Spray Theory and Applications
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Lecture - 02
Spatial versus Temporal Sampling

Hello, at the end of the last class we made a list of few applications of sprays, and what we are going today is sort of zoom out a little bit and look at spray as an entity. Spray is the collection of about a million, billion may be even a trillion drops, and we want to see what sort of measures can I bring to a spray; to gain a quantitative understanding of a spray.

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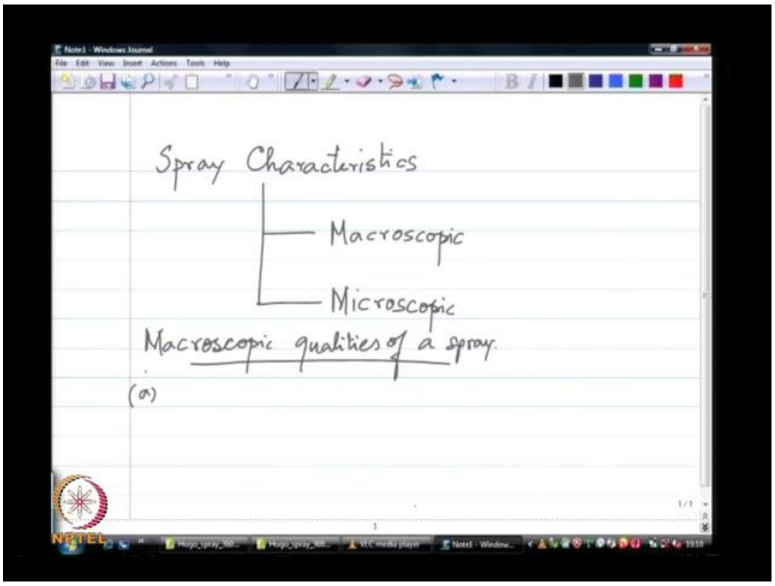

So, we will start to make a list of a few different spray characteristics. Now before we turn to the audience and try to ask this question of them, I want to suggest there are basically two kinds of spray characteristics; one that are macroscopic in nature and second that is microscopic. So, will make a list of a few different macroscopic qualities first and then move on to the microscopic qualities. We will just take a steady spray.

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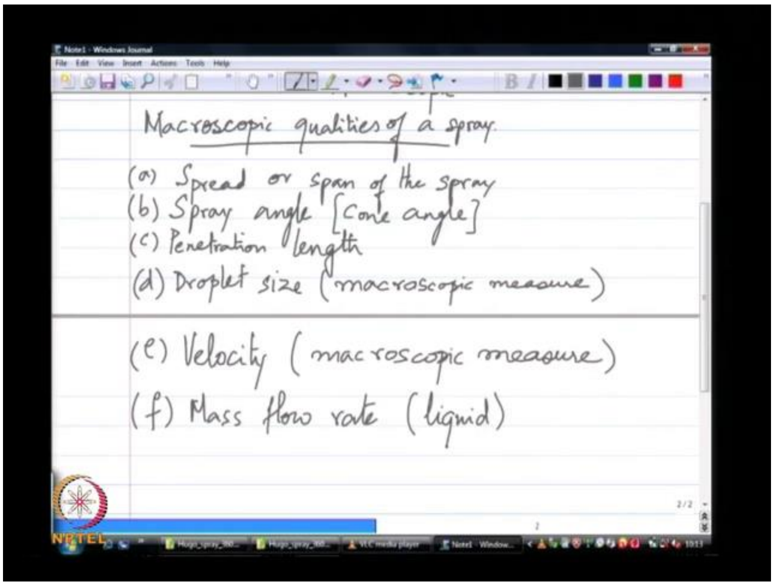

So, as suppose to a little perfume spray; perfume spray is where you push the plunger down, a whiff of perfume comes out there is a start and end to it. So, we are going to talk about a spray that has no start and an end. It is just like a continuous perfume spray. It is just easier to understand these qualitative aspects in that contest. Then we will talk about an intermittent spray also and talk of some qualities of an intermittent spray as well. So, let us start listing a few different microscopic qualities.

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So will you say spread or span of the spray?

Student: Spray angle.

Spray angle, very nice. I will also distinguish this between this and cone angle; we will talk about that also, penetration length.

Student: Droplet size.

Droplet size; we going to list this in the; I will distinguish this because this is going to occur in both the macroscopic and the microscopic list. So, we will talk of this in just a moment similar macroscopic measure any others that you want to include.

Student: Velocity.

