Kinematics of Machines

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Module-3 Part-3

In our last lecture, we have discussed displacement analysis through examples using only graphical method. Today, we continue our discussion on displacement analysis but through the other method namely, the analytical method. Now, when do we use analytical method and when do we use graphical method?

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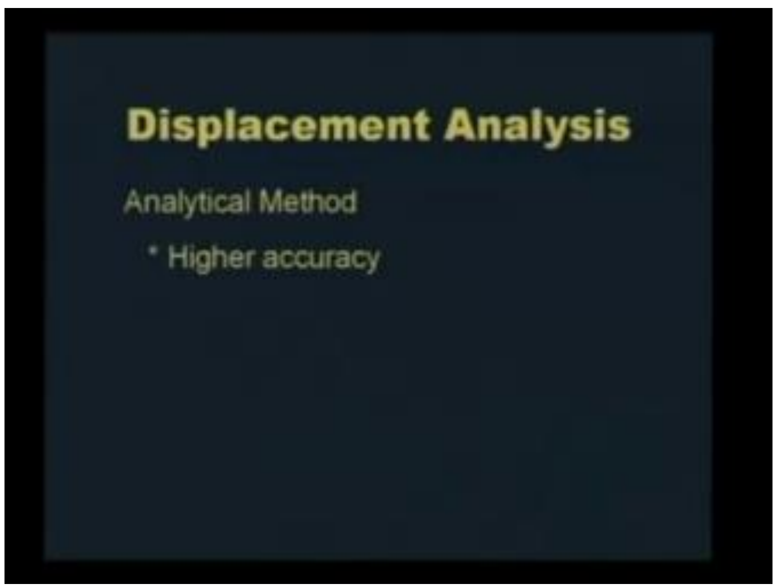

As we see, the graphical method, there is an inherent limitation on the accuracy, because of the scale of the figure and your drawing inaccuracies. So, analytical method is preferred when we want higher accuracy.

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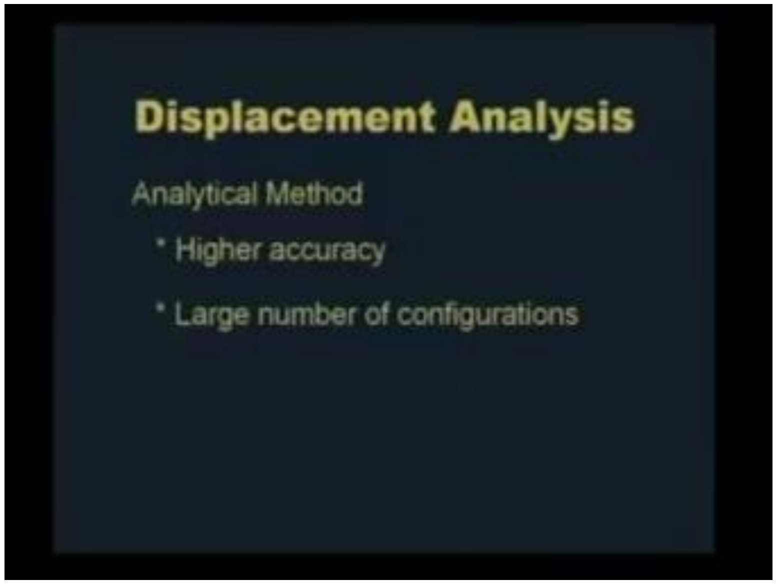

Or, if the displacement analysis has to be carried on for a very large number of configurations and in the graphical method, the picture becomes really cumbersome.

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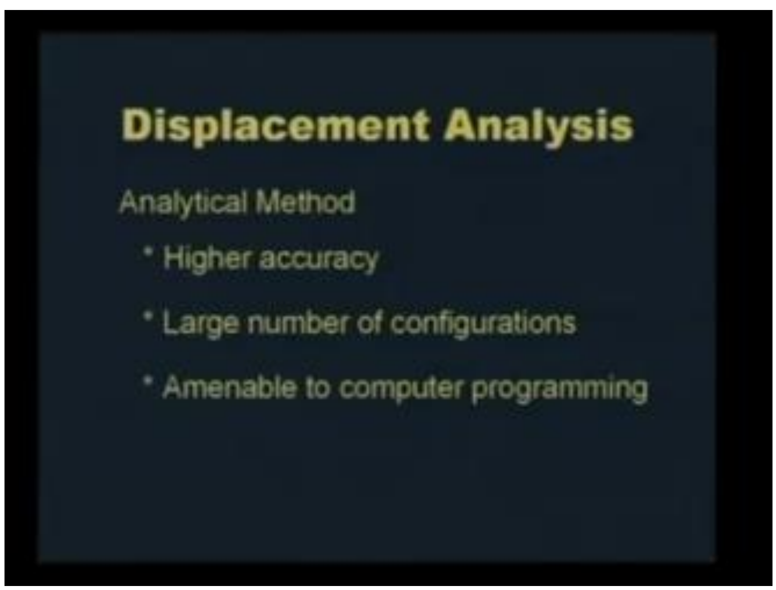

The other advantage of this analytical method is that, it is amenable to computer programming. Now, before we get into the details of the analytical method and solve examples for showing the power of the analytical method, let us go through the basic methodologies, that is followed in this analytical method. As we know, all mechanism consists of closed kinematic loops.

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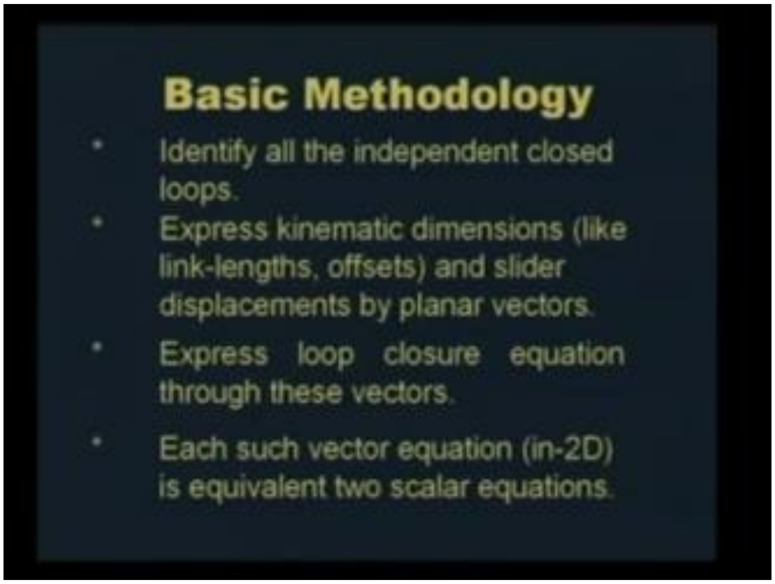

Basic Methodology:

- First task is to identify all the independent closed loops that exist in the mechanism.
- Next, we express all the kinematic dimensions (like link lengths, offsets) and also the slider displacement by planar vectors.
- Then express for every close loop, what we call as a loop closure equation in terms of these vectors.

