Kinematics of Machines

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Module – 13 Lecture - 1

So far in this course, we have discussed straight tooth spur gears which are used to connect two parallel shafts.

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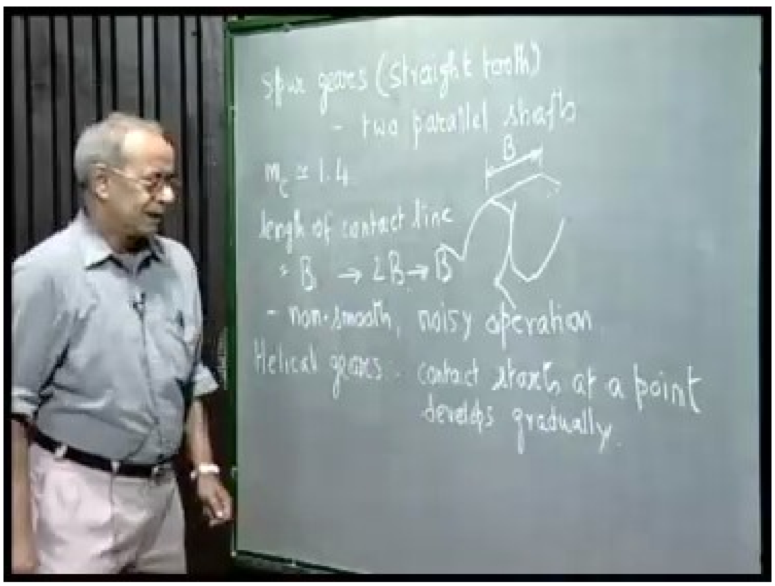

So we have discussed spur gears with straight tooth. These are used for connecting two parallel shafts and the tooth on the gear surface runs parallel to the axis of these shafts. There are other types of gears which are used to connect either parallel or intersecting or skewed shafts and in today's lecture, we will briefly discuss all these other types of gears. In case of spur gears, we discussed contact ratio and contact ratio is always more than 1 and the recommended desirable value is of the order of 1.4. What does it mean? That for some part of the cycle either one pair of gear teeth are in contact and for rest of the cycle two pairs of gear teeth are in contact. So, on an average 1.4 pair of gear teeth in contact.

Now, if we have straight tooth spur gears: suppose these are the two teeth which are engaged at this point. Now because these teeth are running straight parallel to the axis of the shaft, the contact takes place at the same instant along the entire face width. If the width of the gear face is face width, if we denote it by B, then the length of contact line between a pair of gear teeth is B if one pair of gear tooth in contact and when two pair are in contact, this B changes to 2B. Then again when one pair of gear teeth comes in contact, because contact ratio is not 2, it is 1.4 for some part of the cycle, it will again come back to one pair of teeth in contact, so the contact line length will change from 2B to B. So this way, the length of the contact line suddenly changes from B to 2B and again suddenly from 2B to B because the contact along the entire length of B takes place at the same instant. This results in non-smooth, noisy operation. To get rid of this noisy operation, we can go for little costly gears, which are known as helical gears. These are used to connect two parallel shafts like spur gears, but the teeth of the gears are not straight along the outer surface of the gear. They don't run straight parallel to the axis of the shaft, rather they are in the form of a helix and in that case, we call it a helical gear.

Now I will show a diagram of a pair of helical gears and also show in this case, that contact between a pair of teeth starts at a point between two helix and then develops gradually. The contact does not take place simultaneously, i.e., at the same instant over the entire width of the gear. Rather it starts at a point and then gradually develops. As a result, the length of the contact length does not change suddenly and it gives rise to smoother, less noisy operation. These are called helical gears.

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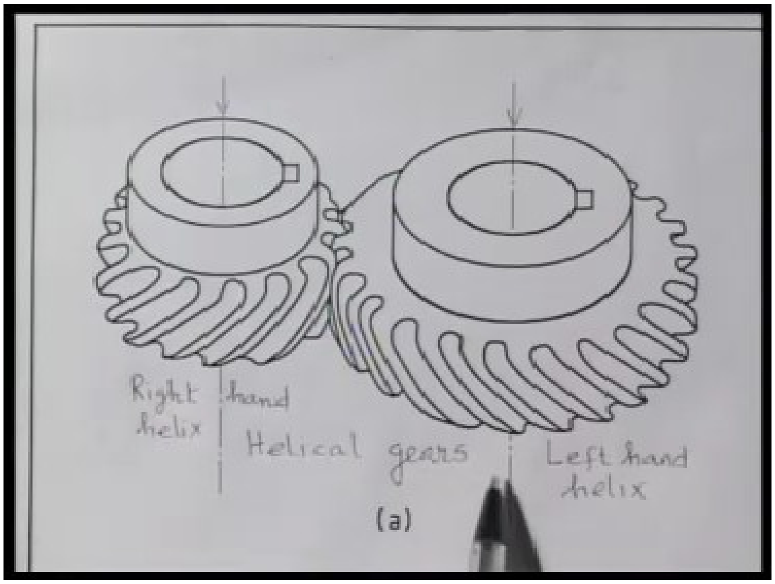

The above diagram shows a pair of helical gears. As we see, this is one shaft, this is another shaft, which are both vertical, that means both are parallel shafts. On this shaft, this helical gear is mounted and, on this shaft, this helical gear is mounted. Here at this contact point, as I said the contact between the pair of teeth, between these two helical gears starts at a point and gradually develops that gives rise to smoother operation. These are called helical gears.