Velocity, so will you say velocity has again a macroscopic measure. So, we already made a list of a few different things here. And I want a add one to which is one that is relevant to many different process is actual mass flow rate of the liquid, we can add a few more, but this is a good enough list.

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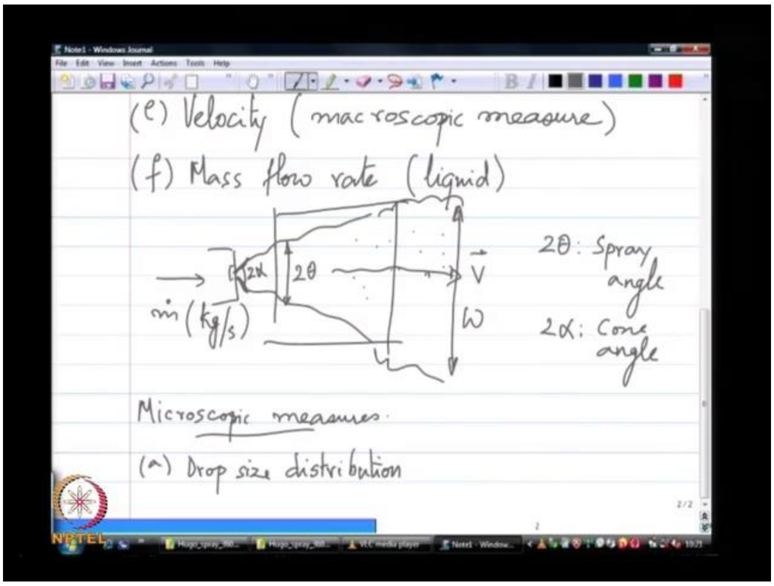

So if I just take a typical spray, let us say I have a spray nozzle from which I have this spray emanating. Now first of all, I have this all these drops in this spray we have seen this many times, the actual flow rate coming out in a steady sense is the mass of the liquid flowing out per second. This is relevant to surely in many applications.

Now talk of velocity there are two kinds of velocities, the reason I wrote down macroscopic measure is because I want to know sort of in a bulk. So, if I take a whole bulk of this spray what is like a velocity vector of that bulk? But really speaking it is not a view to know what this whole bulk is doing; I want to know what different points in this spray are doing. Therefore, there is a need to understand sprays using microscopic measures.

Back to the macroscopic I just want to complete the list here, spread or span like for example, that width at a target surface. Say for example, if you are spray painting a wall; I have the paint can about let us say 10 inches away from the wall, when I push the plunger and this is the continuous spray because I can just walk along the wall and paint the wall. When I am walking along the wall there is a certain width that the paint impact will cover. That is important because I want to know how many passes I have to make of this can. So, if I look at a measure like that; that gives me a width and clearly that is also tied to the cone angle or the spray angle.

So, there are two kinds of angles and we will talk about them may be about a week from now, but essentially if I call that angle 2 theta; 2 theta is what we will call the spray angle. The cone angle is slightly different; we will look at that in some detail. In the sense that the angle very close to the nozzle here would be slightly different for different fluid mechanic reasons and that is often referred to as the cone angle.

So, you have a cone angle that is usually larger than your spray angle. Penetration length, now again some of these measures are all tied to each other because, if I take a spray of a certain mass flow rate and somehow increase the width which also means increase the spray angle, I am naturally going to decrease the penetration length because the drops, sorry the mass of drops are going to slow down. So the penetration length is a measure of that. So, let us put this intuition to work we will look at a little video we made in this in our lab here. This is a simple perfume spray let us going to go from start to end.

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Let us play this video and see what we learn from it. I want to pause it there to point to a few different things, you look at that angle there that is the spray angle. Now as you go further downstream, you can see that the spray angle is becomes less and less defined. So as I go further and further downstream, this spray angle becomes less and less defined because there is no spray edge, the idea of a spray edge is an illusion it is not like the edge of this pen, this edge of pen is well defined. Edge of a spray has to be defined by us and our definition of the spray edge is going to also influence our definition of the spray angle.

So, if I say I want to go as far as I find no drop at all that is going to give me all most 180 degree cone angle and many different sprays and it is going to a very large number in comparison to, where I said I am going to go to a point where the mass flow rate drops below some critical value; so mass flux, that is the rate of mass flow per unit area drops below certain value. So, I will say when there is very, very few drops almost a miss, I am not going to count that in my spray that is a simple definition of a spray edge.

