Kinematics of Machines

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Module 13- Lecture - 3

In our last lecture we have seen that an epicyclic gear train has two degrees of freedom.

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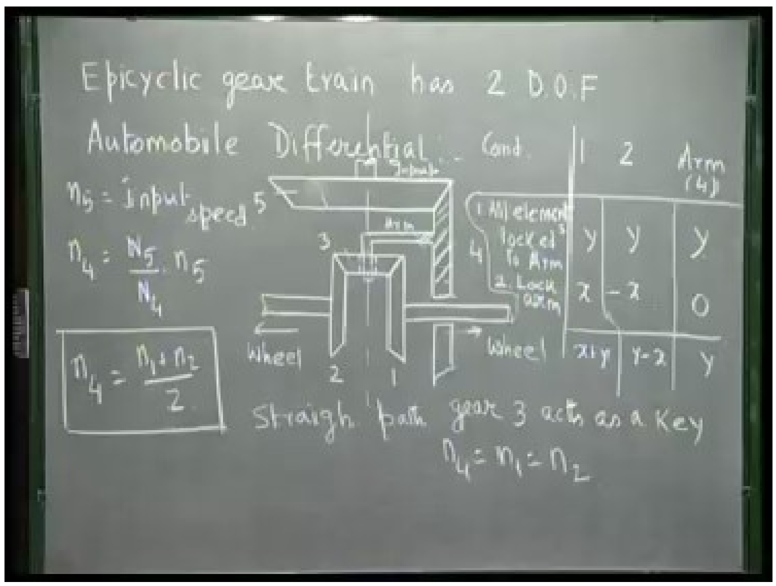

Because it has two degrees of freedom it can create two output speeds from single input speed and that has a lot of applications. For example, commonly used epicyclic gear train, for the same purpose that it can create two output speeds from one input speed is in automobile differential. As we know that when an automobile goes in a straight path both the rear wheels rotate at the same speed. But if the car wants to take a turn, then the input speed may not change, but the outer wheel must rotate faster than the inner wheel. That means from the same input speed, we must create two different output speeds when the car wants to take a turn and that is possible because the input shaft and the wheels are connected by what we call an automobile differential which is nothing but an epicyclic gear train having two degrees of freedom.

Let me draw the differential gear box. These are the two shafts of the wheels, this is say to the one of the rear wheels and this shaft is connected to the other rear wheel. These are bevel gears which are connected to another bevel gear. But this bevel gear is carried by an arm. So, these are two rear bevel gears which are connected to the two rear wheels. This is another bevel gear which carries this arm, that means this bevel gear can rotate about this arm, there is a revolute pair here. That is what we call the planetary gear. This bevel gear is driven by this bevel gear and this is the input.

The input comes to this gear which drives this bevel gear, this bevel gear carries this arm, and this bevel gear is carried on that arm, so there can be relative rotation between the arm and the bevel gear. This bevel gear connects these two bevel gears which are connected to the wheel. Let me say these gears we name 1, 2 and this is 3, this is the arm which is same rigid body as this gear which is 4 and this is 5. So n_5 is the input speed, this small ‘n’ refers to the rpm. This n_5 decides the rotation of gear 4, so n_4 we can find that will be N_5/N_4 ¿ n_5, where N_5 and N_4 are the number of teeth on the gears 5 and 4 respectively. Depending on the direction of this rotation, we can find what is the direction of the rotation of gear 4? Gear 4 is nothing but the arm. To analyze this epicyclic gear train, we follow the tabular method which I said earlier. We talk of condition, and we are interested in gear 1, gear 2 and arm or gear 4, they are the same.

