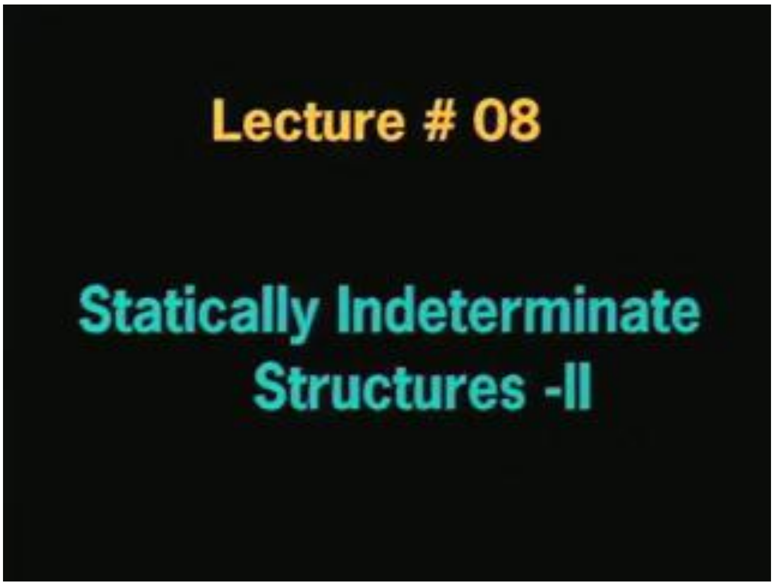

So, we were discussing this problem in our previous class. The starting of the problem was like this. So, fixed beam, load placed little bit away from the center. So, a load, bending moment, at the two ends force, bending moment separately we have applied. And due to the load, the slopes we have written, and due to your MA, MB, you wrote the expression of slope at the two ends. So, next part is just we have to merge the slope, because slope here and slope here; it should compensate, because it is a fixed support; and here also same thing.

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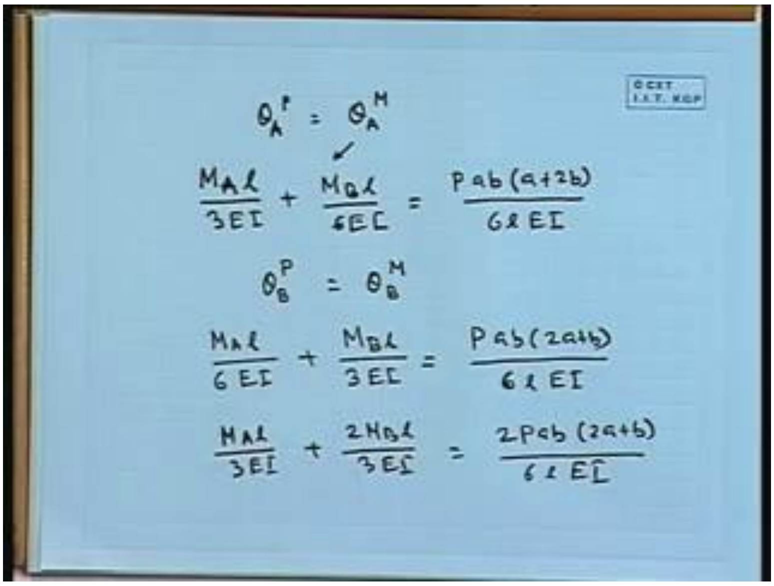

So, we can write theta A due to P should be theta A due to moment. And theta A due to P and theta A due to moment, the expressions whatever we have written, so if we just write it on the left side, it will be your MAl 3 EI MB 6 EI; that was Pab a 2 b divided by 6 lEI. So, definitely, this part on the left side, the other part on the left part here, this is on the right side.

So, similarly, at B, due to the load, and theta B due to the moment, if we equate, if we write here, so it will be MAl divided by 6 EI MBl by 3 EI; that is equal to Pab 2 a b 6 lEI. So, here it is a plus 2 b; here 2 b plus a; here it is 3; it is 6; it is 6; it is 3. So, these are the two equations and two unknowns: MA, MB; MA, MB. Now we have to just process these two equations and find out MA and MB Now the lower equation if we multiply with 2, so this 6 will be 3, and you may part it, we can cancel. So, this equation we can write, we can straightaway multiply or we can write in a separate line. So, it will be MAl by 3 EI; 2 MBl by 3 EI; it will be 2 Pab 2 ab divided by 6 lEI. Now, if we just subtract that equation or from this equation, this equation if we make it minus. So, your MAl by 3 EI part will be cancelled and here MB part will be remaining.