We can find more quantitative definition as well, but our idea of a spray angle is clearly tied to that. So, here let us go on and continue to play this video I want you to understand two or three different aspects here. You can see that there are some big white flashes right where my mouse is pointing, those are in essence big drops. So, I can go back to the start and show you were they started. You see that the drop there that started is going to continue forward. I want you to notice two differences is here; one, of course this is not a steady spray, but this idea also applies to a steady spray. So the concept of having big drops alongside tiny drops is not something that is out of the ordinary. Every spray of any commercial interest will have a range of drops that the spray has. So, typically the range is you is on the order of at least two or it is least two orders of magnitude in span more often it is three orders of magnitude in span.

So, the idea that you have drops that are ranging from nearly let us say fraction of a micron, micrometer in diameter to hundreds of micrometers in diameter is not at all out of the ordinary. In fact, you can see this in this image. Now as you keep going you can see that the perfume has practically stagnated around and there is not much movement of the drops passes that point. So, you spray and the spray goes into the air and sort of the drops reach due to the drag from the surrounding air they sort of equilibrate they may continue to move forward by momentum conservation, but it is defused quite a big, so your spray width start to increase as you go for a downstream.

Now, when I go back and look at that like I said there are many different microscopic properties that are actually more important and relevant to a spray. Like for example, at every point we said you are going have a drop size distribution. So, I said point, but we will qualify that in just a moment.

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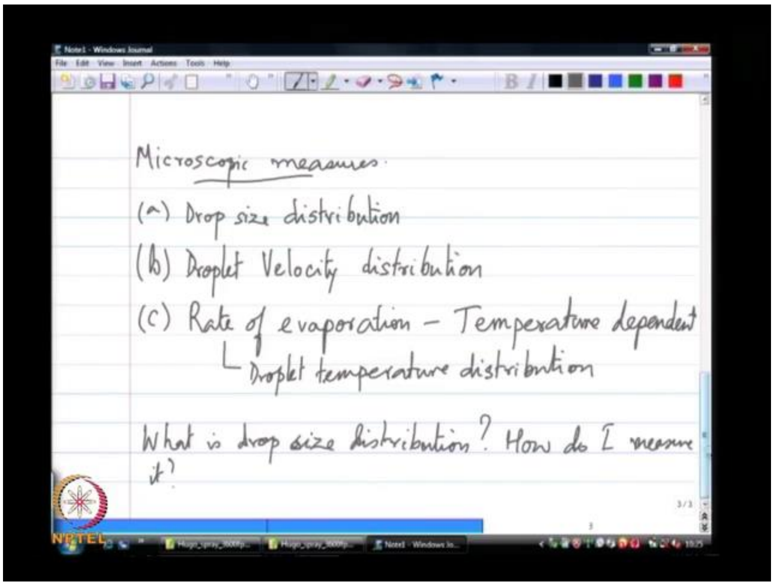

The idea that a spray is composed of several sizes of drops, this is sort of obvious to us by now. Not just that, that there is also a velocity distribution. Again I will qualify this to clearly distinguish it from the air around; we are looking at a droplet velocity distribution.

So, you have drop size distribution and droplet velocity distribution that are all microscopic properties. So, these are now that I am looking at the individual droplet level, I am almost looking at somehow understanding is spray from within as suppose to from without from outside. So, what other microscopic properties can I list here, I can look at in a typical spray like a perfume spray we looked at this yesterday as well in the last class. We are interested in like a rate of evaporation or temperature dependents which is clearly temperature dependent.

So, from there I can say somehow there is also this idea that there is a droplet temperature distribution. So, these are all in some sense individual variables like a drop or a single drop has a size, it has a velocity and it has a temperature, these are all at the droplet level they are properties that you can assign to a single drop. Now at the spray level, how do I convert this droplet level information into spray level information? There are two ways of doing it; one I do just this. So, I sit here play this video or have a spray like this going I pause; I pause the picture and then look at all the drops in this frame. Now this particular video is not particularly good to zoom in, zoom out although you could to some extent it is not quantitative, but clearly that is just a question of camera resolution.

So, if I have a sufficiently high resolution camera and these are not out of the ordinary also I can pretty much image every drop in this frame right now. And from the image of every drop I can reconstruct the size of every drop. So, let us say we did this calculation last time, there are approximately a million drops in this picture right now probably slightly less than a million. I can image every drop, get a size and get million numbers; from that I can construct a histogram of what to do with this.

So, once I go to the million drops in the picture, so sample. So, let us first pose a question and then will answer, what is drop size distribution. So, I want to understand what do I really quantitatively mean by drop size distribution and how do I measure it, that is what I want to get. We will spend quite a bit of time on this how do I measure it thing?

