**Fluid Dynamics And Turbo Machines.**
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**Part B.**
**Module-1.**
**Lecture-1.**
**Integral Analysis.**

Good morning and welcome to the 2nd week of this course on fluid dynamics and turbo machines. So, we will start with the part B of the first module, the first module is on fluid dynamics. The part B of the 2nd chapter for this fluid dynamics part of this course deals with integral analysis of fluid flow. We have already introduced the integral analysis and the differential analysis for fluid flow in the first chapter. Basically the integral analysis deals with a control volume of finite size, whereas the differential analysis deals with an infinitesimal control volume, a very small control volume. And the first one, that is the integral analysis gets an output in the form of gross parameters, like the force, torque, thrust, etc. acting on the control volume. Whereas in the case of the differential analysis, more detailed parameters are obtained as output, for example the velocity field, the pressure field.

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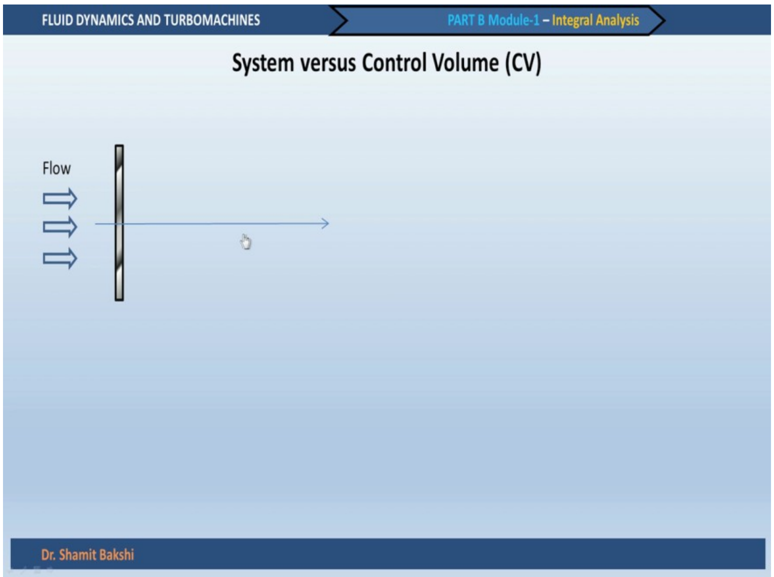


So, let us start our 2nd chapter which deals with the integral analysis. So, this is the first lecture on the, for the 2nd week for the 2nd chapter. We begin with the definition of this term system versus the control volume because we will be doing this analysis for a control volume and we already said that the, the integral analysis deals with a finite size control volume. So, what actually we mean by control volume? For this we also bring the context of system. So, let us see what is a system and what is a control volume. Now this is also defined in different books either as control mass, the system is defined as control mass and the, also it is defined sometimes as close system, as the control volume is defined as the open, an open system. But in our course, we will mainly use the terminology as system versus control volume.

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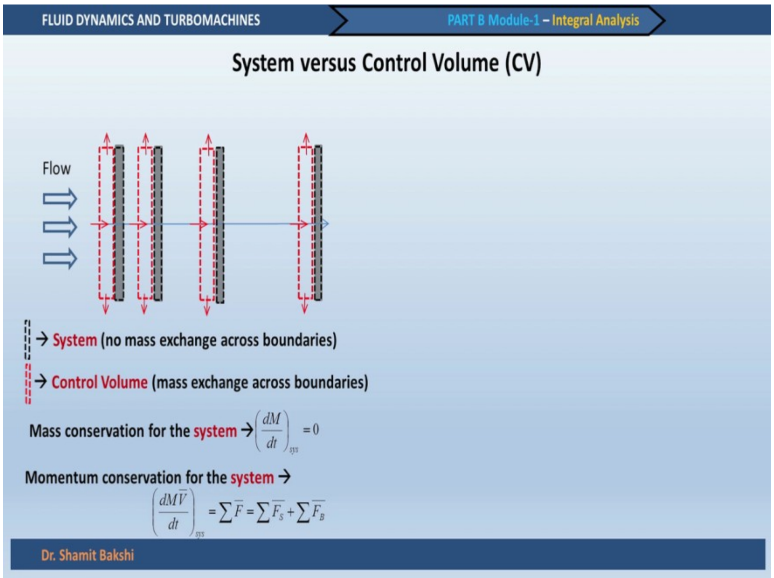


