Kinematics of Machines

Prof. A. K. Mallik

Department of Mechanical Engineering

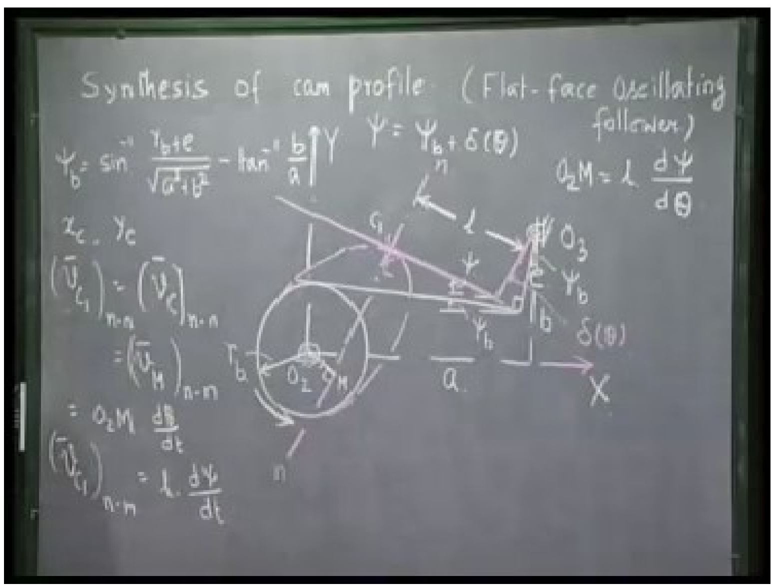
Indian Institute of Technology, Kanpur

Module – 11 Lecture – 3

Synthesis of Cam Profile (Flat-Face Oscillating Follower)

Today we continue our discussion on synthesis of cam profile for a flat-face oscillating follower.

(Refer Slide Time: 00:25)



Let the base circle of the cam be given and when the flat-face follower is in contact with the base circle, that is the lowest position of the follower and the oscillating follower is hinged at this location. This dimension we called 'e'; the location of the follower hinge from the cam shaft was given by these two dimensions, 'a' and 'b'. The angle that the follower face makes with the horizontal at this lowest position, we denote it by ψ_b that is the angle 'e' makes with the or the follower face makes with the horizontal. In our last lecture, we got an expression for this ψ_b which was, $\sin^{-1} [(r_b + e)/O_2O_3] - \tan^{-1}(b/a)$, where r_b is base circle radius and O_2O_3 is nothing but $\sqrt{a^2+b^2}$. Now when the cam rotates from this position by an angle θ, let the follower rotates by an angle, δθ. The follower has rotated by an angle δθ and this is the follower. This is the cam profile. We denote the contact point on the cam as C. This is the common normal within the cam profile and the follower surface. Let’s say this distance is l, which keeps on changing as the cam rotates, because the contact point shifts from this cam surface.

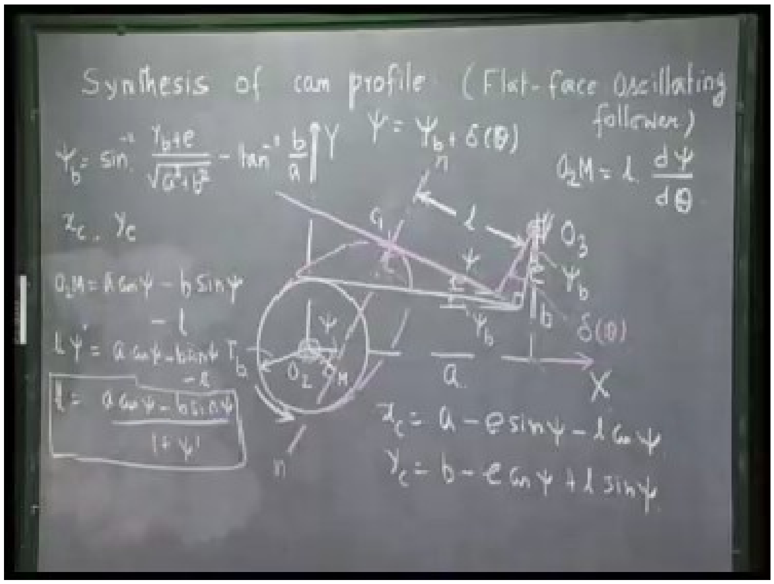
As we have done for the translating follower situation, we take our X and Y axis passing through O_2. Our objective is to find the co-ordinates of the contact point, namely x_c and y_c. To find the x_c and y_c, we will need this quantity or value ‘l’. So, as a first step, we try to find what is this quantity ‘l’. Towards that end, from O_2 I drop a perpendicular to this common normal and let this point be ‘M’. The follower position is at this instant, again measured from the horizontal is given by ψ. And as we noted in our last class, ψ is nothing but $\psi_b + \delta\theta$. We have already got ψ_b; δθ is prescribed for us so we can find ψ. To find this distance ‘l’, we consider a point C_1 on the follower surface, but at this instant coincident with C. C belongs to the point on the cam and C_1 denotes the point on the follower surface which is in contact. To maintain the contact, velocity of this point C_1 and this point C along the direction of n-n must be same. Otherwise, the contact will be either lost or one point will get into the other point, since they are rigid body and that cannot happen.