The thing to note is that when two parallel shafts are connected by helical gears, their helix are of opposite direction. In this gear, the helix is in this direction and on the mating gear, the helix is in opposite direction. Accordingly, we call them either left-hand helix or right-hand helix. To decide whether it is left-hand or right-hand, we follow this convention. We look at this gear along the axis and we see the teeth is inclined, if we view it from this side, the teeth are inclined to the left. The tooth starts from here and gets inclined to the left. So, this is called left-hand helix.

Similarly, if we look at this gear along this axis of the gear, then the teeth are getting inclined to the right. Starting from here, all the teeth are getting inclined to the right. So, this will be called a right-hand helix. Two parallel shafts are connected by a pair of helical gears, one of which is left-hand and the other is right-hand.

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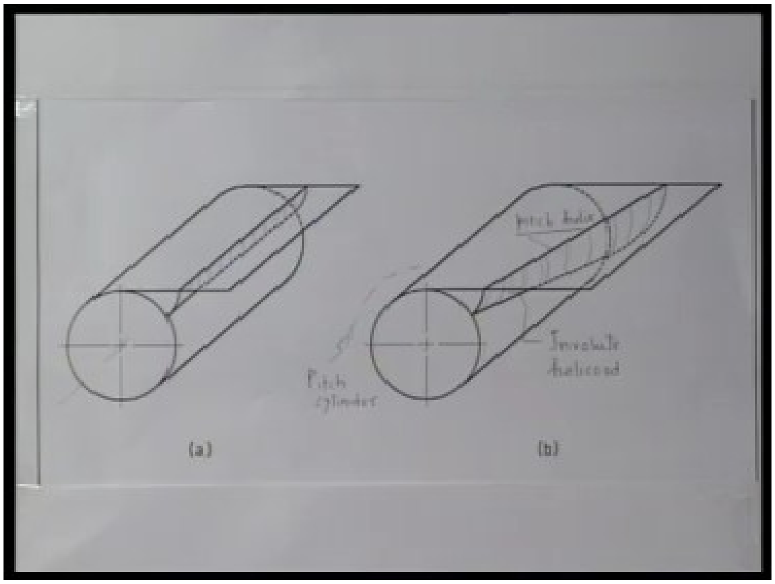

While discussing straight tooth spur gears, we said that one of the most common profiles of the gear tooth was an involute. This is the involute of a circle. Actually, this represents the base circle from which this string is unwinding to generate this involute profile. So if we consider the width of the gear, that is this base circle we can think of a base cylinder, then this strip of tape or paper which was wound on this base cylinder. If it is unwounded, then the surface that is generated by any line of this paper like which was here is generating this involute surface and the gear tooth of a straight tooth spur gear was this involute profile.

Let us see what happens in case of a helical gear. In the case of a helical gear, this is the generating line which was parallel to the axis of the base cylinder. If this is the axis of the base cylinder then this generating line which was generating the involute profile was parallel to the axis. Here, we take this line on the base cylinder which is inclined to the axis. This is the tape or strip which is being unwound from this base cylinder, but the generating line is not parallel. It is inclined on the base cylinder and as this tape unwinds, this particular straight line generates this surface and this surface is the helical tooth surface of helical gears. This surface is called involute helicoid.

So, we shall discuss a little bit of the geometry of this helical tooth surface. To discuss the geometry of the helical gears tooth, let us consider the pitch cylinder, this is the base cylinder, so a little larger than will be that imaginary pitch cylinder rather than the pitch circle. We can think of a cylinder of radius r_P which we will call pitch cylinder. The intersection of that pitch cylinder with the tooth surface is this line which we call pitch helix. Pitch helix is the intersection of the pitch cylinder with the tooth surface. If we think of a pitch cylinder, then the tooth surface if we draw, wherever it intersects that pitch cylinder that is called pitch helix.

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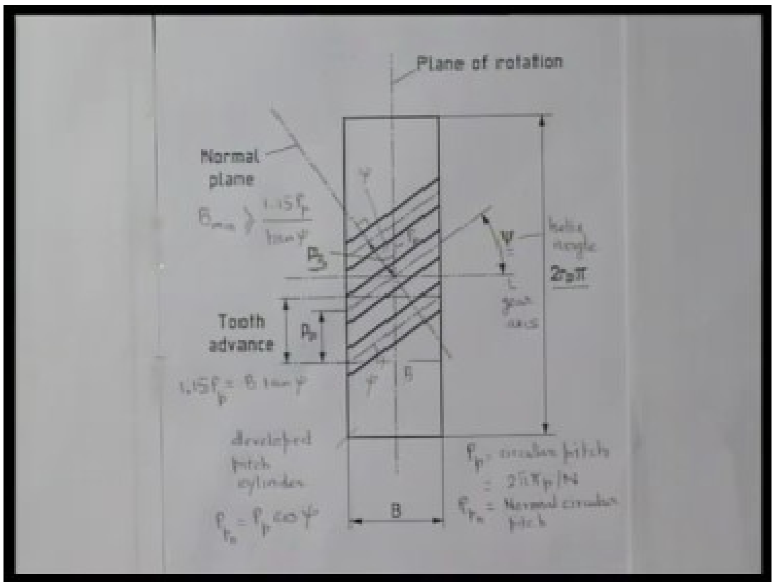