So, let us give a definition to this. Now again we consider this flow, like we started our first chapter and we see how the flow is impinging on the plate. Of course the plate experiences a force in this direction and then in the direction in the positive X direction which is the horizontal direction as shown in the slide and it keeps moving. This is the previous position of the plate and this is a new position of the plate and then it keeps moving further and further. Now, if we look at this particular part, we see that we can draw some, as shown here, we can draw a boundary across the plate, so this dashed line indicates the boundary. So, this is essentially the system boundary. Why is it called a system boundary, because the system boundary is such that if there it does not allow any mass exchange across the boundary, this is true for any system. That there could be energy exchange across the boundary but there is no mass exchange. For example in this case, if we consider the plate as a system, then there is no mass exchange across the plate but there is energy exchange.

We can see the result of the energy exchange if we look at the trajectory of the plate which is shown here. Assuming that these are like snapshots taken as same intervals of time, we can see that gradually the plate accelerates. So, that means a kinetic energy is imparted by the flow to the plate. So the plate, which is a system here, which is defined as the system here, is actually receiving kinetic energy from the, receiving energy from the flow and accelerating. So, there is an energy exchange across the system boundary but there is no mass exchange. It is not that, this mass exchange is only true for a boundary which bounds a solid. You can also have a fluid bounded by boundary and there is no mass exchange across that boundary. We will see that example.

Now in the same case if we consider now a boundary as shown by this red dashed line, so this is essentially the air sticking to the system. So, sticking to the plate here. So, if we consider this as a boundary, then inside this, this bounds a region across which there is mass exchange. So, this is basically a control volume. So, control volume the region in space across which there could be mass exchange, of course there could be energy exchange across the boundary. Now, if we follow this trajectory of the control volume, we will see that it will, we will have the control volume also moving along with the plate for doing the analysis. Now, in this integral analysis, we will be dealing with the analysis of this control volume. We will not deal with the system, we will deal with the control volume across which there is a mass exchange across the boundary.

Now, the aim of this chapter is to actually write the governing equation for this control volume so that we can write the known equation for a system in the form of a control volume. That is basically the objective of this particular analysis. Say, for a system, for example in the system, the governing equation like mass conservation, that mass could be created or destroyed, so this equation is well-known. So, for example, if we can always write for a system, the rate of exchange, the rate of change of mass with time with dm by dt for a system is zero. In fact this is the definition of a system that there is no mass exchange across the boundaries of a system.

Whereas this is not true for a control volume because across the control volume there is a mass exchange. So, we have to see how to write this equation for a system for this governing equation for the system, how can we apply this for a control volume. Another well-known equation is momentum conservation. So, for example if we want to find out the acceleration of this plate, what we have to do is, we have to solve the momentum conservation equation, which says the rate of change of momentum, that is M into V bar which is a vector quantity for the system is equal to the summation of all the forces acting on the system. So, by doing this we can find out how, what is the acceleration of the plate provided we know the forces acting on the plate.

The forces acting on the plate actually can be divided into 2 types of forces, one is the surface forces, other is the body forces. The surface force is basically the force which acts on the surface of the system, that is on the system boundaries. For example, the pressure, the pressure can only act on the surface and thereby it produces a force or a normal reaction acting on the surface of the plate. FB is the body force which acts not only on the surface but on the bulk of the system. So, each fluid region element or each atom within the bulk experiences this force, for example, gravity force. So, you can broadly divide these forces, summation of forces into surface and body forces. Now again the question is how can we write this kind of equation for a control volume. So, this is basically the motivation for studying this particular chapter.

