Kinematics of Machines

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Module-13 Lecture-2

So far, we have discussed gears with reference to a pair of gears, but very often the input and the output shaft are connected by more number of gears and such a combination of gears to connect the input to the output shaft is called a gear train.

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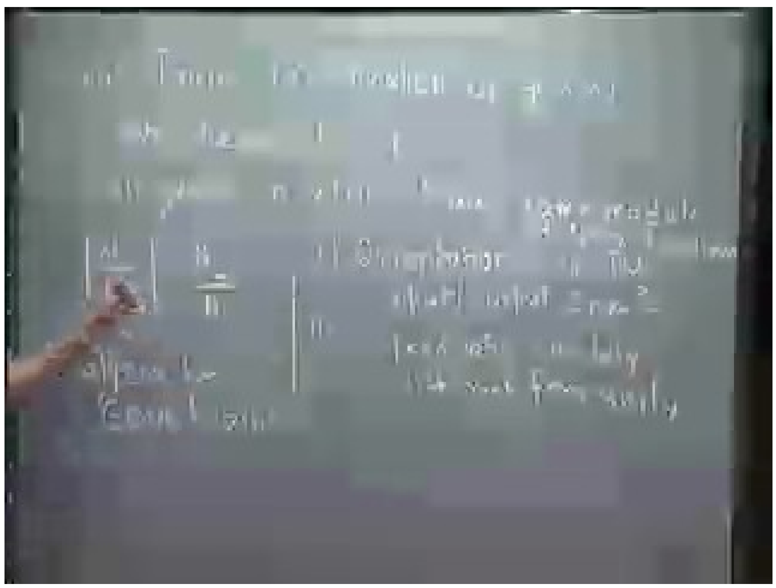

Gear train is nothing but a combination of gears which are used to connect the input shaft to the output shaft. The same gear train may consist of a few spur gears, few helical gears, bevel gears etc. So, a combination of gears which are used to connect the input to the output shaft on the gear train and a gear train may consist of all types of gears or maybe only one type of gear. Unless module is specified, we will always assume that all gears in a train have same module. There are exceptional gear trains, where all gears do not have the same module but in general all gears in a gear train will be assumed to have the same module. If two gears say 1 and 2 are in engagement, then their speed ratio will be given by the inverse ratio of the number of teeth. That is, $|\omega_1/\omega_2|$ =

N_2/N_1, where, ω_1 and ω_2 are angular speeds of gear 1 and gear 2 respectively and N_2 and $N_1$are the number of teeth on gear 2 and gear 1 respectively.

As we know, if this is external gearing then ω_1/ω_2 will have a negative sign and if it is internal gearing, then ω_1/ω_2 will have a positive sign, that we have to keep track of. Now the question is, why do we need more number of gears to connect a particular input shaft to the output shaft? There are various reasons for that. For example, the orientation of the input and output shaft that may dictate that we need to have more than two gears, orientation and spatial location. The input and output speed may vary by a wide margin, that is the ratio of the input and output speed is very much different from input speed. It is much larger than 1 or much less than 1. In such cases also, we may have to use a gear train. That means, the speed ratio or the input to the output speed, is widely different from 1. If the speed is say 60, then we cannot get by using only a pair of gear to connect the input and output shaft, to create a speed ratio 1/60 or 1/50. So under such situations, we need gear train.

In this lecture, we will do the analysis of gear train. That means, if a gear train is given and the numbers of teeth on the gears are specified, we should be able to find out, what is the speed ratio between the output and input shaft. Now I will show a figure of a particular gear train which consists of all kinds of gear.

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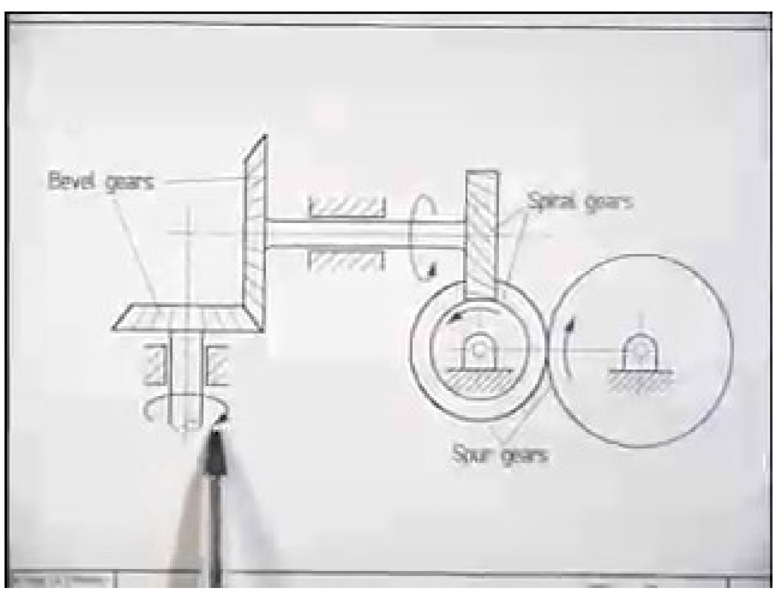