First we say everything is locked to the arm, all these elements or this gear, this gear everything is locked to the arm and then the arm is given some y revolutions, because 1 and 2 is locked to the arm, this whole thing rotates like a rigid body, so 1 and 2 also get the same revolution y. Then second condition is we lock the arm, that means we hold this arm fixed. So arm is locked, so no revolutions to the arm. These two gears are of same dimension as they have the same number of teeth, if the arm is locked, this gear is rotated, the arm is not moving so this is like a simple gear train. This motion is transmitted to this gear through this bevel gear and the number of teeth on this gear is unimportant as it was the intermediate gear and because these two gears are of same size only thing they rotate in opposite direction. If gear 1 is given ‘x’ revolution gear 2 gets –‘x’ revolution. So, the resultant is x + y, y – x and y. So, what we see that if the arm rotates, we can say speed of gear 4, which is y nothing but $\frac{n_1+n_2}{2}$. So, whatever maybe the values of x and y, it is the average of n_1 and n_2 will be the rotation of gear 4.

When the car is moving in a straight path, there is no relative rotation between this gear 3 and the gear arm, so there is no epicyclic action. So, n_4 just drives n_1 and n_2 in the same speed and this gear 3 does not rotate, this does not have any gearing action, this just acts like a solid connection. Then n_1 is same as n_2 is same as n_4. If the car is moving on a straight path there is no relative rotation of gear 3 with respect to the arm there is any epicyclic action, gear 3 does not revolve about its own axis and n_4 is same as n_1 is same as n_2. Whereas if the car takes a turn then this gear starts rotating about its own axis and average of n_1 and n_2 becomes n_4. So, the speeds are different, one is y + x and other is y – x. This is how an automobile differential can create two different output speeds if necessary because of this epicyclic gear train. We shall now solve 1 or 2 problems to show this tabular method for analyzing epicyclic gear trains.

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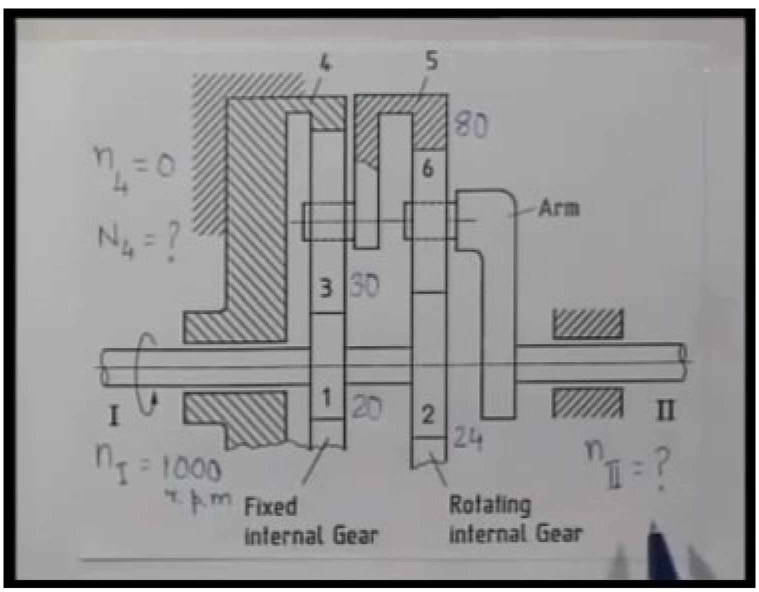

Let me now solve this example of an epicyclic gear train. This is an epicyclic gear train, where the input shaft 1 is connected to this compound gear 1 and 2, that is the speed of this shaft is the same as that of speed of gear 1 and gear 2. Gear 1 is connected to this planet gear, gear 3. Gear 3 is carried by this arm that means there is a revolute pair hear and this arm is this gear 5 which is an internal gear. This gear 5 is an internal gear, this arm is for this planet gear 3, and the gear 3 is also connected to this internal gear 4 which is fixed. The internal gear 4 is not rotating and this gear 3 is moving in space, as this gear rotates the axis of this gear 3 is carried in space. Gear 2 is connected to this planet gear 6, which is carried by this arm, there is a revolute pair between the arm and gear 6 here, and gear 6 is also connected to this internal gear 5 which is rotating. Our objective is, if the number of teeth is specified and input speed is specified we have to find the output speed.