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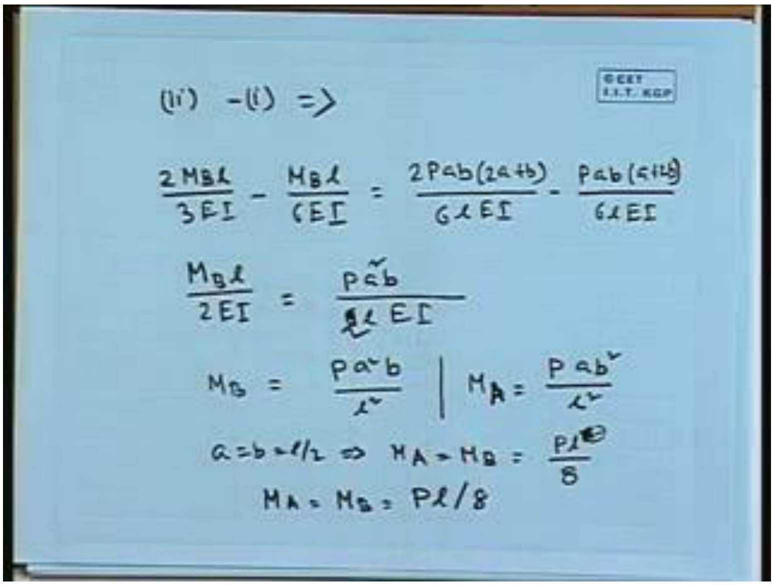

So, if I come to the next page, say, this is your, if I say this is your equation one; if we say it is equation two. So, if we make equation two minus equation one. So, it is equation two minus equation one, if we write. So, this MAl by 3 EI MAl by 3 EI that part will be cancelled. And here it will be 2 Mbl by 3 EI minus Mbl by 6 EI, and here, your 2 Pab 2 a plus b divided by 6 lEI minus Pab a plus twice b 6 lEI. So, straightaway we just put the expressions. Now, there are some common quantities that we will consider in the next step. So, here it is 3 EI 6 EI, we can just take 6 EI below; and it will be how much? So, here it will be 4, 4 minus this side will be 1. So, 4 minus it will be it will be 3 and 3 and 3 denominator will be 6. So, it will be half. So, we can write, say we are writing MB MBl divided by 2 EI. We take common 6. So, here it will be 4 minus 1, 3, 3 6.

So, if you simplify you should get that. and here denominator 6 lEI 6 lEI common; Pab, Pab common. So, here the internal part you have to adjust. So, it will be Pab divided by 6 lEI. So, 2 if you put inside, it will be 4 a, here minus a; so, it will be 3 a; and it will be 2 b; minus 2 b. So, it will cancel. So, it will be 3. It will be 3 a. So, we can put it a into a, it will be a square; and this 3 we can cancel, we can make it 2. So, your MB, it will be Pa square b; 2 2 cancel; El will be cancel; there will be one l. So, it will be l square. So, it will be MB will be Pa square b by l square. Unit also you can check, because a square a square unit will more or less cancel, and P into b, so force, distance. So, it should be MB.

Now, this MB if you put in any one of the equation, equation two or equation one, or again you just rearrange equation one and two, cancel MB; you will get MA. And MA if you calculate, you can check it; so it will be simply MA; it will be Pab l square; the square of a square will be shifted to b square. So, that part you agree with me and if you have some doubt, you can check it; rather you should check it; it will be a good exercise. So, MA will be Pab square by l square. So, square will be also there.

Now, if the a and b, they are identical it should be equal to l by 2. So, if a equal to b equal to l by 2. In that case, your MA will be MB. And it will be ab; it will be equal. So, it will be a cube, you can say, or you can say it is b cube. So, both the side it will be a cube, b cube or it is simply l cube divided by 8. So, l cube denominator l, it will cancel. So, it will be pl square divided by 8; ab will be equal; so, a square b will be a cube; a cube will be your l by 2. So, l cube 2 2 2 will be 8. So, cube this; this will not be your l square ;it will be just l Pl by 8. Because it will be l cube l square, l square will cancel; it will be l. So, it should not be… it will be pl by 8. So, power will not be 2, it will be 1 or you can write again your MA MB Pl divided by 8. So, force, length, moment divided by 8; this is one of the very standard expressions.

If there is a beam; both end are fixed; we sometimes defined that as a fixed beam; that is a problem of fixed beam, means both the ends are fixed and there is a load. At the center if it is acting, so both the moment generate the support, we say fixed end moments. And this fixed end moment is Pl by 8, under this condition. Now, if you take it fully distributed load, in a similar way, we can go ahead, and the value you will get omega l square divided by 12; fixed end moments.

So, some of the statically indeterminate a problem having one degree of indeterminacy, two degrees of indeterminacy - we have handle. Now these problems are basically your beam type of problems.

Now, let us come to a problem, which is little bit different in look, not a beam. So, it is little bit frame type. So, beam, normally, it is a straight member. So, frame definitely it will have some change in orientation. Now, the next step, if we take that type of problem, say, one of the most simple type of frame problem is like this.