The above figure shows a typical gear train. This is the input shaft which is rotating in this direction and the output shaft is located here whose axis is perpendicular to the plane of the board or plane of the paper or plane of the pin. So, to connect the input and output shafts and to get a particular speed ratio, we may need a pair of bevel gear, then a pair of spiral gear or cross helical gear, which are connecting these two skewed shafts and a pair of spur gears to connect this shaft to this shaft. So, this pair of bevel gear connects these two shafts. This shaft and this shaft which are skewed are connected by a pair of cross helical gears and these two parallel shafts are connected by a pair of spur gear. So, this shows you an example of a typical gear train.

In this lecture, we shall analyze gear train and we start our discussion with classification of gear train.

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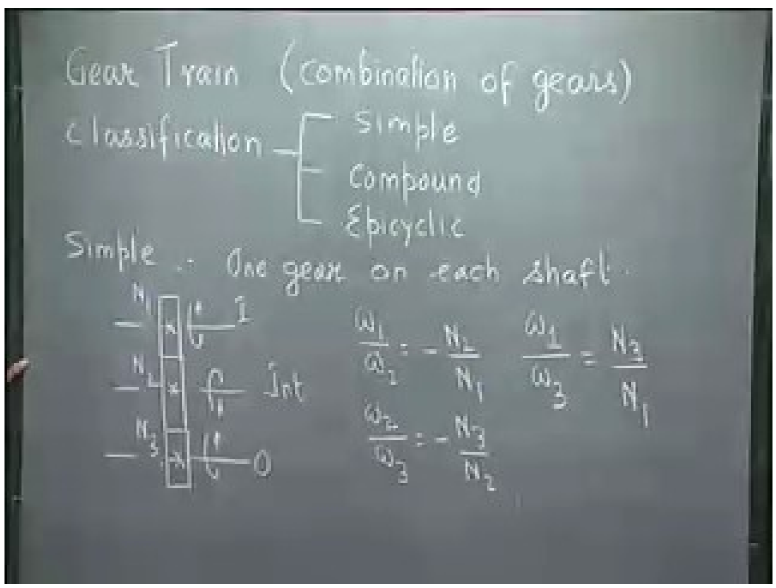

Gear train classification: Gear trains are normally classified under three headings – namely, simple, compound and epicyclic. These are the three normal forms of gear trains. Simple gear train is that gear train where on each shaft there is only one gear. There is input shaft, there is output shaft and there maybe several or one intermediate shaft connecting the input to the output shaft, but on each shaft, there is only one gear. So simple gear train has one gear on each shaft.

For example, this is one gear mounted on this shaft which is the input shaft. This is connected to another gear, which is on another shaft and this maybe the output shaft. This is called the intermediate shaft. Input shaft, output shaft and there is an intermediate shaft, and there is one gear on each shaft and the motion from this input is transmitted to the output. Now, analysis of such a simple gear train is almost trivial. As we see here, these are all external gearing and the number of teeth is say N_1 on this gear, N_2 on this gear and N_3 on this gear.

So, $\omega_1/\omega_2 = -N_2/N_1$, because this is external gearing and this is inversely proportional to the number of teeth. Similarly, angular speed of the intermediate shaft to the output shaft, which will be, $\omega_2/\omega_3 = -N_3/N_2$. From here to there, again there is an external gearing so the direction of rotation changes and in the inverse ratio of the number of teeth. If we multiply these two, we get, ω_1/ω_3 where ω_1 which is the input speed, ω_3 which is the output speed. If we multiply these two, ω_2 cancels and N_2 cancels and so, $\omega_1/\omega_3 = N_3/N_1$. So we see that the direction of rotation of the input shaft is same as the direction of rotation of the output shaft and the intermediate shaft rotates in the other direction. The input speed/output speed is N_3/N_1, so we can get the reverse also if we want output speed/input speed.

The thing to note is that the number of teeth on this intermediate gear is of no relevant. The speed ratio is dictated only by the number of teeth on the gear mounted on input shafts and the number of teeth on the gear mounted on the output shaft. There may be several intermediate shafts, but the number of teeth on those gears will be immaterial, so far the speed ratio is concerned. The number of intermediate shafts will change only the direction from plus to minus. Here, for example, there is only one intermediate shaft, ω_1/ω_3 is positive and if we had another intermediate shaft then, ω_1/ω_4 would have been N_4/N_1 but we have to take negative sign.