For example, let's say the input speed is given to be in this direction 1000 rpm, what is the second output speed? We are specifying one input speed, because it is an epicyclic gear train so we must specify two input speeds, the other input speed is specified as this gear 4 is fixed, so the other speed of gear 4 is 0. So now two input speeds are specified 0 and 1000 rpm, we have to find out this output speed, what is n_{II}? Obviously, number of teeth on various gears should be specified, let me say number of teeth on gear 1 is given to be 20 and the number of tooth on gear 3 is 30, number of teeth on gear 2 is 24, and number of teeth on gear 5 is 80. This is an intermediate gear; the number of teeth is not given and not necessary either. Here also number of teeth on gear 3 though it is given, it is not necessary for any speed calculation, but we notice that the number of teeth on gear 4 which will be necessary, because that is not an intermediate gear, so that can be calculated from this value. That is number of teeth on gear 4 that we need for calculation of various speeds. Whereas here we can calculate the number of teeth on gear 6 from this 80 and 24, but we will never need it. In this epicyclic gear train problem, two speeds are specified 1000 and 0, we have to find out the output speed.

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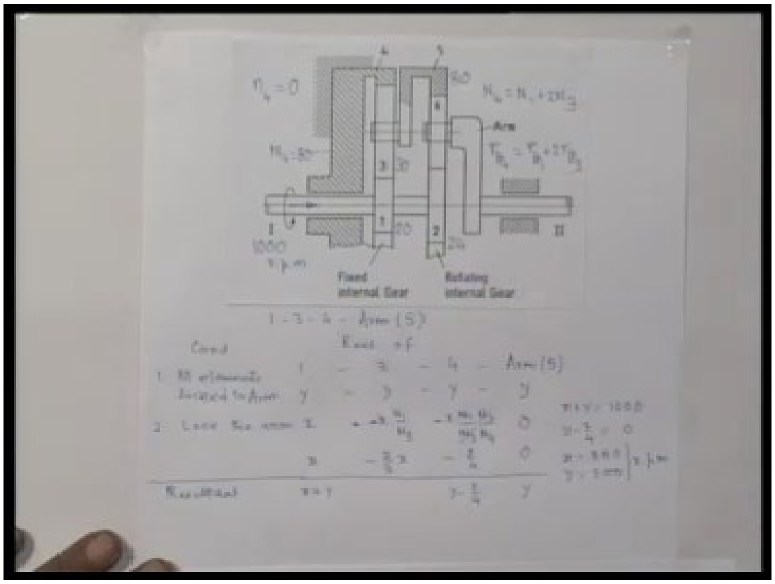

To solve this problem, let us see that there are two epicyclic gear trains in this whole gear train. There are two epicyclic parts because there are two arms, one arm for this, if this is the arm which is nothing but gear 5 and for this portion, this is the arm which is carrying gear 6, and this arm 5 is carrying gear 3. So, let me first consider one epicyclic gear train consisting of gear 1, 3, 4 and its arm which is nothing but gear 5. Number of teeth on 1 is 20; number of teeth on 3 is 30, so we can find out what is the number of teeth on gear 4.

For that, pitch circle radius of gear 4 pitch circle radius of gear 1 is r_{p1} + pitch circle diameter of gear 3 that is 2 r_{p3} , because all the gears have the same modules pitch circle radius is also proportional to the number of teeth. So, from here we can write, number of teeth on gear 4 is number of teeth on gear 1 plus twice the number of teeth on gear 3, N_1 is 20, N_3 is 30 so N_4 is 20 + 2 ¿ 30 = 80. We make the tabular method, we write conditions and revolutions of gear number 1, 3, 4 and the arm which is nothing but gear 5. So, first condition is lock all the elements to this arm. After locking all the elements to the arm, we give y revolution to the arm, because everything is locked so we get the same number of revolutions in all the elements. Next you lock the arm, that is no revolution to the arm and give any revolution x, to say gear 1. If we give x to gear 1, then gear 3 rotates in the opposite direction and it is – x ¿ N_1 ¿ N_3 because this is an external gearing so they rotate in opposite direction and inversely proportional to the number of teeth. Now rotation of gear 3 causes rotation of gear 4, but this is an internal gearing so they rotate in the same direction.