To maintain the contact, velocity of the point C_1 along the direction n-n must be velocity of the point C along the same direction. Now because M and C are two points on the cam which is a rigid body, so velocity of C and M along the line CM that is along n-n must be same. So $[\acute{v}_c]_{n-n}$ is nothing but velocity of the point M in the n-n direction, because the two points on the same rigid body, so the distance CM does not change, that means, the velocity along this direction must be same. And velocity of C and C_1 along n-n must be same; otherwise the contact will be lost.

Now to find the velocity of the point M along n-n is nothing but O_2M into the angular velocity of the cam. So we can write, O_2M ¿ dθ/dt, where dθ/dt is angular velocity of the cam and that is velocity in this direction. If ω is counter-clockwise, the velocity of M is along n-n and O_2M ¿ dθ/dt. Now velocity of C_1 along n-n is nothing but $1 * \acute{\psi}$. So the velocity of the point C_1 ($\acute{v}_{C_1}$)along n-n, where N_1 belongs to the follower surface is nothing but l ¿ dψ/dt, where ψ is measured clockwise.

Because these two are same, I can immediately write O_2M = l ¿ dψ/dθ. Once we get this O_2M, then we can write, as we see if this angle from the horizontally is ψ and this line is perpendicular to this line, this line is also perpendicular to this line. So, these two lines are parallel and this angle is also ψ.

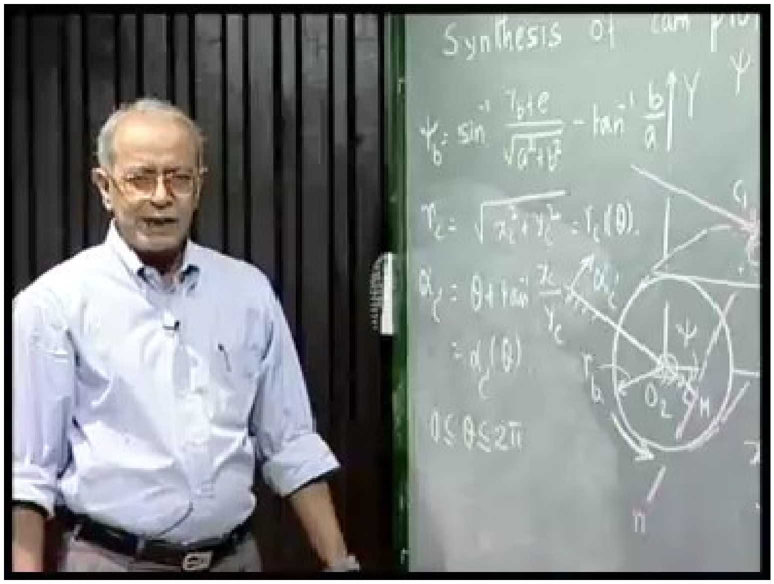
(Refer Slide Time: 09:04)



So I can write O_2M = cos ψ – b sin ψ – l; since e is perpendicular to M, so there is no component. I am going from O_2 to M in this way: O_2 to this point, to this point, to this point, to this point and this point. So O_2M, if we add these vectors or the components, we get O_2M = cos ψ – b sin ψ – l, (e will not give anything, a will again give a cos ψ, b will give me – b sin ψ, l will give again l, but in this direction, so – l) and $O_2M = l\,\psi'$. So that gives me $l = \dfrac{a\cos\psi - b\sin\psi}{1+\psi}$. So we get this distance l, which changes with θ because ψ is a function of θ.