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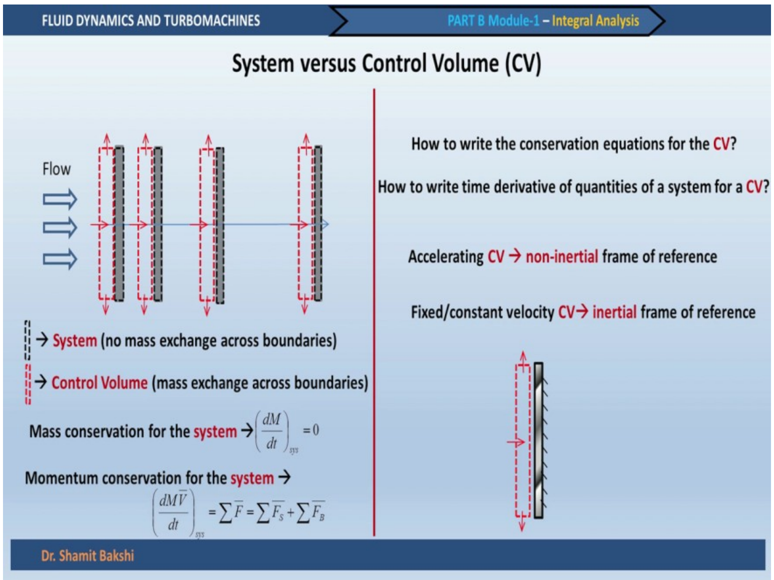


So, this is what we already said that we try to see how to write the conservation equations for the control volume. We know this for the system but we want to see how we can translate it to the conservation equation for the control volume. In other words, it means that how to write the time derivative of the quantities of a system for a control volume. So, mainly you see here, the left-hand side of the equation which is a time derivative of mass or time derivative of momentum for the system, if we can write this time derivative for a system in terms of a control volume, then we can rewrite this equation, this governing equation for a control volume. So, that is the, that is more precisely what we are looking for in this analysis.

Now, another thing before we go ahead with looking at how to write the time derivative of quantities, of a system in terms of that of a control volume, let us introduce ourselves with the concept of accelerating control volume. For example in this particular situation which is shown here, the plate is accelerating and as the control volume is sticking to the plate, it is also accelerating along with the plate. We want to find out the forces on the control volume, so this has to remain along with the plane. So, such a control volume for an analysis of such a control volume, we have to take a reference frame which is on the control volume itself. We have to see by sitting on the control volume what is happening, what are the forces. So, such a frame of reference is called a non-inertial frame of reference.

This is a little complicated subject and we can only discuss about that after we understand how to deal with a fixed or a constant velocity control volume. And if we put the frame of reference on a fixed or a constant velocity control volume, that frame of reference is called inertial frame of reference. So, to be, we start with this integral analysis for an inertial frame of reference. So, this is shown in the figure that we have stopped the plate from moving and now we have a fixed control volume. We can also extend this analysis to the situation by the plate is moving at a constant velocity, it is not accelerating. If it accelerates, then extra force terms will come with we have to deal with separately.

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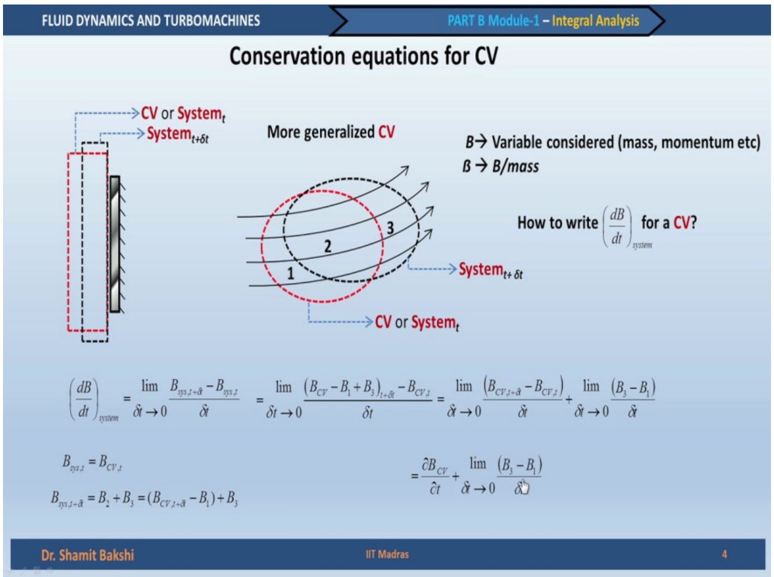