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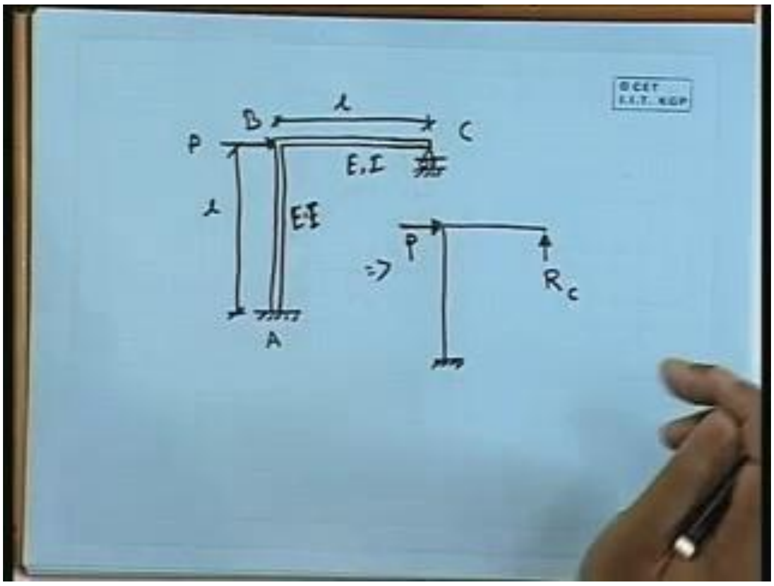

So, there is a vertical member, and there is one horizontal member. See, there is a load here; it is acting; it is P; say this length is l; for simplicity, say, this is also l; and the material is identical; it is E; cross section of your section properties are identical; it is also I; this is also I; it is l; it is l; this part is fixed; there is a roller support; this side there is a load, it is l, and that is l.

So, you just think in that manner; this is a bar. So, we have tried to change its orientation, just. So, it was straight; now it is like that. So, it is moving like this; now changing its orientation. So, it is not just connected by means of a pin. So, it is same member. So, here the connection is a fixed type of connection. We say it is a rigid type of connection. There the idea is the angle initially it is 90 degree; if there is a rotation of this member - vertical member, the horizontal member will rotate by the same angle. So, the relative angle it will be unaffected or unchanged. So, whole thing will rotate by same amount. We say it is a rigid joint.

Normally, any structural joint, we get some plate, some welding, some riveting. So, it is a solid type of connections. And as a whole it will try to rotate in a similar manner. So, we define it is a rigid type of joint. So, here there is a member. So, there is no discontinuity is there; there is no a pin type of connection, so that it can easily rotate. So, it cannot fly and change their relative angle. So, the angle would be identical. And that type of problem we say it is a typical frame problem. So, frame and beam, basic difference is beam members are all in line; frame, the orientation may be different. So, frame we can say it is a three-dimensional frame, two-dimensional frame.

So, if we take a frame, three-dimensional, means member may be vertical, horizontal; it may go to this side; it may go to inclined direction. So, it will be a total three-dimensional general type of frame. So, here at least members are in the plane. We can say it is a plane frame or two-dimensional frame. And this joint is a rigid type of joint. And there is a load, we want to find out the stresses, bending moment, shear force - everything of that particular frame.

Now, first of all, this is a fixed support. So, we have three reaction components, plus there is a roller, and this roller will have one reaction. So, basically one extra unknown is there. If the roller is not there, so due to this load, it should be a rather a cantilever. And this end was just extra. So, it is a fixed one; something is connected; there is a load; it should undergo some bending, the vertical member; horizontal member should be just there as an attachment. But here, it will try to bend, and when it will try to bend, horizontal member will try to follow that; follow that means, it will it will try to move in the right side, that is allowed. But here, there will be some slope; this member when it will bend, there will be a slope at the end; and this member, it will follow that slope, means this end will try to come down, for this member.

When it will try to come down, automatically the reactive force will be generated. So, in the form of physical deform, if you think in a physical way, the deformation of the structure. So, it will try to come down, some reaction will be regenerated. Or other way you can think, there is a force, there will be a reaction force, and this reaction force will makes generate some moment, and that moment some part will be taken care by the support and some part, because there will be equal and opposite reactions, and that part will take care the remaining part of the moment.

So, whole problem is the force system is integrated inside; you cannot say, this will be this one or other part will be 0. So, everything will be associated. Now, the main difficulty here it is a statically indeterminate problem, and any one of the component you can take as a additional unknown, and that part you can find out from the deformation of the structure.

Now, one of the simplest way is this roller part you can remove and put a reactive force. And that reactive force you can find out from the deformation of that point, because it is a support, at least vertical deformation will be 0. Now, if we say if it is A, and if it is B, if it is C. Now, that problem can be drawn in that manner.

So, I am just putting with a line diagram; not giving the thickness of that. So, there will be a P and there will be a reaction Rc. So, this is just represented in that manner. So, here it is fixed; there is a P; there is a Rc. And Rc, this force, is not known; if that is known, we can find out everything, because it is this end is now free end, I think. So, it is just a cantilever; only it is changing its direction.

Now, this part of problem is basically two forces are acting; one is P, another is Rc. So, effect of P and Rc we can handle in a different manner, and we can try to compare the deflection at this end, and it should be equal to 0, because here there is a support. So, these two parts if we take separately.

