Kinematics of Machines

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Module - 2 Lecture - 1

The topic of today's lecture is mobility analysis. By mobility analysis, we obtain the degrees of freedom of a given mechanism. This is accomplished by the counting number of links and the number of different types of kinematic pairs those are used to connect these links.

Let me now elaborate, how we carry out this mobility analysis for planar mechanisms. It is worthwhile to recall that in a planar mechanism each link has 3 degrees of freedom: 2 of which are translational in the plane of motion and 1 is rotational about an axis perpendicular to the plane of motion.

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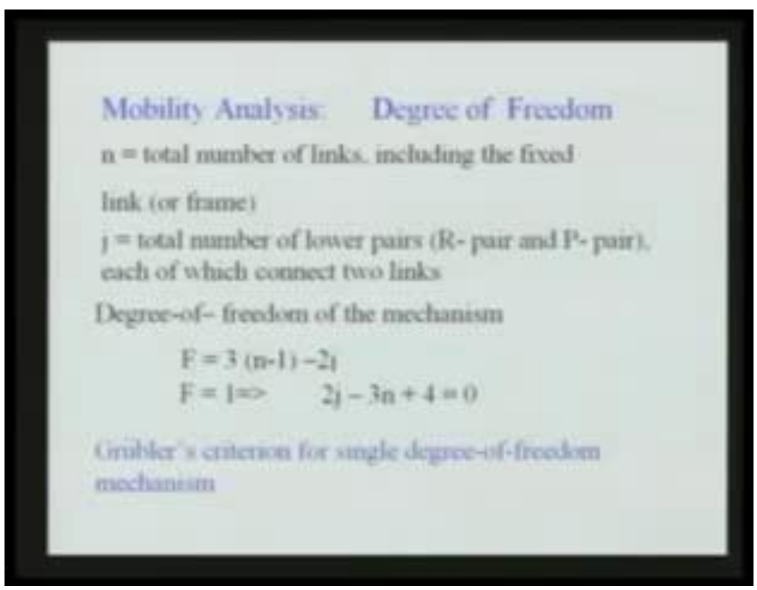

Let there be 'n' number of total links in a mechanism, which includes the fixed link of the frame that means there are n – 1 moving links. When these links are not connected by any kinematic pair then the total degrees of freedom are obviously 3(n – 1). For each of these n – 1 links there are 3 degrees of freedom, so the total degree of freedom of the system is 3(n – 1). Let these ‘n’ links be connected by j number of lower pairs. By lower pair in a planar mechanism, we can mean either a revolute pair or a prismatic pair and each of these kinematic pairs connects only 2 links. We also recall that whether it is a revolute pair or a lower pair, at each of these pairs, 2 degrees of freedom are cuttled and only 1 out of 3 is maintained. If there are ‘j’ number of total kinematic pairs 2j numbers of degrees of freedom are cuttled.

So, the effective degree of freedom of the mechanism is reduced to F, which are the degrees of freedom of the mechanism is 3(n – 1) – 2j. Let us consider a constant mechanism with a single degree of freedom; that is, there exist a unique input-output relationship, where the degree of freedom of the mechanism F is 1. Substituting F = 1 in above equation, we get 2j – 3n + 4 = 0.

For a single degree of freedom mechanism, maintaining a unique input-output relationship, the number of links and the number of lower pairs must be related to this equation that is: 2j – 3n + 4 equal to 0. This equation is called Grubler’s criterion for single degree of freedom mechanism. While deriving this Grubler’s criterion, we assume that each of these lower pairs is connecting only 2 links. However, due to practical considerations some times more than 2 links can be connected at a particular hinge. As an example of a different types of kinematic pairs which connects more than 2 links, let us consider this figure.

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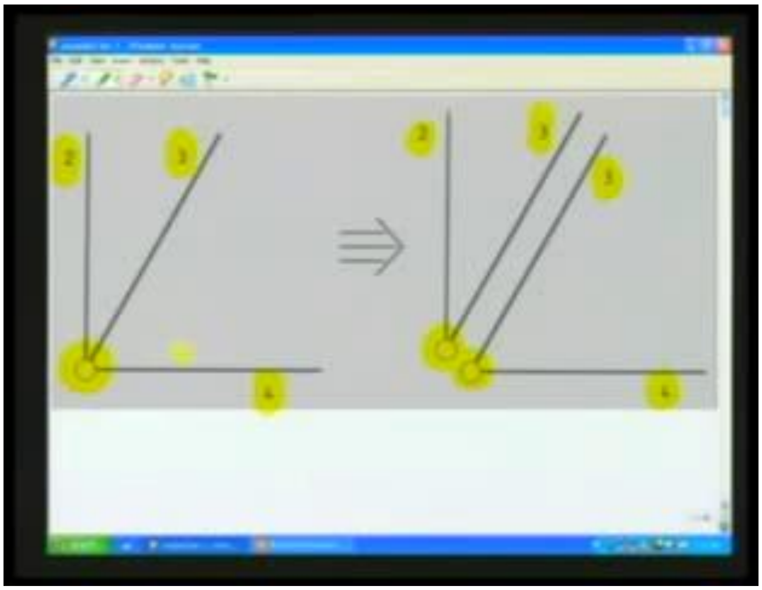

Here 3 links namely 2, 3 and 4 are connected by a single hinge at this location. Such hinges are called compound hinges or higher order hinges. This particular compound hinge is equivalent to two simple hinges as explained in the adjoining figure. For example, this particular hinge can be thought of as 2 hinges. One connecting link 2 and link 3, whereas another hinge connects link 3 and link 4.

Thus, a hinge which connects 3 different links is equivalent to 2 simple hinges. This way we can think of another type of hinge where 4 links are connected and such a hinge will obviously be equivalent to 3 simple hinges. Maintaining this equivalent between higher order hinges and simple hinges, we would like to modify the equation for calculating the degrees of freedom of a mechanism as follows. When higher order hinges are present, the symbol ‘j’ in the equation, we would like to modify as follows.

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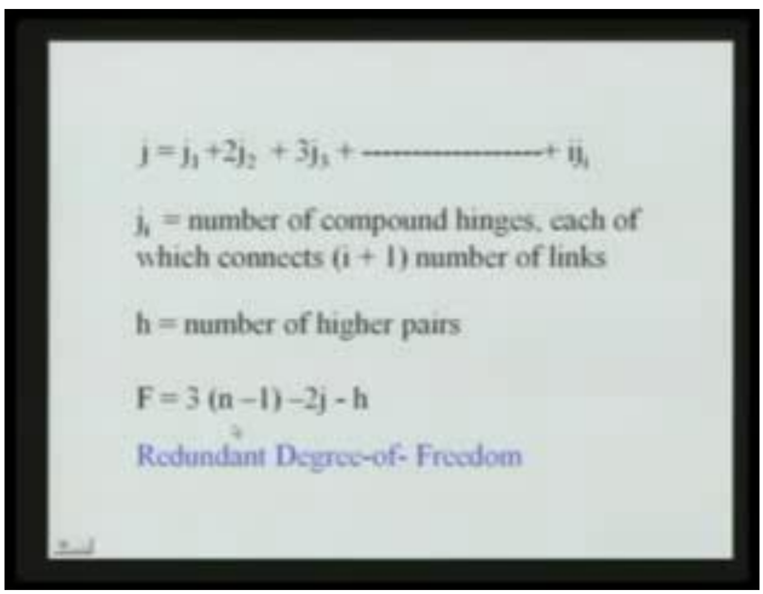

j is equal to j_1, which represents the number of simple hinges, which connects only 2 links plus $2j_2$, where j_2 is the number of hinges to each one of which connects 3 links and so on; that is j_3 represents the number of hinges each one of which connects (3 + 1), that is 4 links and so on up to j_i. that is j_i is the number of compound hinges, each of which connects (i + 1) number of links. In a mechanism, there can be higher pair as well and as we recall, if there is a higher pair then at each higher pair only 1 translational degree of freedom is cuttled that is along the common normal to the point or line of contact. Two other degrees of freedom can be retained. Consequently, at each higher pair only 1 degree of freedom is cuttled.

