Kinematics of Machines

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Module - 3 Lecture -2

As told at the end of the last lecture, we shall continue our discussion on displacement analysis of planar mechanisms using graphical method. Last time we had seen examples in which all were six link mechanisms with single degree of freedom. Today, we shall discuss or take up a little more involved mechanism with more number of links and may be with more number of degrees of freedom and so how the graphical method can be used for displacement analysis. As the first example for today, that is the fourth example in the overall displacement analysis, we consider a nine link front-end loader.

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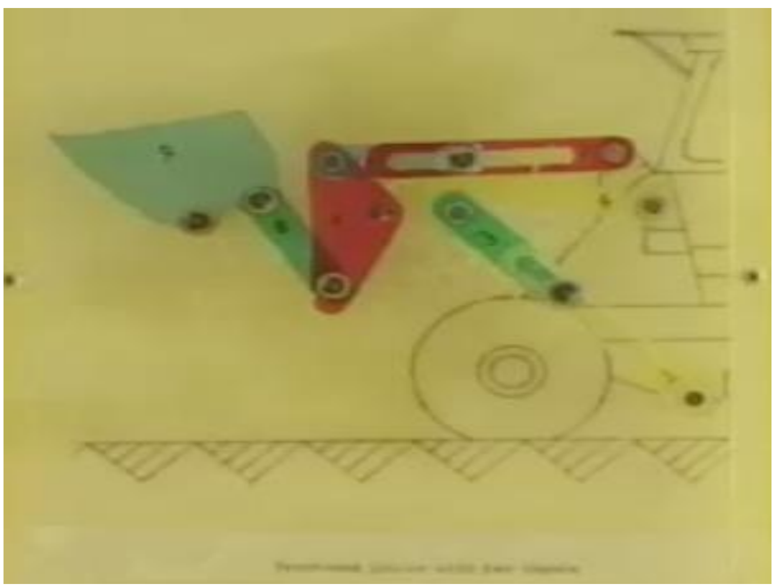

First, let us have a look at the model of this loader. To start with, let us first discuss the model of this mechanism which is used for a front-end loader. As we see there are two hydraulic actuators between link 2 and link 3 and there is another hydraulic actuator between the red link and this link, that is link 6 and link 7. By moving these two hydraulic actuators, we can move the front end of this loader, that is bin, which is link 9.

We will first show, that this particular mechanism has 2 degrees of freedom and with two inputs, which are these two hydraulic actuators. Consequently, this output link cannot go anywhere with any arbitrary orientation. It will have only two independent co-ordinates, which we can specify either the x, y coordinates of the point only and the orientation will be automatically fixed up or only the x-coordinate and the orientation of this bin and the y-coordinate will be automatically decided.

Let us determine the displacement analysis of such a mechanism by graphical method, that will be our next example. Let us now discuss the problem of displacement analysis with reference to this front-end loader, the model of which we have just now seen. As we see there are two hydraulic actuators, one named Z_1 and the other named Z_2.

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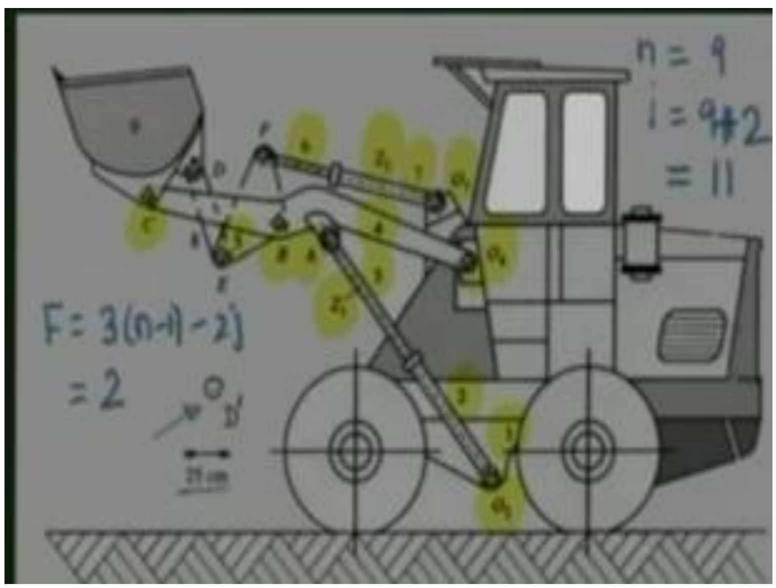

Link 1 is the body of the truck and this hydraulic piston cylinder constitutes link 2 and link 3. There is obviously a prismatic pair between link 2 and 3, which is the piston and the cylinder. Similarly, there is a prismatic pair between link 6 and link 7 where there is a hydraulic actuator, again between the piston and cylinder of this hydraulic actuator. Link 7 one end is connected to this body of the truck, at the revolute pair at O_7, link 4 one end is connected to a revolute pair to the body of the truck O_4 and link 2 one end is connected to the body of the truck by a revolute pair at O_2.