But if it is not accelerating and moving at a constant velocity, then by making small changes in the, in the case for a fixed control volume, we can do an integral analysis. So, we go to the next slide. So, let us look at the conservation equations for the control volume. So, this is our fixed plate and this is a fixed control volume in front of the plate. Now, this is the control volume at time T, we can also define this as a system. That means there is no mass exchange across this boundary. At a certain time we can define this as a system. Now, the 2nd box, the 2nd region which is shown by a bounding surface with black dashed line, this is the system at T plus delta T. T is a small-time, small period of time. So, if we look at the system boundary, what would have happened because of the flow, the mask which was here, the flow, the fluid particles which were in this region has moved into region region nearer to the plate and some fluid particles have moved out of this control volume in this region.

So, basically this is the system at T plus delta T. So, as we can see here, which we have mentioned in the last slide also that we can use, we can define a system even for a fluid, not necessarily only for a solid. Only the mass within that region has to be constant and it has to be the same mass, it has, it has to deal with the same mass at all point of time. So, we define this kind of a situation that we have a control volume, the control volume does not, it is a fixed control volume, it does move with time but the system, just to keep its own definition that the mass within the system remains the same at all point of time, it has to move. But the control volume at T, time T plus delta T is again shown by this red dashed line.

Now this is a this was to show for this simple case, we can take this in a more, look at it in a more generalised control volume. So, let us consider this is a flow, this is a arbitrary flow and we define a control volume within this flow. So, this is shown in terms, like a oval shaped control volume, it could be of any shape. Now, this is the boundary of the control volume or the control surface at time T or this is again defined as the system at time T. So, now after a point of time delta T, that is at time T plus delta T we see that the system has actually moved to a new position because it has to keep the same fluid particles as it had at point of time T. So, it has to move with the particles, it has to move along with the particles, so till now in this new position.

The control volume actually than its same position, of course the properties of the control volume changes. For example, the mass of the control volume can change. Now let us see how to write the time derivative of quantities in this control volume shown here. So, this is a systematic T plus delta T and the system at T which is same as the control volume boundary which is same as the control volume region. Now let us also define a variable B, so this is a general variable so that we can apply it to time derivative of any quantity. So, this general variable could be mass momentum, angular momentum, anything but for the, we make it a general variable and find out what is the time derivative of this quantity B or what is a time derivative of B for a system in terms of the control volume parameters.

Now we also define this parameter beta which is B per unit mass that means this variable, the same property per unit mass. For example if we are defining momentum, then this is beta comes velocity. So, this is the question how to write dB by dt of the system for a control volume or in terms of a control volume. So, we write dB by dt of the system from the basic calculus, we know that we can write a derivative as a limit of as delta T tending to 0, B system at T plus delta T minus B system at T divided by delta T. So, basically this is what we can write, the expression of derivative from first principles. So, whatever B is there, at time T plus delta T within this boundary, minus what was there at time T with divided by delta T is the time derivative.

Now, what is B system at time T? It is same as whatever B was there in the control volume at time T because the system and the control volume are one and the same time, at point of time T. Now, what we do is to look at B system time T plus delta T, we divide this entire region into 3 parts. First part is 1, 2nd part is 2 and 3rd part is 3. So, this entire region which has the first control volume, the first system at time T, the system at time T plus delta T and their intersection into 3 parts. So, part one is the is a subset of basically the control volume or the systematic time T. But it is not a subset of the system at time T plus delta T. That means it is only this region, so this region is basically the region bounded by this control surface by red dashed line and this black dashed line.