I would like to modify the equation, the degrees of freedom of a mechanism,

$$F = 3(n - 1) - 2j - h,$$

where h represents the number of higher pairs, j represents the number of equivalent simple hinges and n represents the number of total links.

Sometimes there can be some redundant degree of freedom of a mechanism. What we mean by a redundant degree of freedom? Due to some typical kinematic pairs and their placement, we may find that in a mechanism a particular link may be moved without transmitting any motion to any other link. Such a degree of freedom is referred to as redundant degree of freedom.

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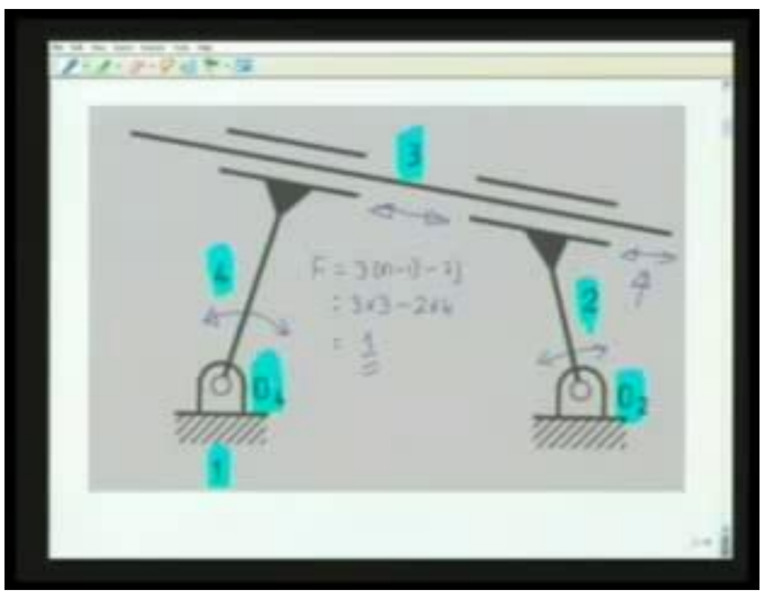

Let me now explain some redundant degrees of freedom and how to take care of that in the equation so that we get the effective degrees of freedom. As an example of a redundant degree of freedom, let us look at this 4-link mechanism, where we have link number 1 which is the fixed link; link 2 which is connected to link 1 through the revolute pair at O_2. There is link 4, which is connected to link 1 through the revolute pair at O_4. And link 3, has 2 prismatic pairs connecting it to link 2 and link 4.

The thing to be note is that the direction of this revolute pair is same; both the prismatic pairs have the direction is along the link 3. Consequently, link 3 can be dragged along the direction without transferring any motion either to link 2 or to link 4. Consequently, this constitutes a redundant degree of freedom. If we apply the formula bluntly, that is $F = 3(n - 1) - 2j$, we get,

$$n = 4,\ j = 4;\ \text{Therefore, } F = 3(4 - 1) - 2*4 = 1.\ \text{i.e., } F = 1$$

It appears according to the formula this is a single degree freedom mechanism implying unique input-output relationship. However, the link 2 or link 4 cannot be moved at all. This is permanently locked which acts like a structure. What is this degree of freedom '1'? That is nothing but, this redundant degree of freedom of the link 3 along this direction of the prismatic pairs.

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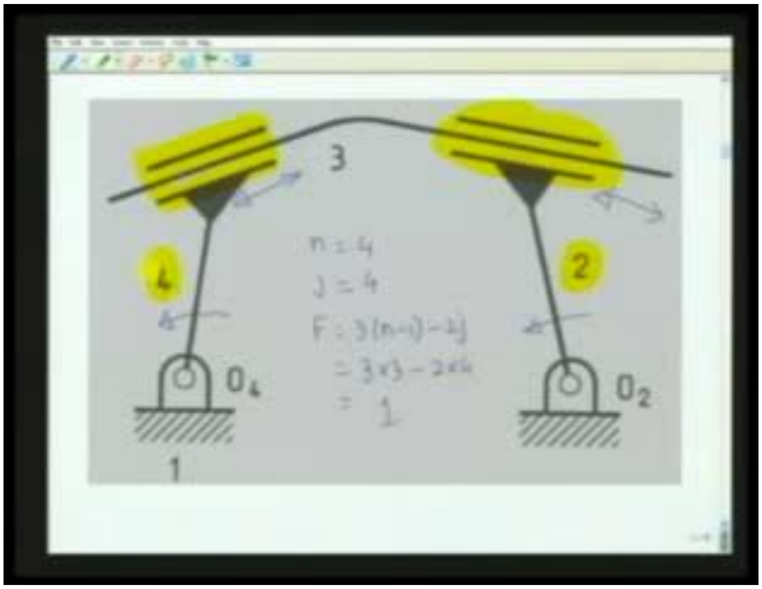

It may be interesting to see what happens if the directions of the 2 prismatic pairs are different. Between 3 and 4, it is in this direction; whereas, between 3 and 2 it is along a different direction. Consequently, here the formula will work perfectly, because, there is no redundant degree of freedom. I cannot move link 3 without transferring motions to links 2 and 4. So, here n is 4, j is 4 as we obtained earlier and F = 3(n – 1) – 2j = 3(4 – 1) – 2∗4 = 1. i.e., F = 1.

Actually, here link 2 can be moved to transmit motion to link 4. A little thought would convince that the rotation of link 2 and link 4 must be identical. Let me explain why. As we see, link 2 and link 3 has a prismatic pair here, which means there is no relative notation between link 2 and 3. Similarly, there is a prismatic pair here between link 3 and link 4. So there cannot be any relative rotation between link 3 and link 4. Consequently, there cannot be any relative rotation between link 2 and link 4, both of which are in translation with respect to link 3. What is the implication? That there is no relative rotation between links 2 and 4. Both of them rotate but they rotate by the same amount, so that, there is no relative rotation.

Let me now take another example of a redundant degree of freedom which is very commonly seen.

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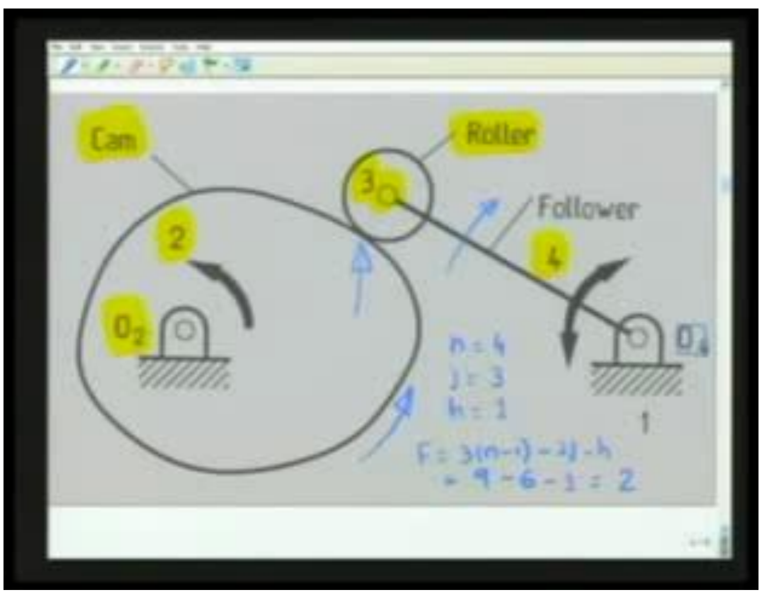

In the above figure, we see what is known as a cam-follower mechanism and we have a roller follower. Cam is input link which is link 2, which is hinged to link 1, the fixed link at the revolute joint at O_2. Follower is link 4 is hinged to roller at this revolute pair; roller is the link 3.