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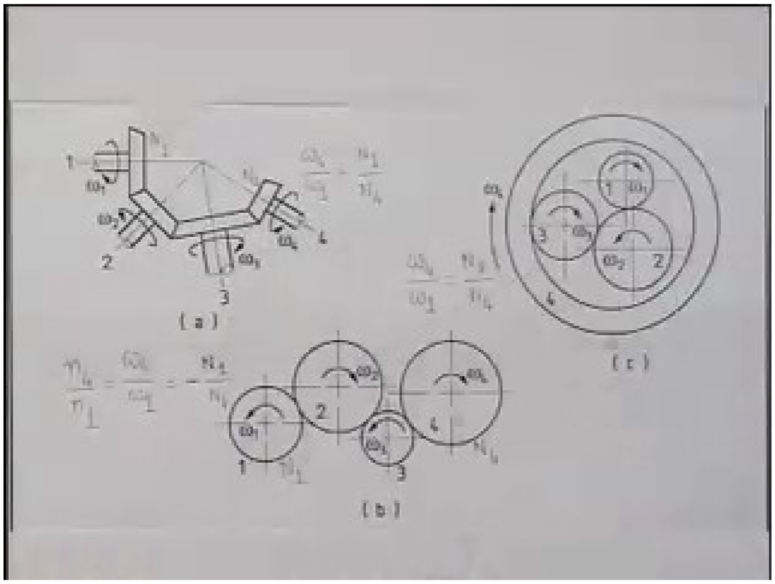

In the above diagram, we show three different simple gear trains. We see there are four shafts in the figure, 1, 2, 3, and 4. Let this be the input shaft and this be the output shaft. These are all connected by bevel gear. Now, the speed ratio as we calculated just in the example we discussed before, the shafts are parallel. So, it was very easy to keep track of plus or minus. At every step, there is a change of sign but here as we see the shaft axis are all intersecting and not parallel, so plus, minus will be continuing. So, if this is the direction of rotation of the shaft 1, then this is the direction of rotation of shaft 2 and this direction of rotation creates this direction of rotation for shaft 3 and that creates this direction of rotation for shaft 4. So, if we want to write the speed ratio, ω_4/ω_1 will be nothing but N_1/N_4, where N_1 is the number of teeth on the gear 4 and N_4 is the number of teeth on the output gear. The intermediate number of teeth are totally unimportant and the direction of rotation, which is as we have shown in this drawing.

Let me now take the example of this train, where again it is a simple gear train. There are four shafts and, on each shaft, only one gear is mounted from input to the output. As we see, the counter-clockwise rotation rotates this gear clockwise, which makes this gear to rotate counter-clockwise, which makes this gear to rotate clockwise. Here of course, the output speed ω_4/ω_1 because there are two intermediate shafts, we get a negative sign N_1/N_4. With one intermediate shaft, they rotate in the same direction, but as soon as there are two intermediate shafts, we get in the opposite direction that is the negative sign, where N_1 is the number of teeth on input gear, N_4

is the number of teeth on output gear and number of teeth on the intermediate gear does not match. Very often this angular speed, we will write in terms of rpm and we will denote by small n. The small n refers to rpm and capital N will refer to the number of teeth on various gears.

Let us now come to the example of this simple gear train. Here again, 1, 2, 3 and 4, this is an internal gear, whose axis is somewhere here, at the centre of the gear 4. The thing to note is that this is a pair of external gearing, this is a pair of external gearing but at 3 and 4 it is a pair of internal gearing. So, this ω_1 chases this ω_2, that chase this ω_3 and because of internal gearing ω_3 and ω_4 are in the same direction. So here again, we can write $\omega_4/\omega_1 = N_1/N_4$.

Though there are two intermediate shafts, but this is plus sign, because there is a pair of internal gearing as oppose to this situation, where there are all external gears. Here 1 and 2 are external gearing, but between 3 and 4 it is internal gearing. So, there is no negative sign here. So, analysis of such simple train as I said is pretty trivial.

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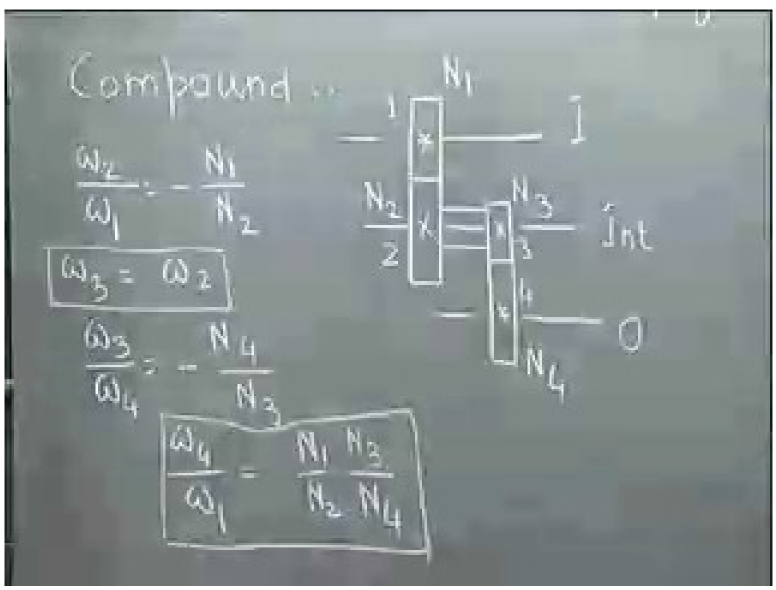

Now, let me discuss how to analyze compound gear train? Let me now discuss what we mean by a compound gear train. In a compound gear train, there will be more than one gear on some shaft, either on the input shaft or output shaft or intermediate shaft, there must be one shaft in the gear train where there will be more than one gear on the shaft.