Now let us see this mechanism does not have all binary links, as we have considered earlier. On the first three examples that we considered, they had either binary link or ternary link. Here if we consider link number 4, as we see it is connected to four different links, namely to link 1 at O_4, link 3 at A, link 5 at B and link 9 at C. So, link 4 which has four revolute pairs becomes a quaternary link and link 5 is a ternary link connected to link 8, link 4 and link 6 and link 9 is the front end of the loader, that is the bin.

So in this mechanism, because we have a quaternary link, that is a very higher order link, I will show how to conveniently use the tracing paper as an over lay as we discussed in the previous lecture. Now let me state the problem, the problem is posed as follows: that if the point D of link 9 comes to D′. This is the scale of the drawing; 25 cm is represented by this distance. Now when the point D comes to D′, can we locate all other revolute pairs? That means, where they move in this consideration when point D of link 9 has moved to D′.

Before that, let me show that this mechanism has 2 degrees of freedom. For that we count the number of links n, which as we have seen 1, 2, 3, 4, 5, 6, 7, 8, and 9, n is 9. Let me count the revolute pairs and prismatic pairs, that is j number of joints. What we see, we have 1, 2, 3, 4, 5, 6, 7, 8, and 9, 9 revolute pairs plus 2 prismatic pairs, that is at these two hydraulic actuators. So, total j is 9 + 2 that is 11. If we remember our formula to calculate the degrees of freedom that is $\{F = 3(n-1) - 2j = 3*8 - 2*11 = 2, \text{ i.e., } F = 2\}$. Thus, it is 2 degrees of freedom mechanism because we have two independent inputs at these two hydraulic actuators, it is a constant mechanism. Because it is 2 degrees of freedom, I can have 2 degrees of freedom for this output bin, that is the point D can go to D′ but at that location the orientation of bin will be automatically determined. I cannot take D to D′ and then again change the orientation. So, we solve this problem, that when D goes to D′, how do we determine the location of all other revolute pairs? That is the first part of the problem.

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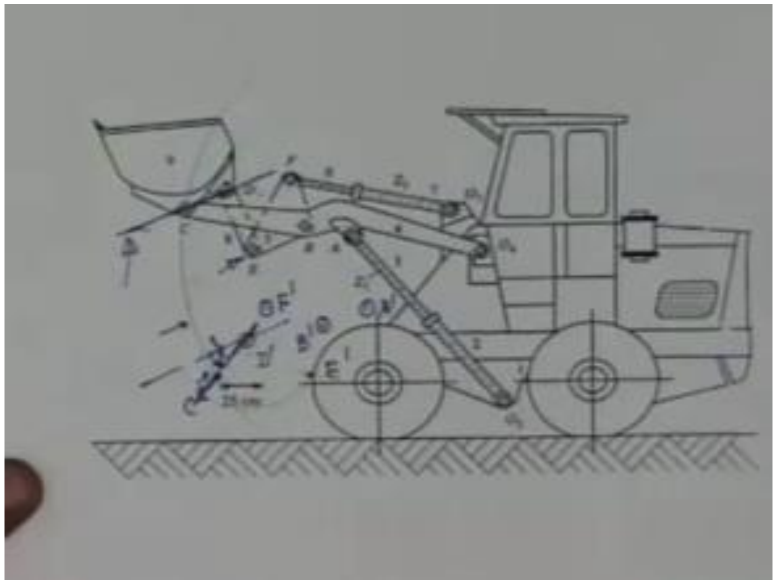

This is the figure of the same front-end loader which we are discussing right now. As I said earlier, the point D has moved to this point, which I call D^1 and the objective is to find the location of all other movable revolute pairs that is at A, B, E, F and C. We should note that O_4C being two revolute pairs on the same rigid link 4, the distance O_4C cannot change. So, C must move on a circular path with O_4 as centre and O_4C as radius. This is the circle on which C must move. Also, D and C being two points on the same rigid link, that is link 9, that is the bin, this distance also cannot change because I know the location of D', I put my centre of a circle with DC as radius with D^1 as the center.

If I draw this circular arc, C must lie on this circle. Thus, we get C which should lie on this circle and also on this circle, that means the intersection of these two circles determine the location of C, when D moves to D^1, C moves to C^1. Now to locate the positions of the hinges, say A and B, which lie on link 4, as we noted earlier link 4 is a rigid link but it is a quaternary link having 4 hinges, one at O_4, other at A, other at B and one at C. When D comes to D^1, C has moved to C^1, O_4 does not move, so, O_4 remains there. To locate the positions of A and B, corresponding to the second position, what we do, we take a tracing paper and mark these points: O_4, A, B and C. The relative positions of these O_4, A, B, C can never change because they lie on the same rigid body.

So what I do is, I move this tracing paper now with O_4 at O_4 and C at C^1. These two red dots, which are the positions of A and B is automatically determined, A and B moves to these new locations. If we can now pierce this tracing paper here and there, I immediately get the locations of A and B. This point will give me A^1 and this point will give me the location B^1. Let me repeat it once more because O_4, A, B, C are four points on the same rigid link, I use this tracing paper as an over lay, mark the relative locations of O_4, A, B, and C because two points define the rigid body completely and in the second position I know the location of O_4 and the location of C^1.

