Kinematics of Machines

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Module – 12 Lecture - 2

Today, we begin our lecture with the advantages of involute profile. Because of these advantages, we find involute profiles are most commonly used in mass produced gears.

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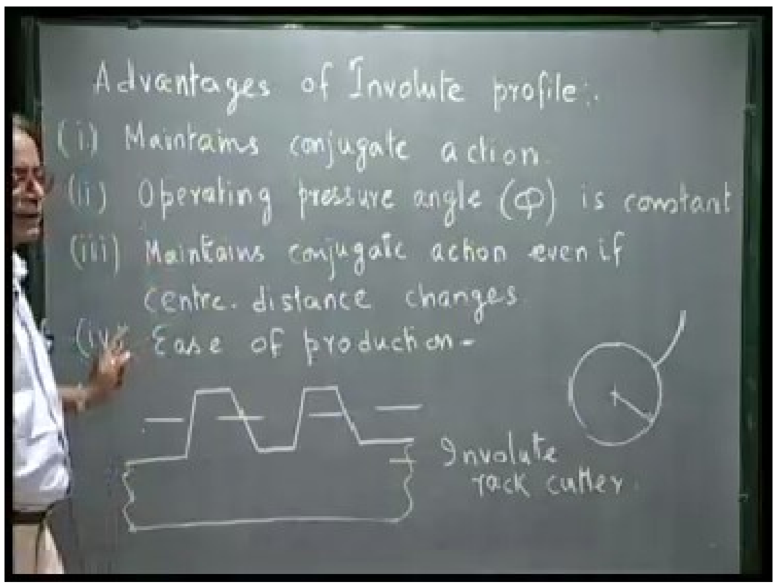

In our last lecture, we have already seen that a pair of involute profiles can maintain the conjugate action. That is, it satisfies the fundamental law of gearing, that is, it maintains constant angular velocity ratio between the two shafts.

Advantages of involute gear tooth profile:

1. It maintains conjugate action.

However, this is not so much of importance because if given any smooth curve as one profile, then one can find another profile which will be conjugate to the given profile. So, this is not the unique choice of involute profiles which maintains conjugate action. There are other advantages of involute profile over and above that it maintains the most fundamental requirement, that is maintaining conjugate action.

2. Operating pressure angle is constant.

In our last lecture, we denoted this operating pressure angle by the symbol φ which is the angle between the line of action and the common tangent to the pitch circles. We will get back to the same figure, which we discussed last time to show what is the advantage if the pressure angle remains constant? or what is the consequence of this pressure angle remaining constant?

3. A pair of involute gears maintains conjugate action even if the centre distance between the gears are changed.

That means, we have a pair of gears, we can mount them with a little different centre distance, even then the conjugate action will be maintained, even if centre distance between the pair of gears changes. These two points as I said, I will explain with reference to the figure that we discussed last time.

4. The most important reason why involute profiles are so common is, ease of production.

Why it is easy to produce involute gears? Because gear teeth are produced by a process called generation. That is, again we use the conjugate action but not between a pair of gears but between a rack and a pinion and it is the conjugate rack of an involute profile is a straight tooth rack. If this is a rack whose tooth profile is straight, then it can maintain conjugate action with an involute gear and such a straight sided rack cutter is used to produce the involute tooth profile on a circular gear blank. I can say it is an involute rack which is used as the cutter and on the cutter, it is much easier to produce straight side than any complicated curve and it is this involute rack cutter which can maintain conjugate action with an involute tooth profile of a gear. So, this is called involute rack cutter.

Why the involute of a rack is a straight line? This point again will be clear in today's lecture a little later, that the involute of a straight line is another straight line. Like we have seen involute of a circle is the curve. So, if this radius of the base circle goes to infinity, when the gear gets converted into a rack, then this also becomes a straight line. All these points, we will make clear in today's lecture. Let me get back to that figure, where I showed the pressure angle and now we will show, what is the advantage of this pressure angle remaining constant in a pair of involute gear teeth profile?

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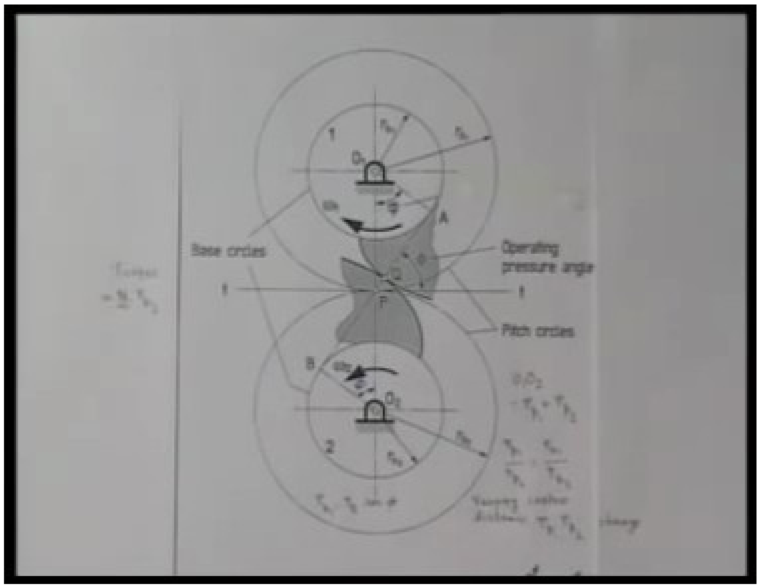

In this figure, these are the two base circles of radius r_{b_1} and r_{b_2}. These are the pitch circles of radius r_{p_1} and r_{p_2} and it is the common tangent to the base circle, which we call line of action. This line of action AB makes an angle φ with this common tangent to the pitch circles. This tt is the common tangent to this pitch circles of radius r_{p_1} and r_{p_2} and this angle is φ, which is operating pressure angle. Now as we have already noted, because the line of action just remains AB, this pressure angle does not change as if the profiles are involute. The pressure angles remain constant during the entire interval. Now, what is the advantage that the pressure angle remains constant? If we neglect the friction force between the gear teeth, if it is well oiled and well lubricated, friction force is not very large. The driving effort is along the common normal. If this gear is driving this gear, then most of the force is acting along this common normal. If this normal force (N), then the torque that is transmitted is N times the base circle radius, because this line of action is along AB and this angle is 90^0. So, the torque acting on this gear 2 is $N*r_{b_2}$.