It is intuitively pretty obvious that if we move link 2, say we give it a rotation then the follower will also have a rotation in the given direction.

There exists a unique input-output relationship, unique rotations of link 2 causes unique rotation of link 4. Let me calculate the degree of freedom. As we have seen, there is a unique input-output relationship depending on the shape of the cam profile, so the degree of freedom should turn out to be 1. But, let me do it by counting according to our formula. We have already seen that there are 4 links; n is 4. There are three revolute pairs: one between 1 and 2, one between 1 and 4 at O_4 and one between 3 and 4, at the roller centre; j is 3.

Now there is a higher pair between link number 2 and 3 at this point; h is 1. If we calculate the degree of freedom F,

$$F = 3(n - 1) - 2j - h = 3(4 - 1) - 2*3 - 1 = 2 \text{ i.e., } F = 2.$$

So, the degree of freedom according to the formula is standing out to be two, because, there is a redundant degree of freedom and that is, roller 3 can be rotated about this revolute pair without transferring any motion either to link 2 or link 4. So, that is the redundant degree of freedom. So F_r if we call as the redundant degree of freedom, F_r is 1. In view of this redundant degree of freedom, let us modify our equation which we obtained earlier.

Now that we have seen there can be some redundant degrees of freedom, let us now modify the formula in view of this.

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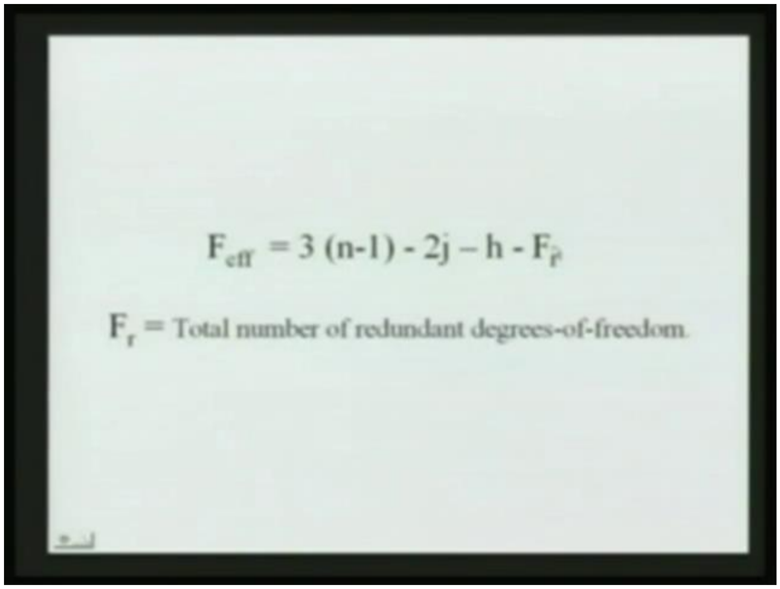

F_{eff} that is the really the input-output relationship is governed by

$$F_{eff} = 3(n - 1) - 2j - h - F_r$$

where F_r is the total number of redundant degrees of freedom. Sometimes due to some other practical considerations, a mechanism may have some redundant kinematic pairs, which means, those kinematic pairs are not kinematically important, but they may be required due to some other considerations. The simplest example is a shaft is normally mounted on 2 bearings, but both the bearings act as 1 revolute pair permitting rotation about the same axis. By counting we may call it 2, but kinematically, that is only 1 revolute pair. Let me show an example of such redundant kinematic pair.

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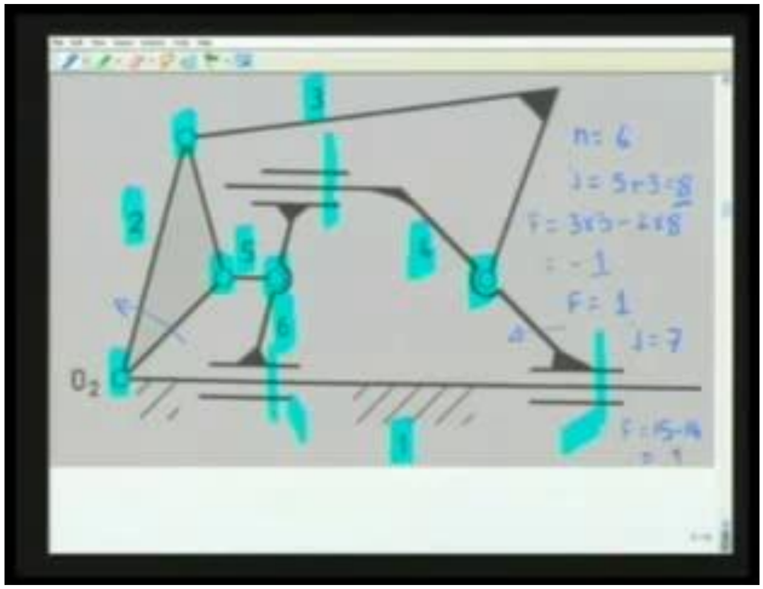

Here, we consider a 6-mechanism. Link 1 which is fixed, which is connected to link 2 through a revolute pair at O_2. Link 3 having a revolute pair between 2 and 3. Link 4 is next link which is connected to link 3 by this revolute pair. Link 5 is connected to link 2 by this revolute pair. Link 5 is connected to link 4 by this prismatic pair here. Link 6 is another link which is connected to link 5 through this revolute pair. And link 6 is connected to link 4 by this prismatic pair. Link 6 is connected to link 1 by this prismatic pair and link 4 is connected to link 1 by this prismatic pair.

Let me apply the formula and try to find the degree of freedom of this mechanism. Here, we have n is 6. All these pairs are simple pairs because they connect only 2 links. So j, we count there are 1, 2, 3, 4, 5 revolute pairs and 3 prismatic pairs, so j is 8. Consequently, the degree of freedom of the mechanism F is given by

$$F = 3(6 - 1) - 2*8 = 15 - 16 = -1 \text{ i.e., } F = -1$$

That means according to the formula, this mechanism is a structure rather a statically indeterminate structure with negative degrees of freedom and no relative motion should be possible between various links. However, as we see shortly, this has degree of freedom 1 and there is a unique input-output relationship that means, if we use link 2 as my input link and rotate it, link 4, which we may treat as output link will have some motion.

Now why is this calculation failing? This is because if we notice these 3 prismatic pairs, we should note that all these 3 prismatic pairs are in the same direction. This prismatic pair is allowing horizontal translation between link 1 and link 6. This prismatic pair here is allowing relative translation in the horizontal direction between link 1 and link 4. This prismatic pair which is there to ensure horizontal translation between link 4 and link 6 may be redundant. Even we can replace, we can withdraw, any of this 3 prismatic pairs because all of these are ensuring horizontal translation between links 1, 4 and 6. Thus, j which we counted previously as 8 is actually is 7 because, kinematically, 1 of these 3 prismatic pairs is redundant. So, I can remove this as a redundant pair and make j equal to 7, which will give me F = 1. Now that we have seen there is a possibility in an actual mechanism to have some redundant kinematic pairs, let us rewrite the formula in the light of such redundant kinematic pairs.