For example, suppose this is the input shaft, this is the output shaft and this is one intermediate shaft. As we see on this intermediate shaft, there are two gears. So, this is a compound gear train. Let me find out, what is the speed ratio in such a compound gear train? Let the number of teeth be N_1, N_2, N_3, N_4, because the gear numbers 1, 2, 3, 4, because gear 2 and 3 are both connected to the intermediate shaft, their speed will be same, that is the speed of this shaft. So, $\omega_2/\omega_1 = -N_1/N_2$ because this is external gearing and $\omega_3 = \omega_2$, because they are mounted on the same shaft. Again $\omega_3/\omega_4 = -N_4/N_3$ because this is again a pair of external gearing.

So if we divide this by this, ω_2 being same as ω_3, that will cancels. So we get $\omega_4/\omega_1 = N_1/N_2$ ¿

N_3/N_4. So, this output speed to the input speed is $\frac{\omega_4}{\omega_1} = \frac{N_1}{N_2}\frac{N_3}{N_4}$.

The thing to note is that the speed ratio from the output to the input or input to the output is decided by the number of teeth on all the gear, intermediate number of teeth becomes relevant. For a simple gear train, it was depending only on N_1 and N_4, but for a compound gear train they are dependent on all the numbers of teeth on all the gears. Very often, such two gears are made out of one rigid body. The gear teeth are machined on a same rigid body, the gear blank teeth are cut here and teeth are cut there, such that this becomes one rigid body which again ensures $\omega_3 = \omega_2$. So, this is how we analyze a compound gear train.

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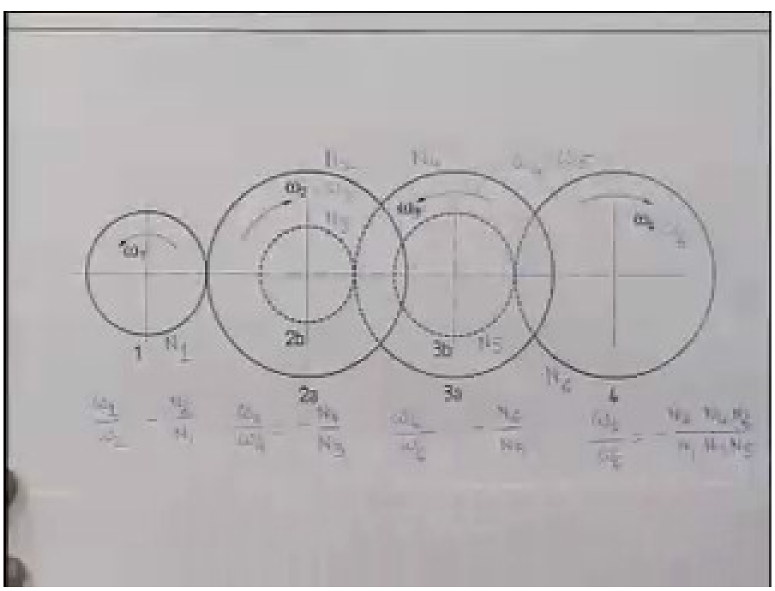

The above figure shows again a compound gear train. As we see, on these two there are two gears, one here and one there. This gear is connected to this gear and on the same shaft there is this gear which connects to this gear and on the same shaft there is another gear which is connected to this gear. From here to here, the speed rotation is being transmitted. These two are intermediate shafts. So, this is a compound gear train.

If we say, the number of teeth on this gear is N_1, number of teeth on this gear is N_2, number of teeth on this gear is N_3, the number of teeth on this gear, which is connected to 3 is N_4, this gear is N_5 and on this final gear is N_6. As we see ω_4 and ω_1 are in opposite direction that can be seen very clearly from here to there, then this rotates clockwise, so this rotates anticlockwise, so this rotates clockwise.

Now we can write, $\omega_1/\omega_2 = -N_2/N_1$. Since ω_3 is same as ω_2, we can write $\omega_2/\omega_4 = -N_4/N_3$. ω_4 is same as ω_5, that is the speed of this gear, so we can write ω_4/ω_6. So, $\omega_4/\omega_6 = -N_6/N_5$.

It is actually $\omega_5/\omega_6 = -N_6/N_5$ and ω_5 is same as ω_4. So I write $\omega_4/\omega_6 = -N_6/N_5$. Now if we multiply all of these, as we see ω_2 and ω_4 cancels, we get the input speed by the output speed $\omega_1/\omega_6 = -N_2N_4N_6/N_1N_3N_5$. So, negative sign implies they are in opposite direction and they are governed by the number of teeth on all of these gears in this compound gear train.

