Seakeeping and Manoeuvring

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Lecture No. # 04
Single Degree of Freedom Motions in Regular Waves

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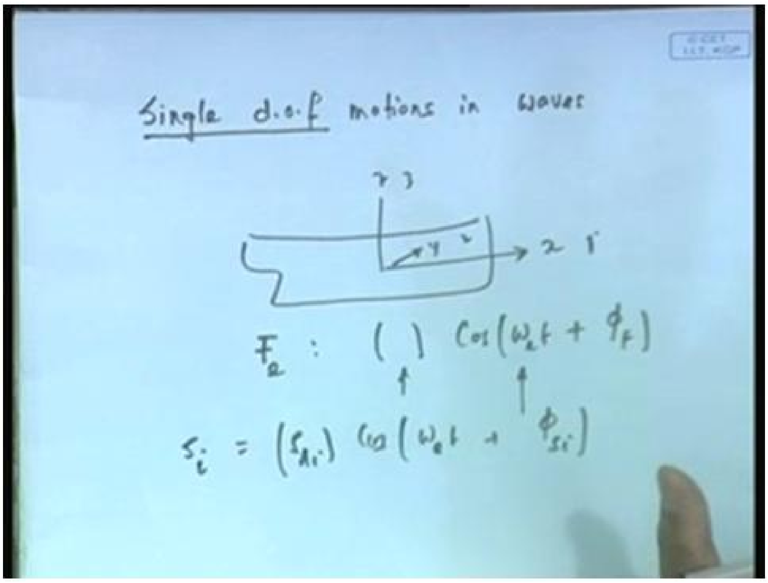

Now, we are going to now talk about the single degree of freedom equations of motion. Let me write it down, because I have a reason d o f motions in waves, regular waves of course, see the here the catch what you will find is single degree of freedom. Now, let me for before going to this, let us talk about the motions itself. Now, you see in the last lecture, we saw that if I have a ship, the excitation F, if I you call x the wave wave induced force would have a periodicity of omega e that is it will be something into cos omega e t plus may be some phase gap. In generally, it will look like that - the excitation.

This is we do not know, this is whatever the excitation is, but we what we know here of course, is that it will have a periodicity of omega e, because at omega e it is coming. So, generally therefore, since it is sinusoidal with one harmonic component, we would expect all the motion displacements to be also of similar form. So, I can expect for example, all the motions if I were to call in a now I can call in a index form let us say x y z, I am calling it as 1, 2, 3 and about x y z, I am calling 4, 5, 6 to be the indexes defining those. So, j this becomes see this become some amplitude into cos omega i e t plus phi psi i something like that. It will look something of this nature.

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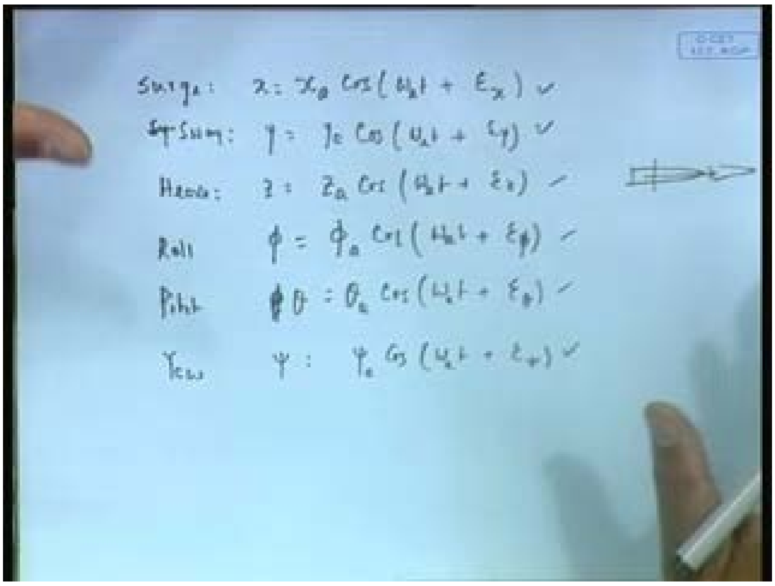

In fact, I will just elaborate that on that for example, we will have surge motion I can call say x a cos omega e t plus let us say a e, these are phase angle sway y can call it this x equal to y equal to y a etcetera. Let me write it down only, heave, roll is normally called phi, this is phi theta. I have a reason for writing this entire thing up, although it is repetitive.

Now, you see here on these modes of motion, which is important for us that is why I wrote all all this stuff. Now think of this, there is a ship here, somehow it got moved in forward, because of a force we need come back, how much it will come back etcetera. Question is that, in horizontal plane that is this, this and this, I have nothing restoring force that is if I pushed it through it do not, the hydrostatic force do not try to pull it back. So, from a point of view of motions, it becomes less important. Because, it in other words the ship is neutrally stable in the horizontal plane

On the other hand, it is heave, roll and pitch. Where, if I for example, heave the body, if I pushed it down then, it will try to come up and there is an oscillation and as you know all there is a restoring force then there is chance of resonance, we will come to that in a in a in subsequently. So, what happened for us an important modes of motion becomes these three. See now, what is happening is, remember that the entire motion is defined if I know two things. One is the amplitude, one is the phase, there I know full thing how we will see that.