- Now, each such vector in 2D, that is a planar linkage is equivalent to two scalar equations that means, if a vector equation is there, that is equivalent to two scalar equations and two unknown quantities can be solved.

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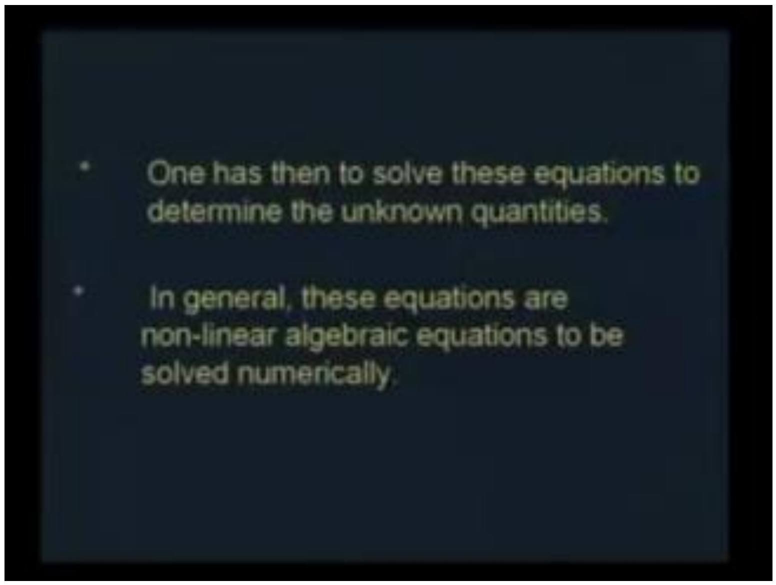

- Now, once all these vector equations are generated using all the independent closed loops, one has to solve these equations to determine the unknown quantities that is relevant to our particular problem.

- In general, these equations are non-linear algebraic equations and can be solved only numerically. However, in simpler cases like 4R mechanism or if there is a four-link closed loop, then we can show that these non-linear algebraic equations reduces to quadratic equation and can be solved analytically.

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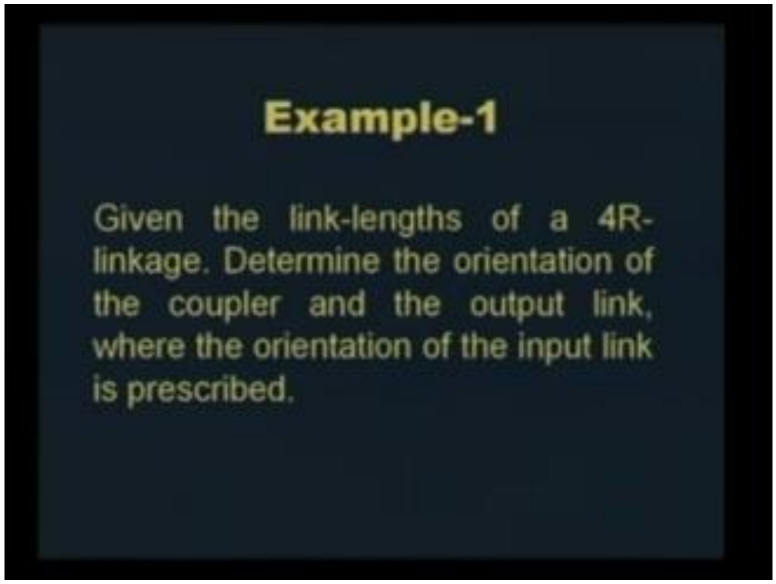

So, we start with an example that is the simplest one. Suppose, we are given all the link-lengths of a 4R-linkage, then determine the orientation of the coupler and the output link, when the orientation of the input link is prescribed. This is a problem of displacement analysis. We are given all the kinematic dimensions and the position of the input link and the task is to determine the position of all other moving links for that particular configuration.

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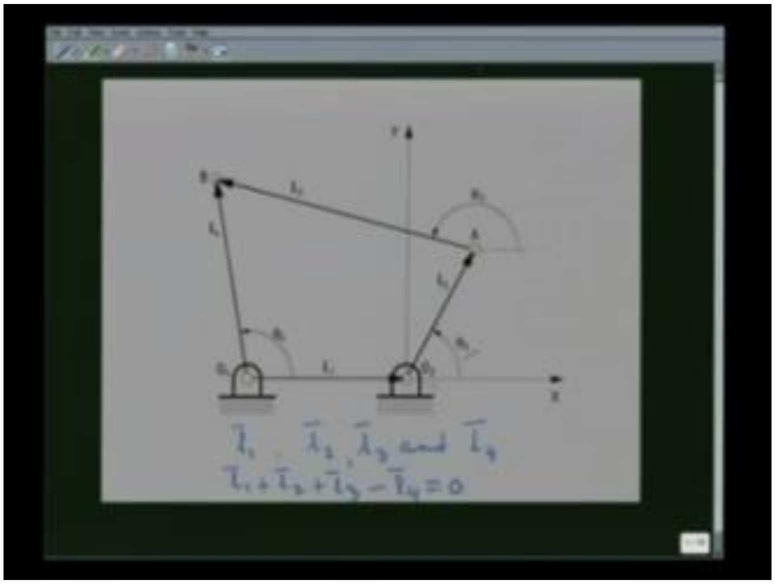

Above diagram shows a 4R linkage O_2ABO_4. As we see, the fixed link O_4O_2 suppose has a length l_1 and O_2A – the second link has a link length l_2, AB – the coupler has a link length l_3 and O_4B is the fourth link with link length l_4. The problem is, suppose this four link-lengths are prescribed and the orientation of one of the links say, input link that is O_2A is given by angle θ_2. Our objective is to determine the orientation of the coupler and the fourth link, that is O_4B.

To do this, first we set up a Cartesian coordinate system XY, with O_2 as the origin. The first task as I said, we have to identify a closed loop, namely – $O_2ABO_4O_2$. Then the link lengths l_1, l_2, l_3, l_4 are represented by four vectors namely – $\overline{l_1}, \overline{l_2}, \overline{l_3}$ and $\overline{l_4}$. These vectors are shown by these arrows in this diagram. Now the angle that these vectors make with the X-axis, we are using as the reference angle to determine the orientation of these vectors, that is θ_2, θ_3, and θ_4. θ_1, that is the angle made by O_4O_2, that is the $\overline{l_1}$ with X-axis is obviously zero. We can express loop closure equation as, $\overline{l_1} + \overline{l_2} + \overline{l_3} - \overline{l_4} = 0$. This is what we say as the loop closure equation, expressed in terms of link vectors.