Now, x co-ordinate of this point C is a; there is no projection of b on the horizontal direction, horizontal projection of e is e sine ψ, but in this direction, projection of l is l cos ψ in this direction. So, we are in a position to write the coordinate of x_c, $x_c = a - e\sin\psi - l\cos\psi$. So that is the co-ordinate x_c. Similarly, we can write $y_c = b - e\cos\psi + l\sin\psi$. So I have got x_c and y_c everything in terms of θ, once these parameters a, b, e are given to us.

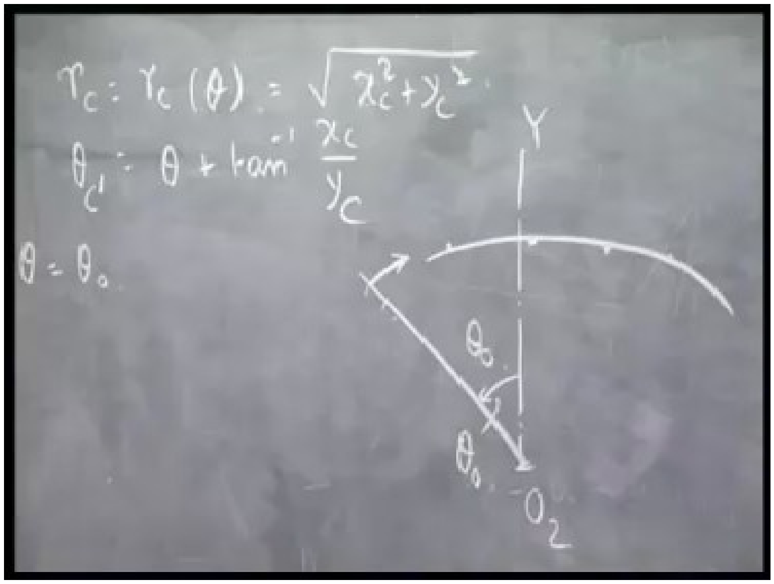
(Refer Slide Time: 12:08)



Rest is as before. Once I get the contact point, I can write $r_c = \sqrt{(x_C)^2 + (y_C)^2}$, as we see this is a function of θ. From any line which I fixed on the cam then if I measure the polar angle from here, if I call it say some α_C, then $\alpha_C = \theta + \tan^{-1}(x_c/y_c)$. So α_C is also a function of θ. This is the parametric equation of the cam profile in polar co-ordinates with the origin here. r_C which we can get for various values of θ, α_C which we can again get for various values of θ, where θ represents the cam rotation. So varying θ from 0 to 2π, we can get r_C and α_C for various values of θ. Then, plotting those values of r_C and α_C, we get the cam profile as we have done in case of translating follower. This portion is exactly as before.

One point I would like to emphasize at this stage that once we get the r_C, that is, the distance from the cam shaft axis and the polar angle for the contact point for all types of follower, translating or oscillating does not matter.

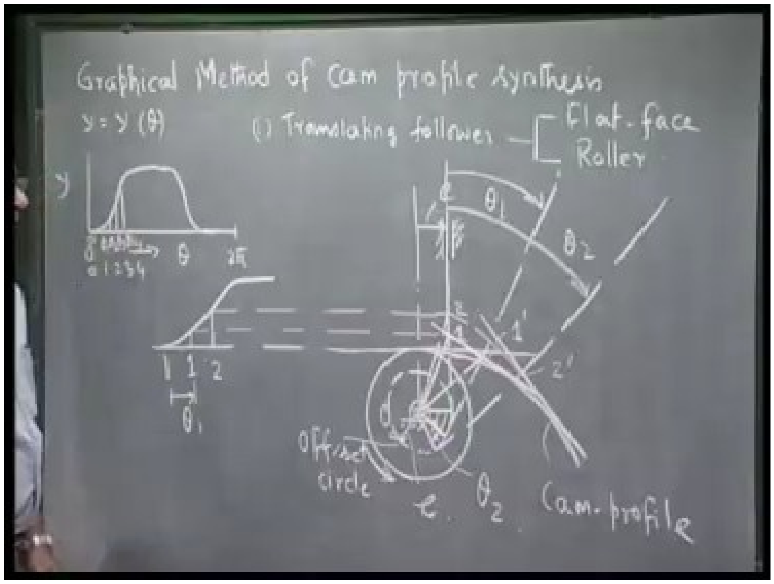
(Refer Slide Time: 13:57)