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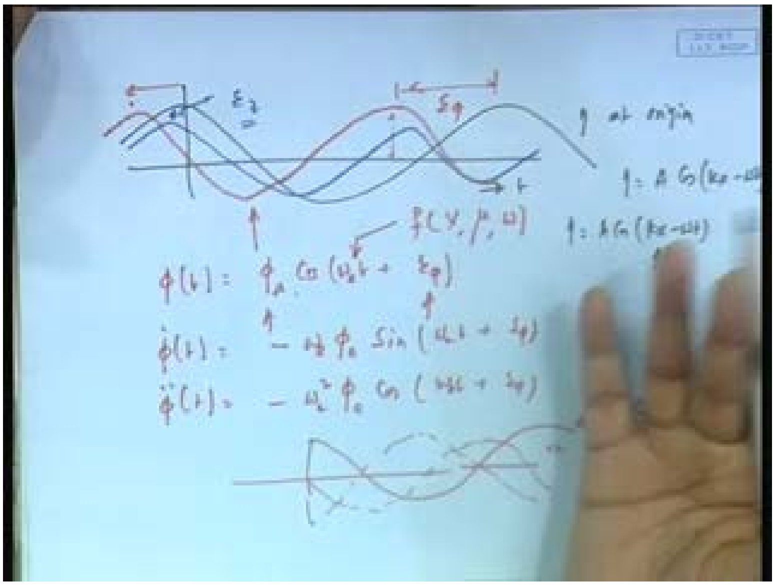

Let us take an example of roll or something. So, what is happening see, remember I have put a phase. So, let us put the first of all wave eta now, we are normally writing eta at origin, this is let us say eta at origin, see remember eta is function of x and t. What is eta, is the function of a cos k x minus omega t or other.

Obviously it depends on x. So, when I try to talk a phase (( )) I draw with respect to t, i must have some x. So, normally what (( )) plotting this at x equal to 0. Let us say that is origin. So, we are always measuring once again this this phase angles. I told you that, it is a only a question of difference between some signals. So, you can measure that with respect to some some signal. Normally, they are going to measure it with respect to the wave crest at origin, which means the wave eta signal at x equal to 0. Let us say it is look see this a cross curve, you know if you a take x 0, it is A cos minus omega t is A cos omega t, (( )) theta is theta.

Now, what would happen this signal therefore, with look something like, let us say this is where this gap, this is a measure I am not writing, this a measure of my phase angle. Now, if this was for example, my roll that is it is my phi a cos omega e t plus epsilon phi.

Then, this is in some sense measure of e phi not exactly same remember that just to be t and all that part (( )) just trying to give a concept of that.

So, you see now the point is that I would now what is my time history, all the points on this, but I know the entire thing if I only know two things. That is the amplitude and that is its location. In fact, this e phi's basically sets me where, the origin of this is, the starting point of the signal this e phi, fixes the position of the signal in the t axis, in principle. In the t axis, it tells me from where it starts, that is my phase and the amplitude tells me the amount of maximum.

So, what is happening, in any case, see this is phi t you know if I know this and if I know this of course, omega e i know because omega e is my input. So, I can find out phi at t at any point, you can run a program at t if the 0 second, one second, two second, ten thousand second, one million second you get the graph. So, only thing is that you need to, but where it comes from is that is what I was trying to tell that e phi face six location, where it is starting from with respect to the wave crest or with respect to some measurement and the other one like that gives you amplitude.

Now, you see let us take me, this is let us called it you know roll now, let us now or some other signal, it can be pitch also, but let me also take now another one, let us say heave. So, so heave will look likes a something like that where, this will be the measure of the e z the phi. So, all of them, all the ship motion if I look at that would have all these components is own amplitude and a phase and between this two, if I want to find out what is the relative phase of heave with pitch it is e z e phi etcetera etcetera

Now, comes the concept of single degree of freedom equation of motion. So, now, we know the motions of course, once before I go again, once I know this motion see it is very easy for us to find out the velocities and accelerations also. How, because any of them if I differentiate that it will simply become you know like omega into phi etcetera. For example, may be I will just a do it here before I go to single degree.

See if I for this, if I were to do roll velocity that is phi dot t, what is what it becomes it becomes minus omega e phi a sin omega e t plus e phi, right. And if I take acceleration, it becomes minus omega e square phi a cos omega e t plus e phi. If you look at this cos curve would have been well in another reference like this this is my displacement velocity would have been a negative sine curve, that is this way and acceleration would have been a negative cross curve that will be this way.

So, you know my point of view therefore, to tell you is that actually if I know this and if I know this, I would have known everything that I need to know as far as roll motion is concerned or a particular single degree motion is concerned in that particular single wave. Remember, omega e is a function of what all thing ship speed v heading mu and of course, wave frequency omega. We have seen that omega square equal to in last class omega square omega e equal to omega minus omega square v by g cos mu.

So, for a given ship speed, for a given wavering angle, for a given regular wave of any length, I know omega e. This is my environment. I have a given wave coming from a given direction in which, my ship is moving at a given speed. So, I know omega e, what I want to know, therefore what is my motion, well I now say my ship was going to move in this fashion and I need to know my amplitude of the motion and the phase. And once I know this, I can find out the acceleration, I can find out the velocity, I know the entire thing there is nothing more I want to know.

How do I do that that is the question number one is how do I do that, which is (( )) will discus, but does it occur singly, that is the another question that we have to answer. What is mean by single degree of freedom equation of motion? Now, comes this question see here, I will give this example right from your heel case.

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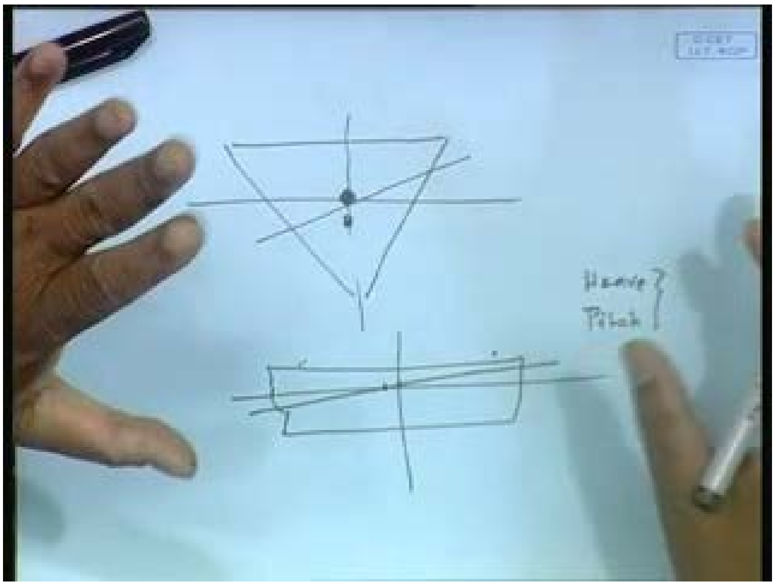

Now, let us take a body here body here. Now, you know say this was my point g or let us take only this point as my origin, it is easier. Now, what is happened if I heeled it, you know all that if the angle is little large, this will not go through pass through it will pass through somewhere else, which would imply that if I were an experimental device, if I fix this point, you know somewhere at some location, if I heeled it this point you will try to either go up or go down, agreed; because under water volume has to remain constant.