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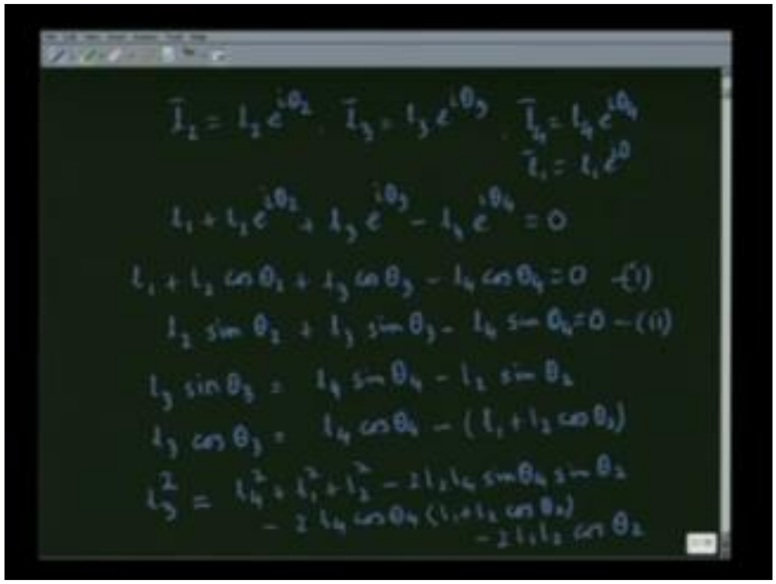

To analyze this equation, we write, $\overline{l_2}$ in terms of complex exponential notation, that is $l_2e^{i\theta_2}$. Same, for $\overline{l_3} = l_3e^{i\theta_3}$ and $\overline{l_4} = l_4e^{i\theta_4}$. Since θ_1 is zero, the $\overline{l_1}$ is simply, $l_1e^{i*0} = 1$. So, the loop closure equation, let us substitute these complex exponential notation, so you get,

$$l_1 + l_2e^{i\theta_2} + l_3e^{i\theta_3} - l_4e^{i\theta_4} = 0.$$

We know, $e^{i\theta}$ is cosθ + isinθ. So, first we substitute for all these angles, $e^{i\theta_2}$, $i\theta_3$ and so on, in terms of cosine and sine and equate the real and the imaginary parts of this complex equation separately to 0. This is nothing but really adding the X components to 0 and adding the Y components to 0, because it is a closed loop. As a result, we get,

$$l_1 + l_2 \cos\theta_2 + l_3 \cos\theta_3 - l_4 \cos\theta_4 = 0 \quad \text{----------------} \quad \text{(i)}$$

$$l_2 \sin\theta_2 + l_3 \sin\theta_3 - l_4 \sin\theta_4 = 0 \quad \text{----------------} \quad \text{(ii)}$$

Thus, a vector equation, because these vectors are two-dimensional vectors from one vector equation, we get two scalar equations such as equation one and equation two. Each vector equation is equivalent to two scalar equations if the vectors are two dimensional.

Now from these two equations, we can eliminate one of the unknowns. Say, either θ_4 to solve θ_3 or we can eliminate θ_3 to solve for the unknown θ_4, if l_1, l_2, l_3, and l_4, which are the link lengths are given and the input angle θ_2 is specified. Now, let me carry out this algebra to determine say, θ_4. To determine θ_4, we write

$$l_3 \sin\theta_3 = l_4 \sin\theta_4 - l_2 \sin\theta_2$$
$$l_3 \cos\theta_3 = l_4 \cos\theta_4 - (l_1 + l_2 \cos\theta_2)$$

From these two equations, if we square and add up, obviously we get

$$(l_3)^2 = (l_4)^2 + (l_1)^2 + (l_2)^2 - 2l_2l_4sin\theta_4sin\theta_2 - 2l_4cos\theta_4(l_1 + l_2cos\theta_2) +$$
$$2l_1l_2cos\theta_2$$

Let me carry out this algebra a little further and now onwards I will write "C" for cos and "S" for sine for brevity.

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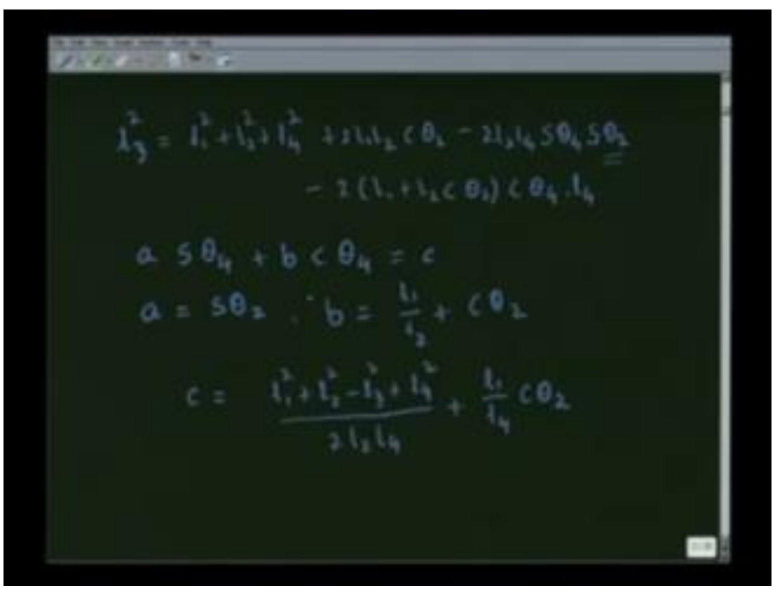

$$(l_3)^2 = (l_1)^2 + (l_2)^2 + (l_4)^2 + 2l_1l_2C\theta_2 - 2l_2l_4S\theta_4S\theta_2 - 2(l_1 + l_2C\theta_2)\,C\theta_4l_4$$

If we divide now both sides of this equation by $2*l_2l_4$. As a result, we will get a simple equation,

$$a\ S\theta_4 + b\ C\theta_4 = c,$$

$$\text{where } a = S\theta_2;\ b = \frac{l_1}{l_2} + C\theta_2;\ c = \frac{(l_1)^2+(l_2)^2+(l_4)^2-(l_3)^2}{2l_2l_4} + \frac{l_1}{l_4}C\theta_2$$

So you see, if θ_2 and all the link lengths are given, these quantities namely: a, b and c are completely known and the only unknown is this θ_4 which has to be solved. But as we see, this is not a linear equation in θ_4, because it involves sine and cosine. To solve this angle θ_4 unambiguously, that is we need not judge the quadrant after we get the value, in what quadrant the angle θ_4 lies, it is always better to replace sine and cosine by the tan of the corresponding half angles.