The 2nd region is the common region between the system at time T plus delta T and time T. So, this is basically the intersection of these 2 positions of the same system. So, this is bounded by this line, this black dashed line and the control surface only. And the region 3 is a subset of system at T plus delta T but not that of the control volume. So, this is the region bounded by this black dashed line and the red dashed line here. So, in other words, this is like this region one is like whatever has come into the control volume at during the time delta T. The property of that, if B is a property, then B of that quantity. This is a region which has moved into the control volume in time delta T. And this is 3 the region which has moved out of the control volume at time delta T, at a, in a duration, during the duration delta T.

And the region 2 is the time, I is the region which has remained within the control volume during the duration delta T. So, one is the, one is the first part which is entered into the control volume from somewhere else and 3 is the region which has gone out of the control volume during the time delta T and 2 is a region which has remained within the control volume during this duration. So, we divided into these 3 parts, now this makes us easy to define the value of B for the system at T plus at time T plus delta T. So, it is given as this, so value of the variable B for the system at T plus delta T, so system at T plus delta T is actually shown by this black dashed line.

So, value of B for that will be value of B in this region 2 which has remained within the control volume plus value of B in the region 3, that is a region which has come out of the control volume. So, that constitute of the value of B, variable B for the system at time T plus delta T, B2 plus B3, of course B1 will not come here because B1 something which has entered the control volume during the time delta T. So, it is not a part of the system, the system can only have the same fluid particles at all instants of time. So, that we have to always keep in mind. It has to keep a mass, same fluid particles at all instants of time. So, one cannot be a part of it. So, it is B2 plus B3, you can write B2 as B for the control volume at T plus delta T minus B1.

So, you can indirectly write it in terms of B1 as the total value of B within the control volume at time T plus delta T which could be different from the value of B at time T because control volume, the value of this variable B which could be a mass momentum, angular momentum, anything, so the value of that quantity at time T plus delta T within the control volume will constantly change. So, this BCV at T plus delta T minus B1, so BCV at T plus delta T is this region minus minus B1 which has entered into the control volume, that will of course be B2 plus B3. B3 is what has gone out of the control volume. So, now by writing it in this form, if we can plug-in these 2 components here, what do we get?

We get the limit of delta T tending to 0, this is the numerator and divided by delta T. So, what we see here is basically the value of B CV at T plus delta T and B CV at T appears in this expression. So, this helps us to write expressions in terms of the control volume, in the sense if we club in, if the club these 2 parameters like B CV at T plus delta T and B CV at time T, then we can write an expression like this which is very similar to the time derivative like we have shown here. So, this is the time derivative from the first principles and this is the rest of the term which is B3 minus B1 by delta T. We will see how to deal with this 2nd part of this expression but right now we can directly write this as partial derivative of the quantity of B in the control volume with respect to time.

B in the control volume could be dependent on time and space, so we write it as a partial derivative. So, partial derivative of this quantity BCV with respect to time plus this 2nd quantity. Let us see how to write the 2nd part of this equation now. Because we have partially written, what we wanted is to write the time derivative of a parameter B for a system in terms of a control volume B for a control volume, we have partially succeeded in that, let us see how to proceed further in deal with the 2nd part of this expression.