Now, I would define this angle to be roll. So, I see that as soon as I or heel rolled or heel there is (()) heave coming up because, the full thing is moving up. So, I cannot have not isolated roll, rolled is necessarily inducing heave, heave may not induced roll because, if I pushed down it is, but roll is inducing heave, this is what is called coupling; that means, when I some of the motions and in fact, strictly speaking all the six motions are coupled if you induced one the other will come of course, what would happen? There are some decoupling that we will see afterwards and because of symmetry etcetera, but in a general sense, there will be coupling between some modes of motion.

Actually a better example is heave and pitch always, you know if I take this body here, since it is somewhere here, you we all all know it also that when when I want to this question I always have we have discussed that even in hydrostatic and the question that I we ask is that supposing I shift a body from here to here, then can you say that there will be no change of draft at all one cannot say that because, what happen the trimming is over l c F position. Draft is measured typically at some other location mid ship or so. There the draft has changed.

So, what what trying to therefore, say is that if I call the draft to be a measure of heave, then if I give a trim, then I also have a heave right and same thing will happen. If I have a parallel sink age as you know it trims because, if I have a parallel sink age, then it will just like this is the question same as when I ship most from salt water to fresh water etcetera, as you know it will only, it will always have heave and trim.

So, in other words if I in a vertical plane, in this vertical plane, if I want to displace the hull. Automatically, both the vertical displacement and the rotation gets induced, this means heave and trim or pitch, I will actually the word trim is a static pitch, heel is a static role. That is if you do not move it, but you know we are now (( )) to a next level where we are time dependent. So, we find that the these two are coupled typically. Now, this is a practical example show that is, there is no way you can decoupled it, I mean strictly speaking even if it is very small. And if you go theoretically, all six modes of motions are always coupled. That is how you should start of course, we can you know like reduce it.

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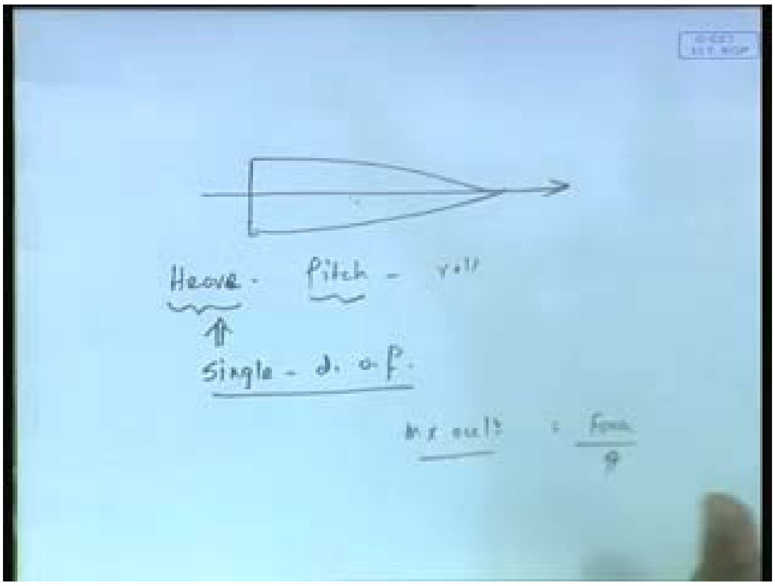

Now, now question comes is like this. Some of the modes we know are not coupled. How? I will tell you an example. Let us say surge, now body is symmetric. So, if I shift it this side, there is not going to be any motion this side. So, my surge will not induce any motion in sway direction. So, surge and sway are decoupled. Similarly, there are modes similarly, surge will not give me heave or trim like that, but we also know some modes are coupled. In particular heave and pitch are very much coupled.

But when we talk of single degree freedom, what we what we are saying is that. In fact, roll also you may add, what we are saying is that look, before I study more complicated couple one, since the coupling is not all that strong, it is only small. We can study the characteristics by assuming there is no coupling. By (( )) that if I make it you know if I were to heel it, the trim is so small, I can ignore it.

So, this is what is meant by single degree of freedom equation of motion and it is good enough because, remember that when we are studying, we keep making rational simplification from the beginning. First of all, we said the waves are small amplitude then, we said the in the defining motion also we said motions are relatively small compare to the length of the vessel etcetera etcetera. So, it is not inconsistent to study by itself a single degree freedom equation of motion.

So, this is the full point therefore, is that when I say single degree freedom equation of motion? We are decoupling it, we are trying to in isolation see we are saying that supposing the vessel has a mode of motion in this particular plane. Then it is not having motion in any other plane or it is not inducing. This is what we are trying to actually say.

So, this is where the concept of single degree freedom equation motion and what is happening is that, any equation of motion you are going to write mass into acceleration equal to force. What we are saying here is that, if I were to accelerate the body in heave for example, it is not going to get a force because of the consequent pitch or trim. Everything is only heave; if it heaves it is only heaves it does not trim. If it trims, it is only trims.