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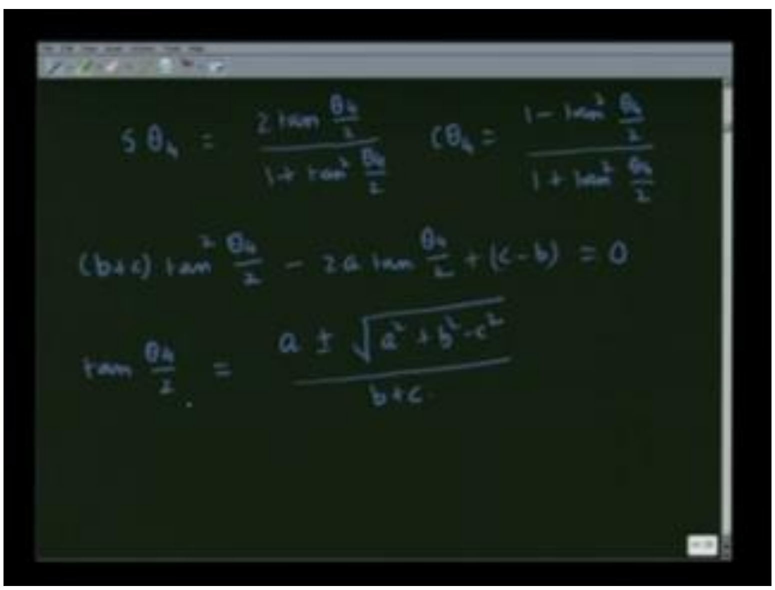

That means, we write

$$S\theta_4 = \frac{2tan\frac{\theta_4}{2}}{1+(tan\frac{\theta_4}{2})^2} \quad \& \quad C\theta_4 = \frac{1-(tan\frac{\theta_4}{2})^2}{1+(tan\frac{\theta_4}{2})^2}$$

Substituting above values in this {a $S\theta_4$ + b $C\theta_4$ = c} equation, we get a quadratic equation in tan(θ_4/2) as follows:

$$(b+c)(tan\frac{\theta_4}{2})^2 - 2atan\frac{\theta_4}{2} + (c-b) = 0$$

So, finally we get this quadratic equation in tan(θ_4/2), where a, b and c are all known quantities in terms of the link lengths and the given input angle θ_2. Now, this is where tan(θ_4/2) is useful, we get a quadratic equation and also we get two roots namely:

$$\tan(\frac{\theta_4}{2}) = \frac{a \pm \sqrt{a^2+b^2-c^2}}{\mathrm{b+c}}$$

So, we get two roots for tan(θ_4/2) and the right-hand side may come out to be positive or negative, of course, it can never be imaginary once the kinematic diagram is okay, that is a 4R linkage has been made, a close loop has been obtained, a, b, c will be of such values that square root can never be of a negative quantity. So, you may get two positive, two negative or may be one positive and one negative.

Now, tan(θ_4/2) as we know, if θ_4 is between 0 and 180^0 and then $\theta_4/2$ will be between 0 to 90^0 and tanθ_4 will be positive. And if θ_4 is more than 180^0, then $\theta_4/2$ will be more than 90^0 and tanθ_4 will be negative. So, depending on the positive and negative values of tan(θ_4/2), I can easily determine, uniquely the value of θ_4. Suppose we are also interested in finding the orientation of the coupler, that is the angle θ_3.

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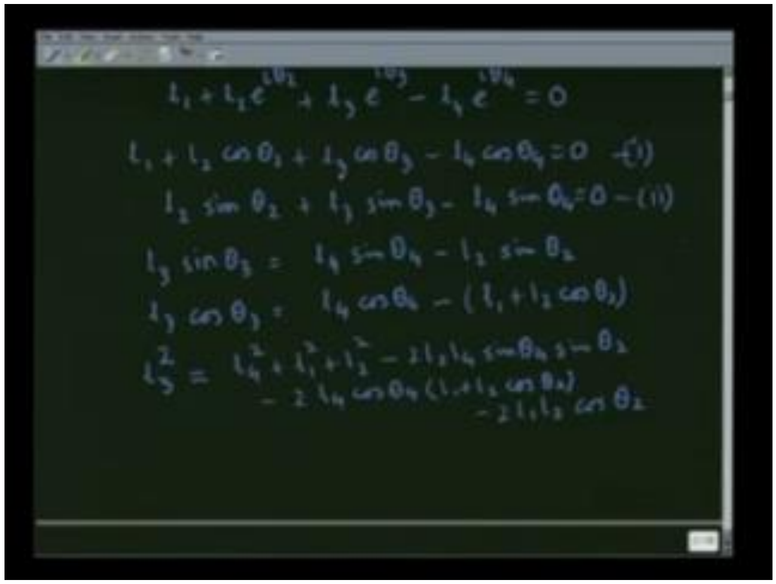

Then if we look at these two equations, we see l_3 and l_4 are appearing exactly the same way, only l_3 is appearing on the left-hand side and l_4 is appearing on the right-hand side. It is even more clearer, if we look at these equations 1 & 2. You can see l_3 and l_4 are appearing exactly similar way, only difference is plus and minus. So, when we have eliminated θ_3 and obtain θ_4. Following exactly the same procedure, I can eliminate θ_4 and obtain θ_3. We have to remember that l_4 has to be replaced by minus l_3, rest of the solutions remains as it is.

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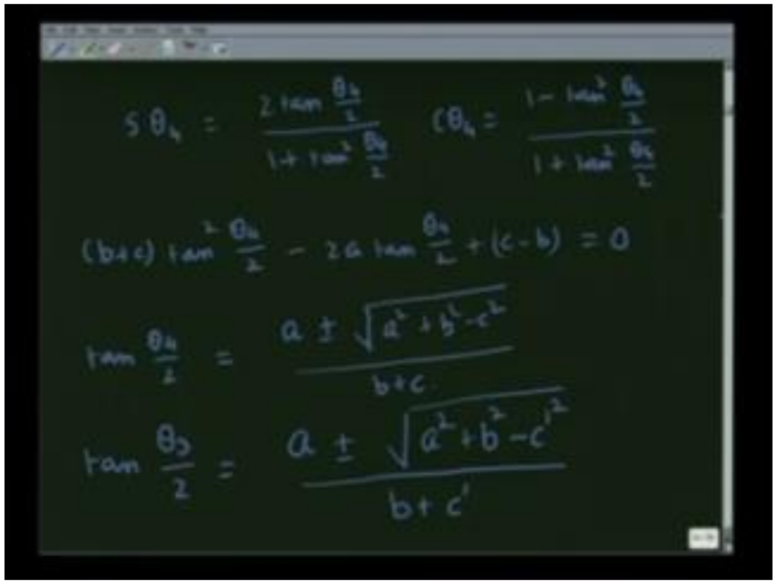

So following exactly a similar method to eliminate θ_4, we can get a θ_3, a very similar equation we get like,

$$\tan\left(\frac{\theta_3}{2}\right) = \frac{a \pm \sqrt{a^2 + b^2 - (c')^2}}{\mathrm{b} + c'}$$

As we might have remembered that the quantities, a and b did not involve l_4 or l_3. a = $\sin\theta_2$, b = $\cos\theta_2 + (l_1/l_2)$. However, the quantity c involved l_3 and l_4. That is why, we have to replace c by c' and expression of c'will also be very similar to c and we have to interchange l_4 by minus l_3 and vice versa.