Now if this torque remains constant, then N remains constant because the base circle radius r_{b_2} is constant. If N remains constant, then we see both the magnitude and direction of this force N is remaining constant, which means the bearing reactions here and here, because there is same N which is acting on this gear also. So the direction and magnitude of N both remains constant, because φ does not change and if we are transmitting a constant torque, then this torque is also constant which means magnitude of N is constant, because φ is constant means the direction of N is constant, which means the bearings are not subjected to any dynamic reaction. The magnitude and direction both of the reactions remain same. So long a steady torque is being transmitted. This enhances the life of the bearings, which are used to mount these two shafts on the foundation. Thus, the pressure angle remaining constant implies that under steady torque, the bearing reactions are not dynamic, they are static. That is the advantage of pressure angle remaining constant.

The third advantage, we discussed it. Suppose this pair of gears, right now the centre distance is $1O_2$. Suppose, this gear remains where it is, only this gear is shifted a little bit upward, then what we will see that it is the same involute profiles will be useful to maintain the conjugate action.

Centre distance O_1O_2 is $r_{p_1}+r_{p_2}$, where r_{p_1} and r_{p_2} pitch circle radius of the first gear and pitch circle radius of the second gear respectively. We have already noted that base circle radius r_b is nothing but r_p cos φ, where φ is the operating pressure angle. So, we know r_{p_1}/r_{p_2} is same as r_{b_1}/r_{b_2} because $r_{b_1}=¿ \quad r_{p_1}\cos\varphi$, $r_{b_2}=¿ \quad r_{p_2}\cos\varphi$.

So what we see, so long the base circle radius of gears remains same, r_{p_1}/r_{p_2} remains same. With the centre distance changing, it is this $r_{p_1}+r_{p_2}$ quantity is varying. If the centre distance varies, the pitch circle radius r_{p_1} and r_{p_2} both will change, but the ratio of r_{p_1} and r_{p_2} that remains same. So it is the same angular velocity ratio ω_1 to ω_2 is maintained. Only thing what changes because r_{p_1} and r_{p_2} is changing means, φ is changing because r_{b_1} is not changing and $r_{b_1}=¿ \quad r_{p_1}\cos\varphi$, where φ is the pressure angle. r_{b_1} is not changing because the gears are not changing but with changing centre distance r_{p_1} is changing, which means φ is changing because this common tangent, if this circle is shifted a little bit upward, the common tangent between these base circles will change, that means this angle φ will change. So, pressure angle changes a little bit, but the constant angular velocity ratio is maintained by the pair of gears even when the centre distance changes a little bit, that was the third advantage.

The fourth advantage as I said, will be explained later, when we will be able to show that the involute of a straight line is a straight line that is for a rack to maintain conjugate action with an involute profile, the tooth profile on the rack will be straight, like trapezium. Because involute profiles have all these advantages as I said earlier, they are almost universally used in mass produced gears. As a consequence of this mass production of involute gears, they have been standardized. There are various standards, but I can mention some typical standards which are more commonly used, like British Standard for involute profile.

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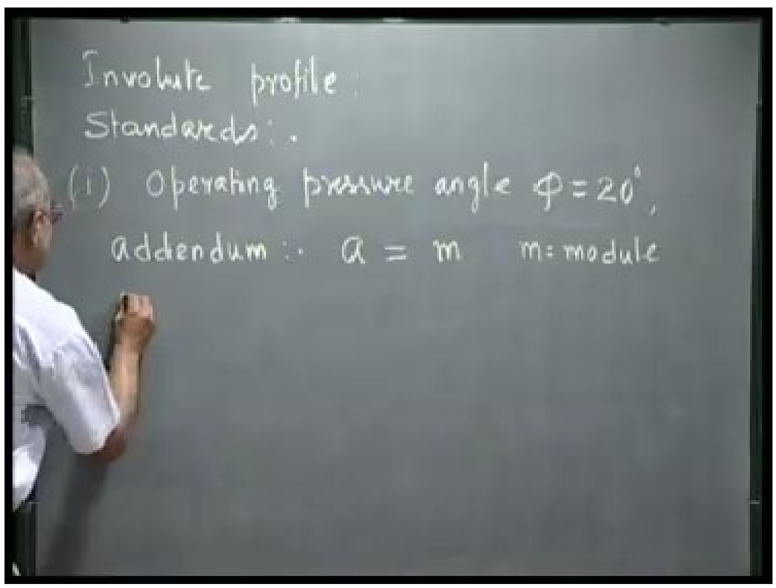

So in involute tooth profile, some standard dimensions are: The most common value of operating pressure angle (φ) is 20^0. Some old gears which were cast, φ had a value equal to 14 $½^0$, but these days most involute gears will have an operating pressure angle equal to 20^0. Similarly, the value of the addendum which we defined earlier ‘a’ is equal to the module, where m is module. The gear teeth are described in terms of the module of the gear teeth and the standard value of the addendum is equal to module.