This is what we have say or; obviously, at the first step you must study this because, the coupling is very weak usually there are cases when coupling is very strong, but you know when you are studying first we study single degree freedom equation, then we study primarily heave pitch and roll. Because, these three are the most important sea keeping motions. Then, we will see whether, we can couple it and we will have an idea regarding its coupled modes of motion. So, this is how we actually kind of proceed. Now I will let us now go to how a single degree freedom equation of motion looks like.

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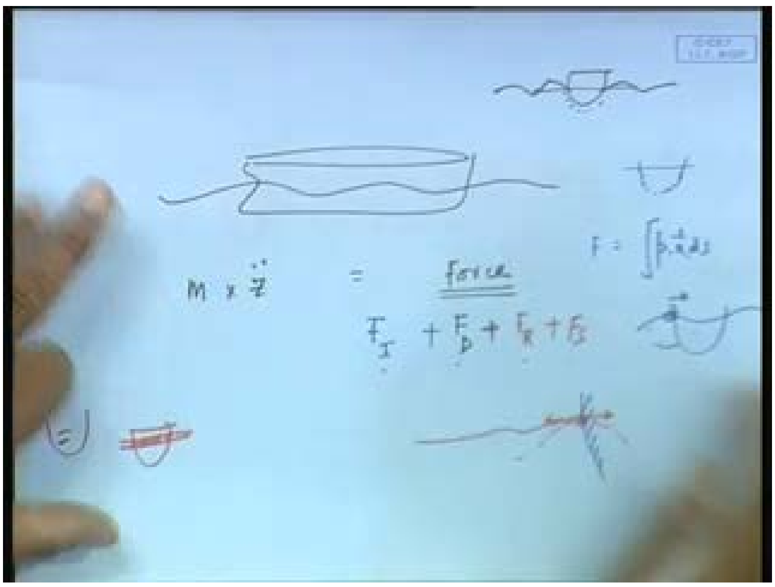

Now, I like to give you little bit of kind of physical scenario of what happens, when a ship moves etcetera etcetera, where the forces and all come from. See now, I will take one more picture the wave here. Number one let us understand this, this is only component build up that I am trying to tell and I request you to pay attention to that because, it is a build up part.

See, Newton has said many years back that and it is all Newtonian mechanics that mass into acceleration is equal to force. So, we have got say mass into; let me take an example heave as an example z or something. So, acceleration is this equal to, what is this force now everything is because of this force coming; now you just see this force. Let us say that the ship was transparent to the wave; that means, this line remains as it is, even if I put the ship.

Let us imagine a case that I have removed the ship from the water. The water line is black line. There is a pressure there and I think there a pressure there in that hypothetical surface. Now I put the body and then I presumed as if the body did not disturb the water, what would happen is that number one a primary force would be on the hypothetical position, let us say this is the hypothetical position in the hull, all these points there are pressure that we have found out yesterday linear water wave pressure.

So those pressures, I add add it all up after all force equal to pressure into n into d s over the surface. Then I will get one component of force, this is one force let me call this force (( )). Now this pressure has two components, but I am talking of the dynamic incident wave pressure. So, I end up getting the incident wave, I am calling this as (( )) again in principle, this would be the force arising because of the incident wave pressure at those locations. That is that rho d phi by d t, where phi is only phi i. Why phi i? Because, we say that the body did not disturb the waves. You know, so I have a body and I simply find out if the body was there this this location what would be the pressure. So, I add it all up. That is one part.

However, what is happened? We all know that the water is fully changed. Reason is very simple. Again, I have this body here, waves was going like that, you know in a in a other. Now, actually this is all going. Now, the point is that, this particle is moving as a circle by it cannot moving a circle because, if is it is moving and if I allowed it then the particle would have penetrated my hull. So, what is happened is that; obviously, that another wave system must have to be created on the other site. So, that it nullified that inward motion to outward motion. So, that my point here remains stationary, not stationary did not move normally inside.

See it is very interesting point to know and this is where the question of this yesterday's 0 equal to 1 minus 1 comes in. See, let us say here, this point cannot have a motion this site, you know normal direction inward I am. Once again, I am only trying to explain the concepts conceptually rather than with a strict (()). See this, let me put it again, this is the my hull, this is a point here. This is a point here, I am just looking now this particular point has at a given instant of time, let us say at a given instant, freed motion this side because, it has the crest location. If the body was not there, the particle would have moved have a velocity this side right, but if the body is there, it cannot have velocity, how much the velocity should be 0 and normal it; that means, how can I get a 0? If I introduced another motion opposite side, how do I do that? Provided I impose another wave on the opposite side. See if I induced another wave opposite side, then it becomes 0, almost 0 again in principle.

So, what is happening; obviously, this would have disturbed and then it 0 means, it I can imagine that it have created one more wave system just opposing the phase of this wave here, so that, the particle cannot go in. So, therefore, this picture would have got changed and I would have got created one more wave system countering it, if I kept it fixed let us say. So, what is happening? This wave I call it diffracted wave, scattered, reflection, it is like wave reflection.

Similar to that, in our in hydrodynamics it is known as diffraction or scattered because, it can be in many direction. This is a same thing as light comes on a particle get scattered. Similar explanation you know why sea is do etcetera etcetera sky do etcetera etcetera. So, there is a wave creation. So, sum effect is 0 for this point. So, now, that wave; obviously, would have altered the pressure filed because, the pressure is now no longer d phi I by d t because, there is another phi. So, that we call it diffracted force.

Remember, in this now there are again two scenarios. I presume that as if I have hold the body fix, I did I did not allow to move, I fixed it, I kept it, somebody is holding it by a hand, wave comes particle cannot move. So, it must have created wave back. This is one part of. So, this gives us to a one component and; obviously, it will add up to one force.

We call this actually say F d diffracted force similar to reflection, but there is more to the story.