So I take these two red dots corresponding to O_4 and C to this new location and where ever these two dots come, they are the positions of A and B corresponding to the second configuration. Now once we have obtained D^1 and B^1, it is very easy to determine the new location of this revolute pair E, because the distance DE does not change. With D^1 as center, I draw a circular arc with DE as the radius. So, E must lie on this circle and BE also does not change. So, if B^1 as center, I draw another circular arc with BE as the radius, where ever these two circles intersect, that gives me the location of E, which I call E^1. Similarly, to determine F, I know the locations B, E and F. These are the three points on the same rigid link 5 so, I can again use this tracing paper as an over lay and mark B, F and E, their relative positions do not change and now I know where B and E has gone, so I can replace this, coinciding this B with B^1, this E with E^1 and where ever F goes, that becomes the location of F, which I call F^1.

I have determined the location of all the moving hinges namely: A, B, F, E, D as A^1, B^1, E^1, F^1 and D^1 of course is given, and C as C^1. We have solved this part of the problem that corresponding to the second position, when D has moved to D^1, where all other moving hinges go. Not only that, I can also find what is the inclination of the bin from the first position. That is this line CD, that was this initial orientation has now become this. So, angle between these two lines CD and C^1D^1 determines the change in orientation of the bin. If I draw a parallel line to through D^1, let us say this is the line which is parallel to CD, then this is the amount of counter clock wise rotation of the bin has taken place, when D has moved to D^1. The problem that we have just solved, let me now restate the problem for your benefit, this was our example-4.

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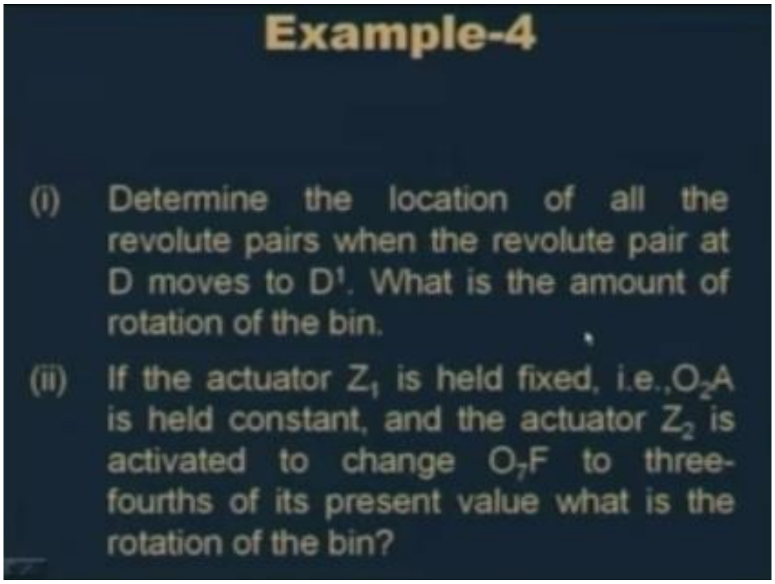

The first part of the problem that we have just solved is, to determine the location of all the revolute pairs, when the pair at D moves to D^1. We have also found out what is the amount of the rotation of the bin corresponding to this movement. Now I will leave an assignment for you to do from the same figure.

Assume the actuator Z_1 is held fixed, that is the distance O_2A is not changing and the actuator Z_2 is activated to change the distance O_7F in the figure to three-fourths of it present value, then what will be the corresponding rotation of the bin. You can follow exactly the same logic and same technique of graphical analysis to solve the second part of the problem.

Let me now go to another example, which is our example-5. Now I will show you a figure which will be simple 3R-1P mechanism to scale.

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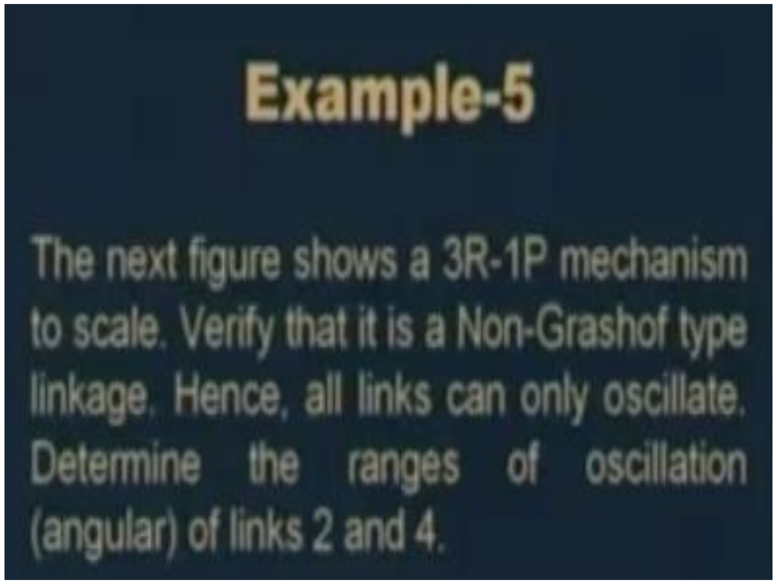

First, we will verify that this is a Non-Grashof type linkage. That means, all the links can have only angular oscillation, no link can rotate completely. The problem will be to determine the ranges of oscillation of these links 2 and 4, in this particular Non-Grashof 3R-1P mechanism. Let me now show the figure of this problem and solve the problem for you.