So, gear 3 has a minus sign, so gear 4 will also have the minus sign and rotation of 3 is $-x\frac{N_1}{N_3}\frac{N_3}{N_4}$. As I said earlier, this intermediate gear number of teeth is not relevant so N_3 cancels. So we get $-x\frac{N_1}{N_3}$ and N_1 is 20, N_3 is 30 so it is $\frac{-2}{3}x$. Now, the resultant motion, you add these two motions so here we get x + y, here we don't need 3, because 3 is not given so we do not write it, gear 4 we get $y-\frac{x}{4}$ and arm we get y. Now input speed is given 1000 rpm which is the speed of gear 1 that is x + y. So we get x + y = 1000, I am taking this direction as my positive direction as the direction which is shown is counter clockwise so this is the positive direction, so x + y = 1000 and $n_4 = 0$, so $y-\frac{x}{4}=0$, which if I solve these two equations we can get, x = 800 rpm and y = 200 rpm and arm 5 which has the speed y is 200 rpm.

When we analyze this part of the epicyclic gear train we know the speed of arm 5 which is 200 rpm and gear 2 again the speed will be known because that is same as shaft one. Gear 2 and gear 1 are one compound gear integral to the shaft 1. In this epicyclic gear train, we will know two speeds that was speed of gear 2 is 1000 rpm and speed of 5 is 200 rpm and I should be able to solve for the speed of the arm as I will show you just now.

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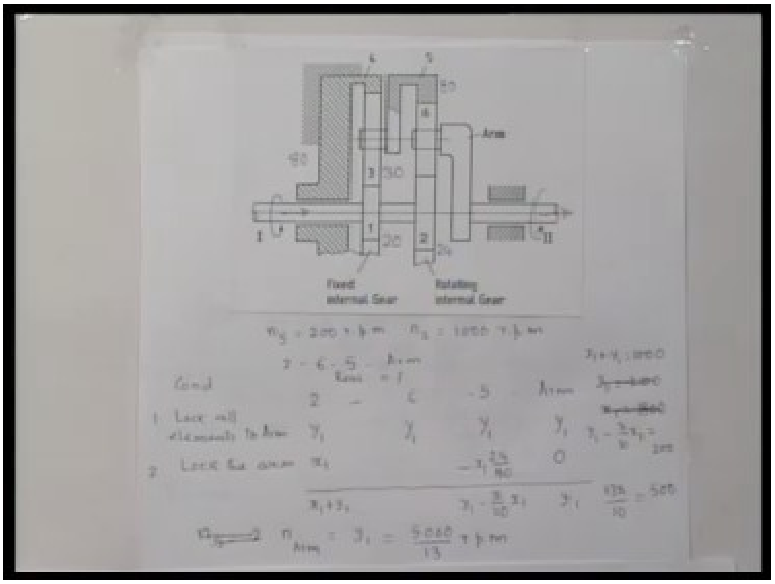

Let me now analyze the second epicyclic gear train consisting of gear 2, gear 6, gear 5, this arm and this arm carries gear 6. Remember this gear 5 was nothing but the arm of the previous gear train which we have just now solved. We have already seen that n_5 was obtained as 200 rpm and n_2 is given as 1000 rpm, and our objective is to find the rpm of this arm which is connected to the output shaft. So gear train is 2, 6, 5 and the arm. As before we write condition and revolutions of 2, 6, 5 and arm. First lock all elements to arm. All these elements are locked to the arm and give y revolution to the arm, so all the elements get y revolution. Then next lock the arm, that is arm has no revolution and let me call it as y_1, so that we do not confuse it x and y we used earlier and lock the arm and give say x_1 revolution to gear 2.