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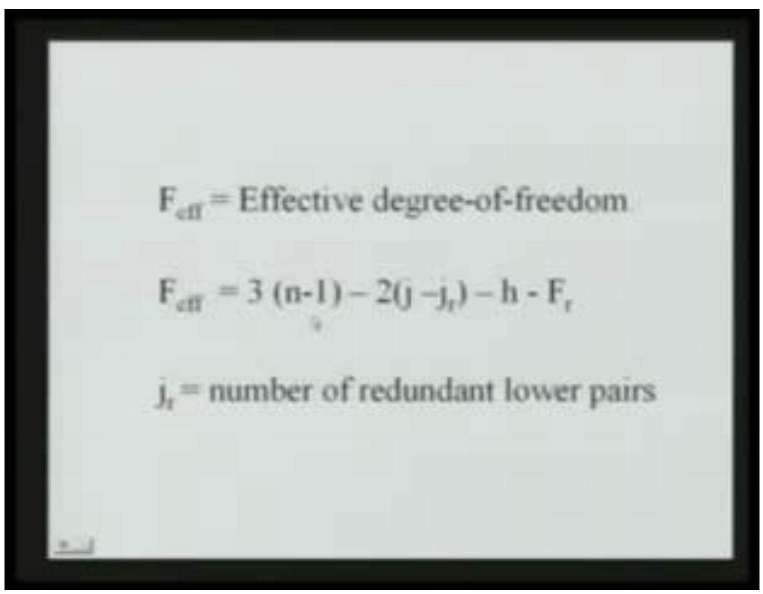

If F_{eff} implies the effective degree of freedom of a mechanism that is given by,

$$F_{eff} = 3(n - 1) - 2(j - j_r) - h - F_r$$

where F_r was the redundant degrees of freedom, h is the number of higher pairs, j_r is the number of redundant kinematic pairs, j is the total number of lower pairs and n is the total number of link. Thus, we arrive at a formula by counting the number of links and considering the different types of pairs and redundant degrees of freedom and redundant kinematic pair, we are in a position to calculate the effective degrees of freedom of a planar mechanism. At this stage, we would like to emphasize a very subtle difference between this revolute pairs and prismatic pairs. So far this formula is concerned, we have not made any distinction between a revolute pair and a prismatic pair, because both types of pairs cutting 2 degrees of freedom and allowed 1 degree of freedom. Let me now point out what is this subtle difference.

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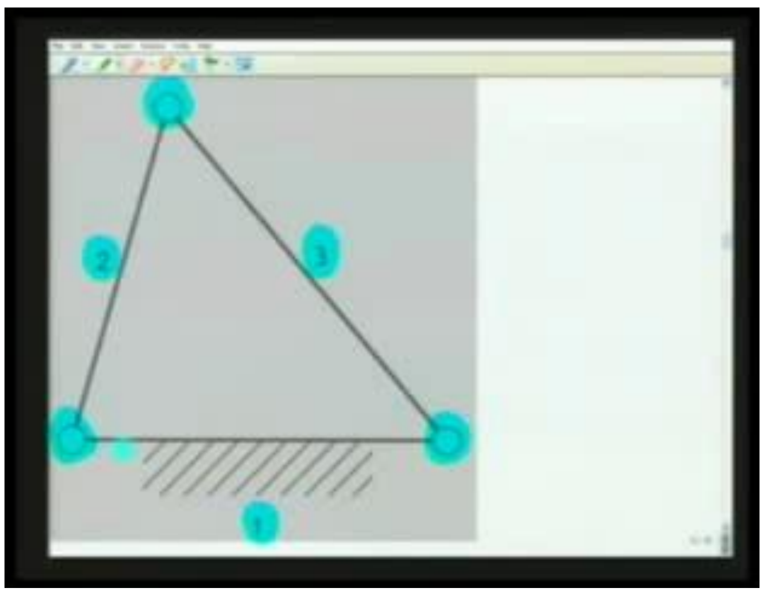

Let us notice this 3-link closed mechanism consisting of only 3 revolute pairs: Link 1, link 2 and link 3 constitutes a closed kinematic chain consisting of 3 revolute pairs. We are already familiar with this and we have seen that it is not a mechanism. It is a structure; no relative motion between various links is possible when all these pairs are revolute pairs. Let us see what happens if all 3 becomes prismatic pairs in different directions.

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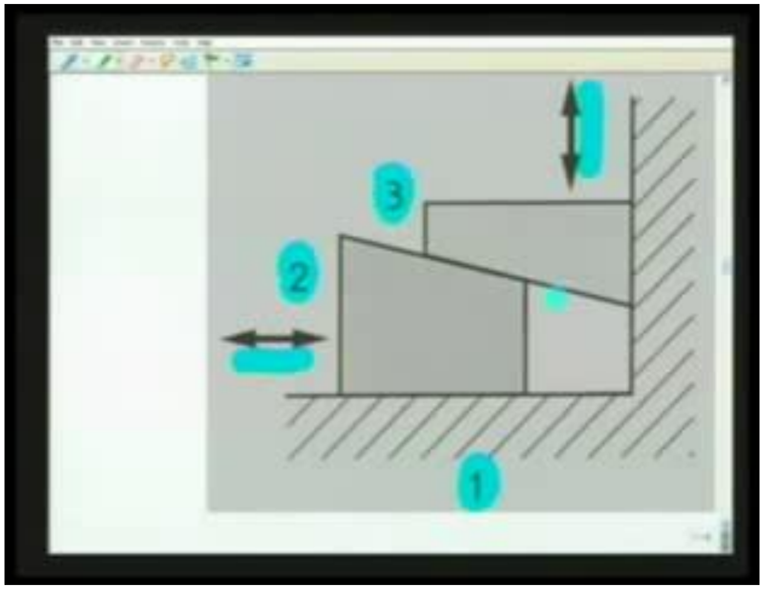

Again there are 3 links: link 1, link 2 and link 3. This constitutes a closed kinematic chain and there are 3 prismatic pairs. One in this horizontal direction between link 1 and 2, one in the vertical direction between link 1 and 3 and there is one in this inclined direction between links 2 and 3.

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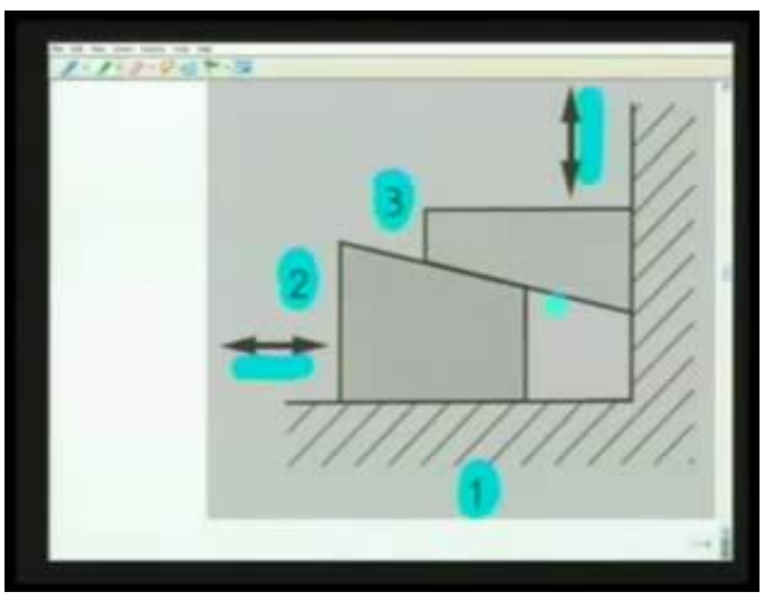

The kinematic representation of this is as follows: There are a three links: links 1, 2 and 3 having 3 prismatic pairs in different directions. It is obvious that here relative motion between various links is possible; it is not a structure, the degree of freedom of this loop is not zero.