Let me now define the typical gear tooth geometry for a helical gear with reference to this figure. This rectangle of this side B represents the width of the gear and the length of side is $2r_P\pi$ that is if a pitch cylinder is developed, then it becomes a rectangle. This is the length of the cylinder and this is the circumference of the cylinder, the pitch cylinder that is why we write the radius as r_P. So, this distance is $2\pi r_P$. This is the developed pitch cylinder. B is the width of the gear and r_P is the radius of the pitch circle. This line represents the gear axis that is the axis of the shaft. These are the pitch helixes. These are the three consecutive teeth which have been shown when you wrap this rectangle on to a cylinder of radius r_P. This gets converted into helix. These lines become helix. The angle that this pitch helix makes with the gear axis is called helix angle and denoted by ψ. This is the axis of the gear and these are the pitch helix. So, the angle of the pitch helix makes with the gear axis, this ψ is called helix angle. This line represents the plane of rotation.

If we measure the distance between the identical points between two adjacent teeth, suppose I consider this point on this tooth and the identical point on the adjacent tooth that is here, the distance measured along the pitch circle is called circular pitch. This we have done earlier, even for spur gears. P_p is circular pitch which is $2\pi r_P/N$, where r_P is the pitch circle radius and N is the number of teeth. If I consider a plane which is normal to this pitch helix, then that I defin as normal plane. This angle is 90^0. Now if we measure the distance between identical points of two adjacent teeth like this point and that point along this normal plane, then that we call normal circular pitch. P_{pn} is nothing but normal circular pitch.

If we measure the distance along this pitch circle or plane of rotation between two identical points of two adjacent teeth, then this distance I call P_p as it is here and if I measure along the normal plane, then I call it P_{pn}. As it is clearly seen, if P_p is this distance and this angle is helix angle, ψ. So, these two are related by $P_{pn} = P_p \cos \psi$, where ψ is the helix angle. Next we define what is tooth advance?

As we see in this particular tooth, if the contact starts here, then the contact ends there. The distance that is covered from the start to the end along the pitch circle is called the tooth advance. The distance measured along the pitch circle from here to there is the tooth advance. We consider the same tooth from the start to the finish. The movement along the pitch circle is called tooth advance. To ensure that the contact starts with the next tooth before it ends with the previous tooth then this tooth advanced must be more than P_p because this tooth starts contact here and this tooth ends contact there and this end is coming later than this. That means it is ensured continuous rotation is transmitted. One pair of teeth come in contact before the previous pair of teeth loses its engagement. So tooth advance should be more than P_p. In fact, to ensure this tooth advance is more than P_p, we must have some minimum value of B. If this is B and this angle is ψ, then $\tan \psi$ is tooth advance divided by B. So this is B $\tan \psi$. Now tooth advance should be more than P_p. Normally, it is recommended the tooth advance should be 15% more than the circular pitch, that is tooth advance should be 1.15 into P_p which means minimum value of B should be greater than or equal to $1.15\, P_p/\tan \psi$, where ψ is the helix angle. So that finishes our discussion with the very rudimentary or fundamentals of the teeth geometry of helical gears.

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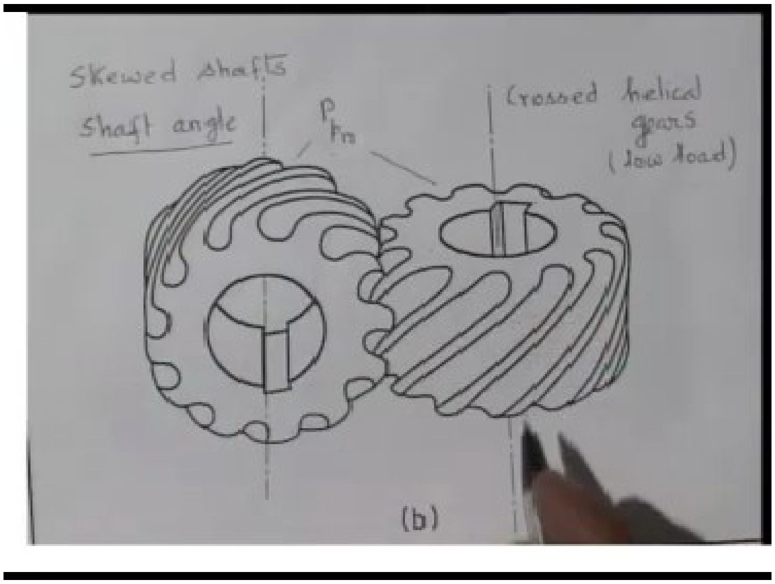

So far, we have discussed spur helical gears. That is two helical gears of opposite handedness like right-hand helix and left-hand helix to connect a pair of parallel shafts, but helical gears can also be used to connect two non-parallel, non-intersecting shafts. A pair of helical gears can be used to connect two skewed shafts. As shown in this diagram, this shaft axis is vertical, whereas this shaft axis is horizontal and they can be connected by a pair of helical gears and such gears are called crossed helical gears. In a pair of crossed helical gear, the contact between a pair of teeth is always at a point, so it cannot transmit too much of load. So they can be used only for very low load application. Since they are incapable of transmitting very high value of torque, they are used only for very low load application like in instrumentation. The thing to note is that these are nothing but helical gears, as soon as they are mounted on two skewed shafts, we call them crossed helical gears. These are nothing but ordinary helical gears and to act as a pair of crossed helical gears, the only condition necessary is that their normal pitch (P_{pn}) of these two gears must be same. They can be of same handedness both can be right-hand, both can be left-hand or one of them right-hand and other left-hand, like that. Handedness is not important of this helix. Only thing the normal pitch of these two helixes P_{pn} must be same. We define what is called shaft angle? The between these skewed shafts, how do I prescribe the angle? To define the angle between two skewed lines, we drew it along the common perpendicular. We drop a common perpendicular between these two skewed lines and view these two lines along that line. The next diagram will make it clear.