If I summarize the analytical method for synthesis of cam profile, we are getting r_C as a function of cam rotation and a polar angle which I denoted by $\varphi_{C'}$ or $\psi_{C'}$ or $\alpha_{C'}$ in various cases. I just call $\theta_{C'}$, which is $\theta + \tan^{-1}(x_C/y_C)$ and this r_c is nothing but $\sqrt{(x_C)^2+(y_C)^2}$. These two quantities I can calculate for all values of θ. And after getting this series of values, we will draw the cam profile, when the cam has rotated say by an angle θ_0. So what I do, I take an origin at O_2, draw a line which is vertical, and then draw a line at an angle θ_0. Then, take these values of r_C and $\theta_{C'}$, but $\theta_{C'}$ measure in the clockwise direction from this line. So, say for some values of θ, I get r_C and $\theta_{C'}$ this point, then this point, then this point like that, and joining these points, I will get the cam profile when the cam has rotated by an angle θ_0. So this is where we end our discussion on analytical method of cam profile synthesis.

Next, we will do the same thing i.e., synthesis of cam profile for various types of follower by graphical method. Now that we have finished discussion on the analytical method of cam profile synthesis, we will do the same problem through graphical method.

(Refer Slide Time: 15:53)



Graphical method of cam profile synthesis:

We assume all the geometrical parameters like base circle radius, offsets, all these quantities are known. And only thing, instead of the follower movement (y) being given in the form of an analytical expression, now we have it in the form of a displacement diagram. Now given the displacement diagram, how can we obtain the cam profile through a graphical method. So as before, we start with translating follower. First we discuss, for the follower is flat-face type, then we will discuss for roller follower.

We first draw the base circle and this is the cam shaft. The offset is also given. So, let this be the follower axis and this is the follower face at the lowest position. This is the offset. In the graphical method, the total rotation of θ is from 0 to 2π, we divide in certain number of station points. Say this is zero. Then we take various values of θ like θ_1, θ_2, θ_3, θ_4, these are called station points 0, 1, 2, 3, 4. How many divisions we take is the question, but obviously more number of points we take, more accurate will be our drawing. But obviously, we cannot take too many points, because then it will be very time-consuming.

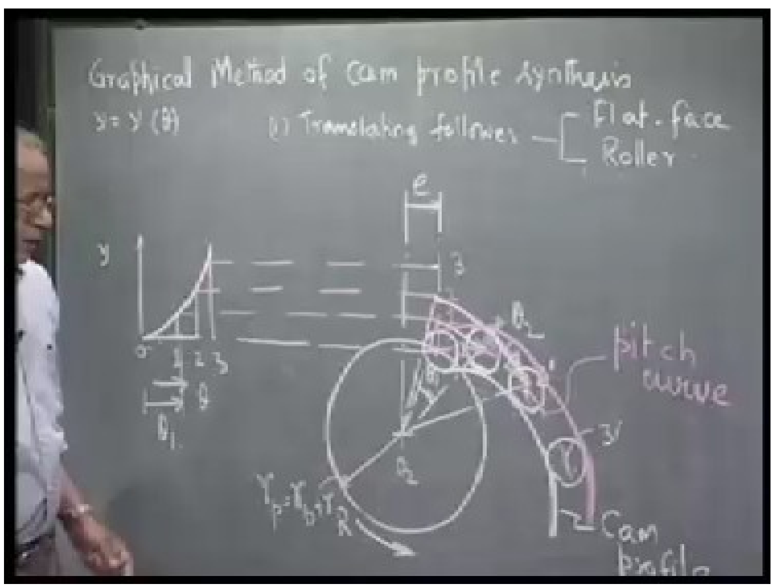
Here I will just illustrate by taking 2 or 3 station points and rest will be same; it can be easily followed. This line represents is the follower axis. Obviously, this is corresponding to station point 0, when the follower is at its lowest position. Now we draw the displacement diagram at the same level. Now say this is the station point 1, this is station point 2. So wat we do, we project these points onto the follower axis. Let us say this is 1, this is 2. So here again I mark this as 1, this as 2. To obtain the follower cam profile, as we have done in case of analytical method, we hold the cam fixed and make a kinematic inversion, such that fixed link rotates in the opposite direction.