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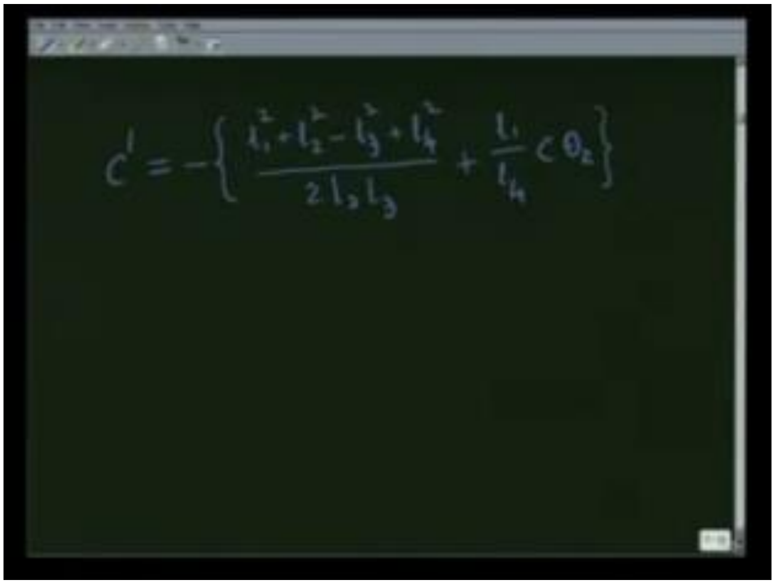

That is, we will get

$$c' = -\{\frac{(l_1)^2+(l_2)^2-(l_3)^2+(l_4)^2}{2l_2l_3}+\frac{l_1}{l_4}C\theta_2\}$$

The thing to note is, for the same set of given length and the same value of θ_2, we have obtained two values of θ_4 and two values of θ_3. This is nothing surprising, if we remember that even by graphical method, we would have got two solutions for the given link lengths and the input angle. This aspect will be very clear if we solve the same problem by graphical methods.

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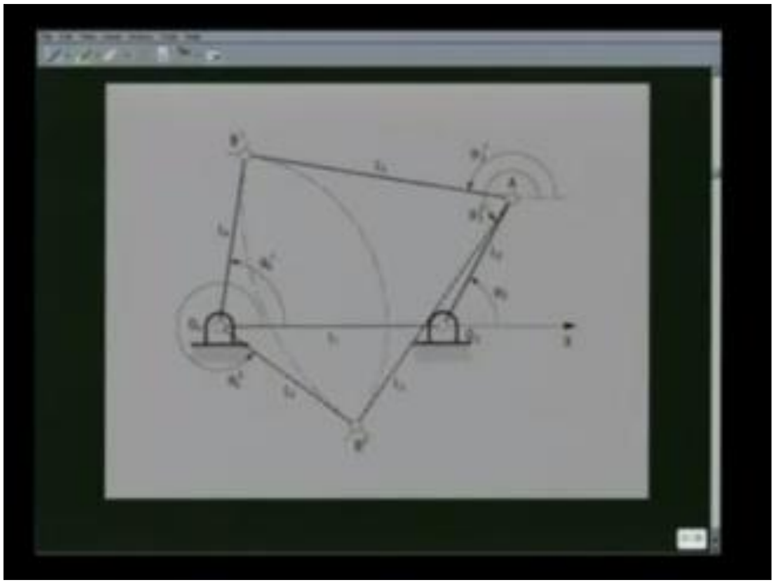

Suppose, we have given l_1 that the length O_4O_2, the link length l_2 that is O_2A, and the angle θ_2, all these are prescribed. So, you reach from O_4, O_2 to A. The link lengths l_3 and l_4 are given. So, we can draw a circular arc with A as center and l_3 as radius, and we get this circle. So the point B must lie on this circle. Similarly, if I draw a circular arc with O_4 as center and l_4 as radius, we get another circle. So, the point B must lie on this circle, if I come from O_4, and point B must lie on this circle if I come from A. These two circles as I said, in general, will intersect at two points namely: B^1 and B^2. So, we get two different configurations for the same values of link lengths and θ_2, one is O_4, O_2, A, B^1 and the other is O_4, O_2, A, B^2. Two configurations are shown distinctly, one by the firm lines and the other by the dashed lines.

Now, we have got two values of θ_3 and θ_4 which are indicated in this diagram as well. For the firm line, that is when the diagram is above the line O_4O_2, this is θ_3^1 and this is θ_3^2. Corresponding to θ_3^1, I have $\theta_4^1$and corresponding to θ_3^2, I have θ_4^2. Now that we have seen how to obtain various values of θ_4 as θ_2 changes. Now, we will show you the typical variations of θ_4 versus θ_2 for various types of 4-bar linkages.

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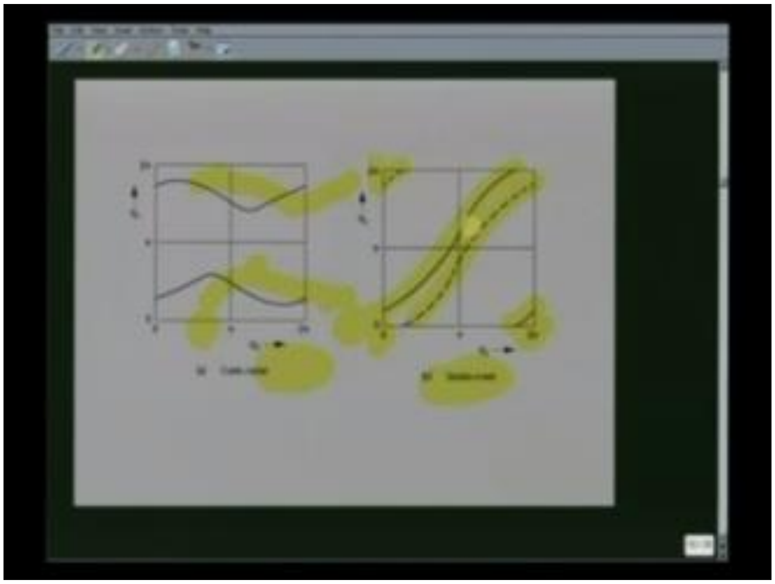

For example, here we see a crank-rocker linkage, that means it is obviously a Grashof linkage and a crank-rocker and θ_2 goes from 0 to 2π because θ_4 is the rocker angle, it only goes from a minimum value to a maximum value and there are two branches, one is here and the other is there. Because it is a rocker, again it rocks either from this minimum value to that maximum value or from this minimum value to that maximum value. As we said earlier that Grashof crank-rocker has two distinct branches or two distinct modes of assembly, that is why these two curves never cross each other. If we are starting on one of the curves you will always remain there, either on this curve or on this curve. You can never go from one curve to the other because each curve corresponds to a particular mode of assembly.

Suppose we have a Grashof double-crank, this is the typical θ_4 verses θ_2 characteristics of a double crank. As you see, θ_2 goes from 0 to 2π and θ_4 also goes from 0 to 2π, because both the input and the output lengths are capable of making complete rotations. But because it is a Grashof linkage, there are two distinct modes of assembly as represented by this firm line and the other mode of assembly is by this curved line and these two curves can never intersect, that is each curve corresponds to a particular mode of assembly.