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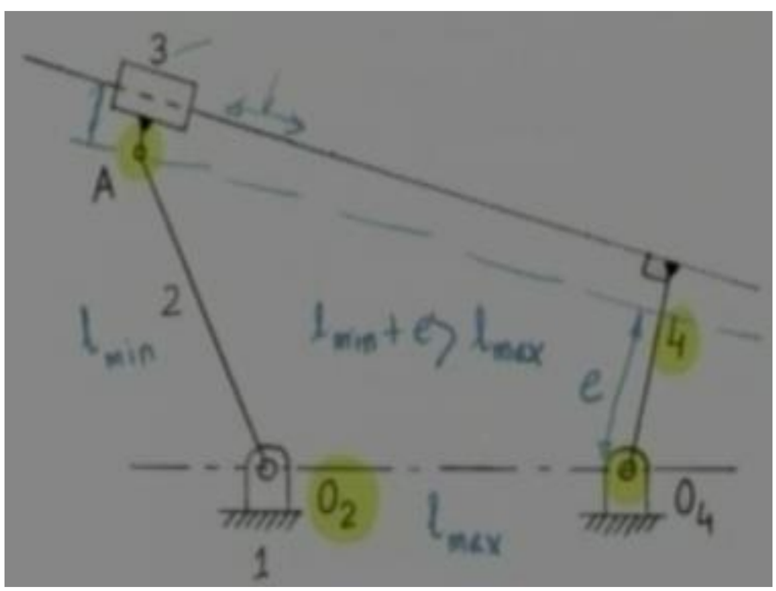

Let us now consider this 3R-1P mechanism, as we see the link 2 has a revolute pair with link 1 at O_2 and a revolute pair at A with link 3. Link 3 has a prismatic pair with link 4 and link 4 has a revolute pair at O_4 with link 1. In this 3R-1P mechanism, there are three revolute pairs at O_2, A, and O_4 and there is a prismatic pair between link 3 and link 4, in this direction. Let's identify, what are the kinematic dimensions of this 3R-1P mechanism.

O_2O_4 that is one link length, let me call it l_{max} and O_2A is another link length which is l_2, let me call it l_{min}. This 3R-1P mechanism has only one more kinematic dimension, that is the offset. One should not be confused by this dimension. This dimension has no kinematic significance because the location of this revolute pair A cannot be changed, but the prismatic pair between 3 and 4, which is along this direction could have drawn it through A. This is the prismatic pair direction and this perpendicular distance from O_2 up to this direction of the prismatic pair, that is the other kinematic dimension 'e', which we call offset.

If we measure, this l_{min} and e, we will find that, $l_{min} + e > l_{max}$. Consequently, this will be a Non-Grashof 3R-1P chain and no link will be able to rotate completely, all links will be oscillated. The question that is asked is, find the angle through which the link 2 and link 4 can oscillate. Let me repeat, it is this distance which is a red heading, which is there only to confuse you because it is the location of the revolute pair O_2, O_4 and A which cannot change and prismatic pair has no location, it is only a direction, that is marked by this line. So, I draw a line through A parallel to this direction of prismatic pair and get the kinematic diagram for this particular linkage. Let me now solve this problem to determine the angle through which links 2 and 4 can oscillate.

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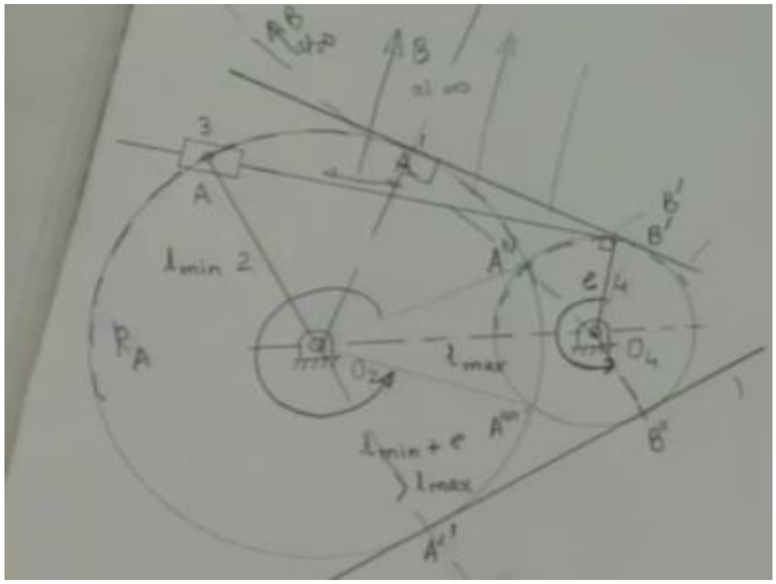

Let us consider now the kinematic diagram of the 3R-1P mechanism we are discussing. This O_2O_4, which we call l_{max} and O_2A, which we call l_{min} and perpendicular distance of O_4 from the direction of sliding passing through A, is what we call the offset ‘e’. By measurement, we can find that $l_{min} + e > l_{max}$. So, it is a 3R-1P Non-Grashof chain and we have to find the angle through which the link O_2A and link 4 that is this line can oscillate. To solve this problem, let us imagine the equivalent 4R mechanism, that is the prismatic pair between link 3 and 4 in this direction. Let us replace it by a revolute pair at infinity in a direction perpendicular to this direction of slide, that is the hinge B of the O_2ABO_4 which is the equivalent 4R linkage, B is on this line at infinity.