Next, we shall discuss the most difficult one where such visualization of rotation of various gears will not be possible and that is what we call epicyclic gear train. In the simple and compound gear trains that we have discussed so far, we say that all the gear axis of the shaft on which the gears are mounted were fixed in space. That means, the gear axis of all the gears were fixed in space, they are not moving in space, both in simple and compound gear train.

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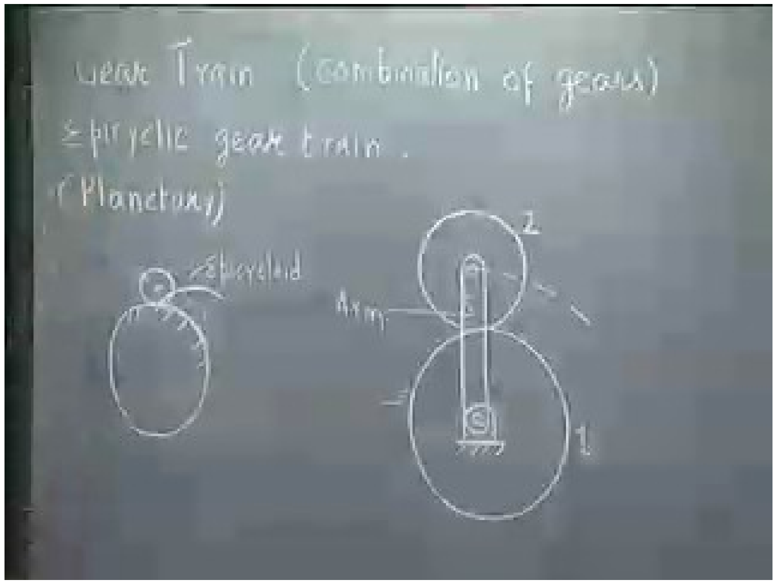

Whereas in epicyclic gear train, we will see that there will be one gear whose axis is not fixed in space, because the gear is mounted on an arm and that arm is rotating and carrying the shaft of that gear along with it. In such a gear train, if there is one such gear, we call it an epicyclic gear train. Some books call it as planetary gear train. Suppose we have one gear here and another gear there. This gear, let me call it 1 and this is gear 2, but gear 2 is carried by the arm which is also hinged here. That the axis of the gear 1 and this arm goes at hinged to the fixed link at this shaft.

When this gear train rotates as we see, the centre of the gear 2 moves on this circle, with this point as the centre. Consequently, the axis of this gear 2 does not remain fixed in space and this constitutes what we call an epicyclic gear train or planetary gear train. In fact, this we call sun gear around which the planet gear is moving, thats why the name planetary. This is called epicyclic because we know, if a circle rolls on another circle, then any point on the rolling circle generates a curve which is called epicycloid.

The curve that any point on this rolling wheel generates as if rolls around this circle is called epicycloid and from there, this name came epicyclic gear train, as a point on this gear will generate epicyclic gear. Let’s not bother so much about the name epicyclic or planetary. What we should note that this is a 4 link mechanism.

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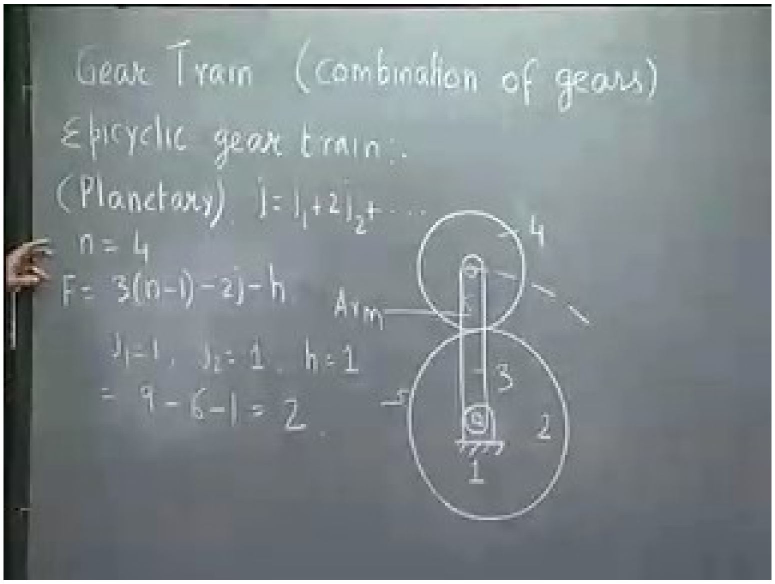

As usual we give them fixed link 1. This gear let me call 2, this arm is 3 and this gear 4. So, this is a 4 linked mechanism, n is 4. First we calculate, the degrees of freedom of this mechanism. Degree of freedom F can be obtained from 3 (n – 1) – 2j – h, where j is the equivalent number of simple hinges that is, $j_1 + 2j_2 + \ldots$ like that and h is the number of higher pair and n is the total number of links.