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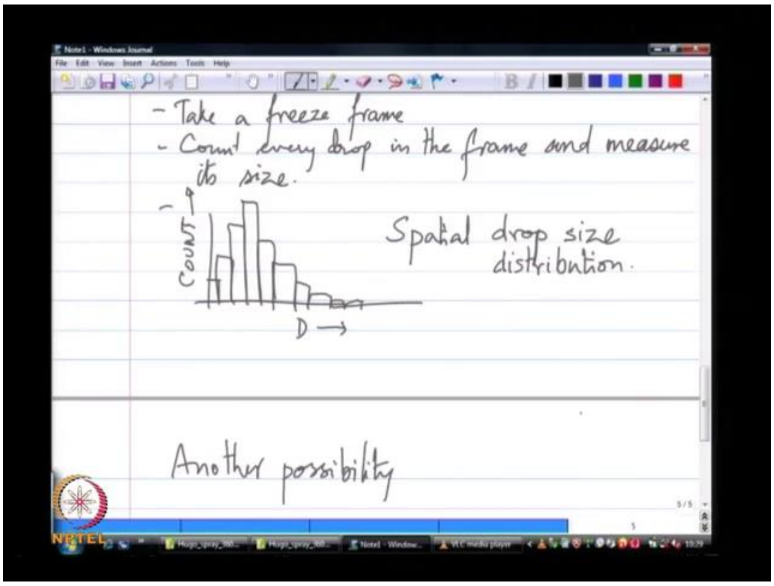

But I want to first start with what is drop size distribution and then that will naturally lead us into how do I measure it. So, I can take a freeze frame, count every drop in the frame and measure its size. This will give us essentially like a million numbers as slightly less than a million numbers and from there I can construct a histogram. We are all familiar with the concept of a histogram. I want to spend some time on that because it is there some very important mathematical tools that we need to acquire to completely understand what real histogram and we can do with the histogram.

So, let us say I do this histogram and it gives me something like this. So, this is diameter and this is count. I can get the count versus diameter of every drop in this frame and that will give me a distribution, clearly this is the drop size distribution in the spray. This kind of information is what we will call Spatial drop size distribution, because my independent coordinate of acquiring this information was x and y space. Then other way of thinking about this is for every drop that I capture or count in the freeze frame, I can also get it x and y coordinates. So, that automatically means I am in the process of acquiring the spatial drop size distribution because, those x and y coordinates would be different for different drops.

Another way of getting the drop size distribution is to sit at a point in the spray, so look at where my cursor is; I will sit at that point in the spray and I have a way of just you know sitting in the launcher and counting every drop that is going by me. So, I am not moving in spatial coordinate, I am just sitting at that one point and acquiring information of the droplet size, droplet velocity; droplet temperature; if I want to really get specific about this spray of every single drop going by me.

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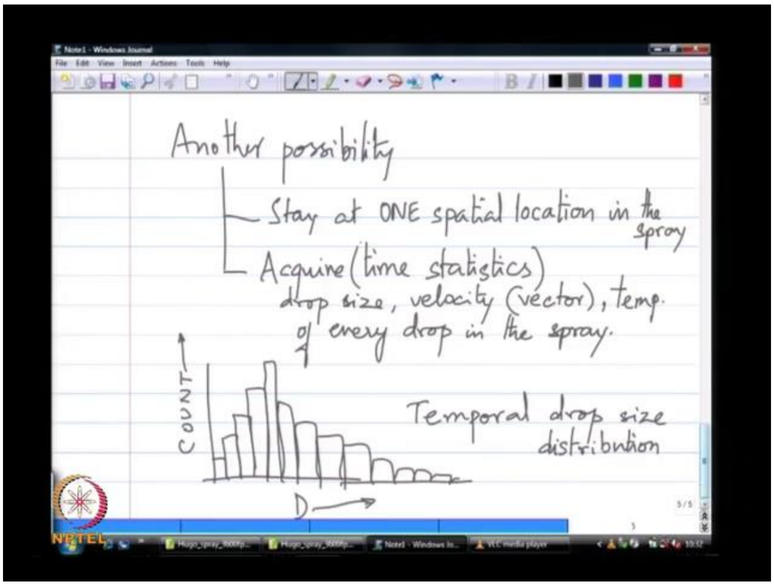

So, let us look at that another way of doing the same thing. So I can look at every drop in the spray that is going by me and get its drop size, velocity; in this case it could be a vector, it could be both x and y coordinate and temperature; if I really want to get fancy of every drop. Now the independent coordinate that distinguishes different drops here is the time of arrival. There are successions of drops coming through. So every drop is going to have a different time stamp of when it arrived at my location. As suppose to the previous way of sampling drop size distributions where I had the distinguishing feature, it was just x and y coordinate of the drop.