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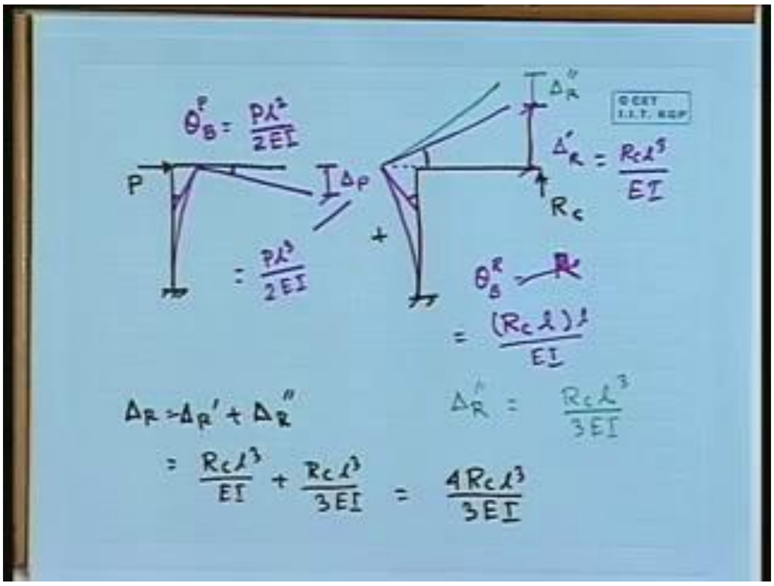

So, this is P, and here, if we add, now our basic problem was like this: this is P and Rc; this is clamped; it is free end. Now, that is only P and Rc, in two steps. Now, if we apply P, what will we have? Only this vertical member will have a cantilever mode of deformation. So, I can draw it in a different colour. So, it will be like this, and this member, it will come like that. So, here, due to this P, there will be a horizontal displacement plus a rotation, and that rotation we know, that rotation is simply Pl by 2

EI. So, that slope will be this slope; this slope and this slope will be identical, because initially it was 90 degree, it will be 90 degree. So, whole thing will be rotated; vertical member will have a rotation about vertical axis and same angle will be produced by the horizontal member. So, this angle and this angle is basically your theta B. And it is your Pl by 2 EI. I think it is Pl square or Pl? So, P into l into l half of that it will Pl square by 2 EI, that will be the theta B, theta B. So, this end we will undergo a vertical deformation; that is your delta, or due to your P, this theta B, it is basically due to P, and delta P will be how much? So, this angle multiplied by length, because by horizontal, vertical - both the members - are having the same length. So, this part should be equal to your Pl cube divided by 2 EI.

Pl square by 2 EI is the slope. So, slope into length, slope into length, will be the vertical deformation. So, it is a vertical member, there is a load. So, this end we have a rotation of theta B due to P of defining is theta Bp Pl square 2 EI and that horizontal member it will just follow that angle. So, slope into l will be the vertical deformation. So, it will be Pl cube by 2 EI. Slope is 2; deflection is 3. So, here the deflection will be Pl cube by 3 EI - here to here - and this slope is Pl square by 2 EI, the slope into that length.

Now the next case, next case this force it will give a cantilever type of deformation of this member. If we assume this is rigid, means there is no deformability of the vertical member, so there will be a cantilever deformation. Now, second level if you take the deformability of that, so this force will be transmitted here plus there is a moment. So, there is a moment; that moment will give the cantilever deformation of the vertical member. So, both the deformation will be combined together and we will get the final deformation here. So, here deformation will be upward; here it will be downward; and ultimately it will match.

Now, the cantilever deformation of the vertical member it will be more or less like this. So, if we take this is a rigid member, so Rc if we cut here, so here we will get a force Rc upward plus there is a moment, and due to that moment, there will be a cantilever of moment. Now, if we take this is, this part is not under deformation. So, here there will be a slope, or about this, there will be a slope; so, this slope will be equal to this slope. And what will be that slope? That slope we can say it is theta at B due to say that reaction. And that reaction will be this Rc into l that will be the moment; that moment will generate some slope. So, moment is your… it will be… let me rewrite here. This will be equal to your Rc into l; that will be the moment. So, it will be, if there is a moment there, so, it will be, slope will be how much? Ml by EI. If there is a cantilever, if there is a moment, so entire beam will have a uniform moment. So, M by EI diagram will be M by EI. So, area will be Ml by EI. So, it will be Ml divided by EI; that will be the theta here; this angle, this angle, and if that is the angle, so this upward displacement will be how much? That into l. So, this value, say, it will be, say, delta; we have written in terms of P, that will be in terms of R, and say it is one part. We have not taken the deformation of that member. So, this part will be this Rc already l l's, it will be l cube divided by EI.

Now, if we add the deformation of the member here. So, you try to get the two deformation. First of all, say, this part is not under deformation, say, assume for the time it is rigid. So, this R will be shifted here in the form of axial force plus irritation; axial force plus a moment. So, moment will be Rc into l. So, if there is a cantilever, subjected to a moment, so slope produce will be Ml by EI.