Suppose this is the rotation of the cam for station point 1, the cam rotation is θ_1 in the counter-clockwise direction. So the follower axis has to rotate, for the same station point the kinematic inversion is that the follower axis must rotate by same value of θ_1, but in the clockwise direction. If I draw a circle with radius e, then this I make θ_1 and draw a tangent. So this follower axis has rotated by θ_1 in the clockwise direction, corresponding to this station point. That is the kinematic inversion that I hold the cam fixed and allow the follower axis of the fixed link to rotate by the same amount, but in the opposite direction. Now the distance of this corresponding station point 1 from O_2 is here, so this is the distance. But now this line represents the follower axis at this distance; so I rotate this radius along this circular arc and intersect here. So this is what I call, give it a name say $1'$ and this is the follower axis. The follower face is perpendicular to that, but at this distance. So I draw a line perpendicular to the follower axis. So this line represents the follower face after kinematic inversion corresponding to station point 1.

Similarly, corresponding to station point 2, I rotate it by θ_2. And this particular point of the follower axis should be at this location, that is so much distance from O_2, so I rotate O_2 and this is what I call $2'$. Then, the follower face is perpendicular to this line; so this is the follower face corresponding to station point 2. This way, we can draw the inverted position of the follower face corresponding to all the station points 1, 2, 3, and so on. The cam profile is nothing but a curve, which is an envelope to this family of straight line representing the follower faces.

We draw an envelope to this straight line, which represents the follower face after kinematic inversion and then draw an envelope to this family of straight lines and that is nothing but the cam profile. So, this is the cam profile which is always tangent to the follower face and these lines represents the follower faces corresponding to station point 1 and 2. And the same construction can be continued for all the station points and we have to draw a curve which is tangent to this family of straight line representing the follower faces. The basic thing is the kinematic inversion. The kinematic inversion is simple, as if we draw these lines tangent to the circle. In some books it is mentioned as offset circle, which is a circle of radius e with center at O_2. But the simplest thing is to rotate this line O_2a_1 by an angle θ_1, then O_22 by an angle θ_2. It is very easy to show from the triangle law that this distance is same as this distance, because both of them are e. Then this distance, this distance if I consider station point 1, this distance is same as this distance. So this particular right angled triangle is same as this right angled triangle. So this whole right angled triangle has been rotated by an angle θ_1 in the clockwise direction. So we can just as well say O_2a_1 rotated by θ_1, I get this point, O_22 rotated by θ_2, I get this point and once I get this point, then I can draw the follower face perpendicular to the follower axis. Next, I will take up a roller follower, but translating. Let me now explain the graphical method for a translating roller follower.

(Refer Slide Time: 26:10)



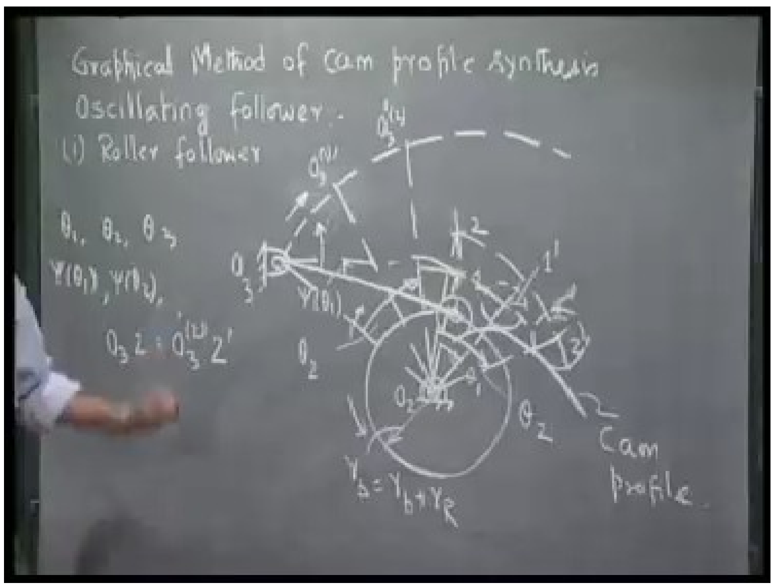
For the roller follower, first let me draw the prime circle. This is the cam center O_2 and this is the prime circle. So, the radius is r_p, which is $r_b + r_R$ and let's say, the offset is e. As we have discussed in case of analytical method, this is the lowest point of the roller center, which is the trace point. The displacement of the follower will be measured from this level. So I draw a horizontal line and draw whatever is the displacement diagram here. So as before, I consider few station points namely 1, 2, 3; this is the plot of y versus θ, this is zero. So now I know, that corresponding to the station point 1, the trace point should be here. Similarly, for the station point three, the trace point should be here and for two, the station point is here. So, to get the inverted position of the station points, we know this offset means, the cam is rotating in the counter-clockwise direction.