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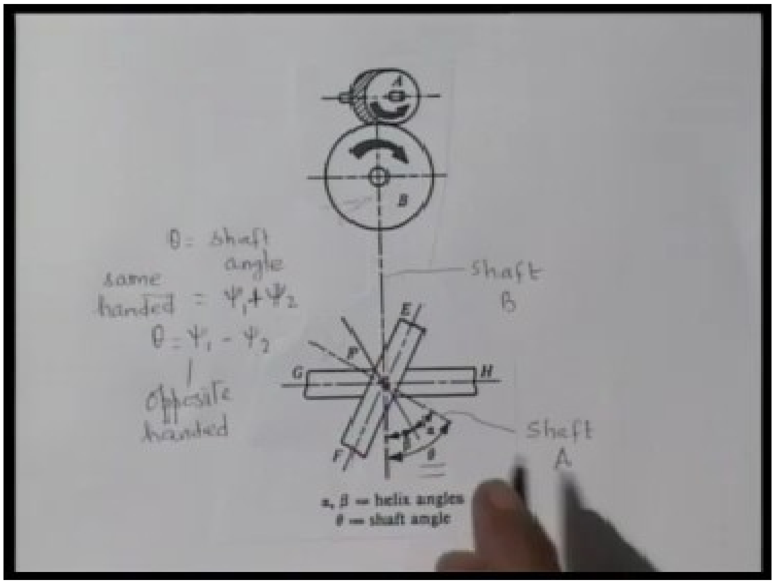

The above diagram shows a pair of crossed helical gears, gear A and gear B. This is the shaft of gear A and this is the shaft of gear B. So, if we view it along the common perpendicular between these lines and this line which is perpendicular to the plane of the paper, then this is shaft of gear B and this is gear A and this line is shaft A. The angle between these two shafts is measured by this angle θ. This vertical line represents the shaft of B which is perpendicular to the plane of the paper in the top view. This line represents shaft of B and this line represents shaft of A and the angle between them, we call shaft angle. This θ is called shaft angle.

If both these helices are of the same handed that is both are right-hand or both are left-hand then I call it right handed. If these two helical gears are of right handed helix, then $\theta = \psi_1 + \psi_2$, where ψ_1 is the helix angle of one gear and ψ_2 is the helix angle of the other gear. This is plus when they are of same handedness and if they are of opposite handedness then the shaft angle $\theta = \psi_1 - \psi_2$. This is for the addition of these two helix angles, if those two have the same handedness, both right-hand or both left-hand and the shaft angle is $\psi_1 - \psi_2$, if these two are of opposite hand, one of them is right-hand other is left-hand, you subtract the smaller from the bigger helix angle and you get the shaft angle.

The shaft angle is that if we rotate one of the shafts such that these two axis become parallel then what is the amount of rotation necessary to make these two shafts axis parallel and these two gears rotate in opposite direction? As shown here, this is rotating in this direction. If we rotate this and make it parallel to here, then that gear will rotate in opposite direction. The amount of rotation that is necessary is what we defined as the shaft angle θ.

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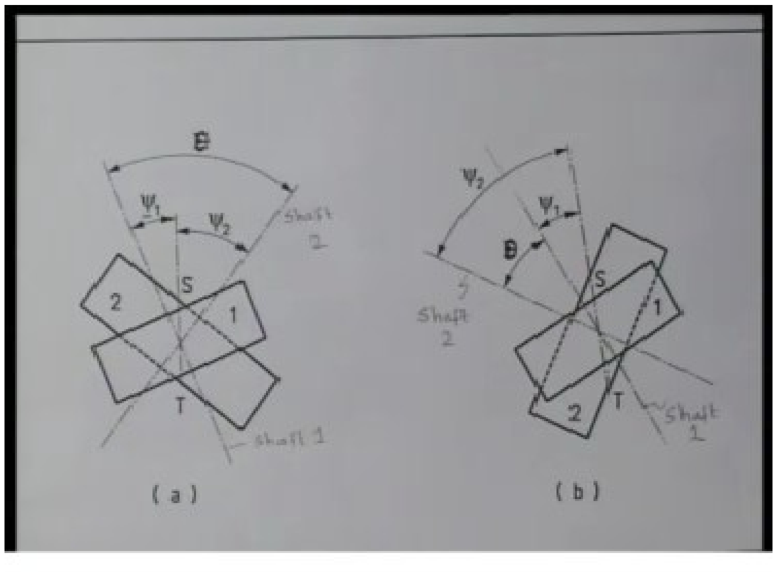

Just now we have explained the shaft angle for a pair of crossed helical gears which are used to connect two skewed shafts. Say this is gear 1 and this is the axis of gear 1. This is shaft 1. This represents gear 2 and this line represent shaft 2. This ST represents the tooth which is just now engaged between gear 1 and gear 2. So the angle between this ST and the shaft 1 is the helix angle ψ_1. Similarly, this is the axis or shaft 2 and this is the tooth. So the angle between them is the helix angle for gear 2 which is ψ_2 and they are of same handed. So the shaft angle θ is clearly seen to be $\psi_1 + \psi_2$. The diagram changes as follows if the two gears are of opposite handed helix. Here this is gear 1, this line is shaft 1, axis of gear 1 and this is gear 2 and this line is shaft 2. So the angle between shaft 1 and shaft 2 is this which is θ and here the gears are of opposite handed. So this is ψ_2 and this is ψ_1 where ST is the tooth. This line represents the tooth in contact. This is ψ_1 and angle between ST and this shaft 2 is ψ_2 which is the helix angle of gear 2 and the shaft angle θ as this clearly seen is $\psi_2 - \psi_1$.