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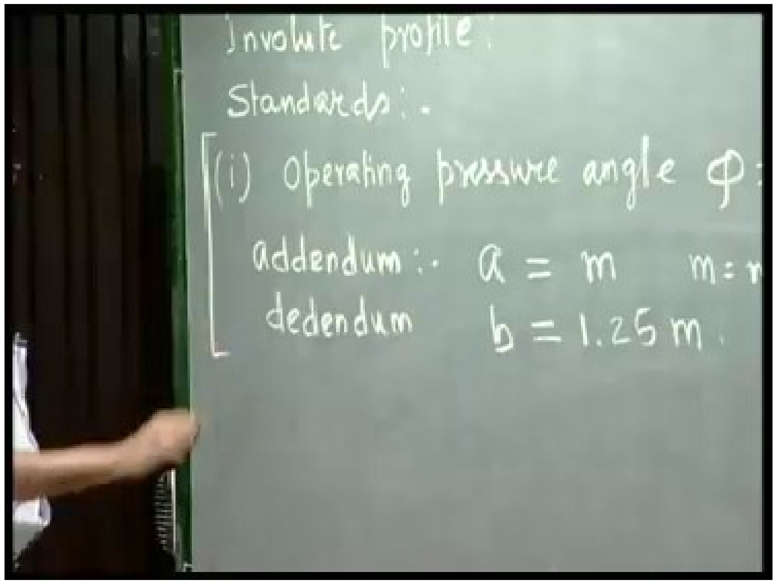

Similarly, dedendum which we denoted by b is always more than a, the most common value is 1.25 times the module, which is expressed in millimeter for metric gears, a is equal to the module m and b is equal to 1.25 m. These three standard values we may take if unless otherwise specified. As I said, involutes are most commonly used and geometry of involute teeth is a very vast subject. We are not going to get into all the details of involuted geometry, but I will give you a little glimpse or a little basic idea of this curve involute, which we called involutometry.

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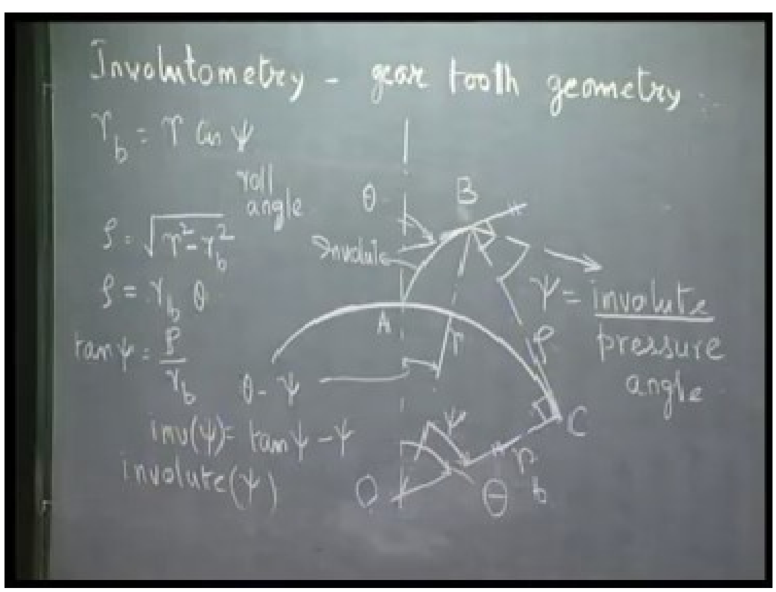

The result that we will obtained from this involutometry will be very useful to determine various proportion of the gear teeth, which are involutes. So we can say application of involutometry in gear tooth geometry. I repeat we will discuss of course only the very basics of this gear tooth geometry. You can always refer to hand book for the details that we may like to know.

So what is involutometry? Suppose, this is the base circle with the centre here and we start unwinding the string from this base circle from this point A and by unwinding the string we generate this involute. This is the string length which is same as this arc length because this string was wound on to this cylinder and now it has unwound up to this point and this is the involute from this base circle. So the string is tangent to the base circle. So this radius r_b is base circle radius and perpendicular to the string.

Now at this point, the involute takes off radially. This radius is tangent to the involute at this instant. And at this configuration, this string which is perpendicular to the involute, so the tangent is perpendicular to the string. This is the tangent at this point, say B. This radial line is tangent to the involute at A and this line which is perpendicular to the string at the point B. This angle between these two tangents, I call the roll angle θ. This is O. This line is perpendicular to the string, this radius is also perpendicular to this string, which is tangential. So this is parallel to this.

So if this angle is θ, this is also θ, the roll angle. That is the string has unwound up to this point B, where the roll angle is θ. This particular point on this involute, at this distance OB is the radius vector, let me call it r. If this represents the base circle of the gear and the gear is rotating about the point O, then this particular point has a velocity which is perpendicular to OB. This is the direction of the velocity and this is the normal to the involute.

At this point, this angle between the normal (it indicates the direction of the force at this particular point 'B') and the direction of the velocity, this angle is called ψ, which we will call involute pressure angle. This is the angle between the line which is perpendicular to OB and the string at this configuration. Please note that this is not the operating pressure angle which we discussed in case of a pair of gears. This is only one gear we are talking of, one involute we are talking of and this particular angle between the direction of the velocity and the direction of the normal to the involute, I call it involute pressure angle. Now this line is perpendicular to the tangent and this line is perpendicular to the direction of velocity, so this angle is also ψ, involute pressure angle. The θ is the roll angle and ψ is the involute pressure angle. θ is the angle between these two tangents at A and B and ψ is the involute pressure angle.