Now, but I am not holding it fixed; the body is oscillating. Now, you take this case either calm water. So, either calm water somewhere here, now I took a body or I oscillated that what would happen? It would create some waves, you can anybody can try that and see, in calm water I took a body and I oscillated, it would create a wave.

Now, here what happen net effect is that the body is moving but if the body did move it would have created one more system of waves around it because any free surface you oscillate it will create one more system of waves now if it created this system of waves again that would created a pressure field. In fact, it is very obvious to see that the energy that you input that creates the waves and that is you know that energy gets dissipated in forms of waves so; obviously, there is a pressure field introduced that is one more kind of waves that comes in this is known as radiated waves you know radiation waves or another wave let me call me it F R I mean when I say F R please understand it implies all effects connected to the wave system created by oscillation of the hull in a otherwise calm water.

So, I end up getting this remember also that there is also hydrostatic pressure standard because you see if I we were to take the ship from sorry from here I pushed it down to this much this extra hydrostatic pressure comes in. So, there is an hydrostatic force also see my ship is having a static draft of 10 meter if it went down to 11 meter I have that rho g a w p into that 1 meter pushing it up that is coming purely because of the hydrostatic pressure rho z. So, there is another hydrostatic pressure rho's this thing.

So, this site therefore, composes of this force this force this force this. In fact, we are able to do this because of linearity because of linearity I can say as I said here 0 is 1 minus minus 1 this why you are saying that this scenario nature will not tell us nature will simply show some wave here we are only saying that this wave is composed of can be broken down into this and this and this we are saying that nature does not say that nature gives me the total force, but we can do that from our force point of view. So, we end up getting this.

Now, comes the interesting points you know what is happening now now I will go the next page and try to work on this part and see how this equation of motion comes about.

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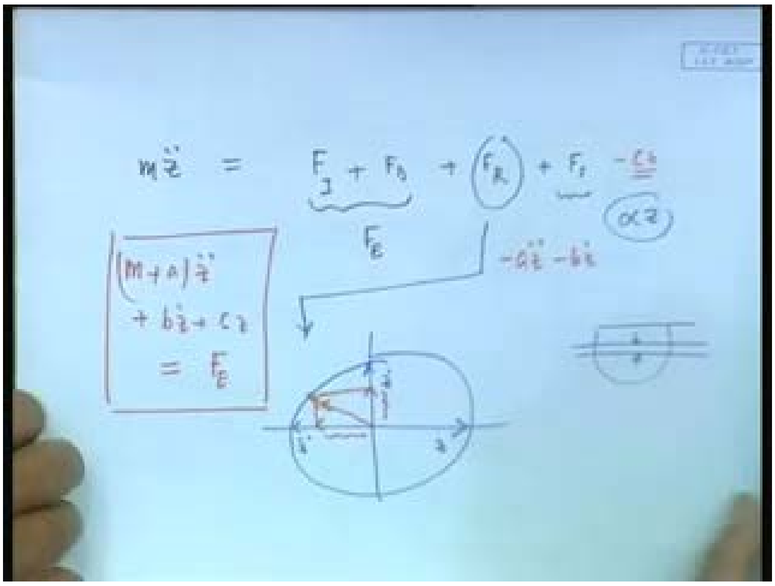

So, we end up getting something like F I plus F d plus F R plus F s F hydrostatic s. So, now, here this is my incident wave force is this? So, this two together quite often is known as F E.

Now, it turns out that let us first talk about this one you see it is it turns out that this force is proportional to z why it is. So, even easily see if I have this body if I pushed it down to amount z there; obviously, the force is rho a w you know a w z into z in if I pushed down one meter it is actually t p c ton per centimeter into z similar to that idea so; obviously, it turns out this is linear with respect to displacement hydrostatic pressure linear with respect to displacement.

Well forget z let us take pitch or rolled other what is roll if I give a phi what is my restoring moment it is rho g v into zee z what is zee? z for small angle g m into theta. So, it is rho g v g m into theta. So, it again depends on theta displacement. So, this is a proportional to z proportional to displacement in this case now it turns out this is this concept we will tell much later on it turns out that this particular force can be represented as a component in proportion to acceleration and a component in proportionate to velocity.

Actually, it is like this if I were to take it a phase circle and if this was my displacement z remember this is my velocity this this is my z dot dot this is a standard harmonic equation now the force turns out in something like this vector this radiation force this what it turns out and one can show show theoretically.

So, what is happen this force has a phase gap at what we can write as you know this to be vectorially can write this and this. So, I can write it this plus this vector ally also it is a vector. So, this force basically is in; that means, I can write this as a force proportional to velocity and as a force proportional to in other words I can write this to be something like minus a minus comes in I will tell you why something like that some proportion this I can write well as minus is a actually always a minus sign come because you know if I do plus z forces in the other direction. So, it looks like that.

So, now I reassemble that equation very interesting I can find out that I can bring it on this side. So, I end up getting an equation like m plus a z dot dot plus b z dot plus c z is equal to F E

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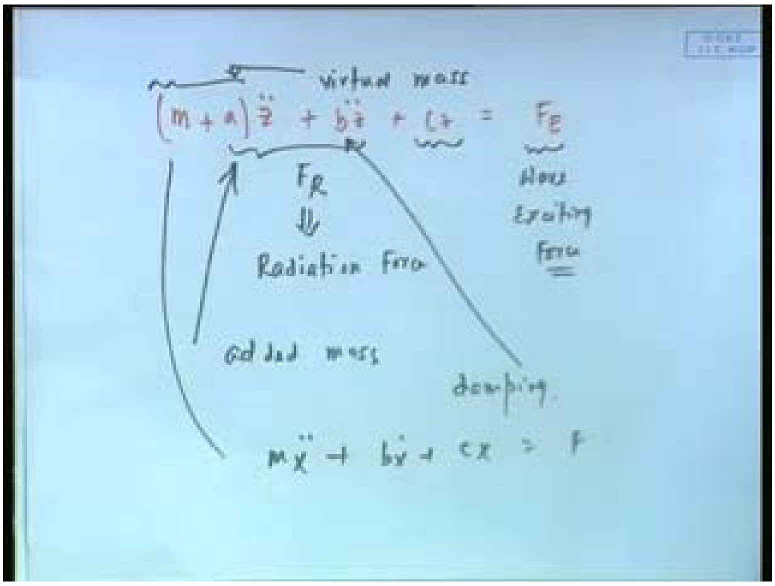