All parallel lines meet at infinity, so I can take any line perpendicular to the direction of sliding and claim that B is at infinity on any of these parallel lines because all these parallel lines meet at infinity. Now to find the extreme position of link 2 or link 4, what I know is that link 4 or link 2 must be collinear with link 3, that is, O_2A and AB must be collinear. And ‘A’ must move on this circle which is centered at O_2 and radius O_2A. This is the path of A, we call it k_A and this offset e, this distance rotates with O_4 as center and ‘e’ as the radius, that is this corner move on this circle.

Now let me draw a common tangent to these two circles, one centered at O_2 and the other centered at O_4. The common tangent between these two circles are these two lines. This is one circle centered at O_4, this is another circle centered at O_2, with O_2 as radius. So, these two lines represent the common tangent to these two circles. Let me see what happens when A occupies rather this point, if I call it B'. When B' comes here and if it oscillates in this direction up to this point of tangency, let me call it as B''. Let me find where is A now.

These are the extreme positions of link 4 because as we will see O_2A and AB has become collinear now because A is here which I call A' and A is here which I call A'', so O_2A' because it is tangent is perpendicular to this direction of sliding and B is also at infinity to this direction of sliding. So, O_2A' and B at infinity has become collinear, same is to for the other configuration, O_2A'' and B which is perpendicular to this line direction of sliding has become collinear. Consequently, link 4 has taken its extreme position. This is the angle through which link 4 oscillates, where O_4B' is common tangent I have drawn and this is the point of tangency, this is the common tangent to these two circles, this is the point of tangency. To find the extreme position of link 2, I have to say that O_4B' and $B'B$ should become collinear.

These two circles intersect at say this I call A''' and this as A''''. Now B is moving on this circle, so B' is also here, this point has moved to here and O_4 if I join this, the B goes to infinity in this direction, because B will be always perpendicular to the direction of sliding at infinity. So what we see, that this link 4, O_4B' and link 3, AB' have become collinear and consequently, link 2 has taken its extreme position. Same will be true for this point of intersection and link 2 oscillates through this large angle between these extreme positions.

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Just now we have seen very large angle of oscillations of the input and output links of a 3R-1P mechanism which is of Non-Grashof type. Here is a model, which is again a 3R-1P mechanism and this is the l_{min} and this is the l_{max} and e is such that $l_{min} + e > l_{max}$. Consequently, this becomes a 3R-1P Non-Grashof type linkage. Let us now observe that angle of oscillations of these links. This is the common tangent that I was trying to draw between those two circles, this line is parallel to that common tangent and consequently this has reached its extreme position, it cannot be further inclined. However, this link can still go and as we see from this extreme position, it is returning, that it is rotating counter clock wise. Though this link is still going clock wise. This is the extreme position of this link and now it cannot go any farther, it has to return.

Again, it has reached that extreme position and it cannot go any further, but this link can go and the red link starts rotating in the opposite direction. This is the extreme position of this link, whereas this is the extreme position of this link. From this point, it has to rotate in the opposite direction and it cannot go any further. And from this it cannot go any further, it has to rotate in the opposite direction.

For this 3R-1P Non-Grashof type linkage, as we see it is a very complicated movement with a very large angle of oscillation for both link 2 and link 4.

As a last example of displacement analysis, let us now consider a really complicated problem as shown in this figure.

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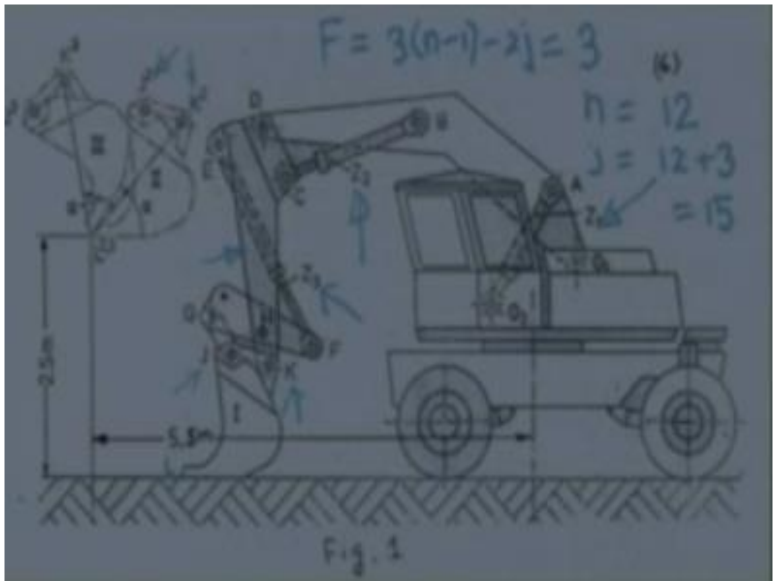