Let us say base circle radius is r_b. And this is, the string length is the radius of curvature of the involute at this point B. If this is the involute, at every instant the radius of curvature is nothing but the string length which has been unwound from the base circle. So, if we call this point C, then BC is the radius of curvature ρ. What we see, $\rho=\sqrt{r^2-r_b^2}$, where r is this radius vector of the point B measured from the centre of the base circle and if we use this angle as the polar angle to define the involute curve, what is this angle? This angle is $\theta - \psi$ and we also see this ρ is same as this arc length AC because this is the length of the string and this was the original length of the string. So, AC is nothing but $\rho = r_b$ ¿ θ and also, we can see $\tan \psi = \rho/r_b$. If this angle is 90^0, this is ρ, this is r_b and this is ψ. So $\tan \psi = \rho/r_b$.

From here, we see ρ/r_b is nothing but θ. So we can write $\theta - \psi$ as $\tan \psi - \psi$. $\theta = \rho/r_b$ and $\rho/r_b = \tan \psi$. So, this angle that the line OB makes with this original radius OA, that angle is $\tan \psi - \psi$. This is called involute function of ψ and we can also see r_b is nothing but $r \cos \psi$. So, we can write base circle radius is related to r as $r \cos \psi$. So everything has been found in terms of this involute pressure angle ψ, which keeps on changing at various points.

The distance from O, I can get as $r_b/\cos \psi$ and this angle, I can get as $\tan \psi - \psi$ which is called involute of ψ. This is very easy to see that if one gives me the value of ψ, I can calculate involute of ψ, very simply. This is called involute function just like sin, cos, we call it involute of ψ. Given the value of ψ, it is easy to calculate involute of ψ, but not the other way around. Tables are available just like tables of sine, cosine, tan and log. I can get the value of ψ if you give me the value of involute of ψ by consulting the table. Now at this stage, I will be able to show that what happens to this radius of curvature of the involute profile as r_b increases. As I said, when r_b goes to infinity, the gear gets converted into a rack. So under that situation, what is the radius of curvature of the corresponding involute profile?

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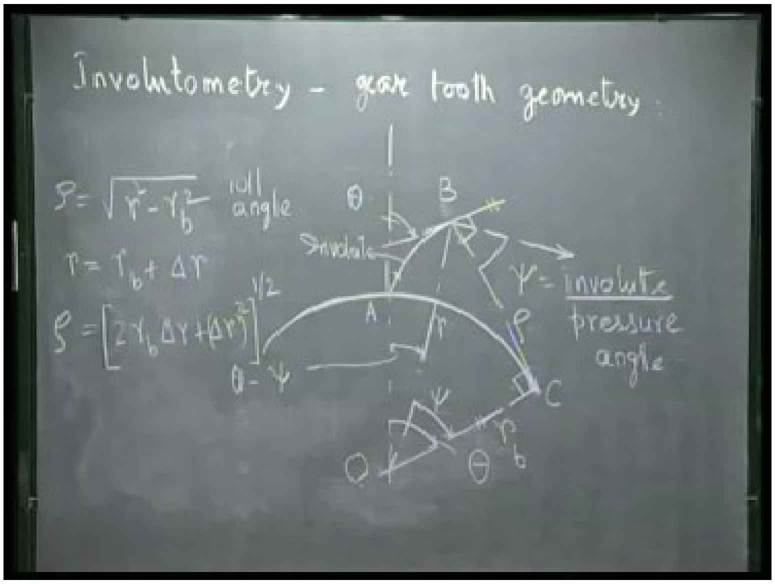

We have already got $\rho=\sqrt{r^2-r_b^2}$, where r is the instantaneous value of this distance. So, if we start from A and go up to some point by changing initial value of r is r_b and then I go to $r_b + \Delta r$. Let's say, as I start from A, when r changes from r_b to $r_b + \Delta r$. If we substitute it in equation of ρ we get, $\rho = [2r_b\Delta r + (\Delta r)^2]^{1/2}$. This is the value of ρ, as we go out from this point A. Now if r_b goes to infinity, it is obvious that ρ also goes to infinity. So if the radius of curvature goes to infinity, which means this profile has become a straight line.

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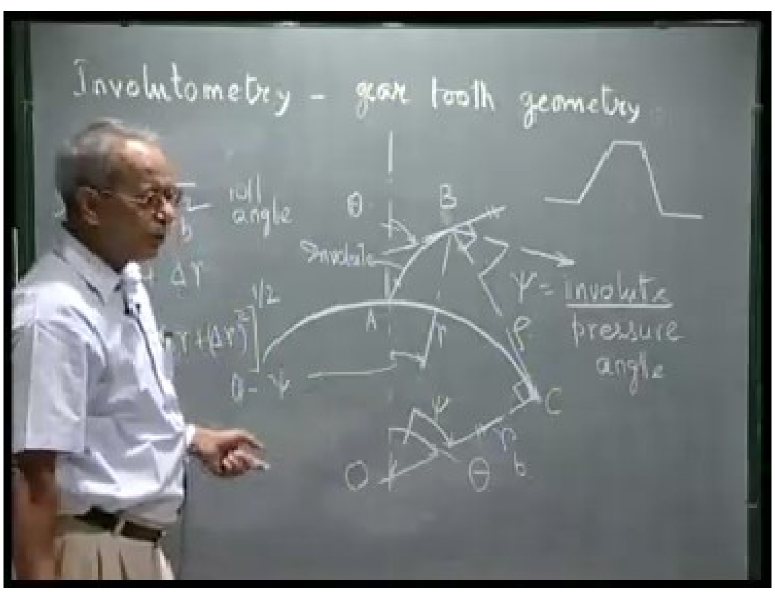

That is what we said that if we have an involute rack, let the tooth profile from the base circle as it come out, it comes out in the form of a straight line. So, the tooth profile of an involute rack is a straight line and which can maintain conjugate action with an involute gear or involute pinion and that gives the advantage that we can use such a rack cutter to generate the involute tooth profile on a circular gear blank.