So, Ml, it will be Rc into l or Rcl square by EI will be the slope here. That slope into distance; that will be Rcl l into l l squared into l l cubed divided by EI; that will be delta R for this moment component. Now, it if it is rigid, it will be like this - a straight line, but it is not rigid; it it has also flexibility. Now, here to here, this load, it will give a cantilever type of deformation. So, that is delta R dash and this delta R dash - this part - will be just your Rc; I think it is l cube divided by 3 EI. So, delta R will be delta R dash plus delta R double dash. So, we can write here delta R.

Now, delta R dash already we have written it is Rc l cube divided by EI. And here, this part, Rcl cube divided by 3 EI. Now, if we add that, this will give, so it is Rcl cube by EI, so one-third. So, it will be 4 Rcl cube 3 EI. So, this delta is the total delta R. So, delta R it is upward and delta P it is downward. So, delta P is due to P and delta R due to Rc. So, both the deformation should be and it should be, because it will be equal and opposite, cancel each other. So, this value and this value, we can just compare, and from there, we will get the value of reactive force.

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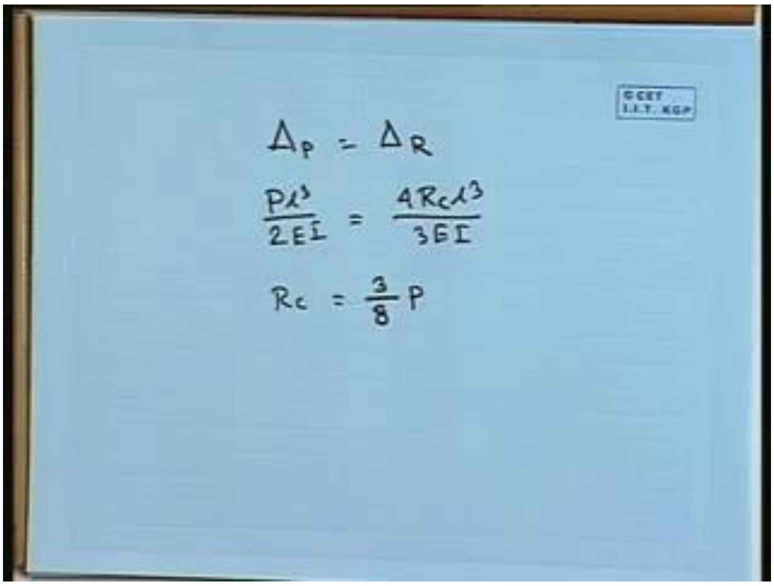

Now, in the next step, so if we take this data and try to match your delta P and your delta R. So, your delta P should be equal to delta R. And delta P we have written here; it is Pl cube 2 EI; that is equal to 4 Rcl cube divided by 3 EI. So, Rc part we can write here 3; it will go there. So, it will be 3 divided by 8, EI will cancel. So, it will be just P.

If I show you the actual problem. So, P if we apply there will be a reaction of three-eight of the P. Now, if there is three-eight of the P, there will be a force downward to balance this. When these two will give a moment and P there will be a reactive force here. Automatically, if you calculate all the moments, that will be the moment here or we can say if you start from there, there is a reaction force you can calculate bending moment at any point.

So, Rc into l that will be the moment here; that is this way; and if you follow that direction minus P into x, some reverse way moment will be starting, generated from this end. And here there will be a moment, it should match with the support moment generated at A. So, bending moment anywhere you can find out; shear force also you can find out, because shear force is this force Rc, Rc here. And here it will be only P, but there will be axial force also in the system, because this reaction, here it will generate axial force, Rc part.

Because there will be a reaction. So, Rc there will be some member. This member there will be no axial force, because there is no resistance in that end. So, anywhere if you cut, there will be no axial force, but vertical member some axial force will be there

So, normally in a frame problem, we get bending moment, shear force, and axial force, and deformation. Normal beam problem, we get shear force, bending moment, and we get the deformation, deflection, and slope, but when it is a frame problem, in a three, in a two-dimensional manner, so any point may undergo horizontal moment, vertical moment, as well as a rotation. So, beam there is no horizontal moment, but here there is a horizontal moment; there is a vertical moment plus rotation. So, there are three displacement components plus there are three force components - bending moment, shear force, axial force. Beam it is only bending moment, shear force, and two displacement component; displacement - vertical displacement plus rotation.

So, we have tried to change our problem a little bit from beam. Earlier, mostly we have handled beam type of problem. In handling a determinate problem through differential equation technique or moment area method, normally, beam type of problem we have handled. Here also at the beginning we have taken all beam. So, this is one of the case, where we are going to take a frame type of problem.

Now, in a real case, you will not get everything in beam. If we take any problem, say, wave frame is also a frame; wave frame at least you have the idea; or any framing system it will have some horizontal components, some vertical component; may be some inclined component. So, it will be basically a structural system where there is no guarantee that all the member will be like a beam. Beam we take, because that is a very simple type of problem to explain the basic understanding. But actual system it will be a frame; most simple case it will be plane frame; much more complicated case it will be a space frame; three-dimensional frame.