Here as we see, there is a revolute pair between the arm and gear 4 here. So j_1 is 1. Here is a revolute pair but that connects three links namely, 1, 2 and 3. So this is a hinge of size j_2. So j_2 is also 1 and between 2 and 4, we have a gear tooth connection which is a higher pair. So, h is also 1 So. if we substitute n = 4, j = 1 + 2 ¿ 1 = 3 and h = 1, we get, F = 3(4 – 1) – 2 ¿ 3 – 1 = 9 – 6 – 1 = 2. So, F = 2. So, the degree of freedom of an epicyclic gear train is 2. Further it implies that, we need two independent input speeds to fix the output speed because it has a degree of freedom 2.

In other words, if we give only one input speed then we can get two output speeds, which will be correlated but they will be non-unique. We can adjust both of them. They have to follow a relationship but they can be otherwise non-unique. So, we can generate two different speeds from the single input speed and that is where we use such epicyclic gear train.

Now, how do we analyze such an epicyclic gear train? It will be very difficult to visualize the direction of rotation of various gears because this gear is not fixed in space. So, let me try to develop a rather mechanical method, an analytical method to follow without trying to visualize which way which gear is rotating as it was possible in case of simple or compound gear train. So epicyclic gear train, we analyze rather mechanically, for that first of all let us derive what is the basic principle of analysis of an epicyclic gear train.

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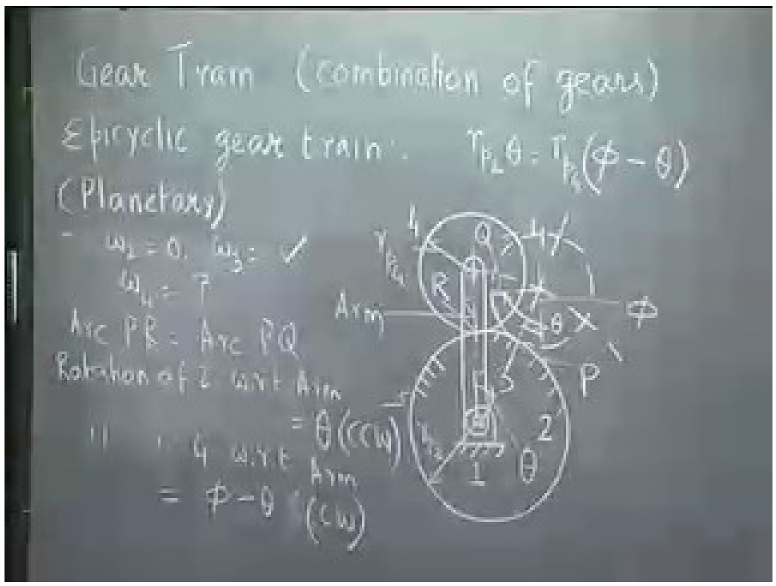

Towards that, we want to study the relative motion between 2, 3 and 4 and as I said, it has two degrees of freedom, so only two input speeds are necessary to fix the output speed. We say we give input to gear 2 and gear 3 and find what is the speed of gear 4? That means, we prescribe the angular velocity of body 2 and body 3 and we want to find that of body 4. So, we can study this relative motion.

To study this relative motion without any loss of generality, say one body to be fixed and let me say this body 2 is fixed, and that means ω_2 will be zero, that is one input and then if we know ω_3, the other input speed, the question is what is ω_4? To do that, we give the arm a rotation θ, link 3 has rotated by an angle θ. The centre of this gear has come here and this gear is taken as this position. This is the vertical line.

Now, this vertical radius after the gear has rotated says from here has become this radius. So, let me say this present contact point we call P and this point we call Q and this original contact point we call R. θ is the rotation of the arm and what is the rotation of gear 4? So, the total rotation of this is how much. φ is the rotation of body 4, θ is the rotation of body 3 and body 2 is held fixed because these are pitch circles, that means one rolls over the other without slip, that means these two arc lengths must be same. This arc length on the smaller gear or planet gear is same as this arc length on the sun gear or larger gear, because they are rolling on each other without slipping because this represents the pitch circles of the two gears.

So arc PR is same as arc PQ, which means, if we say pitch circle radius of this gear 2 is r_{p2} and pitch circle radius of this gear 4 is r_{p4}. If the arm makes an angle θ with the vertical line, then this angle is φ – θ. φ is the total rotation of gear 4 and θ is the rotation of gear 3 and this angle φ – θ. So, this arc length PR is $r_{p2} * \theta = r_{p4} * (\varphi - \theta)$.

Now we shall see that the relative rotation of the gears with respect to the arm still follows the gearing law that is, speed of gear is inversely proportional to the number of teeth with an appropriate sign. Sun gear was fixed, arm gear rotated θ in the clockwise direction. So, rotation of gear 2 with respect to arm is θ, but in the counter-clockwise direction. So, rotation of gear 2 with respect to arm is the opposite of that θ in the counter-clockwise direction. So what is the rotation of gear 4 with respect to arm? Gear 4 has rotated by an angle φ in the clockwise direction, arm has rotated by an angle θ in the clockwise direction, so rotation of 4 with respect to arm is φ – θ in the clockwise direction.