This is the figure of earth-moving machinery, the model of which we have seen earlier. This particular machine has three hydraulic actuators one at Z_1, other at Z_2 and the third one at Z_3. These are three independent inputs. If we count the number of links in this mechanism, I will get n = 12. This will be clear when we draw the kinematic diagram of this particular machinery. If we count the number of pairs, we will see there are 12 revolute pairs and 3 prismatic pairs, so j is 15. Consequently, if we calculate the degrees of freedom of this mechanism: {$F = 3(n - 1) - 2j = 3*11 - 2*15 = 3$ i.e., $F = 3$}.

So, it is a 3 degrees of freedom mechanism with 3 independent inputs. Consequently, it is a constant mechanism and because of 3 degrees of freedom and with three inputs, this bin is the output link which is defined completely by these two hinges j and k can be taken anywhere with any orientation. That means, I can independently take j to say j^2 and k to another position k^2, which will completely define the position and orientation of this output link which is the bin. Second thing to note is that, there is a very high order link, which is connecting at C, D, E, H and K. This particular link has 5 revolute pairs in it and it is a link of fifth order. So, we will show how we can use the tracing paper very conveniently to locate the revolute pairs in this kind of higher order link. Let me now solve this problem, that when j and k move to j^2 and k^2 respectively, how can we locate all other moving revolute pairs, that is at B, C, D, F, H and G and A. That is the problem that we will solve right now.

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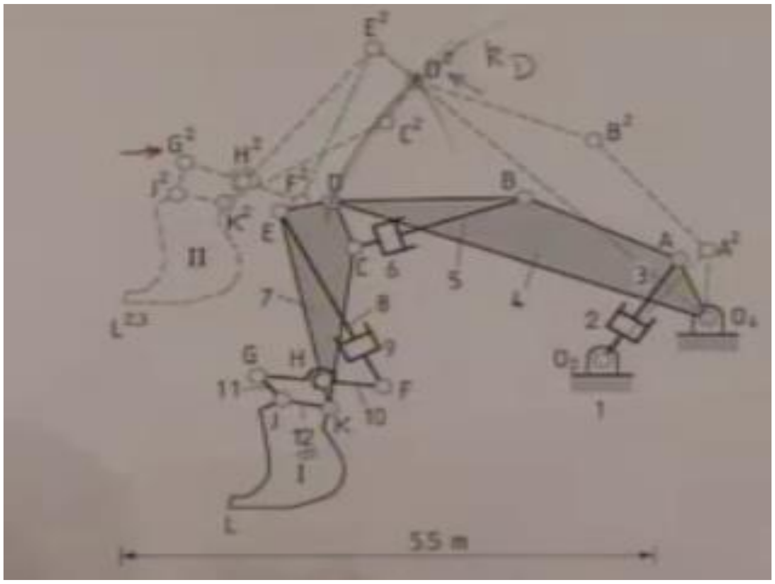

Let us consider the kinematic diagram of this earth-moving machinery which we have just seen. As we have said, there are 12 links and this is the kinematic diagram, out of which link 3 as we see have 4 hinges on it – O_4, A, B, D, so it is a quaternary link. Link 7 has 5 hinges on it – C, D, E, H and K, this is a link of fifth order. Then question is, when this J and k move to J^2 and K^2, can we locate the corresponding positions of all other revolute pairs?

To solve this problem first we note that the point D, because O_4D never changes and O_4 is a fixed point, this D point must move on this circle with O_4 as center and O_4D as radius, it must move on this circle. So, D^2 must lie on this line, let me call it k_D. Now K has gone to K^2, the distance KD also does not change, so I measure this KD and from K^2, I draw a circular arc with KD as radius. So, D must move on this circle and also the circle drawn earlier through D with O_4 as center. So, these two circles one point of intersection is here so I locate D^2. Once I have located two points, that is O_4 and D^2 on this quaternary link 3, A and B I can easily determine using the tracing paper. On a tracing paper, I mark O_4, A, B and D. These four points, their relative positions do not change because they belong to the same rigid link 3.

Now O_4 does not move, so it stays here. When D goes to D^2 and O_4 stays at O_4, where ever these two dots move, they define the location of A^2 and B^2. We have also located the point K which has gone to K^2 and the point D which has gone to D^2. Now link 7 is a link of order 5 and it has 5 hinges namely: K, H, E, D and C. These five points do not change the relative position, but K has gone to K^2 and D has gone to D^2. So, if I now take, K goes to K^2 and let me mark this point, so that will be easy to remember, this is K and this is D. K goes to K^2 and D goes to D^2 and these other mark points gives the location of C^2, E^2 and H^2.

These are five points K, H, E, D and C. I get K^2, D^2, C^2, E^2 and H^2. Once I have determined all these points, rest of the problem is trivial, because this distance is GJ and HG do not change. If I know J, which has gone to J^2, H, I have located, so I can easily locate G^2 by JG and HG these two link lengths. Rest of the points, I leave for the students to complete. What we have demonstrated that the use of tracing paper is very convenient, when we have such higher order links and the displacement analysis can be completed if the problem is well posed. Let me now state the problem that we have just solved which was our example number 6.