Then gear 5 rotates in the opposite direction because of this intermediate gear makes it opposite direction 6 and 5 rotate in the same direction so $- x_1 \frac{24}{80}$. So, the resultant is $x_1 + y_1$, y_1 and $y_1 \frac{-3}{10} x_1$. So, what we see the speeds given are n_2 and n_5, $n_2 = x_1 + y_1 = 1000$, and $n_5 = y_1 \frac{-3}{10} x_1 = 200$. We have to find what is the speed of the arm, n_{arm}? That is the output speed which is nothing but y_1. So, if we solve for y_1 from this, by eliminating x_1 from n_2 and n_5 equations, we get $y_1 = \frac{5000}{13}$ rpm, since it is positive, that means it is in the same direction. So, we get the same output speed as $\frac{5000}{13}$ rpm in the same direction. Next, we shall solve one more problem involving epicyclic gear train.

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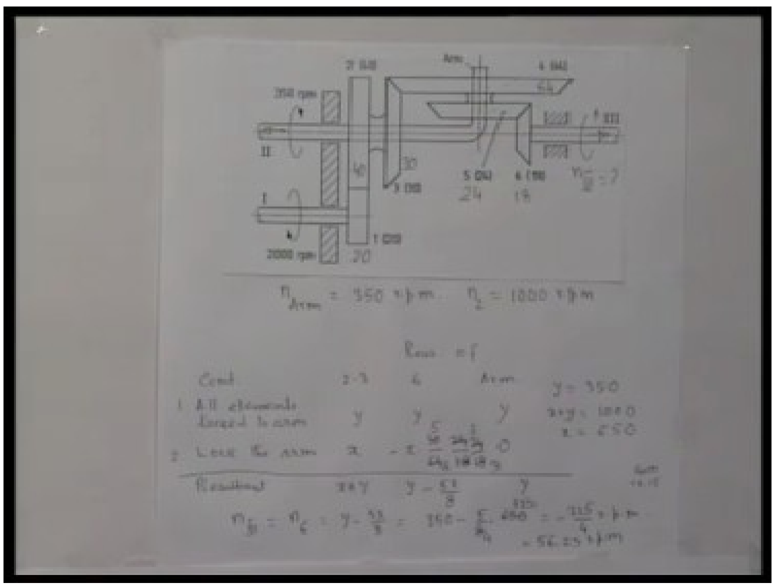

Let me now solve one more problem of epicyclic gear train which is shown in this sketch. This gear 1 is mounted on shaft 1 which is rotating at 2000 rpm in the direction shown. The number of teeth on gear 1 is 20. Gear 1 is connected to gear 2 and number of teeth on gear 2 is 40. Gear 2 and gear 3 forms a compound gear that means they are same rigid body and number of teeth on gear 3 is 30. This shaft 2 which rotates at 350 rpm carries this arm; this shaft is the arm as it rotates this takes these two bevel gears with it. There is a revolute pair here between the arm and these two bevel gears. This bevel gear 3 is connected to bevel gear 4, number of teeth on which is 64.

This gear 4 and 5 are compound gear again they are the same rigid body, and there is a revolute pair between the arm and these two gears which is the same rigid body 4 and 5, and the number of teeth on 5 is 24. Gear 5 is connected to this bevel gear 6 number of teeth on which is 18, and gear 6 is connected to the output shaft. So, these two input speeds are specified that of the arm and that of gear 1 and we have to find out what is the output speed n_{III}? The thing to note in this particular gear train is that, gear 1 decides the speed of gear 2, so if this rotates at 2000 rpm gear 2 rotates at 1000 rpm opposite to this that is in this direction gear 2, because the teeth on gear 1 and 2 is 20 and 40 respectively. n_{II} is known if we take this as positive direction, then what is n_{arm}, we take this direction as positive, n_{arm} is 350 rpm, and n_{II} which is decided by n_I which is 2000 rpm here, so that is 1000 rpm opposite to this that is the same direction as this.

Now we can forget about gear 1 and we have an epicyclic gear train that means this gear 1 is not a part of this epicyclic gear train which is obvious because when you lock all the gears to the arm, we cannot lock this gear because then this gear cannot rotate. This is mounted here in the gearing, but if we remove this gear from this epicyclic gear train rest of it is a simple epicyclic gear train, only thing speed of two is decided by that of speed of one, that is the only difficult point in this problem. Once we have did this stage this epicyclic gear train is simple. So, we can write the same table, revolutions of gears 2 and 3 which is same then gears 4 and 5 we are not interested that is intermediate gear so we write gear 6. Gear 6 which is rotating about the same axis like the gears 2 and 3 and also the arm which is rotating about this axis, so we write arm.