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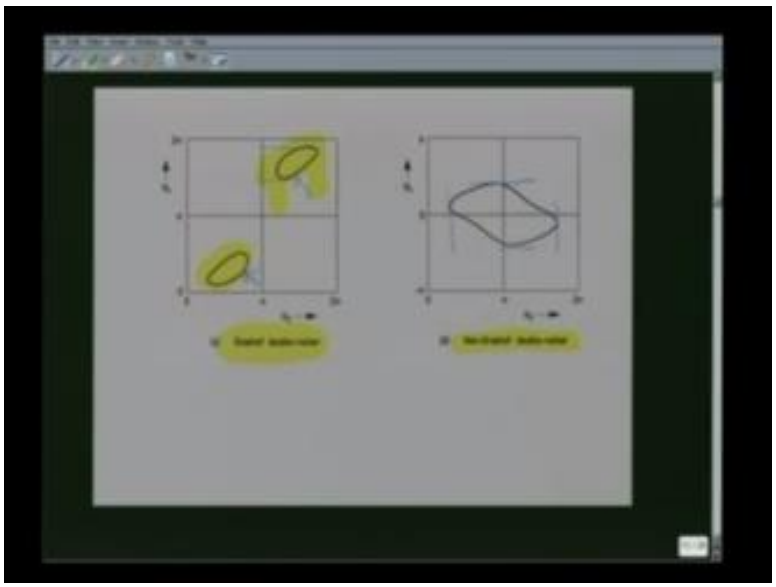

If we have a Grashofian double-rocker, then again, we have two loops, one as we see here and the other there. Because this is double-rocker neither θ_2 nor θ_4 can make complete rotation. So, they are limited between two extreme values for example, for θ_2 from here to there and for θ_4 it is here to there. Again, there are two distinct loops for two different modes of assembly either you are following this curve or you are following this curve, you can never pass on from one mode of assembly to the other.

Now, let us see what happens in a non-Grashof linkage. If it is a non-Grashof linkage then it has to be a double-rocker. θ_4 vs θ_2 plot for a non-Grashof double-rocker looks like this, here again neither θ_2 nor θ_4 can make complete rotation. θ_2 is restricted within this region. θ_4 is restricted within this region, but there is only one curve, because there is only one mode of assembly and as you see, this curve shows that the mirror image configuration can be taken up by the same assembly. You can generate the entire curve by one mode of assembly.

Just now we have seen, how to do the displacement analysis by analytical method for a very simple mechanism that is a 4R linkage which consists of only a single loop. Now, let me show how we can use the kinematic analysis, the same analytical method for a little more complicated mechanism, like the slotted lever quick-return mechanism, which we did last time by graphical method.

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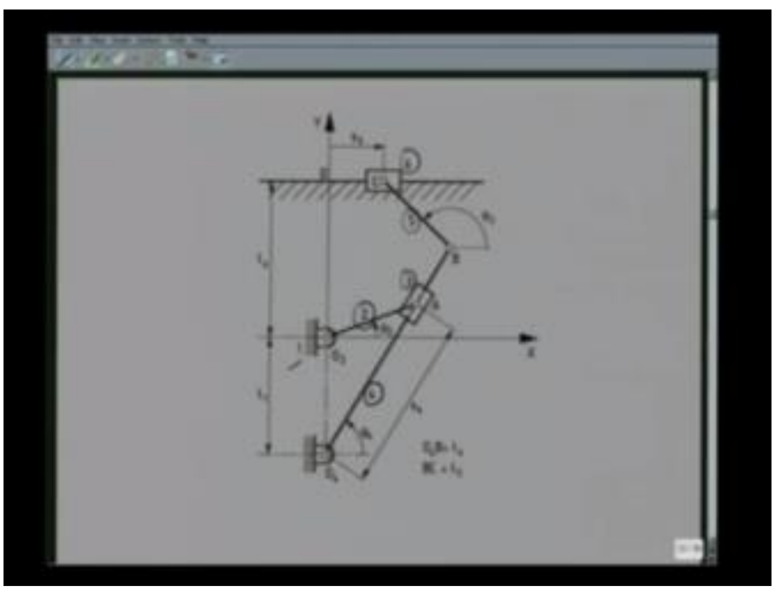

Above figure shows the kinematic diagram of the slotted lever quick-return mechanism. It has six links namely, the fixed link 1, input link 2, which is the line O_2A, then this block which is link 3 and the slotted lever link 4, the tool holder link 6 and the link connecting the slotted lever with the tool holder, that is link 5. So, there are six links with two sliding pairs, one between 1 and 6, and the other between 3 and 4. So to do the kinematic analysis of such a mechanism by analytical method, let us first identify that there are more than one closed loops.

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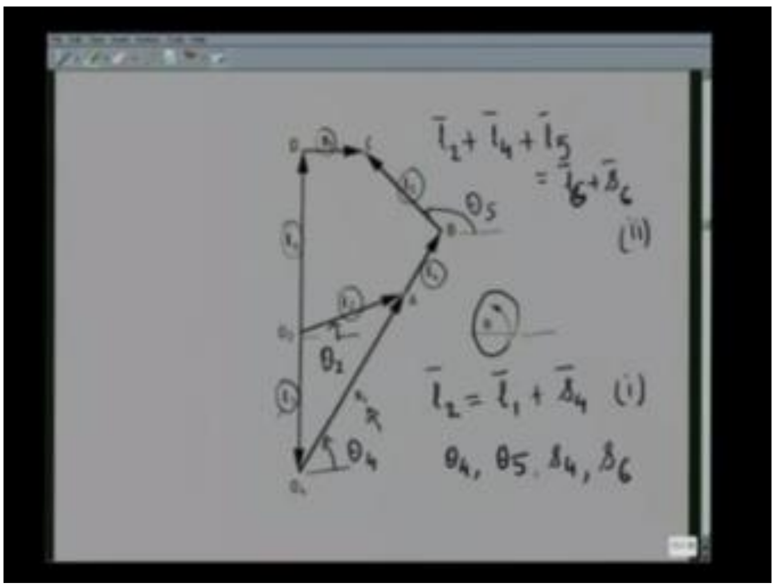

We draw all the link lengths by the corresponding vectors, such as: l_2 is the vector which represents the second link O_2A, l_1 is the other vector which represents the fixed link O_2O_4 and O_4A that is the slider displacement we use another vector namely – s_4. Similarly, the slider displacement we represent by another vector s_6 and O_4B is the other link 4, that is the link vector. l_5 is another vector representing the link BC, that is the fifth link. So, here what we see, that there are link length vectors like l_1, l_2, l_5, l_4 and l_6, that is the distance of the slider movement from the point O_2. So, these five link length vectors l_1, l_2, l_4 and l_6, are constants, they are not changing with time. Whereas there are two other vectors namely s_4, denoting the distance O_4A is changing with time and s_6 which represents the movement of the cutting tool, here it is DC represented by the vector s_6. And all these vectors, the orientation of these vectors, I represent by angle θ, measured from this horizontal line in the anticlockwise direction. So, we can have one close loop $O_2AO_4O_2$, which means $\overline{l_2} = \overline{l_1} + \overline{s_4}$. This is one loop closure equation, in terms of link lengths vector and sliding displacement vector.