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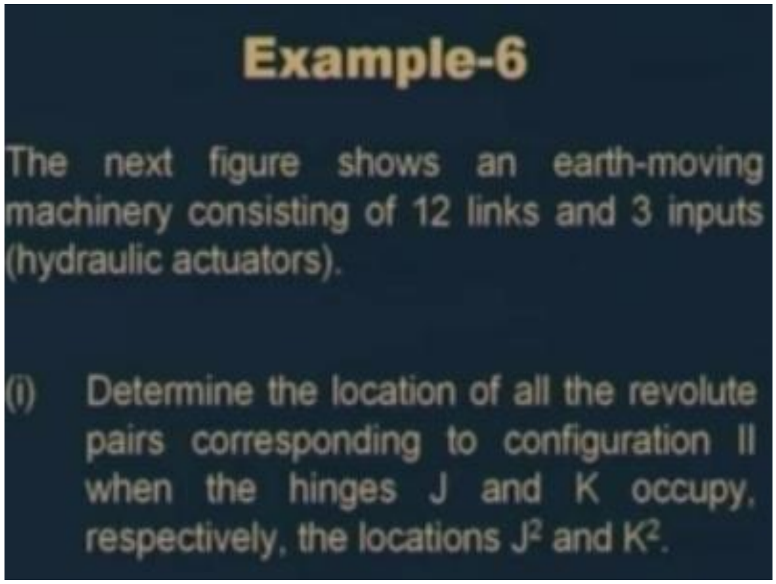

In example-6, we had the figure which had the kinematic diagram of an earth-moving machinery consisting of 12 links and 3 inputs, which had those three hydraulic actuators Z_1, Z_2, Z_3. Statement of the problem was, determine the location of all the moving revolute pairs corresponding to configuration 2, when the hinges J and K occupy respectively the locations J^2 and K^2. I can extend this problem for you to have some practice which are of little tougher.

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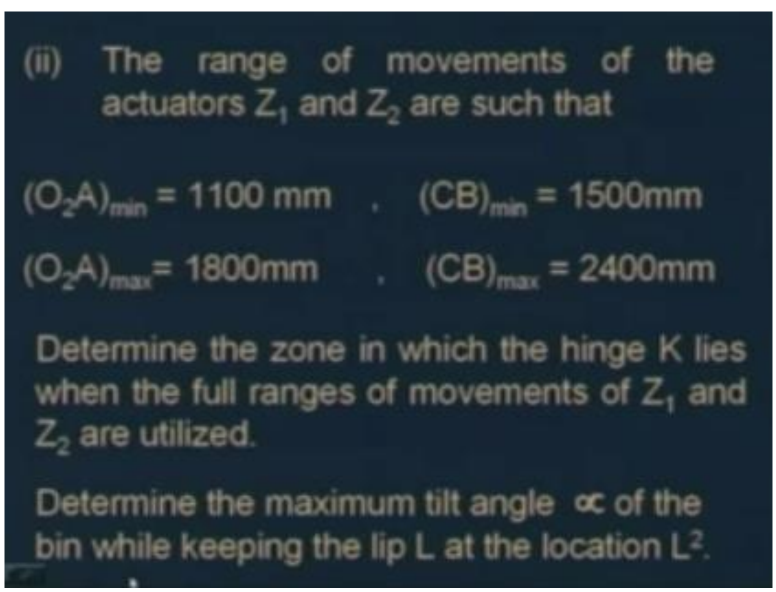

The second part of the problem could be, the range of movements of the actuators Z_1 and Z_2 are such that $(O_2A)_{max}$ = 1800 mm, to the scale and $(O_2A)_{min}$ = 1100 mm. Similarly, the movement of the actuator Z_2 is such that $(CB)_{max}$ = 2400 mm and $(CB)_{min}$ = 1500 mm. The question is to determine the zone in which the hinge K lies when the full ranges of movements of Z_1 and Z_2 are utilized. The third part of the problem could be, determine the maximum tilt angle alpha of the bin while keeping the lip of the bin L at the location L^2. As I said I will not solve this problem in detail just leave you with a little bit of hints.

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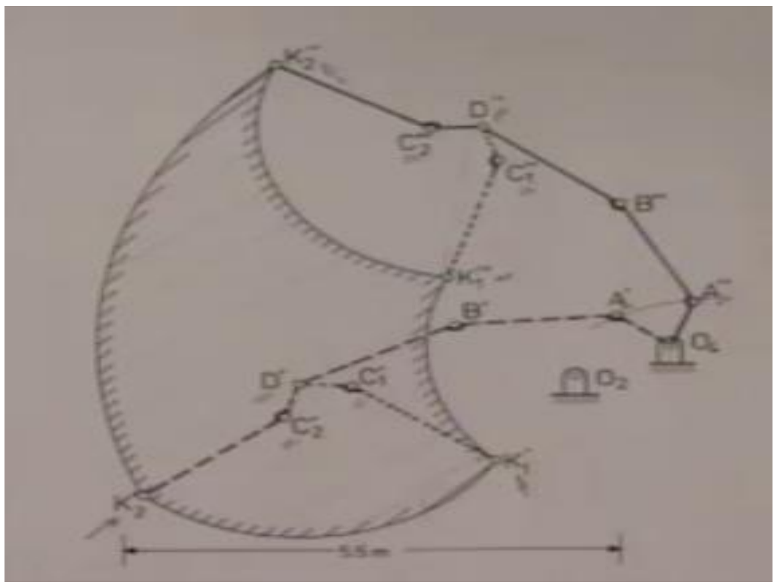