Corresponding to station point 1, the cam rotation is known which is θ_1. So I get the inverted positions of this trace point corresponding to 1, by rotating O_2a_1 to an angle θ_1, but in the clockwise direction. So if we rotate this by θ_1, this we call inverted position of the station point 1, which is 1′. Similarly, for this 2, we rotate about O_2, this line O_22 to an angle θ_2 in the clockwise direction. So θ_2 and we get the inverted position of this trace point as 2′. Similarly we do for all the station points namely 1, 2, 3, 4 like that. These are the inverted position of the trace point. What is the trace point? Trace point is the roller center and the roller radius is given. So corresponding to 0, this is 0; so I draw the roller whose radius is given.

Here again, with this point as center, I draw the roller; with this point as center, I draw the same roller. So this way, I keep on drawing roller, this is the say three′, the inverted position of this trace point corresponding to third station point. Now what is the cam profile? Cam profile is nothing but the curve which is envelope to this family of circles representing the roller after kinematic inversion. So I have to draw a curve which is tangent to the series of circles. If I draw a curve which is tangent to all these circles, that will give me the cam profile. So the methodology is just the same as the analytical method; I make a kinematic inversion with the cam fixed, get the inverted position corresponding to that trace point, draw the family of straight lines or circles representing the flat-face of the roller follower. Then, the cam profile is nothing but the envelope to this family of circles, in case of roller followers or in case of flat-face follower to the family of straight lines, representing the follower face at the inverted positions. So this principle of kinematic inversion is very important that we hold the cam fixed and allow the fixed link to rotate in the opposite direction. The cam is rotating in the counter-clockwise direction; so this follower axis for translating roller follower is rotated in the clockwise direction. That rotation can be done very easily geometrically, by rotating this line O_2a_1, O_22, O_23 by θ_1, θ_2, θ_3 in the clockwise direction. That gives me the location of this trace point on the roller centers.

Once we get the roller centers, we can draw the rollers and cam profile is enveloped to this family of circles representing the roller fixed. In fact, if we can imagine, we can draw another envelope touching the circles on the outer side. This was touching the circles in the inner side. Then, I can use this as the groove in the cam face where we do not need a spring. As I told you earlier that without the spring, for returning the follower we can use a groove in the cam face of the cam and put the roller in that groove. Then cam will always push it up and pull it down, there is no need to have a spring. Actually, this curve is nothing but what we call pitch curve. So this is the cam profile, if we have a spring and this groove is the groove where I have to put the roller, when I do not need to use the spring using a roller follower. Next, we will show how to use the same principle to obtain the cam profile for an oscillating follower either roller or flat-face.

(Refer Slide Time: 33:32)



Let me now explain the graphical method of cam profile synthesis with reference to an oscillating follower. Here again, we shall discuss both roller follower and flat-face follower. First let me take a roller follower. Of course, we assume all the required geometric dimensions like r_b, r_R - all these things are already known and the displacement function or displacement diagram is given to us. So for the roller follower, let me first draw the prime circle. This is the prime circle radius which is as we have noted earlier, base circle radius plus roller radius. When the roller center or the trace point is in contact with the prime circle, that is the extreme position or the lowest position of the follower, from where we measure the movement of the follower. This point is called O_2.

Suppose this is roller center at the lowest most position, this is the roller and the roller follower is hinged here to the fixed link, this point we call O_3. When the cam rotates in the counter-clockwise direction, let's say this follower also goes in the counter-clockwise direction and this rotation ψ as a function of θ is known. The trace point will obviously go on this circle with center at O_3 and this as radius. This will move on this circle. So if we take station points corresponding to θ_1, θ_2, θ_3, then we will know the corresponding values of $\psi(\theta_1)$, $\psi(\theta_2)$, and so on. So corresponding to station point 1, let's say the trace point is here, that means, this rotation of the follower is $\psi(\theta_1)$. Similarly, we can get all the trace points corresponding to station point 1, station point 2 and so on.