Let me see again. Arm has rotated by an angle θ in the clockwise direction, gear 2 has not rotated. So rotation of gear 2 with respect to arm is θ but in the counter-clockwise direction. Rotation of gear 4 is φ, rotation of arm is θ, so rotation of gear 4 with respect to arm is φ – θ in the clockwise direction and because of these two arc lengths being equal, this is the relationship between φ and θ.

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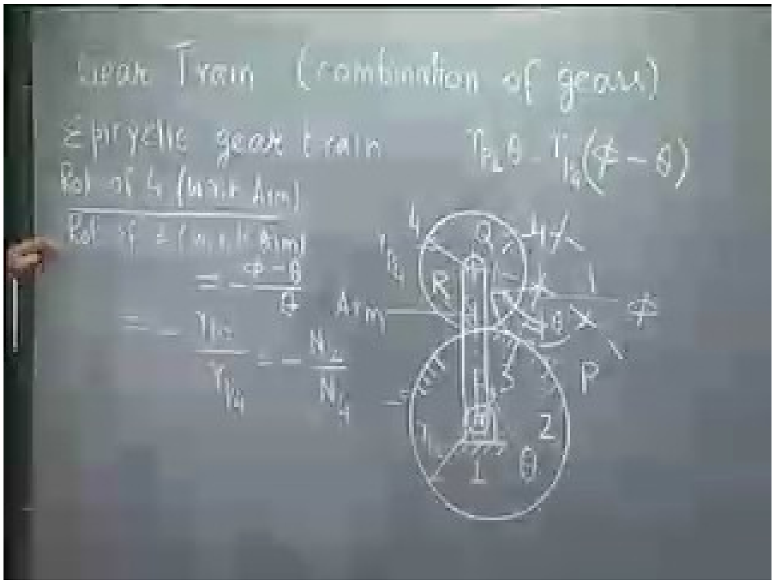

So if we divide this by this, we consider rotation of gear 4 but with respect to arm and rotation of gear 2 again with respect to arm. If I talk of this relative rotation and take the ratio, we are getting φ – θ that is in the clockwise direction, divided by θ that is in the counter-clockwise direction. So I put a negative sign.

So, in this epicyclic gear train, the relative rotation of two gears with respect to the arm is $\frac{rotation\,of\;gear\,4\,w.r.t\,arm}{rotation\,of\;gear\,2\,w.r.t\,arm}=\frac{-\varphi-\theta}{\theta}$, when this is clockwise, this is counter-clockwise, so I put a negative sign. So, from $r_{p2}*\theta=r_{p4}*(\varphi-\theta)$, where we got by having these two arc lengths same, we can write as $\frac{rotation\,of\;gear\,4\,w.r.t\,arm}{rotation\,of\;gear\,2\,w.r.t\,arm}=\frac{-\varphi-\theta}{\theta}=\frac{-r_{p2}}{r_{p4}}$, and this is equal to $\frac{-N_2}{N_4}$ (number of tooth on gear 2 divided by number of tooth on gear 4), because pitch circle radius are proportional to the number of tooth since they have the same module.

In case of an epicyclic gear train, because the arm is moving the absolute rotation of gear 4 and gear 2 are not governed by their number of teeth on them. Otherwise, this is exactly the same train value, only if we consider relative motion of 4 and 2 with respect to arm, not the absolute motion but the relative motion with respect to arm. That still follows the train value which has a minus sign because this is the external gearing and inverse ratio of number of teeth, rotation of 4/rotation of 2 is – N_2/N_4. This forms the basis of analyzing epicyclic gear train in a mechanical way. We have to only remember that it is the relative rotation with respect to arm follows the same value given by the number of teeth.

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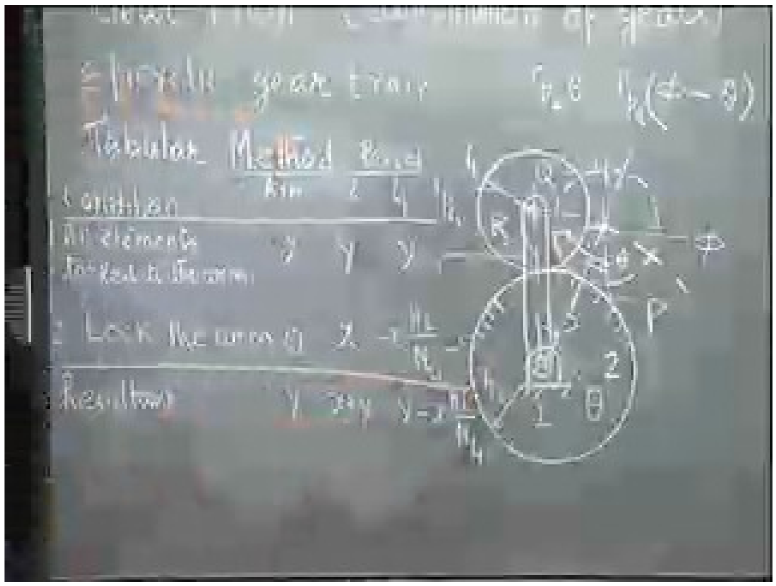