Then we write condition, first condition, all elements locked to the arm, and if arm is given y revolution then gear 6 gets y revolution, gears 2 and 3 also gets y revolution. We are not writing these two bevel gears because as we see the rotation of this gears which is along with the arm and their rotations above the arm are about different axis. These two gears rotate with respect to the arm about this axis, whereas the arm is rotating about the horizontal axis, so we cannot add those kinds of things, and we are not interested, because these are intermediate gears so we don't write it here. Here we write second condition, lock the arm, arm is given no revolution and if we give x revolution to gear 3, then gear 4 rotates due to gearing action. If it rotates in this direction then it goes in this direction so this gear will rotate in opposite direction. These two gears due to this bevel action rotates in opposite direction, and their rotation is decided by the number of teeth on this and this, which is $-x \frac{30}{64}$ and these two rotate in the same direction, then ratio here is, gear 6 is rotating due to this bevel gear 5 is $\frac{24}{18}$. So, this gear 6 is rotating at – x $\frac{30}{64}\frac{24}{18}$.

So now we get the resultant, so it is x + y and this is y, and this is $y-\frac{5x}{8}$. Out of which we are given n_{arm}, y is 350 and gear 2 is 1000, x + y = 1000. So, we get x = 650 and we have to find the revolution of 6 which is n_3, so n_3 which is same as n_6 which is $y-\frac{5x}{8}$. By substituting x = 650 and y = 350, we get this $y-\frac{5x}{8}=\frac{-225}{4}=-56.25\,rpm$. So the output speed is 56.25 rpm opposite to this direction. So that is how we solve this epicyclic gear train, only thing to note that this gear does not belong to the epicyclic gear train and if we have bevel gears, then we should not write it in this table because then we cannot add simply like this because all these rotations are about this axis whereas bevel gear with the arm rotates about this axis, but their gearing action make them rotate about a different axis.

We cannot add rotations about two different axes algebraically. So, this is how we solve the problems of epicyclic gear trains. Now that we have explained how to analyze these epicyclic gear trains so for the kinematics is concerned, that is how to obtain the speeds of different gears, and what is the difference between gear train and gear box?

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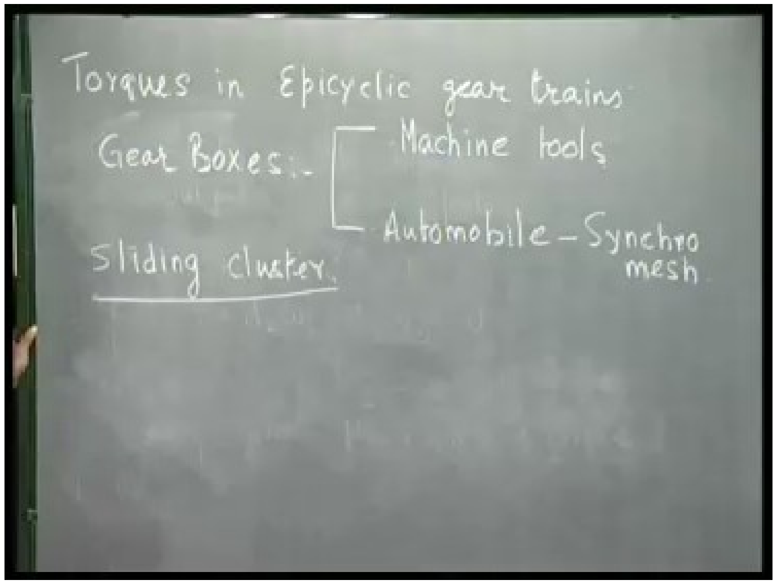

Gear box as we know, there is one input speed, but we may require to get various different output speeds and accordingly, we change the gears within the gear box, the gear engagement we change and using different set of gears within the same gear box we can create different output speed. As we do, in automobile from the same input speed we can go to first gear, second gear, third gear as we want to increase the speed of the car. But there are two kinds of gear boxes, one is, where the gears are changed that is output speed is changed after bringing the entire system to rest. We change the gear engagement within gear box by sliding or something and then switch on the machine and get a different output, as we do in the case of machine tools.