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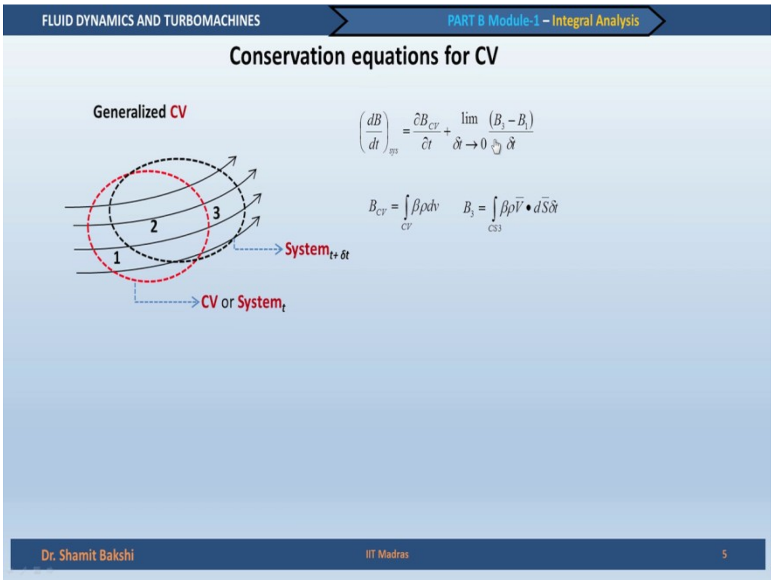


So we come to the next slide, again we have kept the same figure to take help bring the derivation. So, this is dB by dt of the system, we already derived it. This is already derived but we can write that the BCV actually, so this is, let us see what is the value of B CV, we can write it as volume integral, so this is integral over the control volume of the quantity beta rho into dv. So, this v is basically volume, now if we see rho into dv is basically the elemental mass within the control volume because the integration is done within the control volume and beta is the B per unit mass. So, B per unit mass multiplied by elemental mass, if it is integrated over the control volume, will give the total value of B in the control volume. So, that is how we get this, we can plug-in this here and then make it more descriptive in terms of the control volume parameters. Now, what is the 2$^{nd}$, how to deal with the 2$^{nd}$ part?

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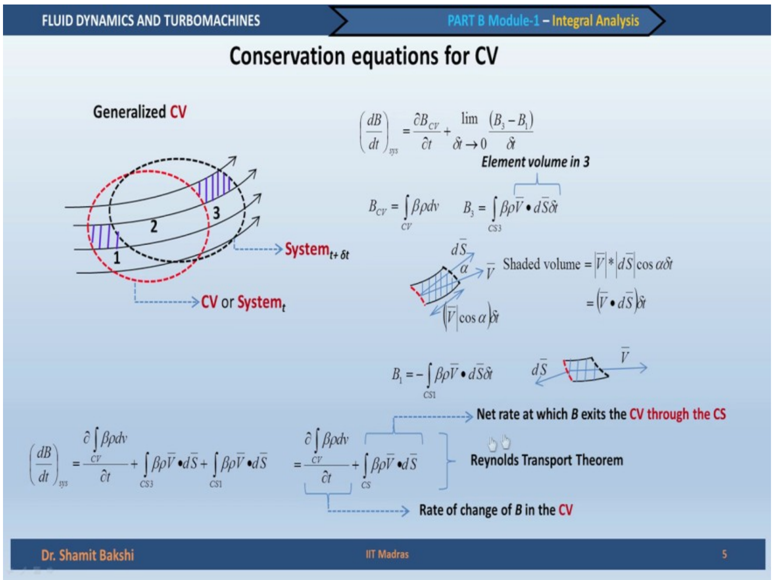


The 2nd part, the first term in the 2nd part is B3, of course delta T comes in the denominator, so B3 can be written in this way, so this is a surface integral. We have just directly written this expression, now we will try to see how this expression is obtained. So, if we look at this expression, this term is again something similar to the term here. So, this is also a volume. So, this is, what volume is this, this is actually volume of an element in region 3. So, we are talking about B3, so this pertains to region 3, so this is volume of an element in this region 3. So, let us see it a little more details. So, of course if we, if this represents, considering that this represents the elemental volume in region 3, then multiplying that elemental volume with density will give you elemental mass which multiplied by B beta per unit mass will give the value of B and if it is integrated everywhere within this region, then across this region C S3, then you get the value of B3.