I can have another close loop namely: O_2ABCDO_2. There is an independent loop and corresponding to this independent loop, I can write, $\overline{l_2} + \overline{l_4} + \overline{l_5} = \overline{l_6} + \overline{s_6}$. So, this is a second vector equation. So, we have two vector equations, which as I said earlier, are equivalent to four scalar equations, that is equating the real and imaginary parts or X and Y components, however you said it, from these two vector equations, I get four scalar equations. So, I can solve for four unknows. Now, let us see in this problem, what are the unknows? What are given are; all the link length vectors, that is l_1, l_2, l_4, l_5 and l_6 and the input angle θ_2, that is the angle that this link 2 makes with the reference this line, that is X-axis. So, these are given and we have to find out the orientation of all other links and also the slider displacement that is s_4 and s_6.

So, the number of unknowns are: θ_4, that is the orientation of link 4, that is the line O_4B. θ_5, that is the orientation of the fifth link, that is BC, that is this angle θ_5. This angle, as I said earlier is θ_4 and also this distance O_4A, that is s_4 and the distance BC that is s_6. So, these are the four scalars unknowns which can be solved using these two vector equations. That is how we carry out the kinematic analysis to analytical method. Later on, we will solve some more different types of problem and show the use of this loop closure equation and the vector representations of link lengths.

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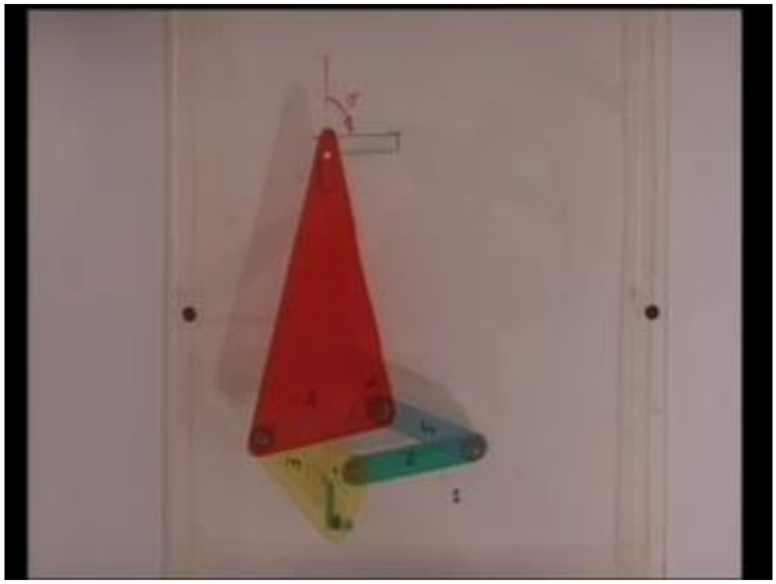

As a second example of this kinematic analysis to the link vectors, let us look at the model of this mechanism which is a copying mechanism or plagiograph. What we see here, that this is a five-link mechanism, the fixed link and there are four moving links: 2, 3, 4 and 5. The dimensions of these links are such that this forms a parallelogram. This link length is equal to this, this link length is equal to this. And these two triangles are similar triangles, that is this angle is same as this angle and this angle is also same as this angle. Now in this mechanism, we will show later that whatever is the movement of this point, that is reproduced by this point, only thing there will be a change in the scale and the orientation. For example, if I move this mechanism such that this point follows this vertical line of this letter 'l' then this point is also following a straight line. As we go around this letter 'l', this point will also draw a 'l', to a bigger scale and of different orientation. The question is that, if all the link lengths and the angle of these two similar triangles are given, then can we find out what is the scale of the reproduction, that is what is the change in the ratio of the size of these two l's. And what is the angle of these two l's? what is the angle, that is the change in the orientation of this letter 'l' to this drawn 'l'?

As we see here the ‘l’ is okay and here it has been rotated by 90^0 almost in the clockwise direction. The question is for this given mechanism, that is the link lengths are given and the angles are given, can we find out what is the ratio of these two figures and the orientation between these two figures? That will be our second example.

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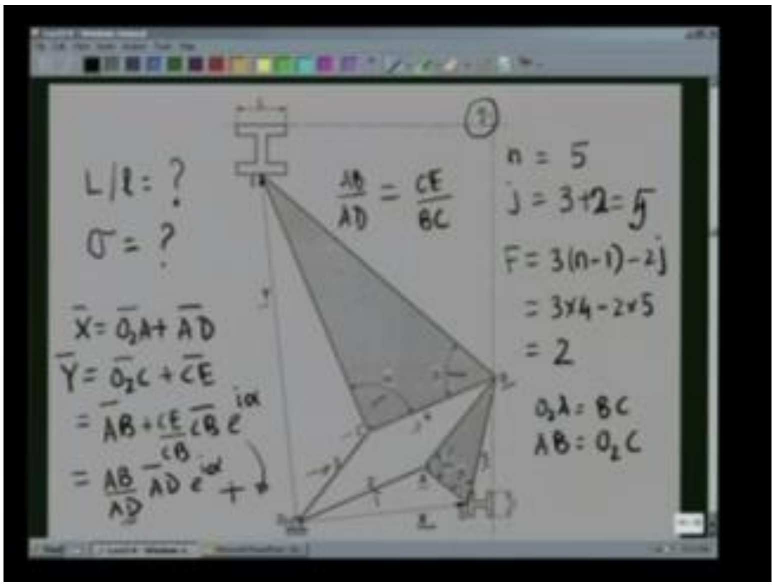

This diagram shows the kinematic diagram of the model of the mechanism which we have just now seen, that is the copying mechanism or plagiograph. As we see in this mechanism, we have 5 links. One is the fixed link, O_2A is the second link, O_2C is the third link, BC is the fourth link and AB is the fifth link. So, it is a five-link mechanism. So, let us say n = 5. Now let me count the number of hinges j. We see there are three simple hinges at A, B and C. So j_1 is 3. At O_2, three links are connected, namely: the fixed link 1, link 2 and link 3. Since three links are connected at O_2, this is a second order hinge, that is of the type of j_2, so j_2 is 1 and $\{j = j_1 + 2j_2 = 3 + 2*1 = 5$, i.e., $j = 5\}$. So, we can calculate the degree of freedom F, $\{F = 3(n – 1) – 2j = 3*4 – 2*5 = 2$, i.e., $F = 2\}$. Thus, we have a 2 degree of freedom mechanism and this point D of this link, can be taken anywhere arbitrarily on this plane, because it has 2 degrees of freedom, it can take any x value and any y value, this point D can draw any figure.

Now let us consider, this O_2ABC to be a parallelogram, that is link length O_2A is same as BC and link length AB is same as O_2C. Thus, O_2ABC is a parallelogram. Not only that, the triangle ABD and the triangle BCE are similar, as indicated that this angle α is same as this angle α and this angle β is same as this angle β. So, these two triangles namely BCE and ABD are similar. Now, we can show that the movement of the point D and the point E will be very similar, that means the movement of the point D will be copied by the movement of the point E. Accordingly, E will also draw a similar figure as D. The only change will be the size and the orientation of the figure. For example here, if the point D writes this letter H, then E will write this letter H but the height of the leg of H is 'L' and here it is 'l'. Similarly, this H is the normal way, that is vertical and this H is almost horizontal that it has been rotated through an angle σ.

