Kinematics of Machines

Prof. A.K. Mallik

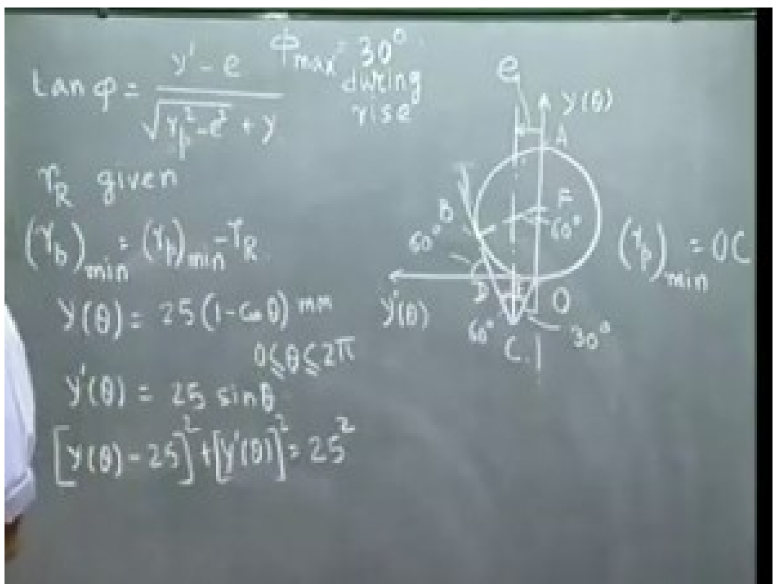
Department of Mechanical Engineering

Indian Institute of Technology, Kanpur

Module - 11 Lecture - 2

We finished our last lecture with the geometrical construction, which expressed the pressure angle for a translating roller follower which is given by this expression.

(Refer Slide Time: 00:23)



Today, we will take an example to show how from this geometrical construction we can get the optimum values of the prime circle radius and the offset. Of course, the roller radius r_R is assumed to be given and if I get r_p the minimum value, then base circle radius minimum $(r_b)_{min}$ can get from $(r_p)_{min} - r_R$.

I take an example, say the follower displacement $y(\theta)$ is given by $25(1 - \cos\theta)$ mm and this expression is valid for all values of theta, that is the entire cam rotation from 0 to 2π. If this is $y(\theta)$, then differentiate it once with respect to θ, I get $y'(\theta) = 25 \sin\theta$.

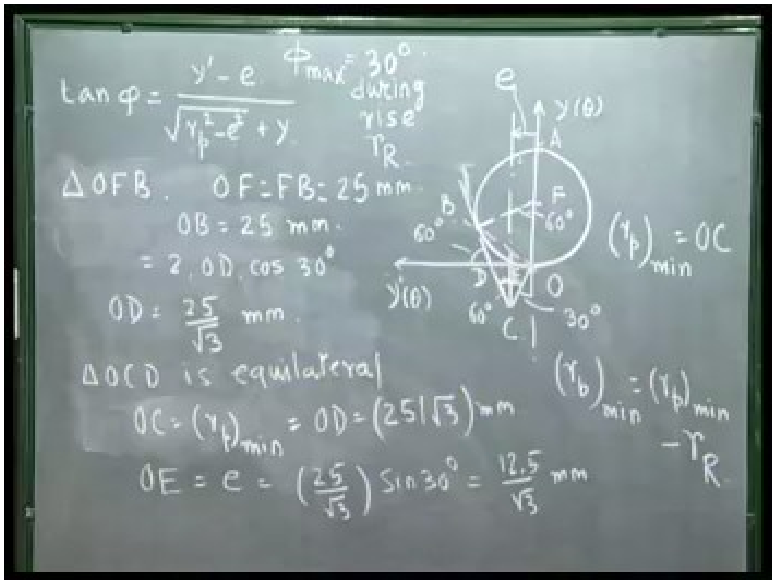
From these two expressions of $y(\theta)$ and $y'(\theta)$ it is very easy to see that, $[y(\theta) - 25]^2 + [y'(\theta)]^2 = 2^2$. The plot of $y(\theta)$ vs $y'(\theta)$ will be a circle of radius 25 and centre at y equal to 25 and y′ equal to 0. If we remember $(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle of radius r with centre at a and b.

If I plot, $y(\theta)$ vs $y'(\theta)$ satisfying this equation and that equation is the circle with centre at y equal to 25 and y′ equal to 0 and radius equal to 25. Now let us say, φ_{max} is prescribed as 30^0, during the rise. We are not bothered about the pressure angle during return. During the rise phase φ_{max} must be limited to 30^0 i.e., maximum value of φ should not go beyond 30^0.