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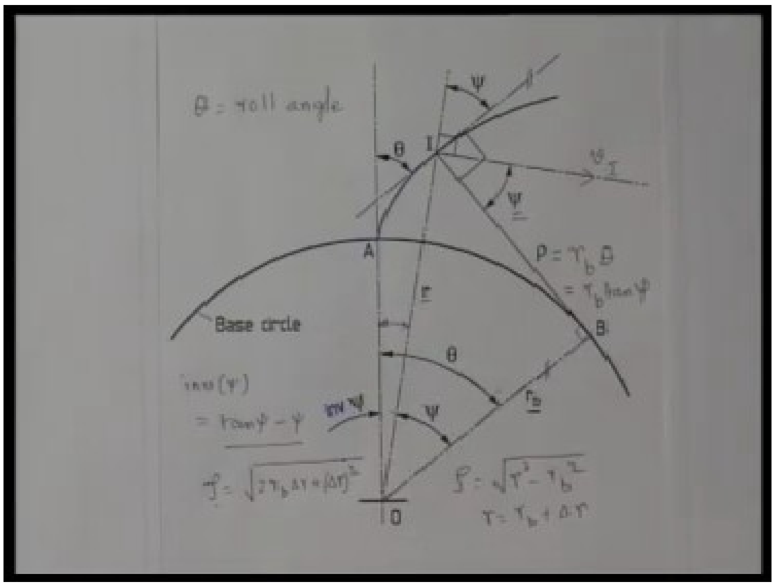

Let me now repeat, what we have just now discussed with reference to this figure. This is the base circle and this is the centre of the base circle. An involute is being generated starting from this point A. If we consider a point 'I' here, then the string at this configuration corresponds to this line IB, which is tangent to the base circle. OB is the radius of the base circle, which we denoted by r_b. The instantaneous polar radius of this point 'I', we denote it by 'r'. The tangent at A to the involute is this radial line and the tangent at 'I' is this line. The angle between this tangent at 'I' and the tangent at A is called the roll angle, denoted by θ.

If this we take as a gear and the gear is rotating about the point O, then the velocity of this point 'I' is perpendicular to OI. Say this is the direction of the velocity of the point 'I', if it happens to be a point on the gear, And the string which is normal to this gear tooth profile or the involute profile, that is the normal to the involute at the point 'I' and the angle between this velocity direction and this normal which is nothing but BI, I call ψ. This is normal and this is a tangent, so this angle is 90^0. This is also perpendicular to the string BI which is tangent to the base circle. So, the tangent at 'I' and this radius OB are parallel so this angle is ψ, then this angle is also ψ.

Similarly, if this angle is θ, then this angle is also θ because this line is parallel to this line. So θ is the roll angle and ψ is this angle. Now the polar angle of this line OI from this vertical line, that is the radius through initial point A, this angle which is θ – ψ, we defined as involute of ψ. This length AB is nothing but r_b ¿ θ and the string length BI is also equal to AB. So this ρ, which is the radius of curvature of the involute profile at I, the string length is the radius of curvature ρ. This ρ is nothing but r_b into θ.

From this triangle, OBI, I can write tan of ψ because this angle is 90^0. Tan of ψ is also ρ by r_b. So we can write, ρ is r_b tan ψ, which means θ is equal to tan ψ. So this involute of ψ, which is θ – ψ can write it as tan ψ – ψ. Then we also found that expression of ρ in terms of r and r_b, which we wrote as $\rho=\sqrt{r^2-r_b^2}$. As we draw this involute starting from this this base circle, the r increases from r_b say by a value Δr. Then substitute it in equation of ρ, canceling r_b, we found the expression of ρ to be $[2r_b\Delta r + (\Delta r)^2]^{1/2}$. So this clearly shows, if r_b tends to infinity, then ρ also tends to infinity. That means, right from the point A, as soon as r changes from r_b to small Δr, the radius of curvature becomes infinity, that is the involute of a straight line. When r_b goes to infinity, this base circle becomes a straight line. That is the gear is converted to a rack and the involute profile becomes a straight line because radius of curvature is infinity, which tells us involute rack tooth profile looks like a straight line.

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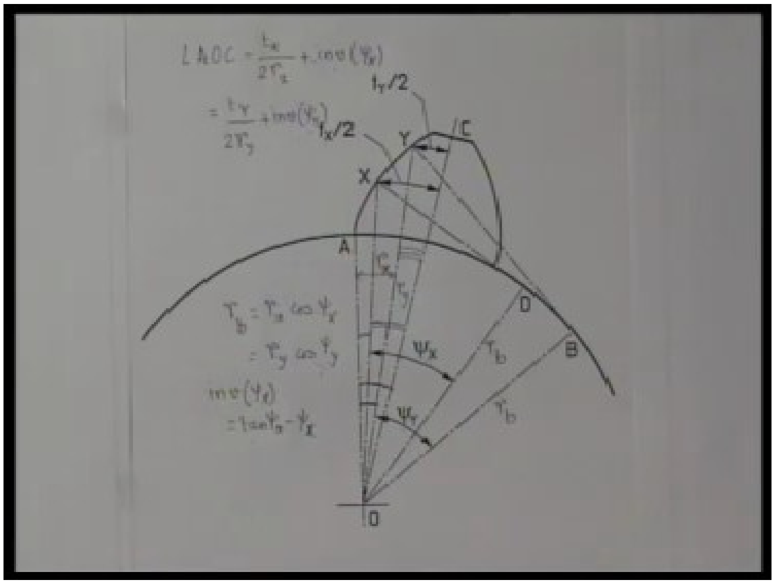