So far we have seen, how to connect two parallel shafts either by straight tooth spur gear or by helical spur gears. Helical gears can also be used as crossed helical gears to connect two skewed shafts. Now we discuss another very common type of gears which are called bevel gears

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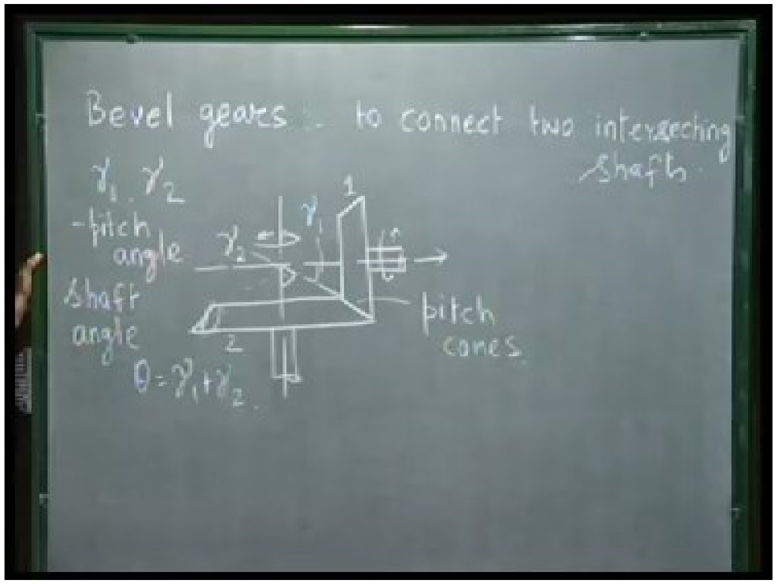

The bevel gears are used to connect two intersecting shafts. Suppose, this is one shaft axis and the other intersecting shaft axis is like this. Just as an example, I'm taking of two shafts intersecting at 90^0. Then they can be connected by a pair of bevel gears. If this gear is rotating in this direction, the teeth contact is here and that rotates this gear in that direction. If we see it from here, this is the direction, so that gear will rotate this gear in about the vertical axis, so it is like this. So these are called a pair of bevel gears.

The thing to note is that for spur gears, we have this pitch surface which are cylinders. Gears teeth were cut on the surface of a cylinder. Here, the gear teeth are cut on the surface of a truncated cone. So, these cones represent instead of pitch cylinder, we have pitch cones and we call this is gear 1, this is gear 2. The semi vertex angles of these cones are called pitch angles. Instead of pitch cylinder, we are having pitch cones and instead of pitch circle radius because there is no constant radius here, we are defining this pitch cones by their semi vertex angle. So γ_1 and γ_2, are called pitch angles or cone angles. The shaft angle that angle between these two shafts which are connected by these two gears are clearly seen to be $\gamma_1 + \gamma_2$.

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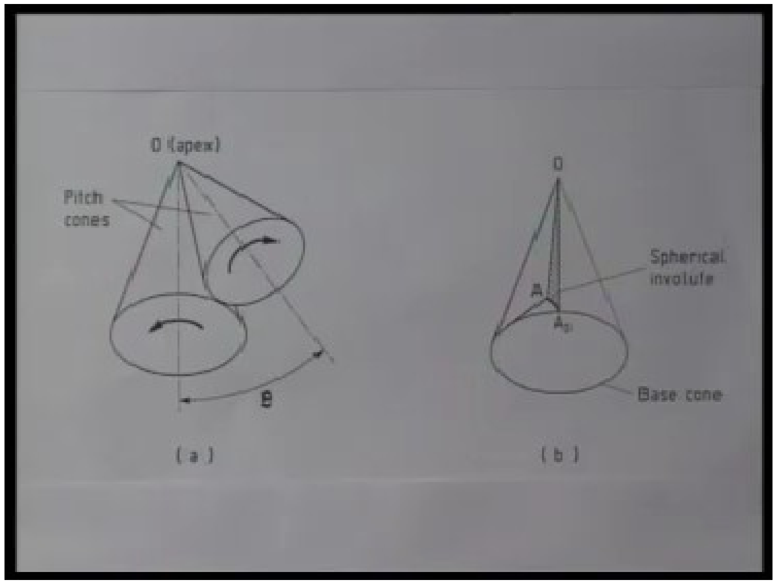

In case of spur gears, we have seen the equivalent motion that is transmitted by a pair of spur gears can be represented by rolling of the one cylinder over the other without friction. Those are called pitch cylinders or in case of two-dimensional representations, pitch circles. In case of bevel gears, those pitch cylinders are replaced by pitch cones. These are two pitch cones and if one rolls over the other without slip then this rotation of this cone will generate this rotation of the other cone, assuming these are friction cones and there is no slip along the line of contact. This is the equivalent motion that is transmitted by a pair of bevel gears.