In this diagram, as we see from O to this point A, y′ is positive, that is the rise phase and this is the return phase when y′ is negative. So during this rise phase, the value of φ should never exceed 30^0. To keep that maximum value 30^0, we draw a tangent to this circle such that the tangent is inclined to the vertical by an angle 30^0. This line is inclined to the vertical by 30^0 that is this angle is 60^0 and from here I again draw a line at 30^0 to the vertical and these two lines intersect at the point C. Then if I draw a vertical line through C, this gives me the optimum amount of offset and OC gives me the minimum value of r_p, because if we recall the last lecture, any point we take on this curve and join it with C, then that line gives you the inclination of that line from the vertical gives you the pressure angle.

So as we see from this point to all other points, the inclination to the vertical never goes beyond 30^0. Similarly, in this phase and this phase, whatever lines I draw the inclination to the vertical never exceeds 30^0. This is also 30^0, so we get this angle as 60^0. Let me now name these points: this point, vertical line and the y′ axis, this point, let me call E and this point, let me call D and this point of tangency, let me call B and the centre, let me call F. As we see, this angle is 120^0. This angle is 90^0. This angle is 90^0. So, this angle also is 60^0.

(Refer Slide Time: 07:01)

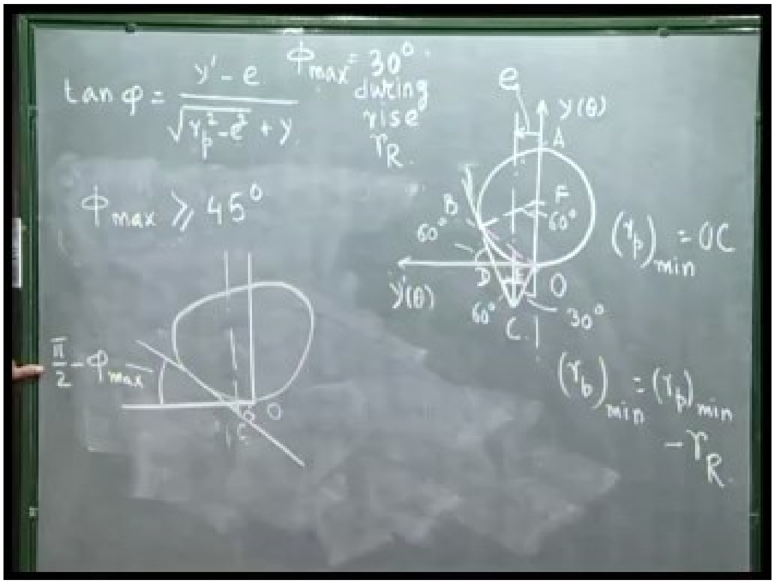


I draw this line just for the sake of construction OFB. In triangle OFB, I get OF is FB is equal to the radius of this circle which was 25 mm and this angle is 60^0. So this is an equilateral triangle. So I can write OB also 25 millimeters. Now if I drop a perpendicular from here, it is easy to see that OB is nothing but 2 OD into, this angle is 30^0, this is 120^0 these two are equal, so 30^0 and 30^0, so OB = 2 OD cos30^0 and cos 30 = √3/2, so OD = 25/√3 mm.

Now here we see, all three angles are 60^0. So this is again an equilateral triangle OCD and all sides are equal. So OC, that is $(r_p)_{min}$ turns out to be same as OD which is 25/√3 mm. And the optimum amount of offset is OE. OE = OC sin 30^0. So, e_{opt} = (25 sin 30^0) /√3 = 12.5/√3 mm. So we got both $(r_p)_{min}$ and e and once we get $(r_p)_{min}$, we can easily get the base circle radius minimum value which is $(r_p)_{min} - r_R$ and the roller radius have to be specified. So this is given and also we have to give the roller radius.

So this is a simple diagram of y(θ) vs y′(θ), which we get a circle. So everything we could calculate analytically, but if y(θ) is a complicated function, it changes during rise and then dwell then again return, then again dwell, whatever it is, we can always calculate for every value of y′(θ) and get a closed curve and do the same construction. Draw a tangent here and take a line exactly at φ_{max} with the vertical.