Now let us see how is that this is a volume. So, we take an elemental volume like this. Now, we take out this volume and reproduce here to look at the different velocity and areas are appearing in the expression. So, the velocity, if this represents these streamlines, then the tangent to that will be the velocity, so the velocity vector will be oriented like this and it is not necessarily oriented perpendicular to the control surface. The red part is basically the control surface taken out from here and this is not necessarily oriented normal to the surface. So, what is oriented normal to the surface, this vector is basically the area vector. So this is ds bar. So, ds bar is always in this convention and in many books also you will find that the general Convention is the area, positive area is oriented outward.

The, of course the direction of the area of any surface area is the direction of the normal to the surface and oriented outward. What is the meaning of this outward? Outward means, so this pertains to a control volume, so there is the region inside the control volume. The area vector will not orient itself towards inside of the control volume, it will orient itself outwardly from the control volume. So, for example if you go to the side, we will see it soon that this side, it will be oriented opposite to what is shown here, it will be in the other direction, normal in this direction. So, anyway, so if you now consider this, then velocity in general, in a generalised form is not oriented along the normal to the surface control surface and there, let us say the angle between these 2 vectors is Alpha.

Then the component of velocity along this normal is V bar magnitude of V bar cos Alpha and if you multiply that with the time duration delta T, what you get is the distance travelled. So, this is basically the length of this control volume. So, if you now see this as the distance and if we look at the volume now, so the shaded volume will be given and V bar into ds bar or cos Alpha Delta T. So, if you bring the magnitude of area here, then you get the shaded volume. So, volume shown in this region or shown in this region. Okay, so this is basically in terms of cos Alpha you can generalise in a generalised way, you can write it as V bar dot ds delta T, so that is how you get this expression V bar dot ds delta T. And if you multiply this with the density, you get elemental mass which with beta, multiplied with beta give the value of B in the entire volume, in the entire region 3.

And this has to be done over where it has to be integrated, it has to be integrated over C S3. What is C S3, CS 3A is basically the control surface of the control volume which is shared with the region 3. So, in this figure it will be like this. So, the control surface shared with region 3. So, for example now, if you sum all this, we take this simple example, so we sum all these volumes, we will get volume of this entire region 3. We have to keep in mind, although it is shown in a little exaggerated way in this figure but this displacement is actually very little because the time which we have considered is elemental time delta T. Okay, so this basically shows us how we get B3. B3 is basically integrated value of beta into the elemental mass, rho into the elemental volume, the elemental volume in this region, in the region three.

And CS3 is basically the part of the control surface, control surface means the part of the surface of the control volume CV, shown by this straight line which is shared with region 3 that is shared with the exiting part or you can say this is basically the path through which the fluid is exiting the control volume. So, this is B3, now in a similar way you can find out B1 because this is the next parameter to be found to get an expression for, a complete expression for dB by dt for a system. For B1, so what is the difference between B3 and B1, B3 is basically the region which is coming out from the control volume and B1 is the region which is entering the control volume, the control volume being shown as the red dashed line. So, B1 is what is entering into the control volume.

So, now for B1 we can again write a similar expression like beta into this is the elemental mass now which can be obtained from a similar analogy like this similar you know expression, explanation like this and so this is a typical elemental mass here, so we have shown the elemental, sorry elemental volume here, so this is the elemental volume. There is one difference here, see there is a negative sign if you have noticed as compared to the previous expression. This B3 has no negative sign but it has a negative sign, why this, where from this negative sign comes, this comes from the fact that if you look at the velocity vector now, the velocity vector is oriented because this is an inlet, the fluid is coming into, so this is an inlet, so the fluid is coming into the control volume through this surface.

These are all the surfaces through which fluid is coming into the control volume. So, the velocity vector a directed like this but the area vector is directed in this way. So this is true for all the surfaces all the regions of the control surface through which the fluid is coming into the control volume. So, if you do a V bar dot ds between these 2 vectors, it will be negative because the angle is more than 90 degree, so it will be negative. To compensate for that, the negative sign is introduced. So, now we can, if we incorporate this value of the one and this value of B3 into this expression, then we get an expression like this. You can see delta T will get cancelled out from the numerator and denominator as it appears here.