Just now, we have discussed the basic geometrical relationship between an involute profile and its base circle. These relations are very useful for studying gear tooth geometry as I said earlier and here we discuss an example. Suppose, this is an involute tooth profile which has been generated from this base circle. Suppose the thickness of the tooth as we see keeps on varying from the base circle up to the addendum circle. Suppose this point X is defined by this distance OX, which is defined by the polar radius r_X and the thickness at this level denoted by t_X.

When this polar distance changes from r_X to say r_Y, I get to this point Y and I want to find, what is the thickness of the tooth at this level? That is t_Y. So t_X is given. I would like to find t_Y and r_X and r_Y are given. To do this first we note, when the involute was at X, the string is DX. This is the length AD, which has become DX and when we come to the point Y, the string is represented by BY. This is the tangent BY same as the arc length AB. So, the angle between this OX and OD, we defined as the ψ. So I write it as ψ_X corresponding to X, XD is tangent to the base circle and the angle between OD and OX I call the involute pressure angle at X, which is ψ_X. Similarly, the involute pressure angle at Y is the angle between OY and the tangent to the base circle from Y which is YB, that is what I call ψ_Y. We have already seen, the base circle radius r_b. This is also base circle radius r_b and this can be written as: $r_b = r_X \cos \psi_X = r_Y \cos \psi_Y$. So if you have given the values of r_X and r_Y, I can get ψ_X and ψ_Y.

Now how t_X and t_Y are related? For that, let me consider this angle AOC. OC is the mid line or the symmetric line of this gear tooth, OC and the angle that OC makes with OA, I call this angle AOC. Angle AOC, I can write as $t_X/(2r_X)$. This angle is $t_X/(2r_X)$ and this angle is nothing but involute of ψ_X. At the point X, the angle that OX makes with this starting line OA, I defined as involute function of ψ_X. So angle AOC is nothing $t_X/(2r_X) + inv(\psi_X)$ and if we remember involute function is $\tan \psi_X - \psi_X$. And the same angle AOC, we can also write as: this angle which is $t_Y/(2r_Y)$ + this angle. This angle that is the angle between OA and OY which is nothing but involute of ψ_Y. So, angle AOC = $t_Y/(2r_Y) + inv(\psi_Y)$.

If r_X and r_Y are given, I can find ψ_X and ψ_Y from this relation. Once, ψ_X and ψ_Y are known, I can find involute of ψ_X and involute of ψ_Y, because involute function is $\tan \psi - \psi$. Involute function of any angle ψ_X is tan of $\psi_X - \psi_X$. Of course, ψ_X must be measured in radian. So from this relationship, if t_X is given, r_X is given, r_Y is given, I can find t_Y which is the only unknown because ψ_X and ψ_Y, I have already found out from here and using the involute function, I can find involute of ψ_X and involute of ψ_Y. So thus, for the gear tooth geometry, if you give me the thickness at any level, we can find the thickness at any other level of the same involute tooth profile.

For continuous transmission of rotation from one gear to another, it is obvious that is imperative that a pair of teeth must remain in engagement, at least until the next pair of teeth comes into engagement. It should not happen, that one pair of teeth has lost its engagement and the next pair of teeth has not come in engagement, because then there will be no transmission. So, to maintain continuous transmission of rotation from one gear to another, it is imperative that a pair of teeth must continue to remain engaged, at least until the next pair of teeth has come into engagement. This phenomenon is studied in terms of a geometrical quantity which we call contact ratio.

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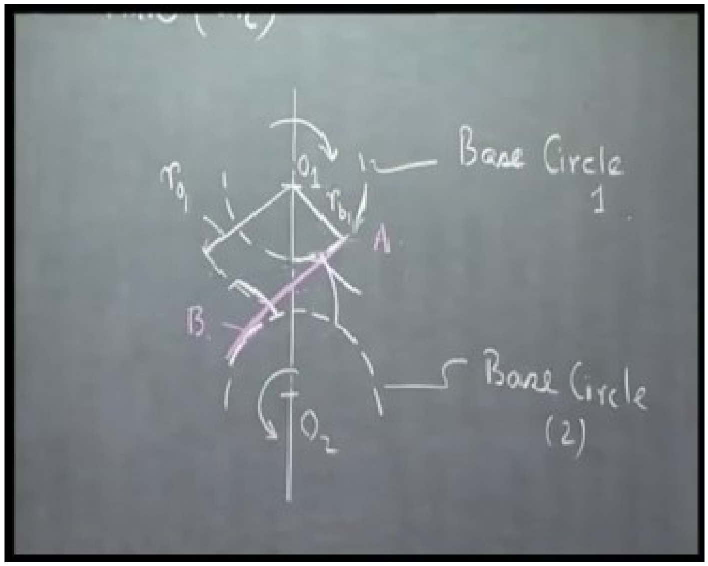

Let me now define the contact ratio and try to get the expression of this contact ratio. We study contact ratio for involute tooth profile, we use the symbol M_c, indicating the contact ratio. Let's say, this is one base circle and the centre of this gear is at O_1, and the centre of the other gear is at O_2. This is the other base circle 2. If I draw the common tangent to this pair of base circles that defines the line of action. This is A and this other point of tangent is B. We know the contact between a pair of gear teeth will always lie on this line AB.