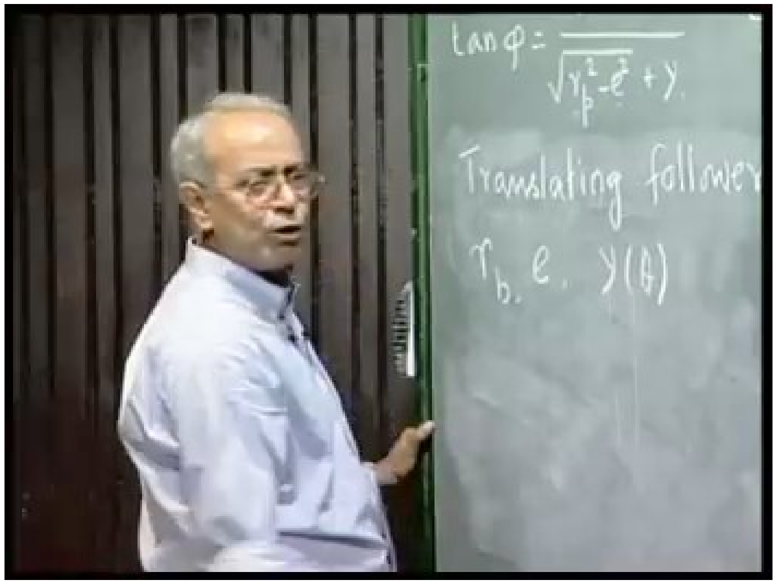
(Refer Slide Time: 11:11)



Now if we consider a very theoretical situation, that is if $\varphi_{max} \geq 45^0$, which normally never is, but suppose if $\varphi_{max} \geq 45^0$ then $(r_p)_{min}$ calculation would have been a little different. If this is the closed curve, then I draw a tangent at an angle φ_{max} to the vertical, so this angle is $\pi/2 - \varphi_{max}$. Then to get minimum value of OC, that is r_p, I just need to draw a perpendicular to this line. If this is O, this point would have been C, because in this case the pressure angle during this part of the rise is automatically guaranteed to be less than 45^0. I have to ensure only that during this phase, φ_{max} does not go beyond 45^0 because if this angle is $\pi/2 - \varphi_{max}$ so this angle is φ_{max} and if this is less than 45^0 and this is 90^0 so this angle is automatically less than 45^0. Since φ_{max} is more than 45^0 so this angle will be automatically less than 45^0.

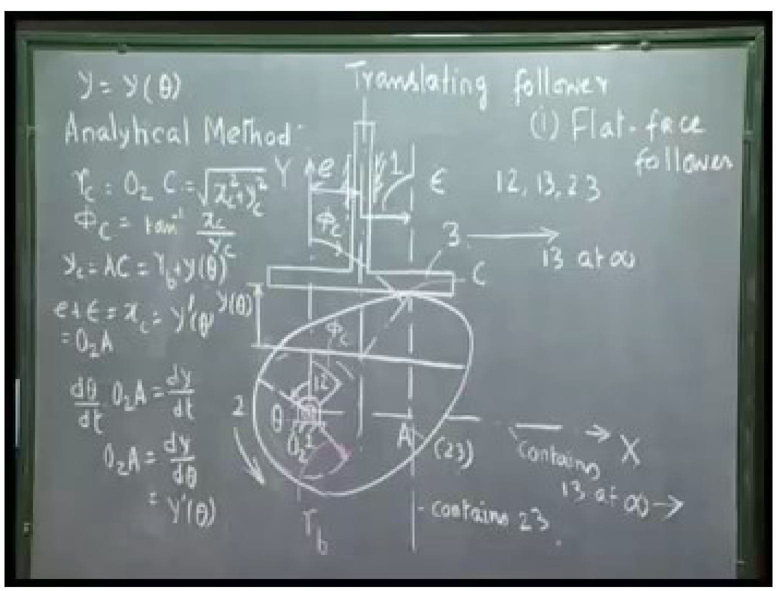
If this angle is $\pi/2 - \varphi_{max}$, then this angle is φ_{max} and if this φ_{max} is more than 45^0, then this angle is automatically less than 45^0. So during this phase and this phase, pressure angle is ensured to be less than 45^0. But this is only theoretical because φ_{max} we never take such a large value. For φ_{max} less than 45^0, I have to draw a tangent which is inclined to the vertical by φ_{max} and again from O I draw a line which is inclined to the vertical by φ_{max} and wherever they intersect that determines the point C and OC gives me the minimum value of r_p and the vertical line if I draw the distance between the Y-axis and this vertical line gives me the optimum offset. So, we have finished our discussion to get the basic dimensions for cam profile for translating follower, both roller and flat face.