So instead of pitch cylinders, in case of spur gears, we have pitch cones for the bevel gears. The angle between the axis of these two cones is called the shaft angle, θ and these shaft axes which are intersecting and meeting at the vertex of these two pitch cones. For spur gears, we have seen from pitch cylinders by unwinding a strip, we generated the tooth surface which was an involute.

In case of bevel gears, we have to unwind from this base cone rather than base cylinder or base circle. Now we unwind the strip which was wound on to this base cone. By unwinding, we are generating this surface as the strip unwinds and this surface will represent the tooth of a bevel gear and this surface is called spherical involute. Instead of involute of a circle as we got from the spur gears, here the radius of the base circle keeps on reducing as a result this surface changes from involute to a spherical involute.

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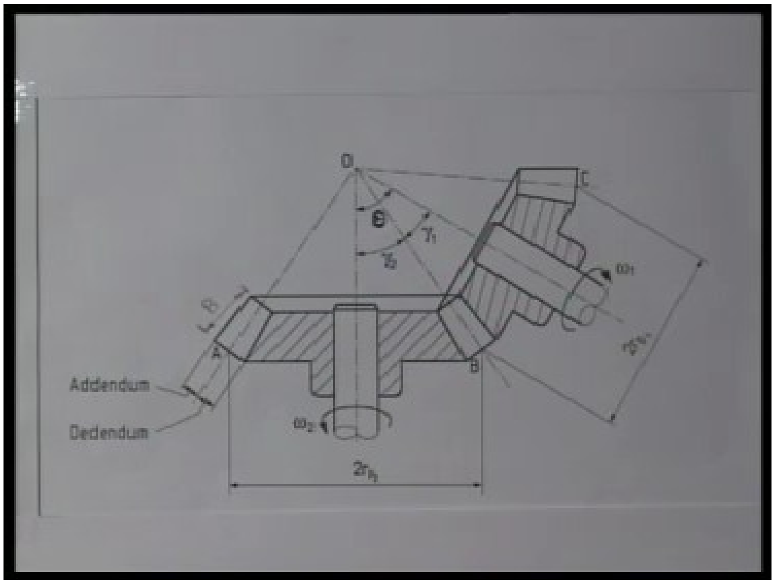

The above figure represents a pair of bevel gears along with its teeth. As we have said, this is one pitch cone. The diameter at the base of this is 2 r_{p1} . This is gear 1, which is rotating with angular speed ω_1 and consequently transferring the motion to this gear of diameter at the pitch cone is 2 r_{p2} and this rotates at an angular speed ω_2. It is r_{p1}*¿ ω_1 = r_{p2}*¿ ω_2 because at the pitch cone, there is no slip. The semi vertex angle of this pitch cones as we see is γ_1 and γ_2 and the shaft angle is $\gamma_1 + \gamma_2$ as I explained earlier. So for the tooth is concerned, we measure above and below the pitch cone level and this is the addendum of the tooth and this is the dedendum of the tooth. Everything is measured at the level of the pitch cone base. BC is the base of the pitch cone of gear 1. AB is the base of the pitch cone of gear 2.

In this diagram, we have just seen the tooth. Only one thing was left out that is this width of the tooth which we call the face width measured along this length at the addendum. We measure this length which we call the face width represented by B.

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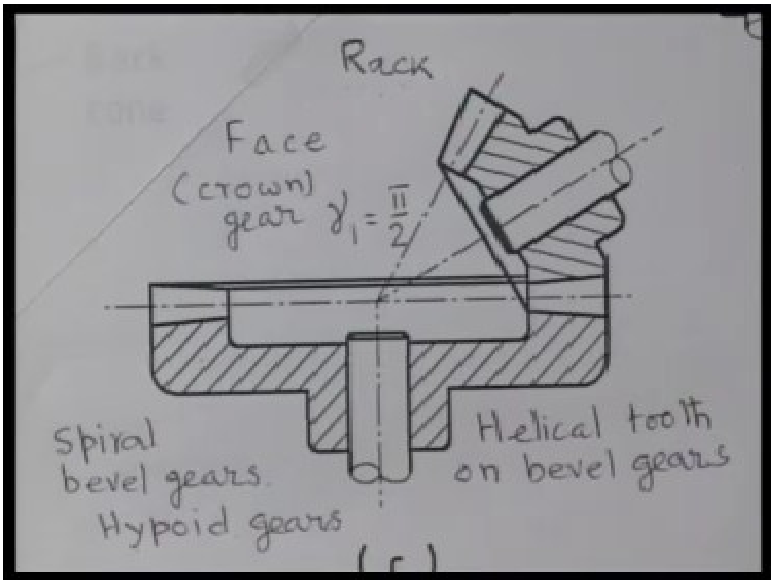

While we discuss spur gears, we talked of rack. Rack was a gear of apparently infinite pitch circular radius then it become a straight. A straight rack is nothing but a gear of pitch circle radius infinity. The analogous seen in case of a bevel gear, what we call a face gear or a crown gear. Face or crown gear which is analogous to rack, in case of spur gears. These are a pair of bevel gears but this we call face gear or crown gear. The thing to note is here the pitch circle radius was infinity and in case of crown gear, the pitch cone angle is 90^0. That means the teeth are cut on this circular track rather than on the surface of a cone, because this cone has a semi vertex angle $\pi/2$. So all these teeth of the crown gear or face gear lies on a circular track and this is analogous to rack in case of a spur gear.