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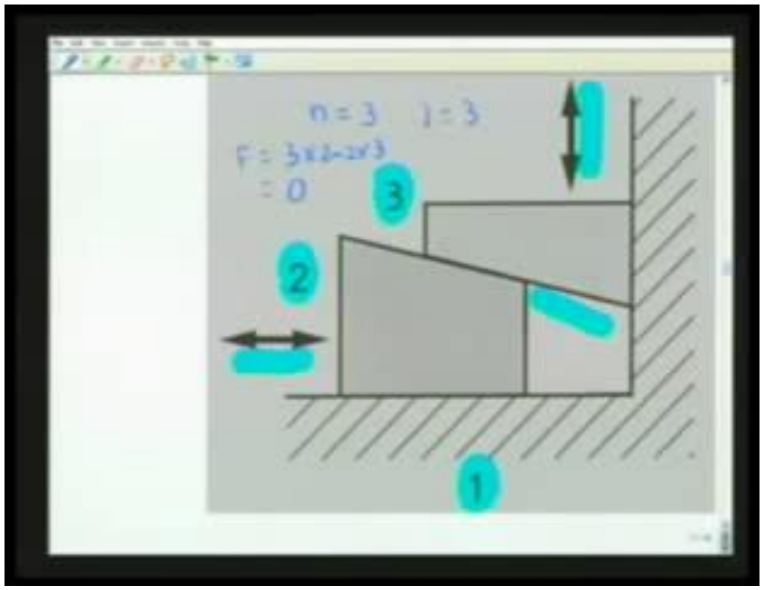

As we can see link 2 can be moved in the horizontal direction to produce a unique vertical movement for link 3. Thus, for this particular closed loop mechanism, n is 3, j is also 3. So according to the formula, we should have {$3(n-1) - 2j = 3*2 - 2*3 = 0$, i.e., $F = 0$}, which is true for the revolute pairs, but not true for the prismatic pair. In light of this difference between revolute and prismatic pair, let us modify our formula for calculating the degrees of freedom. In view of this single degree of freedom closed loop, which is possible by 3 prismatic pairs connecting 3 links, let us modify the formula for calculating the effective degrees of freedom.

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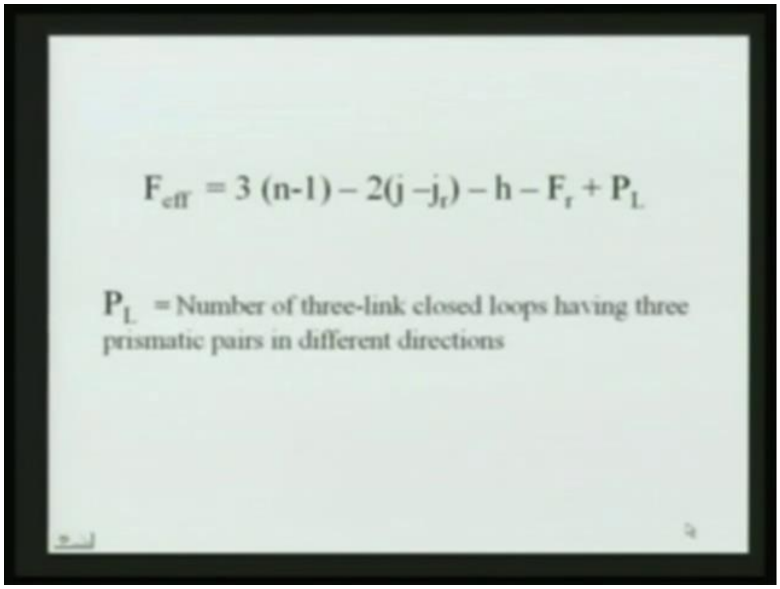

Therefore,

$$F_{eff} = 3(n - 1) - 2(j - j_r) - h - F_r + P_L,$$

where P_L is the number of 3 link closed loops having 3 prismatic pairs in different directions. While deriving this formula, we have not bothered with the kinematic dimensions of the mechanism. So, this formula may have some exceptions for some very special kinematic dimensions, which we shall see shortly through a number of examples. We have already said that due to some special kinematic dimensions the formula that we derived may give wrong result.

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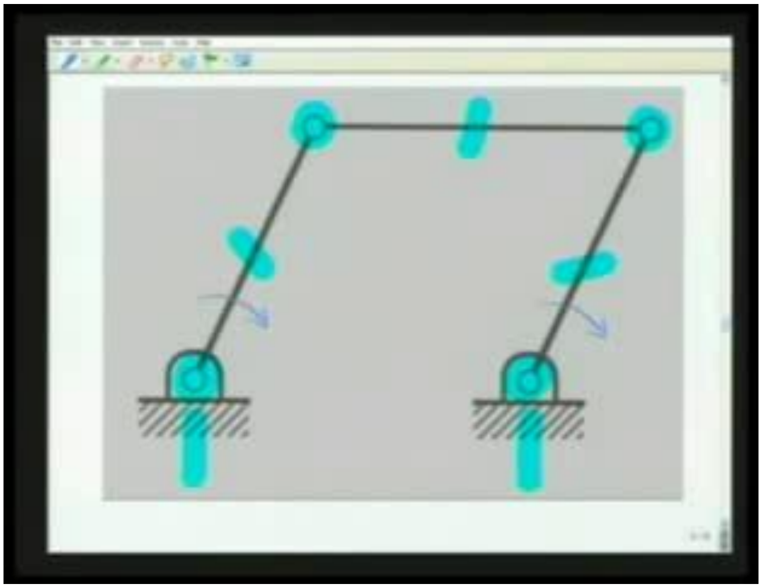

As an example, let us talk of this parallelogram linkage. It is a 4-link mechanism with 4 revolute pairs, but the opposite sides have equal lengths. These 2 links are of same length and this coupler length is equal to the frame length, that is, the distance between these 2 fixed pivots. Obviously, this is a 4R mechanism, which is degree of freedom 1 and it can transmit motion from this link to that link. During this movement, the opposite sides always remain of same length; so a parallelogram remains a parallelogram.

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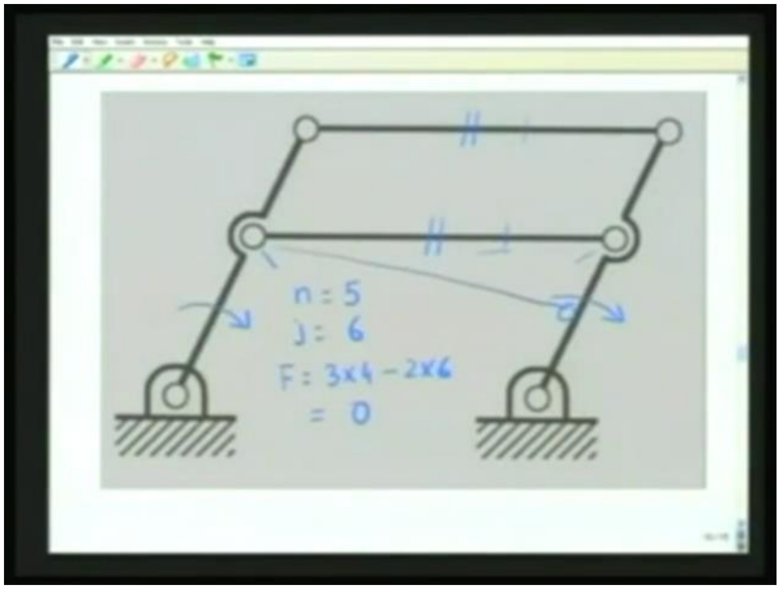

In this parallelogram linkage, if we add an extra coupler which is parallel to the original coupler then what happens? As we see now ‘n’ has become 5 and due to this extra coupler, we have introduced 2 revolute pairs at its 2 ends. So, j has become 6. Consequently, from the formula, we get, F = 3(5 – 1) – 2∗6 = 3∗4 – 2∗6 = 0, i.e., F = 0.