So, you get and because of this negative sign, now it can be written as, this was the first part, first part is written by plugging in the value of BCV, that means the value of variable B within the control volume, a partial derivative of that and the 2$^{nd}$ part has these 2 terms. The first term pertains to the value of B, the sum of the value of B which exits the control volume through the exiting surface, that is CS 3, control surface and this is what is coming into the control volume through the incoming surface. This can be actually written together because CS 1, like we have written here and here, it is this region, it is this surface through which fluid is coming into this control volume. So, if you see, this is, this part CS 3 is this region through which fluid is going out of the control volume.

This part is same, this expression is same in both these integrals, so you can club these and you can do a cyclic integral over this control surface itself. So, you can write it as partial derivative of this and cycling integral over the control surface of this quantity. So, that means you have, to begin with you had the time derivative of a quantity B for the system whereas you have written it as the time derivative of the same quantity in the control volume and the time derivative of the same quantity on the control surface, in terms of the integration of the quantities on the control surface.

So, let us get an idea about the physical meaning of this expression which we have obtained. The first term is basically the rate of change of B in the control volume. So, this means that lets take an example of mass, it is easy to visualize that. We say that dM by dt for a system is zero because the mass does not cross the system boundary. But here if you see this, in that case this will become Dell Dell T of M, mass of the control volume. This is not necessarily zero, even though this is always zero, if we take B as mass but this is not necessarily zero. So, that means the mass in the control volume is changing with time and what is this parameter? This parameter is basically net rate at which B exits the control volume through the control surface.

So, for this specific example of mass, this is the net rate at which mass exits the control volume through the control surface. The term exits also accommodate inlet because inlet will mean a negative value of this parameter. So, now let us see, if it is not negative, if it is if it is positive, so what does it mean that the mass is exiting the control volume. The control volume is actually is shown by this red dashed line and rate of change of, the rate of change of mass of the control volume which is shown by the first part and the $2^{nd}$ part is essentially the net rate at which mass exits the control volume through the control surface. So, if it is positive, it mean rate of change of mass has to be negative because if this quantity is positive, then and the sum of these 2 is zero, okay, so rate of change of mass plus net rate at which the mass exits the control volume, that is equal to 0.

So, rate of change of mass will be negative, which is also true if the net rate of mass exiting the control volume is positive, that means mass is only, the net flow if you consider, it results in a loss of mass from the control volume and so the, mass of the control volume will decrease with time. Similarly if this is negative, if this quantity is negative, it means that dM by dt of the control volume, this is essentially dM by dt of the control volume, if we consider B as the mass of the, B as a parameter like mass, then rate of change of mass of the control volume will be positive because this is a negative quantity and the this minus this will be zero. So, that means is there is net inlet of mass within the control volume, the mass of the control volume will increase with time, so that also makes sense.

So the statement of this expression is very simple. Of course there is a mathematically rigourous way to get to this expression, to make it applicable to a very generalised situation. So, basically the statement of this particular expression is actually stated as the theorem called Reynolds transport theorem and this equation is called Reynolds transport equation. So, again just to repeat, the Reynolds transport equation converts the time derivative of any quantity for a system to do an expression for the similar expression for a control volume and time derivative of any quantity for a system, in terms of control volume can be written as a summation of 2 particular terms, the first is the rate of change of that quantity in the control volume and the $2^{nd}$ term is the net rate at which that quantity exits the control volume.

So, basically that is the Reynolds transport theorem. So, in this lecture, this is the first lecture for integral analysis, what we did is, we started with the definition of system and control volume and we know that the governing equations for the system, all the equations are well-known, the task of this analysis to begin with is to write the similar equation or translate the equations for a system to that of a control volume. For that, what was essential was to write the time derivative of any quantity for a system in terms of that for a control volume. So, we did the derivation of that and we basically came to **to** the end of, at the end of this lecture we came to the expression for the Reynolds transport theorem. In the next lecture we will apply the Reynolds transport theorem to some specific case for conservation of mass and momentum. Thank you.