And I will once again say this because this looks very similar to what you would have done anywhere this is a general form of the equation we will work on that later on in here please understand once again this two are F R and they are called radiation force this is my hydrostatic restoring force not hydrostatic force by itself because, remember that integration of pressure up to mean water line is balanced by weight Archimedes said that long back it is only that displacement additional part that has given additional pressure that is why it is called hydrostatic restoring force, but not the Archimedes or buoyancy force per say.

This is my we call it wave exciting force and this on again classically this a is called everybody knows this added mass and this b is known as damping and this some people call it virtual mass m plus yes virtual mass you know mass plus added mass is virtual mass you can call it depends on how you want to call it. So, now, we end up getting basically an expression which looks very very very similar to what you would find in any vibration equation mass into acceleration plus damping into velocity plus you know stiffness into displacement is equal to exciting force. So, this is a single degree equation freedom equation motion.

Now, you may ask that where is the idea of you know like multiple degree freedom equation of motion it comes from here again see if I have to write F R F R is variation force, but it depends on acceleration velocity, but it may also depend on acceleration other modes of motion in other words it may be a one z a z dot dot plus b z dot plus a one phi dot dot plus b phi dot plus a two theta dot dot plus b two theta dot etcetera etcetera we in other words this could have arise in because of other modes of motion.

In other words, if I oscillated that in direction one it can be having some pressure on the other direction. So, if I added all these up similarly F s F s is what we have seen earlier that you know that if I give a roll it has a heave. So, if I have a parallel sink age it also has a trim. So, you know that if I give a parallel sink age I have got rho g v into g m l, but there is you know that is a trim and I also have this rho g w into z. So, these are the coupling.

The coupling comes out from here if I were to add the full expression and bring it back what would happen there will be more and more term coming; that means, in order to for me to solve for z I will have an equation will have will have z phi theta all together appearing in together well when you do not have you have isolation this equation. So, what I end up getting, but that is one point, but what I showed here in principle how we are getting a kind of an equation of this form. So, having said all these part what is happen we end up getting equation of this form which of course, looks very similar or it is same as any vibration equation you know n x dot dot plus b x dot plus k x equal to F that is any forced vibration equation.

Absolutely, straightforward as force vibration equation you know like this is same as m x dot dot plus b x dot plus c x equal to sum F we will discuss this in isolation for the specific cases where is the hydrodynamics lying we will see that this is also important for us the hydrodynamic lies in this because this is my radiation force and it lies in this. So, if I look at this expression if I break it down threadbare m z dot dot is my mass inertial force coming from rigid body mass I say if I know mass I would know it all c z coming from hydrostatic depends only on the geometry of the hull rho g etcetera where is the hydrodynamics it is here radiation force it is here diffraction force these are the parts that is where hydrodynamics comes in that means, further break it down added mass damping in here if you break it down it will be diffraction force and we can one can show they are connected.

So, this is where the you know like hydrodynamic comes in, but what we will be doing is we will presume as we go along that as if we know this and we will talk about solution of this you know solution for this equation for the isolated cases of heave pitch roll is what we will discuss and even the practical emphasis one of the reason is because you actually would know solution for this pretty well everybody knows this whoever have studied vibration in any vibrating system.

We normally do not like to call in ship motion as vibration although the equation looks same people call it oscillation because the periods are actually are lower normally the word vibration give a impression of very high frequency you know like even ship hull vibration, but oscillation is slower. So, this is only nomenclature remember the equation looks same is the question of numbers how they numbers look different and all that.

So, this is what we will talk about you know from I think I will talk about this from next class, but what we will do the next ten minutes for this class is that we will try to go through a simple problem of what we did it in last class on encounter period just a simple problem because that would be more logical see I wanted to start now in isolated heave pitch roll spending one more hour of lecture with all these thing.

So, having said that now we understand the weight comes we will start with that this is the single degree of freedom we will see how we can couple also and add it eventually, but for a class it is extremely important to realize the component where it comes from how it comes from because you see if you are able to guess a b and F others are much easily available hydrostatic tells you this mass is known then you know the solution. Solution for that will turn out to be fairly straightforward. So, complexity knows this we which we will discuss you know eventually.

Having said that let me just switch back uh because we also should kind of do some kind of problem as you go along let me just briefly go through a small problem on an encounter period problem because encounter period is always important since in all this again the oscillation period is always omega e. For example, here if I want to write I should write this as something into cos omega e t plus something. So, omega e is very important it is that period. So, let us work it out these things.

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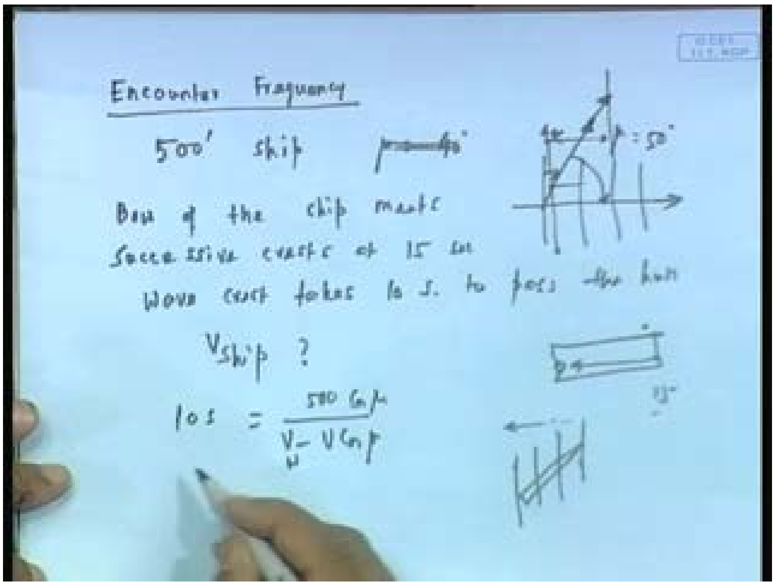

So, it in encounter frequency problem. A problem on this now it says there is a 500 foot ship well in ship many of the you know like dimensions are still used in feet although we should strictly used in s i n e, but it is only for making a point for example, knot will always be there nautical mile no matter what you say kilometer per hour everybody will use knot or nautical mile.