So the formula tells us that, this is structure, but intuitively we can realize that this extra coupler has not imposed any extra constant and the mechanism still retains its single degree of freedom and this moves like a parallelogram as before. Of course, failure of the formula is only because these 2 couplers are parallel and the original diagram was a parallelogram. If this extra coupler which is introduced in an inclined fashion, say starting from this point to this, then the formula will be correct and the assembly will become a structure. In fact, such an extra coupler is normally used to drive a parallelogram mechanism.

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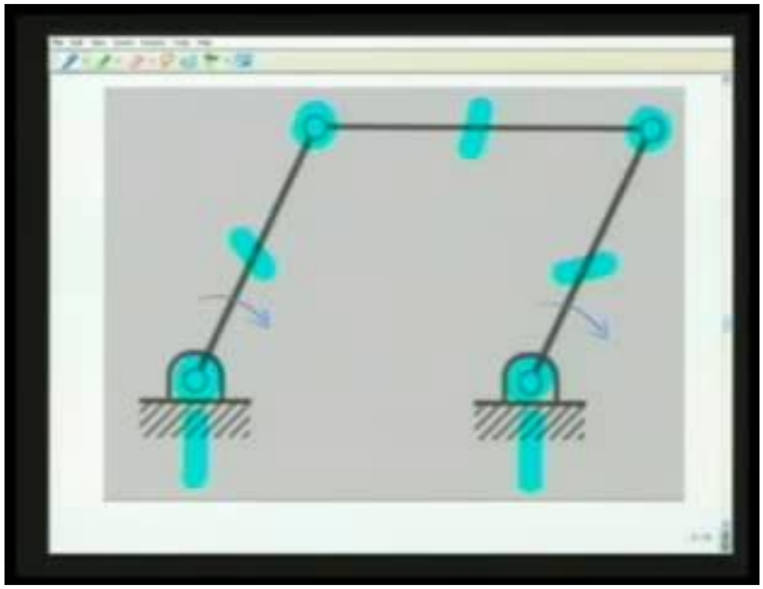

As we shall see in a model that, when the parallelogram moves, there is a configuration when all the links become collinear and that mechanism loses its transmission quality. In fact, it can go into a non-parallelogram or anti-parallelogram configuration. To ensure that a parallelogram always remains a parallelogram such an extra coupler is necessary.

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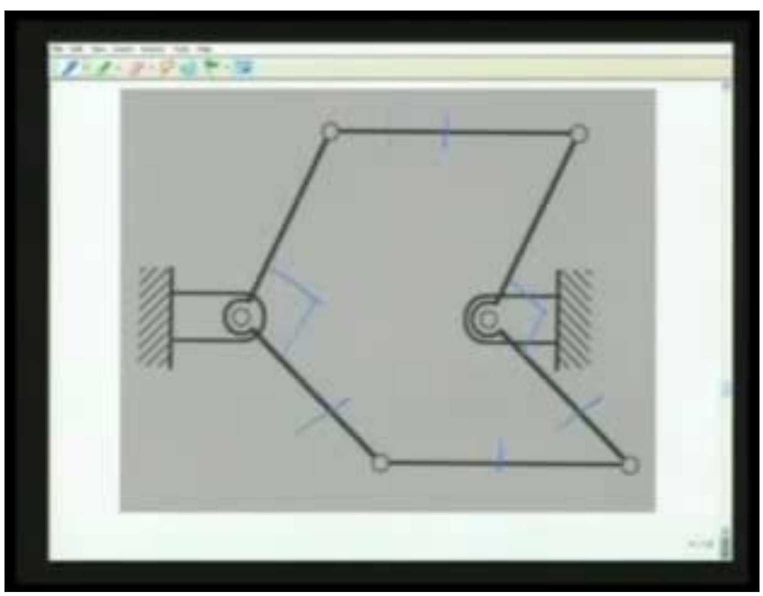

In fact, to maintain the good transmission quality at all configurations, these 2 extra couplers are connected to the input and output link by making a 90^0 angle between the extensions of the input link and the output link, such that, when this particular coupler is collinear with the line of frame, the other coupler is parallel to the line of frame, this portion of the links become perpendicular to the line of frame. This point will be much clearer when you demonstrate it through a model.

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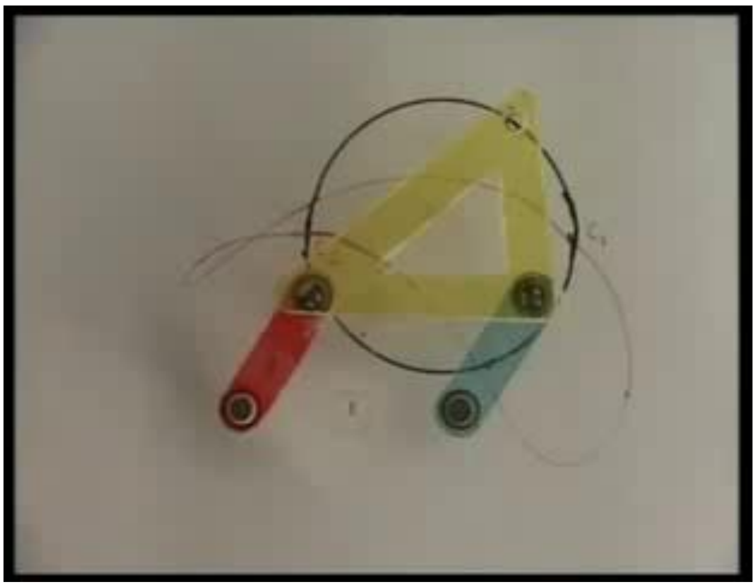

Let us now look at the model of this parallelogram linkage. Here red link and blue link are of equal link length. The coupler which is yellow link has the same length as the fixed link or the distance between the 2 fixed pivots. As we see, this parallelogram linkage when it moves always remains a parallelogram. However, when all the 4 links become collinear, there is a possibility that it flips into anti-parallelogram configuration and it does not move as a parallelogram linkage. Again, here, if sufficient care is taken, one may transfer it to a parallelogram linkage. To get rid of this uncertainty configuration, it is better to have an extra coupler as explained earlier and we shall demonstrate it through our next model.