Now, this period and last period, we are more or less trying to handle a problems statically indeterminant type of structure. And we have seen the number of reactive force are more and we are trying to utilize the deformation of the structure. And whatever understanding we had, for the deformation of beam under simple cases - a cantilever, a simply supported case, a load fully distributed load, or a point load at the end, or at the middle - we are trying to get standard relations to or get the overall deformation of the structure from there we are trying to manipulate deformation due to this force, deformation due to the reaction, try to compare, from there the reactive force we are trying to find out.

Now, the activity, whatever we have done here with two members, now you try to think, if we add another member here, or if there is a inclined member, so whole problem will be much more complicated, because the effect of reaction, so it is shifted in two manner. So, deformation of that, plus it is shifted in the forward moment, then some slope, slope will be reflected in the form of displacement. So, if you go on adding the members, it will be very difficult to handle in that manner. So, we have to, theoretically speaking, we are talking about different methods, and we can think with one method I can solve everything - theoretically you can solve everything; but it will be very, very difficult job to solve everything with a single method.

So, depending on the problem, some method may be much more suitable, and we are supposed to take that type of method, because it will be much more convenient to handle that type of problem, with this typical method of technique.

Now, in that respect, if we try to handle little complicated problem, a frame type of problem or beam little bit more, and if you do not want to remember all this - Pl cube divided by 3 EI and all those, there is one technique called energy method. And energy method is a very, very powerful method. In many cases, we try to use that energy method, and try to get the deformation of the structure, because statically indeterminate problem, means you have to handle the deformation of the structure.

So, this part, I have clarified earlier also. In some cases we want to know what is the deformation at that point, but that type of requirement is not very frequent; rather, we are much more interested for finding out the bending moments, shear force stresses. But statically indeterminate problem, we cannot find out the bending moment, shear force, unless we get the reactions. To handle that we have to apply the knowledge of deformation of the structure. So, that was the basic reason for going through the deformation of the structure. Indeterminate form, then we just utilize that for finding out the reaction for indeterminate type of case.

Now, I want to take the energy method. Energy method is much more useful, much more rather suitable for handling little different type of problem - little complex type of problem. And there one thing it may be beneficial for you, that you need not remember all this standard expressions. So, you have to just write down the expression of the bending moment for the different segment. You have to perform some integration there. You have to get the energy expression, you have to take the derivative; from there, you can get the deflection slope at different places.

So, our next job we can start energy method, but first we will try to apply with a statically determinate determinant type of structures, but we have some understanding which we can handle easily. So, once that understanding will be there, that problem can be extended to a statically indeterminate type of case.

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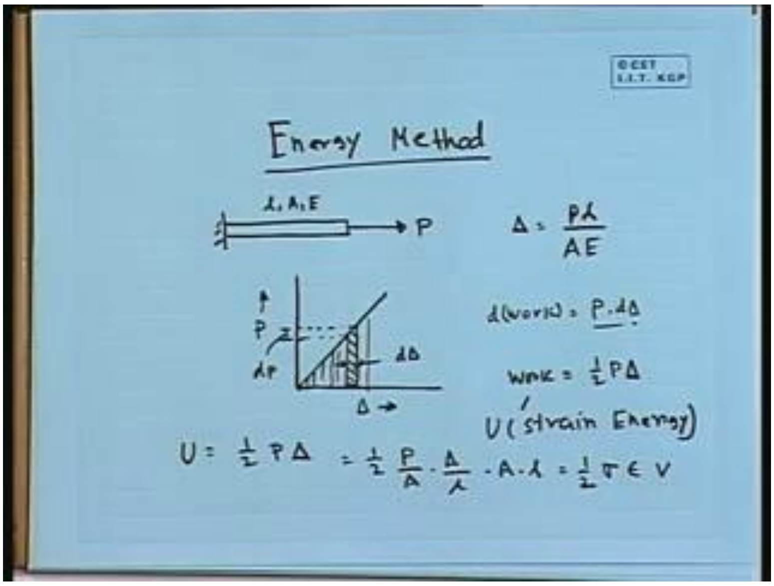

Now, this energy method is our next objective. Now, what is this energy? Basically, here we will be handling strain energy. Now what is strain energy? So, let us discuss little bit, very basic component of energy. Now, I can take a simple bar. I am sure you have the idea of that. If you apply P, if the length is l, area is A, material property is E, so there will be a elongation of that member; that elongation will be Pl by AE. So, this is P; this is l by AE; that will be the elongation.