The bevel gears which we have shown so far all have straight teeth, but we can also have bevel gears with helical teeth. We can have helical tooth on bevel gear. If we have helical tooth on bevel gears, then those are called spiral bevel gears. We are not going to discuss all the details of these spiral bevel gears, but there is one type of spiral way bevel gears which are very commonly used in automobile are called hypoid gears. These are used in automobile and we will show the diagram of a pair of hypoid gears in the next figure.

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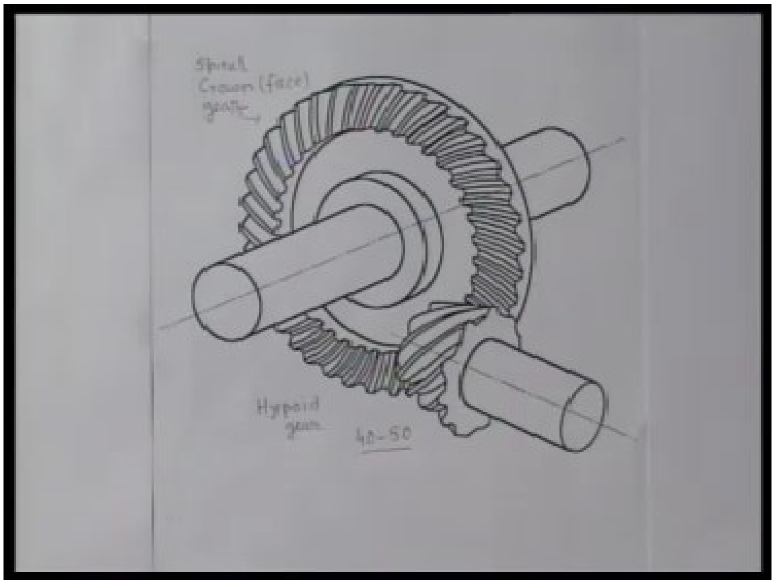

The above diagram shows what I called a hypoid gear. The thing to note is that they look like bevel gears and the gear teeth are in the form of helix rather than straight. So, this is a crown or face gear because the semi vertex angle of this cone is 90^0 and the gear teeth are lying on the circular track. So this is a crown gear but with spiral tooth. So we can call it a spiral crown or face gear. As I said, these are used in automobiles with this as the input gear and this as the output gear and this is called a hypoid gear. The thing to note is that the axis of this shaft and the axis of this shaft are not intersecting. Rather, they are having an offset. So, these hypoid gears are very similar to bevel gears but not exactly bevel gears because bevel gears are used to connect two intersecting shafts, but here there is an offset.

So, this is very similar to a pair of bevel gears and this gear is very similar to a crown gear or face gear but with spiral teeth and this type of gears are called hypoid gears, which are used in automobile. The advantage of hypoid gear is by a single pair of gears, a very large reduction is possible of the order of 40 to 50 speed reduction by this value. Ratio of the speeds by 40 to 50 can be achieved by this hypoid gear.

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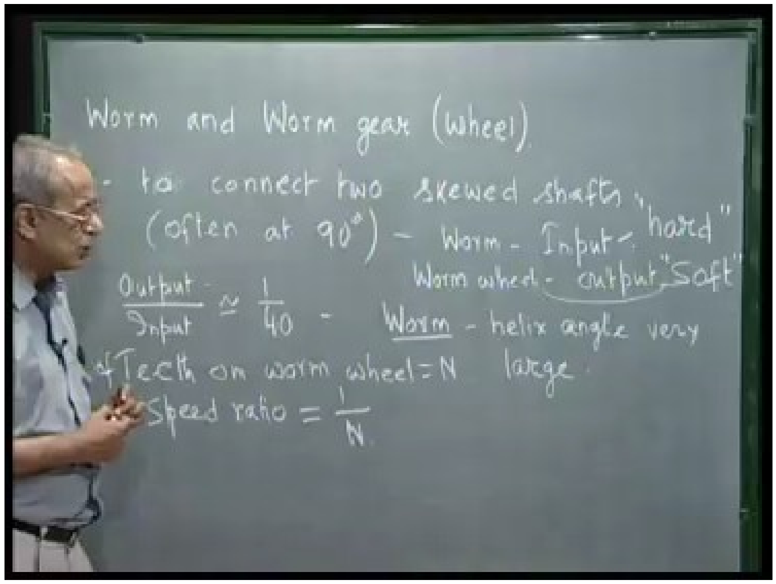

Today we have discussed different types of gears like helical gears, bevel gears, spiral bevel gears, hypoid gears and we finish this lecture by mentioning a very commonly used another type of special gear which are known as worm and worm wheel or worm gear.

These are normally used to connect two skewed shafts. They are vary often perpendicular to each other but non-intersecting, they are one above the other but the angle between them is 90^0. These are used for a very high-speed reduction. Worm is the input shaft and worm wheel or worm gear is connected to the output shaft and the speed ratio i.e., output speed by input speed in one stage can be as high as 1 by 40.