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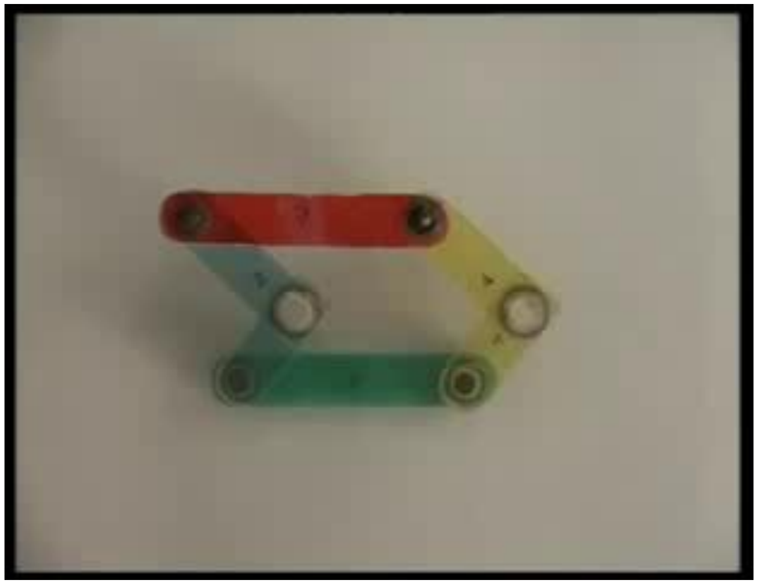

Let us now look at the model of above parallelogram linkage with a redundant coupler. As we see these 2 links are extended at 90^0 and there are 2 parallel couplers. Consequently, here we shall be able to maintain the parallelogram configuration throughout the cycle of motion. It can never flip back into anti-parallelogram configuration.

As we have just seen that for very special kinematic dimensions, the formula for calculating the degrees of freedom may fail. In fact, when the formula was telling that the degree of freedom is 0, we are getting single degree of freedom mechanism. For special kinematic dimensions, when the degree of freedom calculation fails, according to the formula, such linkages are called over closed linkages.

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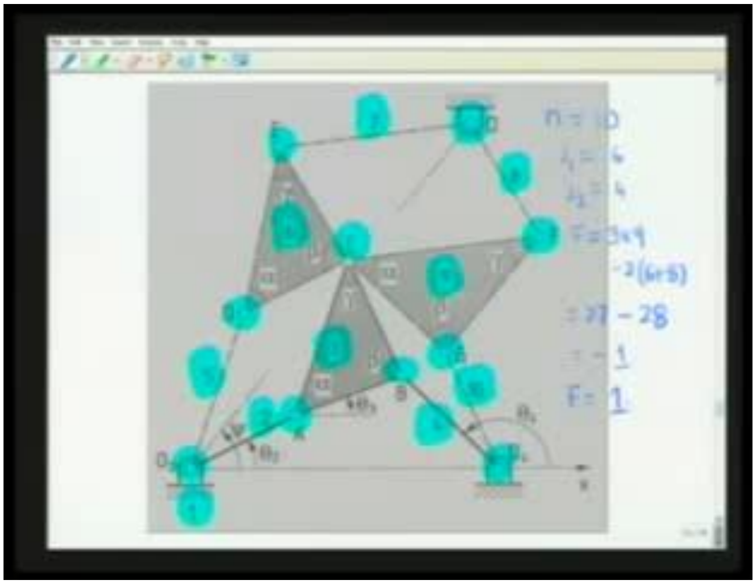

As a further example of over close linkages, let us look at this 10-link mechanism. Here, we have link 1, which is the fixed link; link 2 connected to link 3 connected to link 4 which in turn is again connected to link 1. That means, we get a simple 4-bar mechanism. There is another 4-bar mechanism: link 8, link 9, link 10 and link 1. There is a third 4-bar mechanism consisting of: link 7, link 6, link 5 and link 1. All these 4-bar mechanisms are connected at this revolute pair C.

So, in all we have 10 linked mechanisms and let me also see, what typical revolute pairs are there. There is a revolute pair at O_2 which connects 3 links namely 1, 2 and 5. There is a revolute pair at O_4 which again connects 3 links namely 1, 4 and 10. There is a revolute pair at O which again connects 3 links namely 7, 8 and 1. There is a revolute pair at C which connects 3 links namely 3, 6 and 9. Thus, we have 4 such hinges of j_2 category. There are simple hinges at A, at B, at G, at F, at E and at D.

Let us try to calculate the degrees of freedom of this particular mechanism. We have already seen ‘n’, which is the total number of links are 10. j_1 - that is the number of simple hinges which are at A, B, G, F, E and D that is j_1 is 6; number of compound hinges each one of which connects 3 links, that is j_2 is at O_2, O_4, O and C, that is j_2 is equal to 4. Degree of freedom of this mechanism according to the formula is

$$F = 3(10 - 1) - 2(6 + 2*4) = 27 - 2*14 = -1, \text{ i.e., } F = -1$$

So, without any special dimensions this assembly is a structure with degree of freedom –1. However, if we look at this figure what we see that O_2ACD is a parallelogram; O_4GCB that is another parallelogram and OFCE is another parallelogram. Not only that this ternary links that is 3, 9 and 6, all these 3 ternary links are similar triangles as indicated by the angles alpha, beta and gamma. Due to these special dimensions, we will find that the degree of freedom of this assembly will become equal to 1. That means, this is another example of an over closed linkage, where some of the constants may be redundant, but this will not be highlighted in this lecture. We will just show you the model of this particular mechanism.

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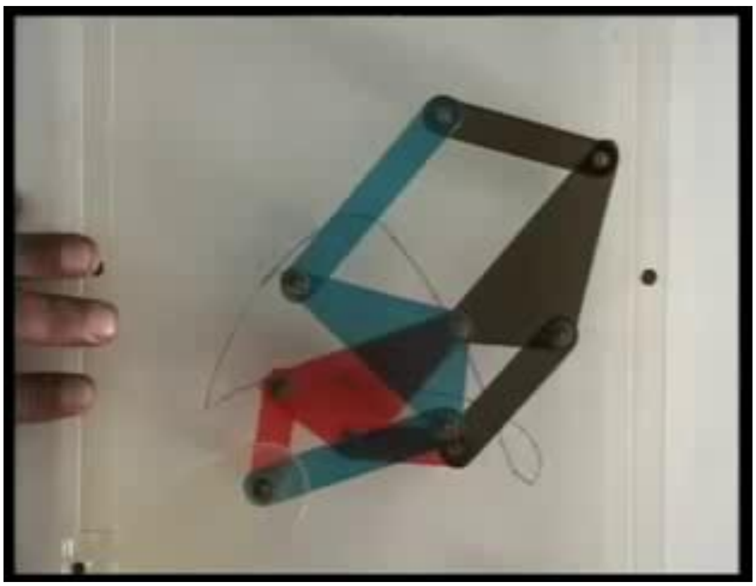

Let us now look at the model of this 10-link mechanism which has just been discussed. As we have seen according to the calculation the degree of freedom should have been –1, but notice that these 4 hinges constitute a parallelogram; so does these 4 and these 4 hinges also constitute another parallelogram. These 3 triangles, the ternary links are similar to each other. Consequently, this constitutes a single degree freedom mechanism, which is an over closed linkage which has mobility; it is not a structure.

As the further example of an over closed linkage, let us consider 8-link mechanism which is known as Kempe Burmeister focal mechanism.