The motor of the lathe machine rotates at a constant speed, but if we want to get different output speed at the head stock or we want to rotate the job of the machine at different speeds depending on the diameter of the job. So, in machine tools we use a kind of gear box, where we bring the entire system to rest change the gear engagement and then again switch on the machine. Whereas, in automobiles we need to change the gears while the car is in motion or the input motion is on. We cannot switch of the car and then change the gears. We have to change the gear while it is in motion for that these gear boxes are more complicated, and we have to use what is known as synchromesh. This is a complicated system where a clutch action is needed such that gears which are going to engage are speeded up and get into the proper speed before they get into engagement. Whereas this machine tool gear boxes which are easier to design, easier to analyze and uses what is known as sliding clusters. We shall end this course by having a little discussion on one of this sliding cluster gear boxes which are used in machine tools, that is, from one input speed we want to get several output speeds by changing the gear engagement. I will show you through figures now such a sliding gear box which are used in machine tools.

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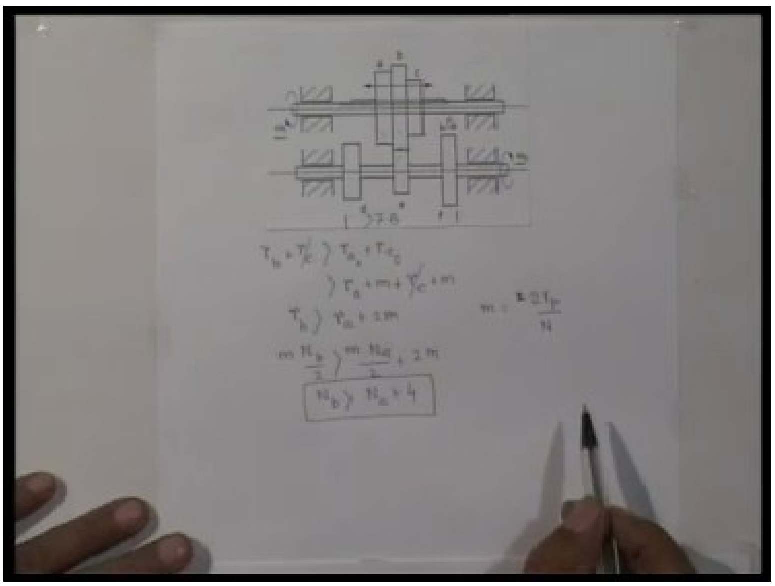

The above picture shows what we just now said is a sliding cluster gear box which are used in machine tool application. This is the say the input shaft and this is the output shaft. For the same speed of the input shaft (ω_i), we may like to get three different output speeds (ω_o). To do that, we have a compound gear, this is the same rigid body and this whole cluster can be moved along this shaft, this shaft is a spline shaft and on this spline shaft this whole cluster can be moved. Right now, this gear b is in engagement with this gear e, so it transmits a particular speed ratio. If we want a different speed ratio, we stop the entire machine then slide this cluster such that this gear c comes in engagement with this gear f and depending on the number of teeth, they will transmit a different output speed. Again, if we want to have another third different speed we will stop the machine and move this entire cluster such that this gear comes in engagement with this and depending on their number of teeth they will transmit a particular speed. So, this is a three-speed gear box using 6 gears and this is the sliding cluster, this is one rigid body but number of teeth is different on these three gear bodies a, b, and c.

The thing to note is that before this gear comes in engagement with this gear, the gear which was already in engagement must be totally clear. It must leave this engagement before its starts engagement. Similarly, when we move in this way before this gear comes in engagement with this gear, this engagement must be fully clear. Suppose the gear face widths of all these gears are same and we call it as B. If every gear the face width is B then this total distance as we see is B + B, because if it has to clear that movement before it comes into engagement is B. So B + B + B + B + B again this gap must be B and B, so this whole distance becomes at least 7B. It should be more than 7B. One more thing to note is that the difference in the number of teeth between these two gears must be at least 4. Otherwise, while moving in this direction this gear will foul with this gear.