If we start from a helical gear, if the helix angle is so large such that one tooth makes a complete revolution on the pitch cylinder, then helical gear becomes a worm. Worm is a helical gear of very large helix angle; helix angle is so large that it makes a complete revolution on the pitch cylinder and then it gets converted into a worm and it looks like a screw. It is this screw drives the worm wheel. I will show the complete picture a little later. The things to discuss is that as we will see the ratio of the speed reduction can be very high at single stage by using a worm and worm gear and it is ⅓rd of the worm which comes in connection with a large number of teeth of the worm wheel. If the number of teeth on worm wheel is N then speed ratio output by input speed will be 1/N because it is the same side of the worm which comes in engagement with N number of teeth on the worm wheel. It is always necessary to make this worm of hard material like steel whereas the worm wheel is made of soft material like brass such that they wear out almost at equal rate and the motion transmission remains uniform. If the varying is rate is different on the worm and worm wheel then the engagement will be non-smooth. To maintain equal wear rate on the worm and worm wheel, worm wheel is made of softer material like brass, whereas worm is made of a harder material like steel. Now I will show you the diagram of a worm and worm wheel.

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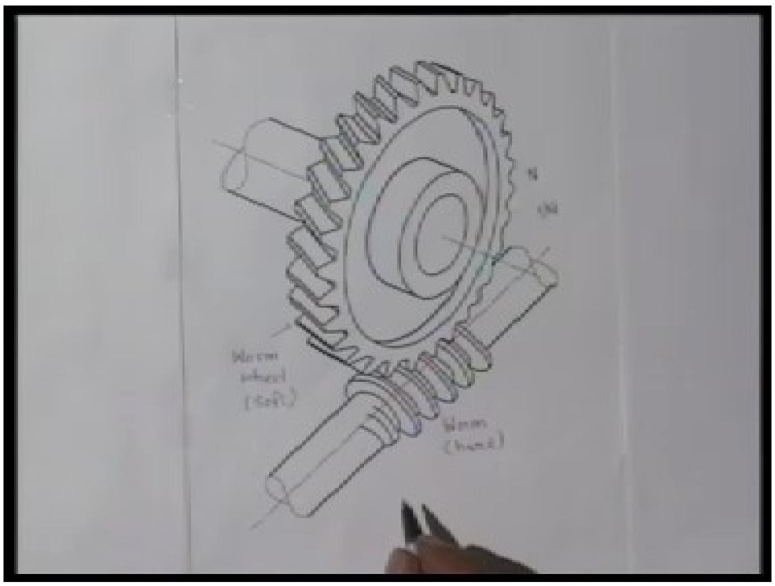

This is the diagram of a worm and worm wheel. This screw like figure is called worm which is always the input speed. The worm wheel can never drive the worm. It is the worm which drives the worm wheel and this is what we call worm gear or worm wheel. If the number of teeth on this worm wheel is N, then the speed ratio, output speed by input speed is $1/N$. So, if we have 40 teeth on the worm wheel, in single stage we can get a speed deduction of 40 and that is not very uncommon. This worm as we said is made of harder material of the two and worm wheel is made of softer material, because it is the same worm thread is coming into engagement with all the worm teeth. So, the wear rate is lower here and more here. To make uniform wear rate on both, this is made of harder material and this is made of softer material. As we see here the angles between these two shafts are 90^0, though they are skewed.

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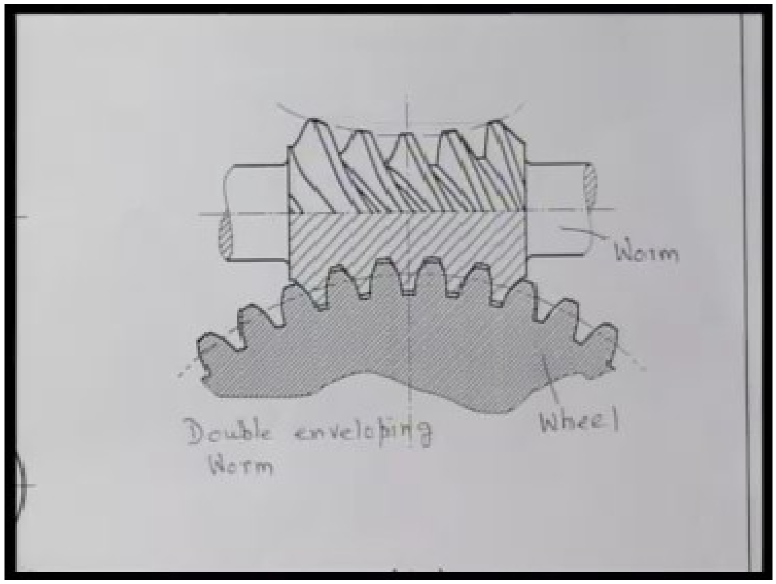

We conclude this lecture by show you this picture of a better design of a worm and worm wheel which is known as double enveloping. This is the worm and this is the worm wheel. The thing to note is that the threads of the worms are not straight, rather there is a curvature and this curvature is exactly same as the curvature of this worm wheel, such that there is much better contact between the thread of the worm and the teeth of the worm wheel. So, it is doubly curved and that is why it is called double enveloping worm. This is nothing but a worm and worm wheel with a better design. This is obviously costlier. Of course, we have a single enveloping worm where the worm threads are straight which means the curvature is not provided on the worm.

So, we conclude only the glimpse of different types of gears. We have discussed in detail the spur gears and in our next lecture, we will discuss the analysis of gear trains.