This is proceeding it says if this is proceeding through a regular wave at an angle mu equal to forty degree heading angle of 40 let me emphasis this part of heading angle you know in some reference book you will find heading angle is defined between minus x axis that is opposite of the wave direction and the ship. So, that zero degree becomes head waves may some books we are using opposite we are using a consistent system where direction of v ship and direction of v wave angle between that is mu that is why for us one eighty degree is this thing.

Now, here it does not say mu equal to 40 degree actually I will tell you what it says it says it is travelling with an angle 40 degree to the line of wave crests. So, the wave crests are there it is travelling with an angle 40 degree to the line of wave; that means, it is travelling with this as 40 degree, but remember the waves are moving this side. So, therefore, the mu is actually 50 degree.

The bow of the ship it says the bow of the ship it meets successive crests at every 50 second at 15 second; that means, the crest takes 15 second to meet the bow now if you are an observer you will see that every 15 second there is a bow coming the other thing is it says that the wave crest takes 10 second to pass the; that means, it is now something like that you are standing here you find that every 15 second a wave is passing by and this crest is taking 10 second to pass the length find out the speed of the ship. What is v ship.

See, here interestingly what is given see in principle vertical we will find out it has given in a sense the wave speed because it tells it takes 10 second to go, but not of course, in a direct line because the wave speed would be measured in a relative angle that is wave is going to pass through this at a speed of v w one and v cos mu. So, I have here v w one and v cos mu is the relative speed of the wave with respect to the ship. So, mu means 15 degree here and this speed at this speed it takes 10 second to cover this length; that means, what we end up getting is that this by here.

Now, here what would happen again is that this is going to be yeah no no here actually the thing is that no there is a there is the length is not actually it is not this way because, see the ship wave is coming by that. So, what is happening that wave is passing from this line to this line here. So, what happen again I have to take this length this length that is taken is this much it is not 500, but it is 500 into cos mu. So, essentially it is 500 cos mu that is what I will tell you why because what is happening the ship is like that crests are going like that passing. So, it is passing by this side and; obviously, different speeds there, but this distance pass is this much. So, it is basically this equal to 10 second.

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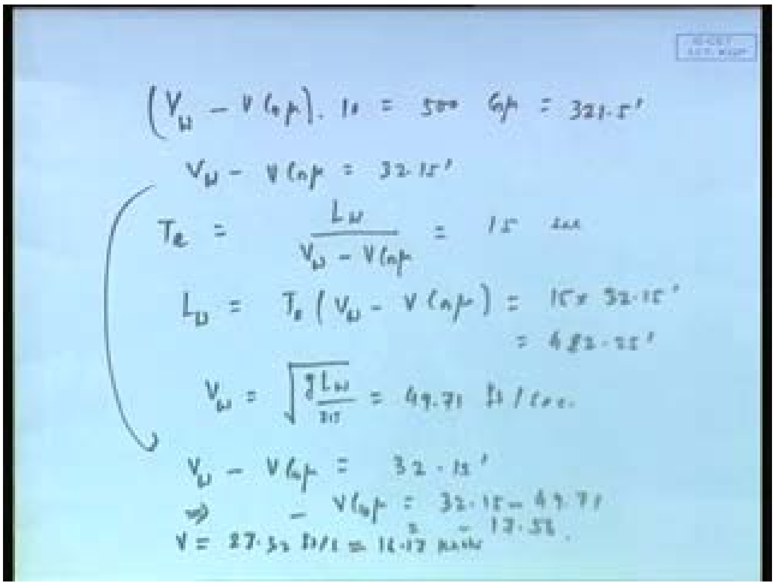

So, from there we will end up getting if I use this expression we end up getting v w minus v cos mu into 10 equal to 500 which will turn out to be feet. So, v w minus v cos 32.15 feet. I am just trying to go slowly and T e encounter period T e is l w by v w minus v cos mu this is 15 see actually the two information, one information is successive crest passing that is kind to T e the other information is the speed essentially you have that two information given T e and v. So, from there you have to just figure it out.

So, this is this relation and that tells us from here l w equal to T e v w minus v cos mu this is 15 into this is 482.25 feet. So, v w equal to g into l w by two pi this is going to give you give us 49.71 feet per second. So, this is v w now we need to find v ship we have v w minus v cos mu equal to 32. that is we had here this relation 15 feet. So, from there minus v cos mu gives you this gives me I think it is the space here v equal to 27.32 feet per second is equal to 16.17 k not.

I meant it is very straightforward problem what i wanted to tell is two things here you know that is in this kind of problem you have two information one was the encounter period T e one was the speed all that one has to do is to figure out the relations properly and therefore, you can work it out. Now a similar thing may occur for example, many times the questions are on this on this or on this there is omega e verses omega.