So, I can now take this same; let us say I sit there for 10 second and in 10 second like in a typical perfume whiff is like let us say 2 seconds and in 2 seconds, I was able to sample let us say again fraction of a million drops and I have now statistics of size, velocity of every drop coming through there. So, I will just for now ignore the other quantities and look at only the size to sort of illustrate the point. So I can now do the same histogram. Again the independent co-ordinate on this histogram is the same draw and counting a number of drops in a certain bin. Now this distribution is what we will call the temporal drop size distribution.

So, we started to talk of microscopic measures of which drop size distribution is one measure and when will listed this here, it was actually a fairly simple thing to list; you know list different size drop. We want to understand some information about the size drop. We quickly found out that there is not one, but there is two different drop size distributions. Now we are left in a limbo, are they the same or under what condition can we expect them to the same, under what condition will be totally different, because they contain different pieces of information.

So, if before we completely understand under what condition are they the same and under what condition are they different. I want to sort of make the equivalence between these two ways of sampling the same physical system. You are essentially measuring something about the physical system. There are two ways of understanding any physical system.

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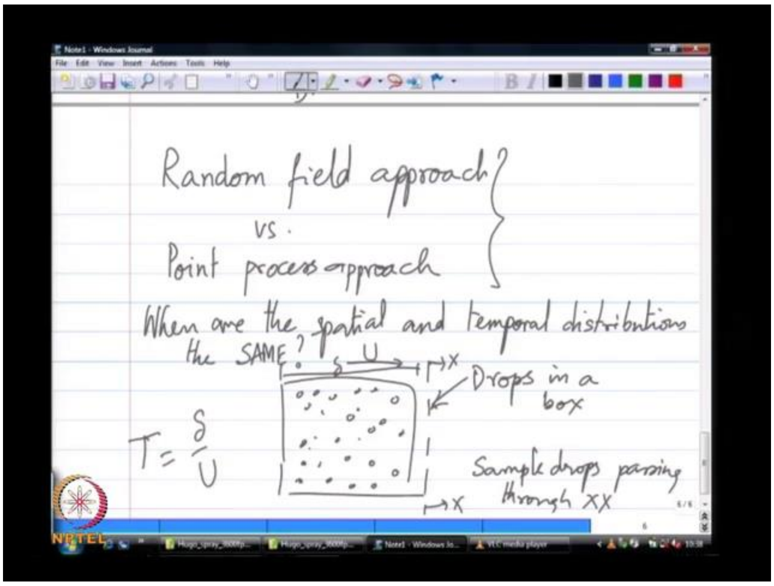

One is what is called a random field approach. Especially a statistical system like a spray that as like some sort of an uncertainty, some sort of a your you have to resort to probabilistic measures at some point and under those situations there are two ways of doing it; either random field approach or what is called that point process approach. We will talk about this in some details, but I want to sort of draw the equivalence between a field approach that is where I am looking at the spatial distribution of some quantity and the temporal distribution of another quantity as being two completely different ways of looking at the same system.

So, in general we do not expect them to be the same, they are completely different. I will present one simple situation under which they will exactly be the same. So, if I say for example, when are the; same is a very restrictive criteria. I want to start with that because it is easiest to understand, they are exactly the same. When you do two things; one if I take a spray and in this case I cannot do a spray like a usual spray, I take drops in a box. So, this is the box that as all these drops in it; let me not clutter the box with it there are different size drops, if every drop in this box was moving to my right with exactly the same velocity.

So, all the drops are moving to my right with exactly the same velocity then, if I had a way of staying at this location I will call this x x sample drops. So, I am not sampling drops at a point, but I am sampling drops passing through a certain cross sectional area and this is also possible measurable and quit routinely done. So, if I am able to sample drops passing though a certain cross section and I construct a drop size distribution from the sampling at a certain cross section. This is like my temporal drop size distribution and if all the drops are moving that exactly the same velocity then, essentially in the time, let us say if the width of this box was sum delta, delta divided by u is the time taken for all the drops to pass though my cross section.