(Refer Slide Time: 13:45)



Our next task will be to design or synthesize the cam profile, when we are given or determine r_b and e. To generate a particular follower motion, what should be the cam profile? That is the discussion that we will take up next.

(Refer Slide Time: 14:29)



Now that we know how to size the cam, that is how to choose the value of the base circle radius and the proper amount of offset, the next task is to synthesize the cam profile to generate the desired follower motion given by $y = y(\theta)$. We start with analytical procedure. For the time being, we are restricting our discussion to translating follower and first we discuss for flat face follower.

This is flat face follower. This is the cam with the cam shaft here. This is called the offset. Suppose the cam has rotated by θ in counter clockwise. If we draw the base circle of this cam, this is the base circle radius (r_b). So when the follower touches the base circle that is the lowest position of the follower, this tangent at the top most point of the base circle is the lowest position of the follower.

This is the contact point, when the cam has rotated by an angle θ and consequently, the follower has moved by this distance which is $y(\theta)$. The contact point let me call C and through this cam shaft O_2, let us take X-axis and Y-axis. First of all, our objective is to find the polar coordinates of the contact point C. To find the polar coordinate that is O_2C and say I measure the angle from Y-axis in the clockwise direction that is this angle which we call φ_c. So the polar coordinates of this contact point r_c is O_2C and the other coordinate is φ_c. As we see φ_c is measured from the Y-axis in the clockwise direction. To find this r_c, we will first find what are X and Y coordinates of the point C. This is the distance which we called eccentricity of the driving effort ε and this point let me call A. So y_C is AC and that is easy to see that $AC = r_b + y(\theta)$.

If we recall yesterday's lecture, this distance O_2A we found out as $e + \varepsilon$ which is x_C was shown to be $y'(\theta)$. The x-coordinate which is O_2A. It can be shown very easily that this distance is $y'(\theta)$. Suppose, I use the Arnold-Kennedy theorem of 3 centers this is the fixed link. There is a guide 1, cam is 2 and the flat face follower is 3. It is a three link mechanism consisting of the fixed link, cam, which I have 2 and this follower which have 3.

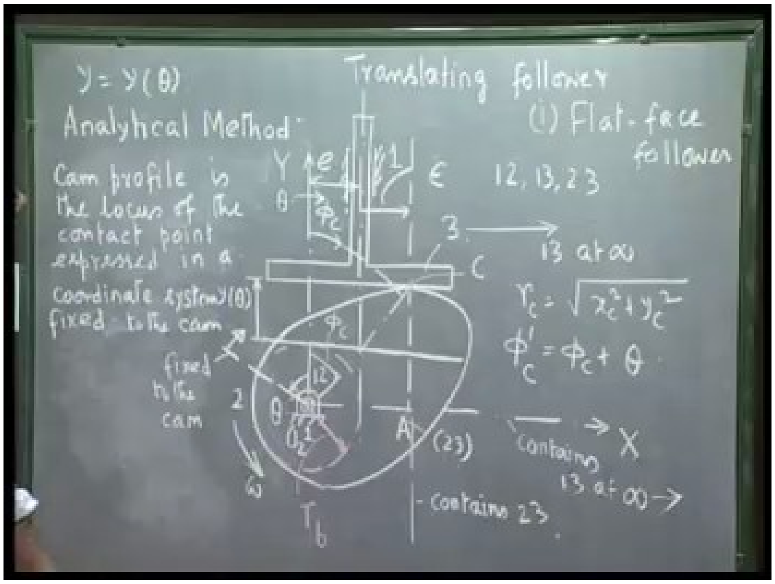
Out of which 12 is at O_2 because there is a revolute pair, the cam can rotate with respect to the fixed link about this axis. This O_2 is nothing but 12, 23 is on this common normal or that is on this vertical line. So this line contains 23 and because 3 is in vertical translation with respect to 1, 13 is on the horizontal direction at infinity. So if I draw a horizontal line through O_2 that is X-axis, this contains 13 at infinity and now I know these 3 relative instantaneous centers, namely 12, 13 and 23 are collinear by Arnold-Kennedy theorem. So 12 is here 13 is at infinity, 23 is on this line. So this point A is nothing but 23. That is the velocity of this point A, if I consider it to be a point on the cam, that is on body 2, whatever velocity I get, it will be the same velocity at this point for the body 3, that is the follower.