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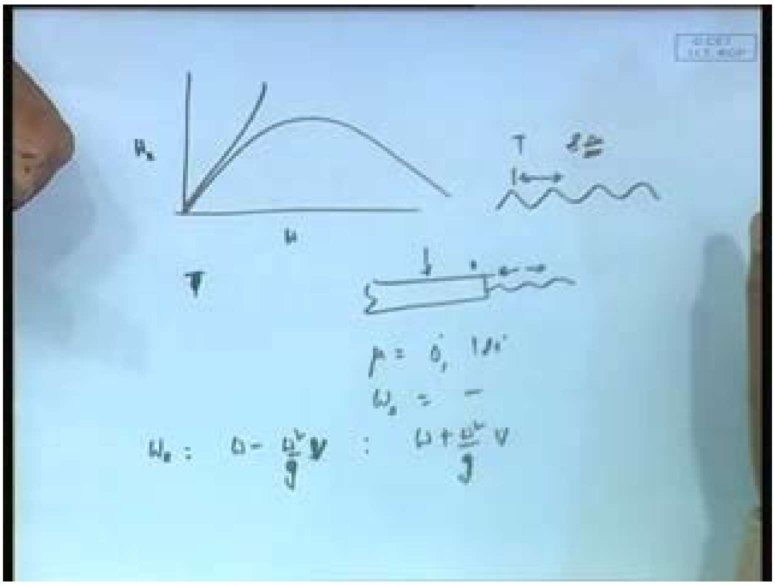

See for example, we may say that you know like simple question is of course, the simplest one that if I have t given or normally what would happen we may say that ships and waves are travelling in the same direction say the; that means, mu is either see their sometime problem like that if you are standing here you are finding out that wave crests are you know like passing by you at certain speed. So, in such problems I had mu given well actually may pass by or may go this side we you do not know sometime the problems are given that way. So, it can be 0 degree or 180 degree now omega e is given find out omega.

Well, we go to this relation omega e equal to omega minus omega square by v g and cos mu is given here it is negative. So, it well cos mu is e that too cases it can be this or it can be sorry g v here g v here and since there quadratic equation you end up finding out two solution for omega and then your once you know the omega you may figure out lambda then you can tell what is the length. So, these are very straightforward problems though you know, but one thing we must realize this excitation frequency that is what I am emphasizing it if you have an instrument here you are measuring excitation.

So, you what is happening I end up getting finding out that my body has an excitation of this period T e some t say each second or whatever plus understand please understand that this 8 second might arise if it is a following wave from multiple scenarios because, if it is a following wave I could have two omega which could have given rise to 8 second.

So, unless I know that I really cannot figure out the motions properly. So, there there is multiplicity in following wave conditions same excitation may occur from more than one wave condition. In fact, typically what happen one at a shorter wavelength one a very long wavelength similar results will come we we keep doing these problems we could have also done this here.

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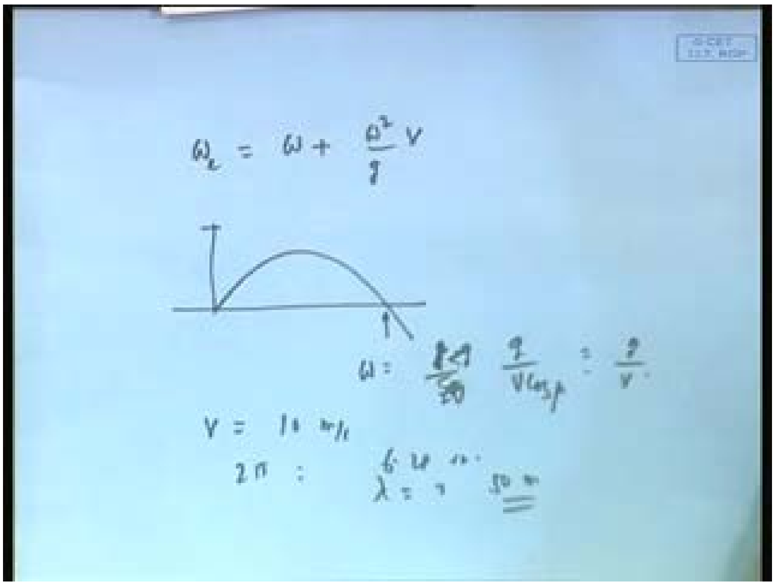

See, the quadratic equation relation right see omega e is omega plus let me write it down this v y g into v. We can easily solve it by putting some number and see there are two feasible omega number now what would happen what happens in this kind of in a problem what happen in this kind of cases I give an omega let us say omega e which transfer to be somewhere here then of course, what will happen if you will solve you will find out your imagery numbers of omega means you will tell that such omega you cannot occur simple as that that is one thing.

Now, we have found out also let us put this number what was omega this omega was g by sorry sorry it is omega was let me let me just tell because we I want to spend this this few minutes on this number here this was g by 2 v cos mu g by two v g this was g by v cos mu yeah sorry sorry this was g by v hah cos mu.

Now, now see just simple thing say cos mu is equal to 1. So, it is just a g by v in a following wave condition what can be the possible number at which this occur that is very important just physically understanding take v v equal to let us take some feasible number v equal to say uh twenty meter per twenty knot ten meter per second say ten meter per second G will be about 10. So, g by v becomes about 1. So, omega becomes one. So, what is t 2 pi by omega? So, about two pi second.

What is 2 pi second is about 6.28 second how much is the length of that 1.56 into 6 square that is approximately equal to 36 into 1 and half about 50 meter. So, you see for a 50 meter long wave for this particular case you end up getting for following wave 0 encounter frequencies. So, it is very feasible range; that means, 50 meter wave length is possible and there can be a small going at 20 meter per second.

So, in other words in realistic waves do exist coming from following wave which can lead to a negative encounter frequency which can lead to 0 encounter frequency it is not outside my practical bound. This is important which means I cannot ignore it if there are following wave coming it is a practical scenario waves are there 50 meter 100 meter 200 meter etcetera.

Anyhow, So we must realize this, but having said that once again I tell you that as importance is more on head waves we are going to give less focus on following waves as such as far as motions are concerned with that I am going to end this today's class I will go tomorrow on this individual modes of motion thank you.