So, whether I take a snap shot at some time t equal to 0 of this box or and then construct a spatial drop size distribution from all the drops in that box or whether I sit at location x x and sample all the drops come through that cross section and then construct the same distribution, I will get exactly the same answers. So, really speaking there are two assumptions I made not one; one assumption is that all the drops are moving with the same velocity. I am going to show you where I cheated you into another assumption.

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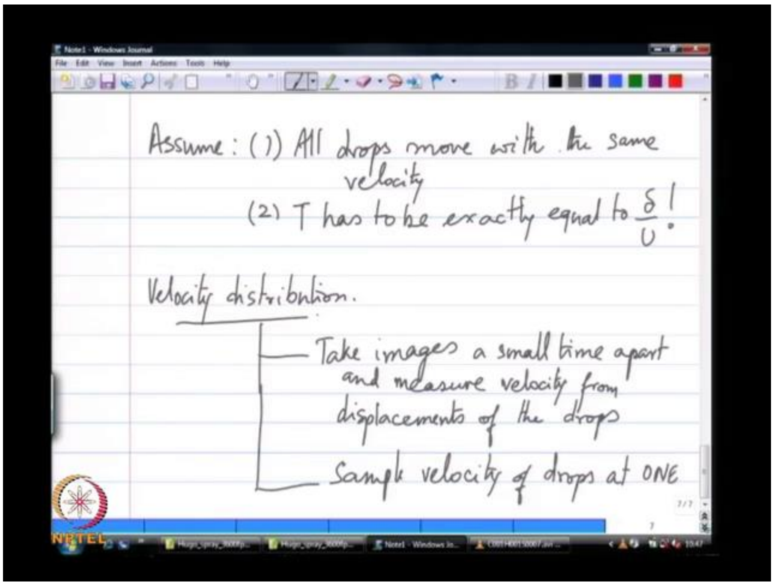

So one assumption, the second assumption is this; let us say this box is my frame correct. This is the frame that I used to construct my; a image frame that I used to construct my spatial drop size distribution and then I am sampling for a certain time T; capital T that is going to tell me the time over which I can sample to get the temporal drop size distribution.

So, if delta is the width of this frame and capital T is the time of sampling, it is only when capital T is exactly equal to delta divided by u that I get this. If I choose capital T to the some other number other then delta divided by u, let us take the simple situation where capital T is less than delta divided by u; what happens if capital t is less then delta divided by u. I would have only sample a part of this frame; I would not have sample the other half of the frame or other part of the frame, which means now is there a guarantee that the drop size distribution I construct from half the frame is the same as the drop size distribution the full frame, there is no guarantee.

I could have had a completely different mix of drops in the back half of the frame; I do not know that, right. So this criteria; that capital T as to exactly be equal to delta divided by u, so in other words if u made a freeze frame of a spray and you reconstructed a drop size distribution, even if all the drops are moving with the same velocity; I am constraint to sample at this capital T only to get the exact same distribution, which is again very restricted. So T has to be exactly equal to delta divided by u, this is not so obvious.

Now clearly the first assumption and the second assumption are different in the basic philosophy. Assumption number one, I have no control over what actually happens in the spray; all drops have to move at the same velocity I do not get to control that, but the second looks more like a measurement thing; the second looks more like. So, even if all drops are moving with approximately the same velocity, can I make this equivalence?

The only way to make that equivalence is, if somehow even if you make spatial drop size distribution measurements with some delta, I make with another T; I do not know what delta you used, but I make it with some other T, what sort of delta and T will give me approximately the same information or at least on what sort of delta and T relationship will give me information, will allow the two kinds of distribution to tend towards each other. The answer to that is, as I keep making my delta larger and larger, so your initial special size distribution is composed of a very large frame and you had enough resolution to actual size of the drop in the spray and then I can take a time sampling that is also very large. So, I can sit there forever and ever and ever and then sample all the drops, so if delta became infinitely large and T was also infinitely large.

So, all I need to know is then they are going to be approximately the same given that the drops are all moving with the same velocity. So, essentially the second criteria is; an arty fact of the idea that we are constrained with finite domain sampling and finite time sampling. The fact that I cannot sit there and measure for ever and ever, I have to do my sampling in a finite time likewise my camera cannot zoom out to the entire region of the spray, I am only constraint to a small part of a spray.

So, these are the sort of the assumptions that underlay the equivalence between the spatial and the temporal drop size distributions. Now what about velocity, let us talk a little bit about velocity as well and then will move on to the third part. So, will look at what velocity distribution is; again if I go back to my image, I can pause the image, this is the paused image, I can take another freeze frame just like this a short term thereafter and use some fairly simple algorithm, simple or sophisticated to find the displacement of a drop.