So if I consider it to be a point on body 2, the angular velocity of the cam is $d\theta/dt$ O_2A in the vertically upward direction and if I consider to be a point on the follower and follower is in perfect translation in the vertical direction. So all points of the follower have the same velocity. So this same velocity is of the follower, which is dy/dt. So that clearly tells me, O_2A is nothing but $dy/d\theta$ which is $y'(\theta)$. For given $y(\theta)$, I can find both the x - coordinate of the contact point and the y - coordinate of the contact point, x_C and y_C. $y_C = r_b + y(\theta)$, when r_b is known, $y(\theta)$ is known, we can find y_C. If $y(\theta)$ is given, I can find $y'(\theta)$ and $x_C = y'(\theta)$. So $r_c = \sqrt{(x_C)^2+(y_C)^2}$.

What is φ_c? This angle nothing but, $\tan^{-1}(x_C/y_C)$ because this is angle measured from the vertical. So $\tan \varphi_c = x_C/y_C$. This is x_C, this is y_C and this angle I am calling φ. Do not confuse it with the pressure angle, that's why I'm writing φ_C, C is the contact point. Now, this is the polar coordinates of the contact point in the fixed frame of reference in this XY axis, which is fixed in space that is, that belongs to this fixed body 1.

But what is the cam profile? Cam profile is nothing but the locus of the contact point, but it has to be expressed in a coordinate system fixed in the cam. So, I fix a coordinate system in the cam. Suppose this line in the cam, I take as my fixed line.

(Refer Slide Time: 24:34)



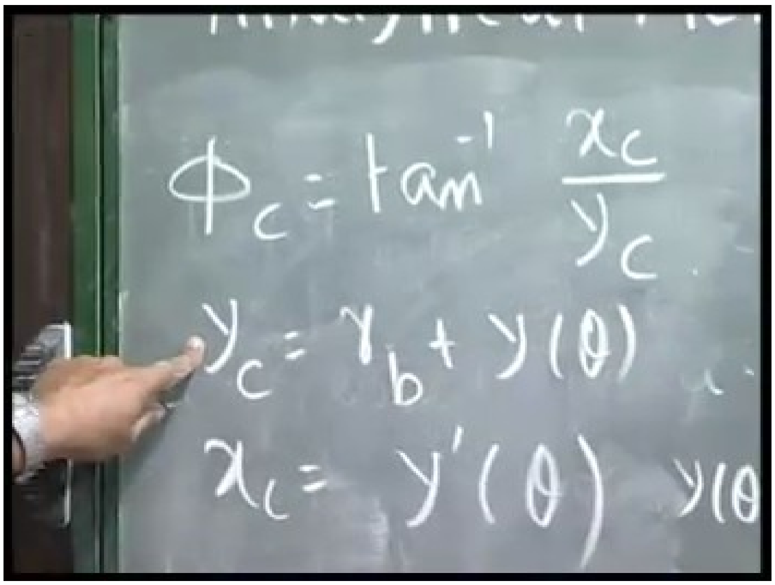
Cam profile is the locus of the contact point for various values of θ as the cam rotates, contact point expressed in a fixed coordinate system to the cam. Now how do I express these coordinates of the point C in a cam fixed coordinate system? If I fix the cam, then it is a nothing but a kinematic inversion. So the fixed link 1 rotates in the clockwise direction with the same value of ω. If the cam is rotating with constant angular velocity ω and if I hold the cam fixed the same kinematics is maintained.

If the fixed link is rotated in the opposite direction by the same amount, that is kinematic inversion. This is the relative motion of the cam rotation with respect to the fixed link in the counter clockwise direction. Fix the cam, that means kinematic inversion implies the fixed link 1 will rotate in the clockwise direction with the same angular velocity. That means this line fixed in space will rotate in the clockwise direction by theta.

So what is the angle of this, if I measure it from this line fixed to the cam. This angle is θ and from this Y-axis which is rotating, so this angle will be always measured by θ because this line has rotated from the fixed line by an angle θ in the counter clockwise direction. If I fix the vertical axis, Y-axis rotates in the opposite direction, with the same value of θ. So if I want to use the cam fixed coordinate system, then I write $r_c = \sqrt{(x_C)^2+(y_C)^2}$, but I write $\varphi_C^{\square}$, which is the angle measured from this line fixed to the cam, for this line O_2C, that is $\varphi_C^{\square} = \varphi_c + \theta$ and it has to be measured in the clockwise sense.