As we see pitch circle radius of this gear b + pitch circle radius of this gear r_e that is the centre distance but when this gear is sliding over this, this must be greater than the outer radius sum of the outer radius of a and b. Otherwise, a will interfere with e, the outer radius of a + outer radius of e, the center distance should be more than that, such that this should not interfere with this when we are sliding it to the right. If we remember, the outer radius is pitch circle radius + the module, so outer radius is pitch circle radius + the module. So if we cancel pitch circle radius of this gear ‘e’, so pitch circle radius of b must be greater than pitch circle radius of a + 2m, and if we remember the module is nothing but pitch circle diameter by number of teeth. So r_b we can write, module into number of teeth on gear b divided by 2, $mN_b/2$ should be greater than $mN_a/2 + 2m$, where m is the module. If we cancel m on both sides, we get N_b should be greater than $N_a + 4$. So, for designing such sliding cluster gear boxes one has to ensure that the minimum number of teeth between these two adjacent gears on a cluster is at least 4 or it should be more than 4 and this width should be sufficient such that total disengagement takes place be from this pair before the other pair this way or that way comes into engagement.

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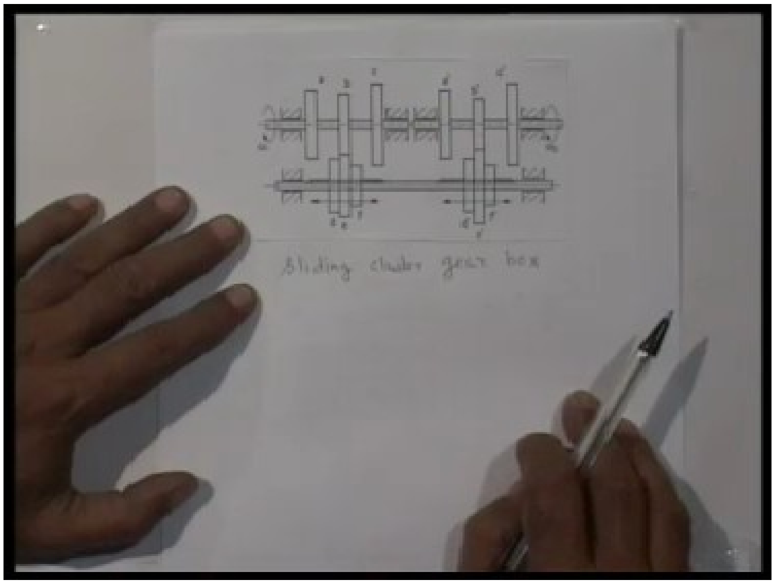

Using the same thing we can get a nine-speed gear box. If this is the input shaft and this is the output shaft, and this intermediate shaft we make it spline and here we keep two sliding clusters. So, depending on where the connection is taking place, we can get from the same input speed nine different speeds. For example, right now this and this is in the engagement and this and this is in engagement. So, depending on the number of teeth on these 4 gears we get a particular output speed. Now we slide it, this comes out of engagement and this becomes in engagement and here it is the same. So, depending on the number of teeth on these two gears we get a different speed. So that way we can get three here and three here and by combining them, three into three we can get nine different output speeds from the same input speed. This is what is called sliding clusters gear box which are used in machine tools. This is a nine-speed gear box, from the same input speed we can get nine different output speeds.

The thing to note is that because spline shaft is much costlier, so we have put both the sliding clusters on the same shaft, input shaft, intermediate shaft which is a spline shaft and this is the output shaft. Input and output shafts have fixed gears, whereas this intermediate shaft which is a spline shaft has sliding gears. So, this is a case of a nine-speed sliding cluster gear box, but as we said for when the gear change has to take place while the whole system is running, that is much more difficult to design and we need what we call synchromesh and that is beyond the scope of this course.