If we look carefully at the original diagram, we can see that distance O_4A never changes because they belong to the same rigid body. Consequently, A lies on a circle with O_4 as center and O_4A as radius. The maximum distance between O_2 and A, which is given to us, locates this point. Minimum distance between O_2 and A, which is again specified in the problem locates this point.

So I can locate position of A, for the extreme movements of this actuator Z_1. O_4, A, B and D, all four belong to the same rigid body, so using the tracing paper technique that we have just now we have seen, I can locate the corresponding position of D as D′ and D′′. For the extreme positions, one can see that link 4 and link 7 get rigidly connected and that distance DC also does not change. Corresponding to the maximum position of DC and minimum position of DC, depending on the movement of the actuator Z_2 corresponding to D'', I get two locations for C. Same way corresponding to D', I get two locations of C, corresponding to the maximum and minimum distance of BC. Now D, C, K, they belong to the same rigid link. So corresponding to the D, C, I can locate corresponding positions of K. Same way, corresponding to D', I get the extreme positions of K.

Now the question is, what is the range in which the hinge K lies when the maximum movements of these two actuators, between O_2 and A, and between B and C is utilized. This is the maximum distance BC, this is the minimum distance BC. One can easily see that when the two actuators have taken the extreme positions, the distance DK does not change. Consequently, this is a circular arc with D'' as the center and this as the radius, same way this as the radius. So I get these two circular arcs. Same way one can say that the distance from O_4 does not change again because if the actuators are maximum locations, link number 4 and 7 get rigidly connected and K moves on a circle with O_4 as center.

So this is another circular arc, joining $K_1''$and K_1' with O_4 as center and another circular arc with O_4 as center and radius O_4K_2'. So these four circular arcs, determine the region in which the hinge K can lie when the maximum and minimum distance between O_2 and A, and B and C are utilized by the hydraulic actuators. So in the full range of operation, the hinge K lies within this region. With this hint you should be able to complete the problem.

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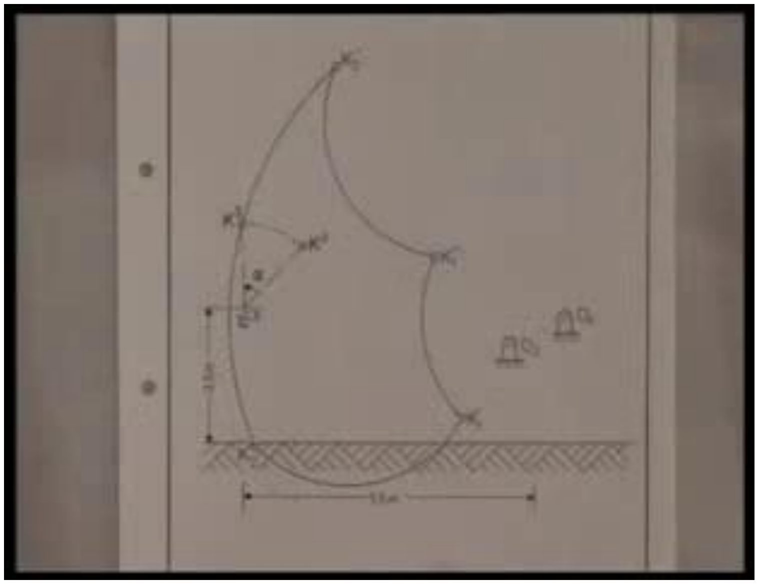

To answer the third part of the problem, that is the tilt angle of the bin when the lip is kept at this location L^2, we solve it as follows: We have already determined that the region in which the hinge K lies when the total range of movements actuators are used. The hinge K can never go outside these regions consisting of four circular arcs. But L and K are two points on the same rigid body, so their distance cannot change. If the lip is kept fixed at L then I rotate this line LK^2 to reach the boundary of this region, that is K^3 and this angle gives the angle of rotation that the bin undergoes keeping the lip at L. It cannot rotate any further because then K will go outside this range which is not possible for the range of movement of actuators which are given.

Let me now summarize what we have learned today. Today, we have solved three rather intricate or I would say more advance problems of displacement analysis of planar mechanism through examples having more than 1 degree of freedom, that is the front-end loader and the earth-moving machinery and the other is a 3R-1P Non-Grashof linkage. What we have seen, that this graphical method is very powerful and clever use of the tracing paper as an over lay can become very handy to solve even more complicated problems of displacement analysis. In our next lecture, we shall discuss the analytical method of displacement analysis.