So, we put in a mechanical tabular method. First, we lock all the elements of a planetary gear train to the arm, all the elements are locked to the arm that means, the whole thing becomes one rigid body and then give some y revolution to arm. Then gears 2 and 4, because they are locked to the arm under the condition, if we give arm y revolutions, we will get y revolution to all the gears. The second condition is, lock the arm. They don't allow the arm to move, that means arm is not given any revolution and give any x revolution to any of the gear, say we give x revolution to gear 2. Then because the arm is locked it is not an epicyclic gear train anymore. It is just a simple gear train. So, rotation of gear 4, we can easily get as – x ¿ N_2/N_4. minus because it is an external gearing and inverse ratio of the number of teeth. Then add these two rows and we get resultant motion. So here it is y, here it is x + y, here it is y – (x ¿ N_2/N_4).

As I said, it is a two-degree freedom mechanism, so all the output speeds can be determined only if two definite inputs are given, that means two of these speeds must be known then only we can determine the third. So, we get two equations by equating the given speeds and we can determine x and y and once we know x and y, we know the speeds of all the elements, because they are all in terms of x and y. Of course, the number of teeth is given. This is the method that we shall follow and I shall solve few examples to show how to use this tabular method for analyzing epicyclic gear train.

So let me now conclude today's lecture, what we have learnt is simple, compound and epicyclic gear trains. To analyze such gear trains, we found that analysis of simple and compound gear trains are pretty trivial. Only thing we need to be a little careful about the direction of rotation if there is an external gearing or internal gearing and there are several numbers of intermediate shafts.

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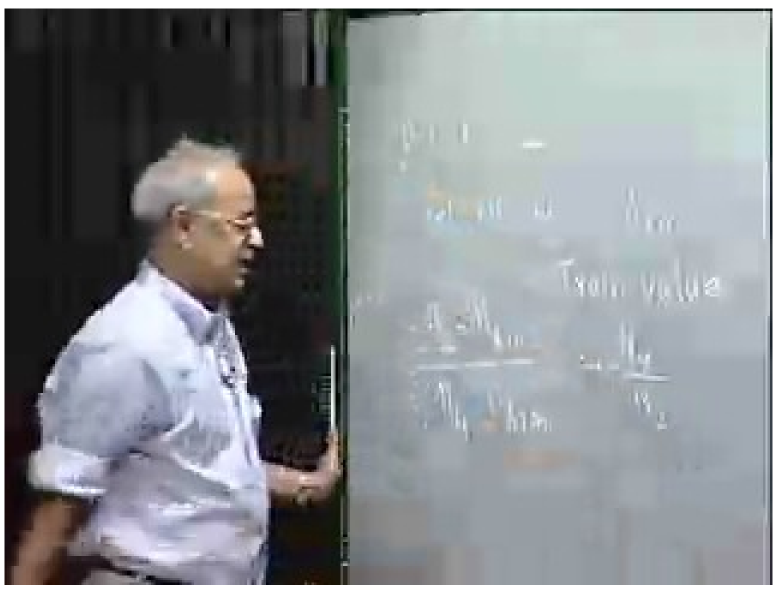

But when we come to epicyclic gear train, we have noted that epicyclic gear train has two degrees of freedom, which means, two input speeds need to be specified to determine all other output speeds. Conversely, if there is only one input speed then we can get two outputs which can be adjusted because there we have to follow one relation but we cannot determine exactly what will be the output speed and it has lot of applications as we will see later in our next lecture.

To analyze the epicyclic gear train the only thing to note is that it is the relative rotation of the gear with respect to arm is equal to what we call the train value. That is, as we get normally in a simple gear train or compound gear train using the number of teeth. So, it is the relative rotation with respect to arm is given by the same value not the absolute rotation ratio of two gears. So, if we write, speed of gear 2 minus speed of arm by $n_4 - n_{arm}$, where small n refers to the rpm, 2 was the sun gear, 4 was the planet gear and n_{arm} refers to the speed of the arm, then for external gearing was given by N_4/N_2, i.e., $\frac{n_2 - n_{arm}}{n_4 - n_{arm}} = \frac{-N_4}{N_2}$.

We shall use this for a tabular method which already explained, we talk of two conditions – First, we lock all the elements to the arm then give arm a particular revolution then lock the arm and give one of the gears some other revolution and calculate what are the revolutions of all the elements. Then adding these two, we get equations in terms of two unknowns and by two input speeds specified, we will be able to solve for those two unknowns and all other speed are expressed in terms of those unknown. So, we should be able to determine those speeds. This we shall see in our next lecture through examples.