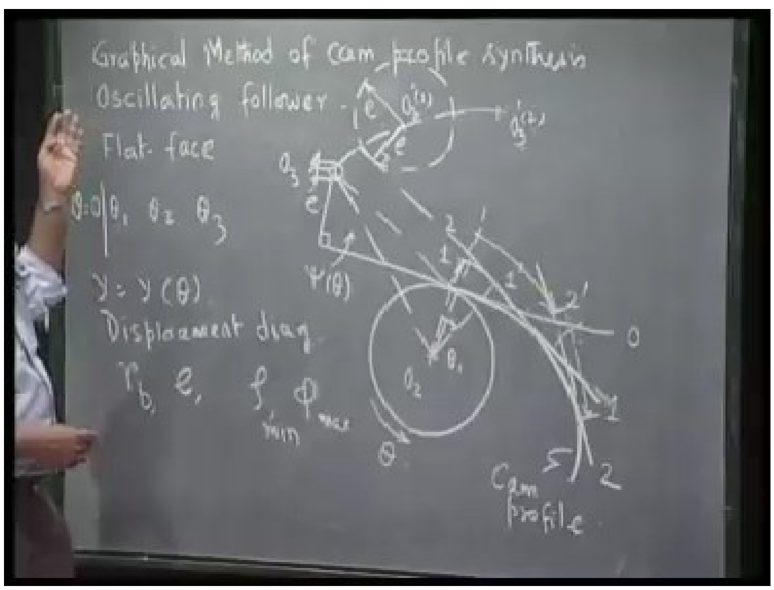
Now as before, we make a kinematic inversion. To draw the cam profile, we hold the cam fixed, which means this fixed link O_2O_3 has to rotate in the clockwise direction. So corresponding to station point 1, the inverted position of O_3, this is the circle with O_2 as center and O_2O_3 as radius. And after the kinematic inversion with the cam fixed, this O_3 will go on this circle. So corresponding to station point 1, when the cam has rotated θ_1 in the counter-clockwise direction, this fixed link will rotate in the clockwise direction by an angle θ_1. So I can say O_3 corresponding to one and the inverted position. Now the distance of the trace point from this hinge does not change. So distance of O_31 is should be same as $O_3^{\square}$ 1′ and the distance of 1 from O_2 must be such that this required or desired follower movement is reproduced.

So to get the inverted position of one, I draw a circular arc with O_2 as center and O_11 as radius. And from $O_3^{(1)}$, I draw a circle with O_31 as radius so and where these two circles intersects; that is nothing but the inverted position corresponding to station point 1, that is the roller center. So at this point, I can again draw the roller whose radius is given to us. Let me again illustrate it with corresponding to station point 2, we draw a circle with O_2 as center and O_22 as radius. 2′ will lie on this circle and $O_3^{(2)}$, the inverted position of O_3 corresponding to station point two is here, on this circle such that this angle is θ_2. Now O_32 is so much, so from $O_3^{(2)}$, I take the same radius as O_32 and draw a circular arc. So that let it be here. The distance O_32 is same as $O_3^{(2)}$ 2′. So this is the inverted position of the trace point, that is the roller center. So here again, I draw the roller of same radius. And now if I draw a curve which is tangent to this family of circles, that gives me the cam profile.

In fact, very simple geometrical consideration can show that this point we call $1'$, I can easily get if I rotate O_21 in the clockwise direction by θ_1, I get to $1'$. Similarly, if I rotate O_22 through an angle θ_2 in the clockwise direction, I get to $2'$. This can be very easily proved from congruence of triangles. We consider a triangle O_2O_31, and $O_3^{(1)}1'O_2$; we can show that very simply. Instead of going through this by drawing two circular arcs, I can as well rotate O_21 by θ_1 in the clockwise direction to get $1'$; O_22 by θ_2 in the clockwise direction to get $2'$ and then draw the rollers and then draw an envelope to this family of circles.

This I did to explain the kinematic inversion. It is actually the fixed link which is rotating, that means O_3 is going along this circle in the clockwise direction, when I hold the cam fixed. But, from congruence of triangle, geometric construction can be much simplified, by just not doing all these; only rotating O_21 through this θ_1 and O_22 through θ_2. Next, we will take up the case of a flat-face follower.