Now, this load P, if we apply from 0 and gradually if we increase. So, it is starting from 0, 1, 2, 3, 4 like that if we increase. So, delta initially it will be 0; gradually it will increase. So, if we plot the relationship between P and delta, so P is load; delta is the deflection. Load-deflection curve for a linear system, we are supposed to get a relationship like this. So, this side is P and this side is delta. So, if P equal to 0; delta will be equal to 0. So, you increase the load, delta will increase, because lAE - these are fixed parameters. So, you go on increasing the load, it will follow a line. So, at any instant, say, that is the P, and under this load P, the deflection is delta. Now, if I slightly increase that load from P to delta P. So, delta will have a little change of d delta. So, here, say, it is d delta and here also it is dp. So, here to here it is P, it is dp; and here delta, it is d delta.

So, under P deflection was delta. So, if I just increase little bit. So, there will be a slight change in deflection. So, there P will do some work. So, at the end there is a P. So, if you slightly increase this end, we will slightly shift by d delta. So, what will be the small work? That d work, we can say, that small incremental work, it will be P into your d delta. P is already there; delta is already there. So, there is a further moment. So, this further moment is P is the force and d delta is the moment. So, that will be the work. And this Pd delta is nothing but the area of this particular state, because it is d delta and that part is P.

Now, you can say this point is P; this point is not P; it is P plus dp. So, which one we should take? Someone may suggest we should take the average one: P plus dp… P plus P plus dp divided by 2. So, it will be P plus dp by 2; but this dp normally we take this incremental concept in a very limiting manner, means they will tend to 0.

So, compared to P, this dp or dp plus dp by 2 will be a very small quantity. So, if it is take 100 and 100.00005 or 1, so that 00001 that part we can drop. So, it is in a incremental manner more or less is a area under the curve. So, what we do? We take a strip. So y into dx. So, y; this end and that end there is a difference, but within limiting manner that is tending to a point. So, it is also like this; that is, there is a P and there is a d delta. Now, if we just increase by another small amount, there will be another strip; another strip like this. Or if we start working from the beginning, there will be a number of strips like this. So, that will be, basically, our work. So, it is the total work; it will be half P into delta, say, at any P deflection is delta, it will be half P into delta, because this is P, and this is delta, this is triangular part will be half of that.

Usually we can think the load is P and deflection is delta. So, what should be P into delta? So, it is not like this, because P it is not applied from the beginning its full value; it is starting from 0, say P is 10, and delta is 2. So, 10 it is not applied at the beginning. So, it is starting from 0 to 10. So, average value is 5; and that average value, that displacement we are getting 5. So, we can say 10 into that value displacement divided by 2 half, means we are basically averaging it, because it is following a line. So, here, it is a line, if you take a non-linear curve, basically area under that curve will be that total work, and that work will be stored in the form of energy, and that energy will stored in straining the member. So, we say it is strain energy. And if we release that load, that strain energy will be released. So, it will help to bring back to the original level. So, this part we say it is U or strain energy. So, U - this total work - we are defining as U and this is called strain energy.

So, if there is a load, there is a deflection d delta half will be the energy, because the load will act gradually from a 0 value to a certain value. Now, we can make an argument that - who is going to put the load in a very slow manner, in a gradual manner? So, you just put there; suddenly applied load. If there is a beam, suddenly if you apply a load, so whole problem will be a dynamic problem. So, if there is a beam, put the load there. So, what will be happening? Suddenly there will be a force and there will be no deformation. So, some equilibrium will not be maintained; automatically some motion will be initiated. So, it will come to the equilibrium. Then some velocity will be there. So, it will cross that limit; go beyond that. After that it will come to rest, but at that level equilibrium is violated. Again it will start come back. So, some vibration will be there in the system.

Now, due to the damping and all those, ultimately it will die out, come to the static stage, but external loading you may apply suddenly, but internal stresses it will generate gradually. Any structure if you put the load suddenly, stress will not be generated from 0 to 10 suddenly. So, it will gradually take. If it is a static problem, dynamic problem, we are trying to put this energy, P is the load, and delta is the elongation. Actually this is the external phenomena, that work will be stored in the form of strain energy inside.

Now, let us write in that manner, say, U is equal to half P into delta. So, this part it is written in this manner, say, half P by A delta by l A into l. So, it will be, we can write, half sigma epsilon into volume. P delta, P we have divided by A, we are getting stress, delta divided by l is the strength. So, A and l unnecessarily we have taken at the denominator; numerator we have to compensate that. So, A into l is basically volume.

So, half stress strain into volume; that will be the total energy or this half into sigma into epsilon that is called as strain energy density; means strain energy per unit volume. So, multiplied by the volume will be the total energy.