So, it is like we are going to get into this in some detail towards then we talk of measurement techniques, but essentially I can image a drop, image a set of drops now and the short term later and from those two images I can reconstruct velocity field. So, one way is take images; a small time a part. Another way of doing the same thing you see how this is equivalent to the spatial information, what distinguishes every particle here is it is original x and y coordinate in the image. So, if the original x and y coordinates are of every particle in this; in the original frame are different like we started do with that we discussed with the drop size distribution.

The second way of doing it, is exactly like the previous temporal size distribution which is I sit at one point and measure the velocity of every point coming through, every drop coming by me, so I can sample the second way. So, these are two different ways again like we discussed with drop size distribution and it is not obvious that they have to be the same. In fact in our case of velocity, there is no reason to even believe that they will be the same because drops at different points are different.

So, in other words I cannot even start to ask question under what condition will lead to be the same, because there two completely different pieces of information. Temperature and other like we discussed towards the end of the class last time we talked of concentration as being a parameter that qualifies each drops. So, if I have a fuel that I am spraying that has multiple components, concentration of a certain component in a given single drop is a scalar measure that is associated with that drop, it is like a scalar property not a measure scalar property associated with that drop.

So, I can have this temperature is also another scalar property associated with the drop, velocity is a vector property associated with the drops, size is the scalar property. So I can get into lists of vector and scalar properties associated with these drops, but the point is all of those properties can be measured in a spatial sense or in a temporal sense. Now the basic requirement for equivalence between these two is that we will only focus on the first one that all drops have to be moving with approximately the same velocity if not exactly the same velocity; that principle is very often violated in a real spray and therefore, these two measures are completely different and that violation is quantified in terms of what is called as size velocity correlation.

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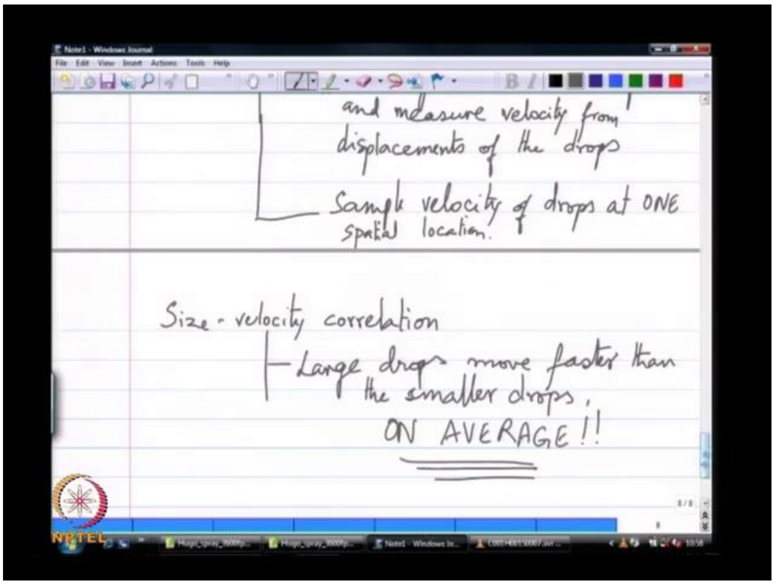

Correlation the word has a meaning that there is somehow, if not a causality meaning somehow there is a sense that a certain size drops can be expected to move with a certain velocity that is known a priori, that is the idea of a correlation. So, I can now take this spray for example and when I play it, you can see in here that the larger drop for example, right in the middle here are moving with the slightly higher velocity compared to the smaller drop, but exactly the same spatial location or very nearly the same spatial location.

So at least as far as this spray is concerned, larger drops are moving with the higher velocity in comparison to the smaller drops and that is basically information that I can take to, if I measure that in this perfume spray, I can transport that information to another perfume spray that is somewhat similar in construction and I can apply this information to other processes as well where even if nozzle design was not the same, fluid mechanically they were similar then larger drops in that other spray can also be expected to move with higher velocity in comparison to the smaller drops and in general in many, many different sprays, spanning many different nozzle designs at any given point the larger drops will tend to be moving slightly faster than the smaller drops on average.