Our task is, given the link lengths, let us find: what is the scale of the reproduction? That is what is the value of L/l and what is this change in orientation? That is, what is the angle σ once we are given the link lengths and the angles α and β completely. To do this, we use the link lengths as vectors. Suppose the position of the point D, that is represented by this $\bar{X}$ and the position of the point E, that is represented by the $\bar{Y}$. What we can see? We can easily write $\bar{X} = \overline{O_2A} + \overline{AD}$. Similarly, $\bar{Y} = \overline{O_2C} + \overline{CE}$. Now we see, that the $\overline{O_2C}$ is same as the $\overline{AB}$, because $\overline{AB}$ and $\overline{O_2C}$ are parallel and are of equal length. So, you write $\overline{O_2C}$ is same as a $\overline{AB}$. And the $\overline{CE}$, as I see is a vector which is rotated from the $\overline{CB}$ through an angle alpha in the counter clockwise sense and magnified from $\overline{CB}$ to $\overline{CE}$. So I can write, $\overline{CE} = \frac{\overline{CE}}{\overline{CB}}\,\overline{CB}e^{i\alpha}$ (this form takes care of the magnification by a factor $\frac{\overline{CE}}{\overline{CB}}$ and to represent the change in orientation between the $\overline{CB}$ and $\overline{CE}$ through an angle α, I have to multiply it by $e^{i\alpha}$).

Now $\overline{AB}$ can write in terms of $\overline{AD}$ exactly the same way, $\overline{AB} = \frac{\overline{AB}}{\overline{AD}}\,\overline{AD}e^{i\alpha}$. So, I have the $\bar{Y}$ as,

$$\bar{Y} = \frac{\overline{AB}}{\overline{AD}}\,\overline{AD}e^{i\alpha} + \frac{\overline{CE}}{\overline{CB}}\,\overline{CB}e^{i\alpha}$$

Now because these two triangles ABD and BCE are similar triangles, it is easy to see that $\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{CE}}{\overline{CB}}$. Therefore,

$$\bar{Y} = \frac{\overline{AB}}{\overline{AD}}\,e^{i\alpha}\,(\overline{AD} + \overline{O_2A}) = \frac{\overline{AB}}{\overline{AD}}\,e^{i\alpha}\,\bar{X}$$

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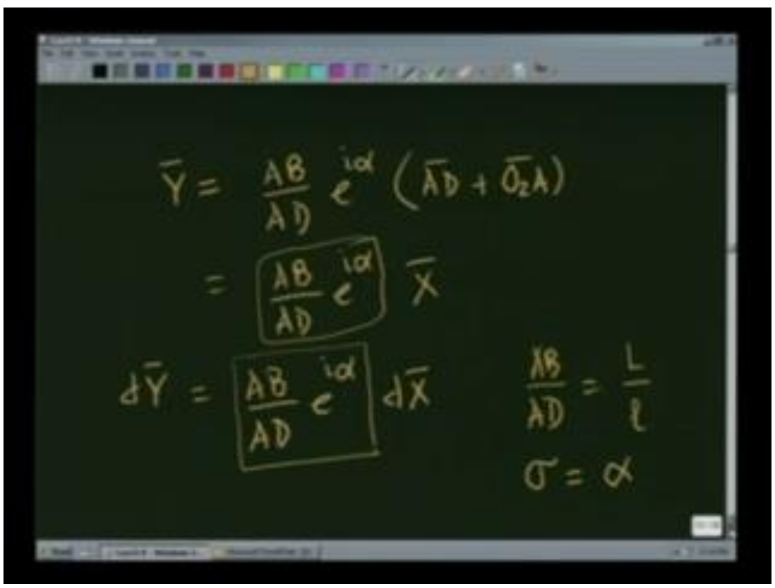

So you see that the position of these two vectors, that is the point E and the point D is given by point $\bar{Y}$ and $\bar{X}$ and they are related through this equation. $\bar{Y}$ is changing with time, $\bar{X}$ is changing with time but the link lengths AB, AD and the angle α of that triangle are not changing with time. So, these factors are constants i.e., independent of time. So, movement of the point d$\bar{Y}$ can be written as,

$$d\bar{Y} = \frac{\overline{AB}}{\overline{AD}}\, e^{i\alpha}\, d\bar{X}$$

So you see, the movements of $\bar{X}$ and $\bar{Y}$ are very similar, but for this factor. That means the movement is magnified by a factor $\frac{\overline{AB}}{\overline{AD}}$, which is the scale of reproduction, which I wrote earlier L/l and there is a rotation through an angle α in the counter clockwise sense so σ that is the change in orientation of the reproduction is equal to α.

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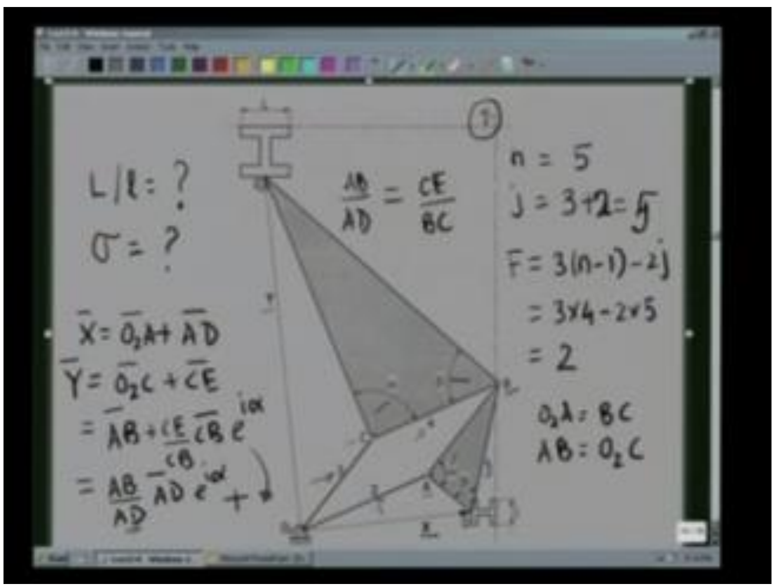

To make it clear, let me go back to the previous slide, $\frac{\overline{AB}}{\overline{AD}}$ that is the factor of reproduction, that is this length by this length, that is this letter H is reproduced to a bigger scale namely by the factor: $\frac{\overline{AB}}{\overline{AD}}$. Not only that, which is vertical like this has been oriented through an angle σ and we have already shown the σ is nothing but the angle α. So, this is the copying mechanism that any arbitrary figure, drawn by the point B or followed by the point D, can be reproduced at the point E at a different size and different orientation.

Let me now summarize what we have learned today. We have shown, how we carry out the displacement analysis of planar linkages by analytical method. In the analytical method, we represent all the kinematic dimensions (like link lengths, off-sets or slider displacements) by planar vectors. Using the closed loop that exists in the mechanism, we generate a loop closure equation and then by solving this loop closure equation, we can finish the kinematic or displacement analysis of planar linkages. We have given the example of a simple 4R linkage and also a six-link mechanism, where we have more than one closed loop. Finally, we have shown an example that shows the power of this vector representation for link lengths to analyze the copying mechanism or the plagiograph.