(Refer Slide Time: 43:27)



So, let me now explain the graphical method for an oscillating follower, which is flat-face type. Let's say this is base circle radius and the flat-face is tangential to this base circle radius at the lowest point. And this is where the follower is hinged. In the analytical method, this dimension is what we call e. As we see, this is one rigid body and as this thing rotates this angle always remains 90^0, which means the flat-face is always perpendicular to this line.

Let me use this particular contact point as the trace point and as the follower oscillates in the counter-clockwise direction, this particular trace point of the follower moves on a circle with O_3 as center, this as radius. This trace point moves on this side and where will it be? It will be decided by the rotation of this, this is what we call O_2. So once this is prescribed, the movement of the follower again I consider station points θ_1, θ_2, θ_3, so on. This situation is shown for θ equal to zero. When it is θ_1, the cam has rotated by θ_1, this trace point goes here. So I call it as one, similarly for two, it comes here at two.

To draw the cam profile, I make use of the same principle of kinematic inversion holding the cam fixed. As I explained earlier for the roller follower, the inverted position of one can be easily obtained by rotating $O_2 1$ through an angle θ_1 in the clockwise direction, because this O_2O_3 moves in the clockwise direction. So this is what we call 1′. The next question is how I draw the flat-face passing through this point 1′, that is flat-face in the inverted position.

For that, O_2O_3 I rotate as before, the O_3 inverted position is say here, which I call $O_3^{(1)}$. Now here I draw a circle of radius 'e' with point $O_3^{(1)}$ as center. This is of radius 'e' and the trace point is here. We know the follower face will be always perpendicular to this offset e, which means this will be tangential to this circle, offset e is same and from 1′, I draw a tangent. This represents the follower face after kinematic inversion. Similarly, I can rotate $O_2 2$ through an angle θ_2 to get the inverted position 2′. Then I go to $O_3 2$ wherever that is, by rotating O_2O_3 through θ_2 in the clockwise direction.

Then, again we draw this circle of radius e and from 2′ we draw a tangent to that circle. Procedure is same; I go from 2 to 2′ by rotating $O_2 2$ through an angle θ_2 in the clockwise direction; that gives me this point. I rotate O_2O_3 through θ_2 in the clockwise direction that gives me $O_3^{(2)}$. Then, with point $O_3^{(2)}$ as center and e as radius, I draw a circle. From this point, we draw a tangent to that circle and that tangent let it be this, that represents the inverted follower face. Now a curve which is tangent to all these inverted positions of the follower face, an envelope to these inverted positions of the follower face, this was corresponding to one, this was corresponding two, this is corresponding to zero.

So once I get the inverted positions of the follower faces which is a family of straight lines, then draw an envelope to this family of straight lines, that is nothing but the cam profile. What we should note that here, the contact point is one, but here the contact point is not 1′, the contact point has shifted on the follower face as we have seen in case of while we discuss the analytical method. Similarly, this is the inverted position 2′, but the contact point is here. So if we measure what is the maximum distance of the contact point from this trace point on this side, that will give me the required follower which on the right hand direction. To the right of this contact point, how much the contact point moves, so that we must have the required follower face.

Similarly, if we continue, we will see, now it will be to the left of this also; right now it is to the right. For these three positions, 0 it is alright; 1 it is here, but the contact point is here; 2 the point is here, but the contact point is here. So, this is the shift of the contact point on the follower face in position 2. If we complete it, you will see that this contact point shifts in one direction, then returns and then shifts in the other direction and the maximum distance that is needed for the contact point to move on the follower face, that decides the minimum width of the follower face.

So that brings us to the end of cam-follower mechanisms and let me summarize what we did. We first explained the different terms which are needed to describe the cam follower mechanism. After that, we described what are the basic follower movements. We had both the analytical expressions and the graphical construction for the displacement diagram. Follower motion, either y expressed as a function of θ or through displacement diagram.

Then, we showed how to get the basic dimensions like r_b, e, etc., so that, certain constants like minimum radius of curvature or maximum value of the pressure angle, all these conditions are satisfied. We would always obviously like to go for the smallest value of r_b so that we can get away with the small cam size. And after getting these basic dimensions, we discussed both the analytical and graphical methods to obtain the cam curve or the cam profile and that is the synthesis problem. Of course, given the cam curve, we can also find what should be the follower movement; that is what we call analysis of the follower movement. But that is purely a geometrical problem which is quite simple; even we can do analytically or graphically and we are not going to discuss that in this course.