Now let us talk of one tooth of this gear. Suppose this gear is rotating this way and to the teeth action the other gear is rotating in the opposite direction. So, if we consider a tooth on this gear, the first teeth come into engagement at its outer circle or the addendum circle and because the contact point must lie on this line, when the addendum circle comes here in contact with this line AB, that is where the contact starts on this face of the gear tooth.

Now, how long this tooth will remain in contact with the tooth on the other gear 1? So long, the outer circle of this gear is in contact. This is the addendum circle of the gear 1, let me call it radius r_{o_1}, that is the outer radius of gear 1 and this is r_{b_1}. As this tooth rotates, this tooth comes here and when the tooth on this gear which is pushing it, let's say here. This tooth comes and rotates, and when this outer circle leaves this line, the contact is lost with this particular gear. So, if we draw this tooth here, this is where the contact is lost.

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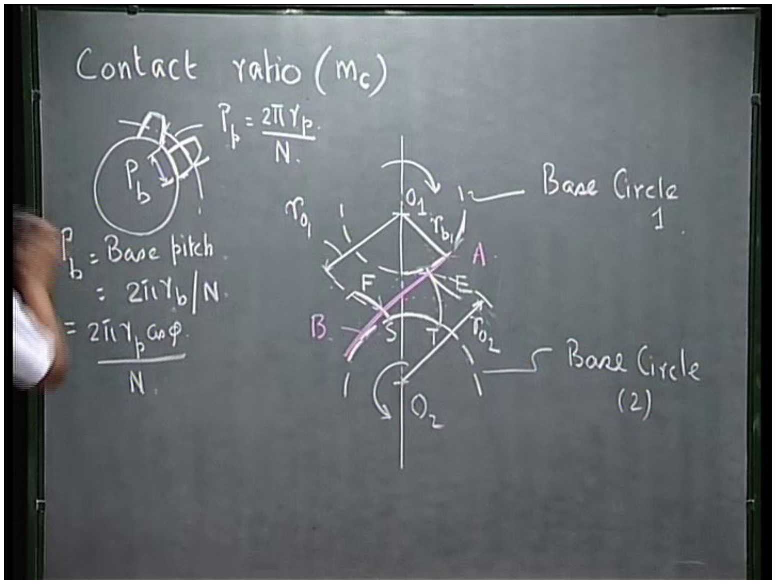

So, this particular pair of teeth is in contact, as this contact point moves from this point say E to this point say F. The contact between this pair of teeth, this one and its mating tooth, starts when the addendum circle of this lower gear, gear 2 (this is the addendum circle of gear 2 and this is the addendum circle of gear 1), intersects line AB at the point E, the addendum circle of gear 1 intersects the line AB at F. So, this pair of teeth is remains in engagement from E to F. What we should ensure that before this point is reach, the next tooth in this gear must come in engagement, must come in engagement that is the outer circle must intersect here for the next tooth.

To do this, let me define what we call base pitch. We consider, this is the same tooth we are considering, but now let me consider two consecutive teeth on one gear, two consecutive teeth on one gear. If we remember that we define the circular pitch on this pitch circle going from one point to another identical point on the adjacent tooth, measured along the pitch circle this is what we defined as P_p which is $2\pi r_p/N$, where r_p is the pitch circle radius.

Similarly, we can define base pitch, that is this point and the identical point on the next tooth, but on the base circle. This distance is called the base pitch, we call it P_b, base pitch; that is $2\pi r_b/N$, where r_b is base circle radius. If we remember r_b is nothing but $r_p \cos \varphi$, where φ is the operating pressure angle.

Now, what is the movement along the base circle of this particular teeth from the start to the end of the engagement, that is given by this circular arc. Measuring on the base circle, this is the rotation that is the distance covered by this particular tooth from the start of its engagement to the end of its engagement, the rotation is such that it covers so much distance on the base circle.

To maintain contact between two pair of teeth that is, before this contact is lost the next pair must come in engagement, this distance must be more than this base pitch. This distance if I defined is at P_b, then this distance must be more than P_b such that the next tooth has come up to this position. Because the distance along the base circle between this tooth and the adjacent tooth is the base pitch. When this point comes here, that point must come above this point; that means, this circular distance must be more than the base pitch. Let me call this point S and T.

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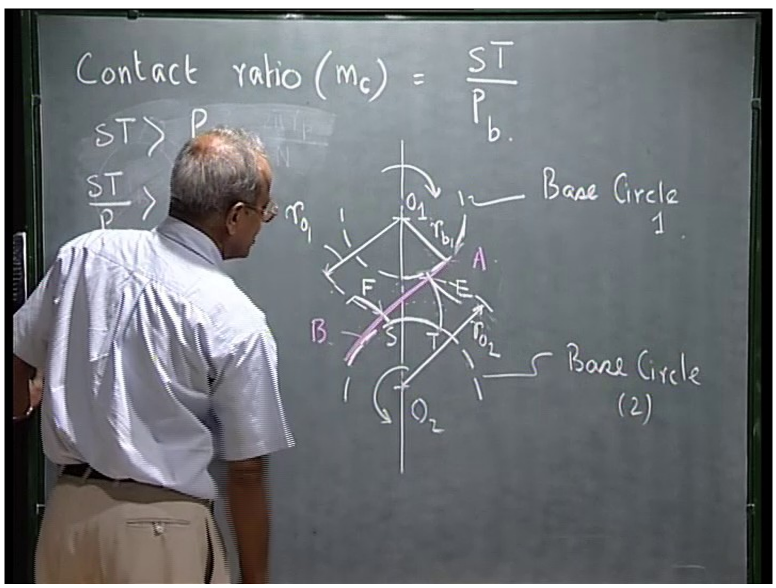

So, to maintain contact all the time between these two pair of gears, ST must be more than base pitch. And it is this ratio ST by base pitch, which we have defined by P_b must be more than 1. We define m_c as ST/P_b. What is ST? It is the movement of this tooth measured along the base circle from the start to the end of the engagement. To calculate m_c we proceed as follows.