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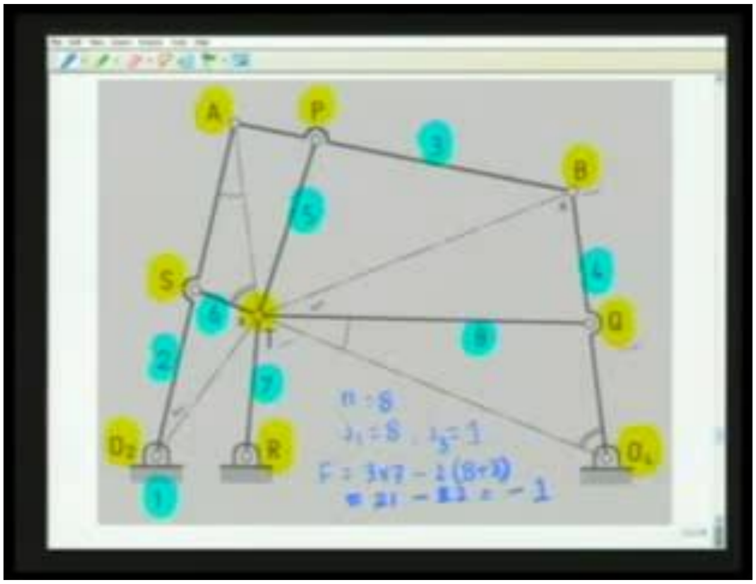

As we see, there are 8 links: link 1, link 2, link 3, 4, 5, 6, 7 and 8. These 8 links are connected by revolute pairs one at O_2, at S, A, P, B, Q, O_4 and R. There are 8 simple hinges and there is a higher order hinge at this point 'T' where 4 links namely 5, 6, 7 and 8 are connected. So, if we calculate the degree of freedom, we see n is 8, j_1 is 8, j_2 is 0, but there is a j_3 at T, where 4 links are connected so j_3 is 1. The degree of freedom F is calculated below:

$$F = 3(8 - 1) - 2(8 + 3*1) = 21 - 22 = -1, \text{ i.e., } F = -1.$$

According to the formula, it should be a structure. However, for very special dimensions, as indicated by these similar triangles BTQ with O_2TS, this angle is equal to this angle and this angle is equal to this angle. Similarly, there are other similar triangles in this figure. For such special dimensions as we see in our model the degree of freedom will turn out to be 1. F will be 1, that means it will be a constant mechanism with single degree of freedom.

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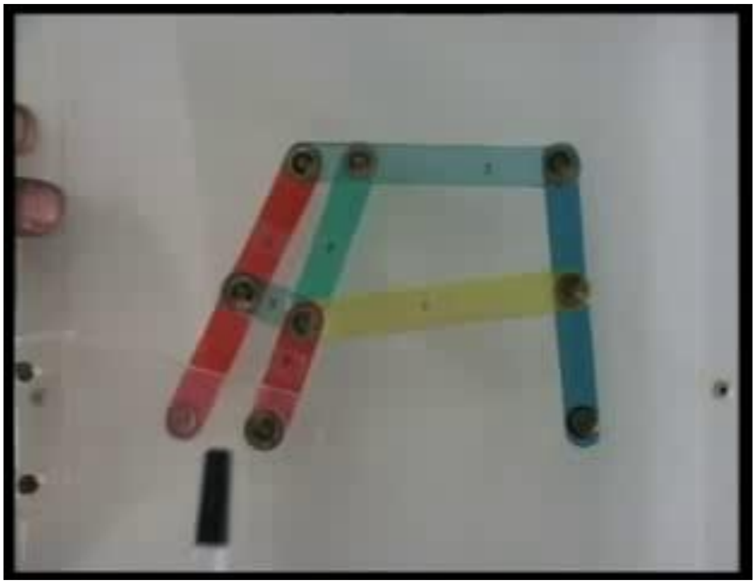

Let us now consider the model of this Kempe Burmeister focal mechanism, which we have just discussed. As we see including this fixed link, we have 8 links: 2, 3, 4, 5, 6, 7, 8 and this is a hinge where 4 links are connected and all other hinges are simple hinges. Accordingly, the formula said the degree of freedom should be minus 1. But however, as we see this mechanism can be moved very easily and there is a unique input-output relationship. That means, the effective degree of freedom of this mechanism is 1; that is only because of the special dimension. If we change any of these points a little bit, this will really become a structure and no relative movement would be possible.

As a last example of an over closed linkage, let us look at this 5-link mechanism which is known as cross slider trammel.

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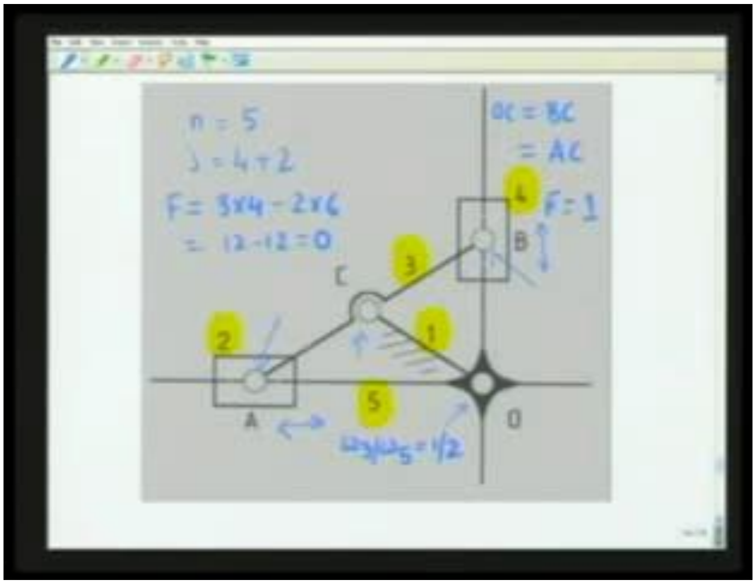

Here we have link 1 which is the fixed link. Link 3 which is connected to link 1 and link 3 is connected to link 4 and link 2. Link 4 and 2 are having prismatic pairs with link 5. Link 2 has a prismatic pair in the horizontal direction with link 5. Link 4 has a prismatic pair in the vertical direction with link 5. Thus, we have n equal to 5; we have 4 revolute pairs here and here and here and here. So, j is 4 revolute pairs plus 2 prismatic pairs. Thus, F turns out to be according to the formula, $\{3(5 - 1) - 2*6 = 3*4 - 12 = 0\}$. So, the effective degree of freedom of this mechanism according to the formula; F_{eff} is 0.

Without any special dimension, it will be a structure and there should not be any mobility any relative movement. However, for these special dimensions when we make OC is same as BC as same as AC, then we will see that there will be an effective degree of freedom of this mechanism will turn out to be just 1. In fact, as we see the angular velocity of link number 3 to that of link number 5 which are both rotating with respect to fixed link 1 will be exactly half for these special dimensions. This is known as cross slider trammel and we would like to encourage the students to show that why it moves by starting from the elliptic trammel that we have discussed in an earlier lecture.

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Now, we shall demonstrate cross-slider trammel through a model. Let us now look at the model of cross-slider trammel. This is that link 3, which has a revolute pair with fixed link here. This is that link 5, which has a revolute pair with fixed link here and there are 2 sliders: 2 and 4 which are hinged to link 3, here and here and these 2 sliders move in these 2 perpendicular slots. For the special dimensions, as we see this has degree of freedom is 1 and rotation of link 3 produces unique rotation of link 5. In fact, we can see that 2 revolutions of link 3 produces 1 revolution of link 5. That is, one can show that ω_3/ω_5 at all instance remain half.

Let me now summarize, what has been covered in today's lecture. What we have seen how we can calculate the degrees of freedom of a planar mechanism by counting the number of links and different times of kinematic pairs. Attention has been also drawn to the fact that there is a possibility of some redundant degrees of freedom that has to be accounted for.

Then, we have also seen there may be some kinematic pairs which are redundant in the sense they do not serve any purpose so far kinematics is concerned, but they maybe there due to some other practical considerations.

At the end we have seen, that these formulas which are derived only from the count without any consideration of any kinematic dimensions may fail when there are some special kinematic dimensions. We have also seen some such over closed linkages through the models, how they move, though the formula says they should be structures.