So, this is the idea that; this is the very important word because I cannot draw any conclusion on any two given pares of drops, if I give you two drops at a particular point and ask you the question is the larger drop moving faster, can you for sure predict if the larger drop is going to move faster. You cannot make that prediction, all you can say is on average in a fairly large sample of similar drops; similar large drops, similar small drops, the similar set of large drop is expected to move slightly faster than the similar set of small drops that is the only prediction you can make and that is the only idea that you can take away from a size velocity correlation.

Now all of the measures that we talked about, all of the spray characteristics both macroscopic and microscopic that we listed this far, are only sort of a steady spray characteristics, so these are we discussed them this far in the context of a spray that has no beginning and ending in time and that is relatively unchanged in time. So, let us sort of make sure we completely understand what we mean by steady because it is a very important. We know we say we are making the steady flow assumption all the time, let us understand what we mean exactly by that word steady spray in the next few minutes.

Let us take this spatial distribution as our means of understanding sturdiness. So, I take a picture and I have reconstructed this drop size distribution, I take a second image and a short time later, let us say I am 1 second later and these two images are exactly alike now they clearly they would not be exactly alike, all I have to say is they are alike. So, without going into the microscopic detail of which drop is at which x and y location in the image, if I can take two frames that I took sometime later and I am unable to tell which one was taken first. So the time stamp is indistinguishable, the time stamp is not encoded into the picture itself that is the case of a steady spray.

So if I go back to my old high speed video, if I took this picture and then this picture, the first image had the spray only about that far, the second image has a spray all most for the downstream, that is the nature of how a perfume spray works. This is the clear case of violation of my steady definition, but so I drag this further down into my high speed video and now play it, I took this image or I will go back, I took this image and then another image a short time later.

So, this is still a transient spray as far as my eye is concerned when there is a started and at an end, but there was a time in the middle say about 1 second when I could not tell two images that came a hundred mille seconds apart. So, these in that time span there was a time span in the case of a perfume spray when I can treat the spray to be relatively steady.

So, our basic definition of steady for the case of this class is that, if I take two pictures of a spray and I cannot from all macroscopic measurements, I am starting off with the less constrictive criteria first then will take of the microscopic case. I cannot tell which one came first then I am beginning to make the case that they are steady then if I make microscopic measurements of the drop size distribution, so I am now recovering the statistics of the spray be temporal or spatial.

If I can still claim that I cannot tell the difference as to which one came first or which one came next from the microscopic measurements of distributions, I am still; I can still call it a steady spray. When I can begin to tell the difference like the first example we saw with the startup I can form a macroscopic measurement of a penetration length, tell you that there not the same, I can tell which one came first, so that is not; that is the clearly unsteady spray.

So, if you take a clear case like a diesel engine where you have your injector operating a few hundred times a second, can I treat, can I use any information from all this literature concerning steady sprays for diesel sprays; the answer is yes. The answer is pick up the time window where you cannot tell the difference and there is a time window as long as you restrict the application of these models to describing the spray in that time window, you are doing quite.

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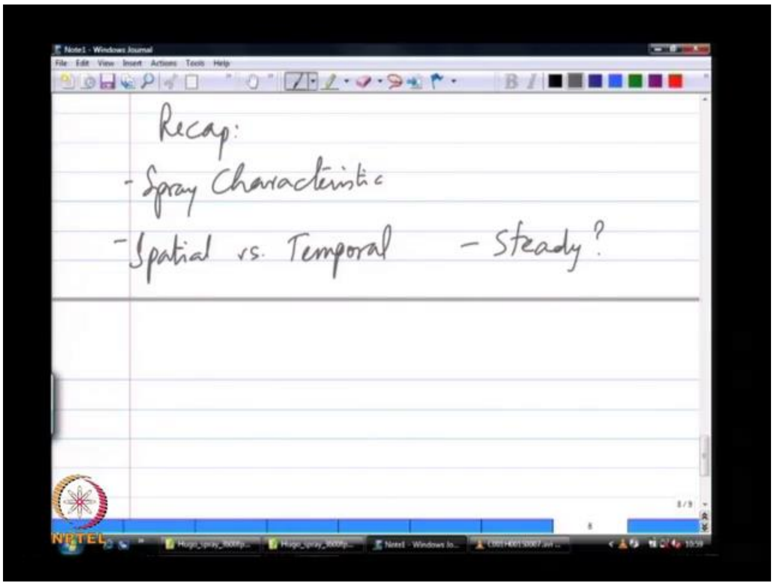

So just as a quick recap we talked of different measures or different characteristics of spray and then we talked of spatial versus temporal measurements and the equivalence thereof and then finally we made the case for a steady, what do we mean by steady spray.

We will continue this discussion in the next class.