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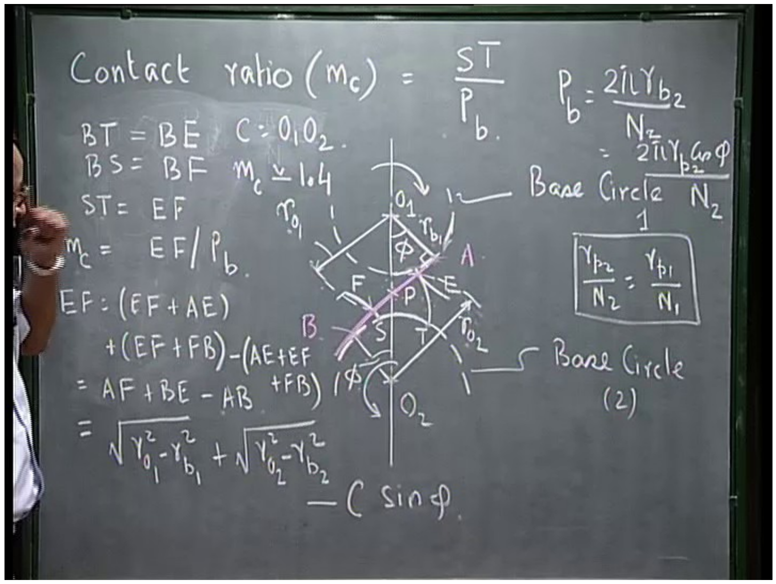

This is called the pitch point P, where the common tangent the line of action intersects this line of centers O_1O_2. Now, because this is the involute and this is the tangent to the base circle BT, is nothing but BE. BT, that is the distance along the base circle is same along the length of the string; BT is same as BE. Same way we can say BS is same as BF, this is the distance along the base circle and this is the length of the string. So, we can say BS is same as BF. If we subtract, we get ST is same as EF.

The contact ratio m_c which was defined as ST/P_b, we can write as EF/P_b. The expression of P_b we have already written, P_b is $2\pi r_b/N$, where N is the number of teeth and r_b is the base circle radius, which is same as 2π pitch circle radius cos φ, where φ is the operating pressure angle. That is, this angle and also this angle.

So, we have got the expression of m_c in terms of EF/P_b and P_b is given by that expression. Now let me try to write EF in this way as EF + AE + EF + FB. I have added AE, EF, FB so I subtract all these three, so, – (AE + EF + FB); EF is this EF, I have added AE, EF, FB I have subtracted AE, EF, FB.

EF + AE is nothing but AF. Similarly, EF + FB is nothing but BE, and AE+ EF + FB is nothing but AB. What we can see; what is AF? This angle is 90^0 because this is tangent, this is radius. So, AF we can write $\sqrt{(O_1F)^2-(O_1A)^2}$ and O_1F is nothing but r_{o_1} this is the addendum circle, that is how we determine the point F.

So, O_1F is nothing but r_{o_1} and O_1A is r_{b_1}, so, AF we can write r_{o_1} ¿ ¿ r_{b_1} ¿ ¿ ¿ √¿. The same way we can write BE, what was E that was the addendum circle of this gear. So, O_2E is nothing but r_{o_2} and this is r_{b_2}. So, BE we get r_{o_2} ¿ ¿ r_{b_2} ¿ ¿ ¿2 ¿ √¿, AB is this angle is φ. So, $O_1P \sin φ$ is AP. Similarly, $O_2P \sin φ$ is BP. So, $O_1O_2 \sin φ$ will give me AB. $O_1P + O_2P$ is O_1O_2, that is the centre distance and AP + BP which is AB if you take $O_1P \sin φ$ you get AP, $O_2P \sin φ$ you get BP. So, this is the centre distance which we write as C is sin φ, where C is O_1O_2.

So, we get EF in terms of base circular radius and addendum circle radius of both the gears, center distance between the gears and the operating pressure angle and this divided by P_b which again we expressed in terms the operating pressure angle, number of teeth and the pitch circle radius. This we should have taken second gear. So, $2\pi r_{b2}/N_2$. So, $2\pi r_{p2}/N_2$. And r_{p1} and N_1 this is the same. So, $r_{p2}/N_2 = r_{p1}/N_1$. So, it is the same expression we have to take the pitch circle radius and put the number of teeth of the same gear. So, that gives me the expression for the contact ratio.

The recommended value of the contact ratio for a pair of involute gears is 1.4. What does that mean? This is a not an integer, what does it mean that during some phase of the contact only one pair of teeth is in contact, for certain other phase of contact at least two teeth is in contact. Because there must be a region when two teeth must be in contact so that continuous transmission is ensured and on an average 1.4 pair of teeth remain in contact if we consider the entire cycle. That is the physical meaning of this contact ratio as 1.4.

That is on an average within the entire cycle 1.4 pair of teeth remain in contact; that means, sometimes there is only one pair of teeth in contact, and just when the contact is being lost within one pair, other pair comes into contact before that. So, for some time at least two pair of teeth remain in contact. So, this contact ratio gives you the average number of pair of teeth that remain in contact during the entire cycle.