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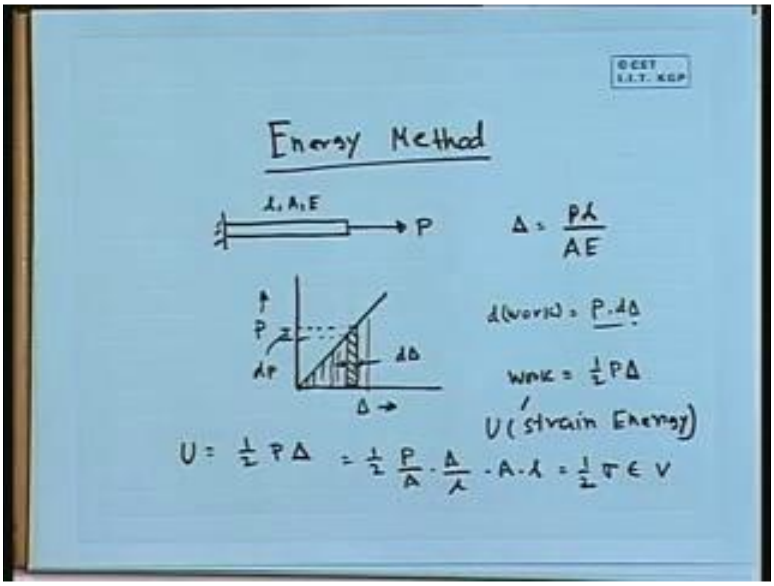

So, in a general case, we can write U is equal to half sigma dv integral. So, this is a problem where stress and strain are uniform throughout. So, we are simply taking it is half sigma epsilon into v, but sigma epsilon may vary point to point. So, in a general case, we can say it is sigma epsilon dv. So, it will be general integral, and this part we say it is intensity of the strain energy, and whole part is the strain energy. So, I think we can drop it here. Just we have initiated the strain energy in the form of simple extension of a bar. That idea will be much more generalized and it will be applied to a structural problem in the subsequent classes.

Preview of the Next Lecture

Just we have started talking on this. The technique is energy method; it is based on, basically, the concept of strain energy. The strain energy we are trying to explain in the form of a simple bar problem under tension.

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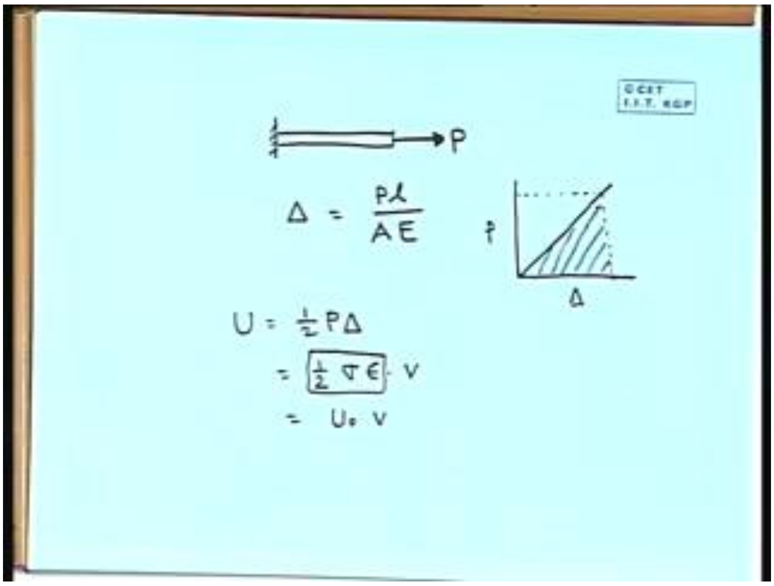

So, we took a bar in that form and applied a load P. Now this is a case of a very simple stress problem, because entire bar will have identical stress and strength. So, anywhere stress is P by A and strain will be your total elongation divided by length of the member. So, it is a case of uniform strain, uniform stress, whatever you can say.

Now, the elongation we wrote in the last class, it was Pl by AE, and we have drawn a curve like this, your load versus elongation - straight line, because whole thing is a linear system. And for any load, if you start applying load from 0 gradually increase go up to a load P, that elongation will increase from 0 to delta, and the area under this load deflection curve, we have found that it is, basically, the work carried out the force P.

Now, it is not straightaway P and delta, because P is not constant throughout, because P is starting from 0 to that P, and it is delta is starting from 0 to delta. So, total work. So, that we have put in the form of half P into delta. So, that work, it will be stored in the form of some energy; we said strain energy; that strain energy will be equal to this work done half P into delta. Now, here load, why we are putting in a gradual manner? Because we want to maintain the static condition. If we apply suddenly, a load whole thing will be a dynamic problem.

So, that will be much more difficult case. So, we want to understand with a very simple type of problem. So, load in a static manner, if we want to apply, we have to apply gradually. So, always there will be balance between internal external force system.

Basically, this strain energy though it is expressed in the form of P into delta, it is rather the measure of some internal quantities; internal quantities, means there will be a deformation that we have defined in the form of strain, and due to the strain, some stress will be there. So, whole thing is a measure of the strain and stress inside. And you must have remember we have written this one as this sigma epsilon into v, because that P we have divided by area, epsilon divided by l. So, it became stress, it became strain, and this l and A it becomes v, that is the volume or sometimes we write it is U 0 v. So, U 0 is this part. We sometimes say it is strain energy density. So, half stress into strain is the strain energy density multiplied by the volume will be the total energy. Now, this is a case of uniform stress and strain. So, everywhere your sigma and epsilon is constant. For a general case, we have written U equal to your half.