We already have the expression of r_c and φ_c. So I get the cam profile in the polar coordinate system. I plot all these points by measuring from the O_2, this distance r_c and measuring the angle of that line O_2C, from this line fixed in the cam, by this $\theta + \varphi_c$, where φ_c is nothing but tan inverse x_C/y_C which I wrote earlier.

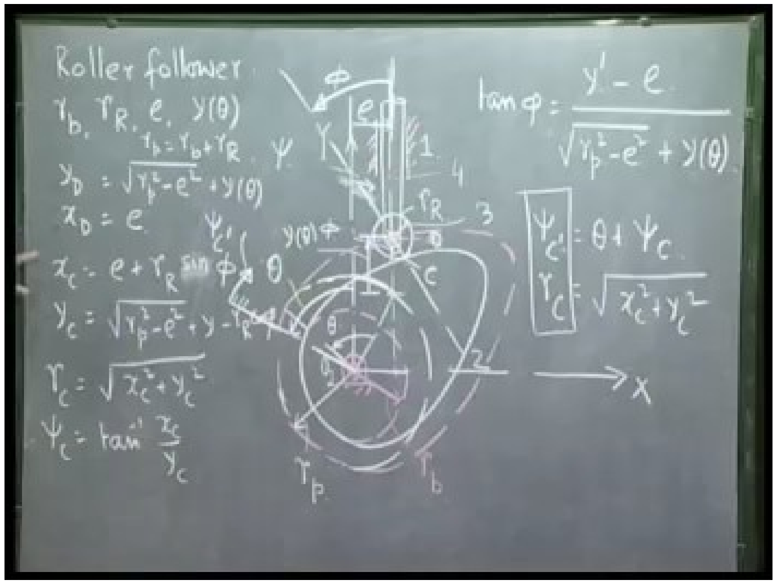
(Refer Slide Time: 28:02)



$\varphi_c = \tan^{-1}(x_C/y_C)$ and if we remember $y_C = r_b + y(\theta)$ and $x_C = y'(\theta)$. For given r_b and $y(\theta)$, we get x_C, y_C and we get φ_c then we can get r_c and $\varphi_C^{\square}$ and to get the polar coordinate representation of the cam profile with O_2 as the origin. I will continue this same technique for the roller follower.

Just now we have discussed the analytical method of cam profile synthesis for a flat face follower. What we did, we obtained the polar coordinates of the contact point and expressed those polar coordinates in a coordinate system fixed with the cam and that gives you the representation of the cam profile. We will extend exactly the same technique for a translating roller follower.

(Refer Slide Time: 29:04)



We have been given r_b, r_R, e and the desired motion of the follower y(θ) and we have to obtain, what is the curve representing the cam surface or the cam profile? We first draw the cam. This is the cam shaft axis O_2 and this circle is as usual represents the base circle of radius r_b. Suppose, the roller follower is offset that is the roller centre is here. The roller radius is given to us as r_R. This is the guide of the fixed link 1, this is the follower 4, this is the roller 3, this is the cam 2.

We have also seen that we can draw, what we called pitch curve, and this is the prime circle with prime circle radius, r_p. As before, we take X and Y-axis through O_2, measure φ in the clockwise direction. To not confuse this φ as the pressure angle, we use a different symbol here, so the polar angle, I call ψ. This is the roller centre. This contact point, let me call C and the roller centre, I call D. Our job is to find, what is the polar coordinates of this contact point C?

So through C and the roller centre if I draw a line, that is the common normal and this angle we define as the pressure angle φ. When the roller centre intersects the prime circle, that is the lowest point of the follower and this is the movement of the follower, when the cam has rotated to θ. So this is y(θ), and this angle is θ.

So let me first, try to get the x and y-coordinates of this contact point C. For the roller centre, I can write coordinates of y_D and x_D. First let me get to the roller centre. x_D is very simple, that is nothing but the amount of offset. This distance is e. So x_D = e and y_D is this distance + y(θ). This is a right-angle triangle with hypotenuse r_p and this horizontal side e, is nothing but $\sqrt{(r_P)^2+e^2}$. So the vertical side is square root of $r_p^2 - e^2$. So this is the vertical side + the roller centre has moved from here to there by amount y(θ). So $y_D = \sqrt{(r_P)^2+e^2}+y(\theta)$.

Now if we get to the coordinate of D, then that of C, I can easily see $x_C = e + r_R \cos \varphi$. As this angle is φ and this is r_R, so extra horizontal distance is $r_R \sin \varphi$ and y coordinate is below that level D and by $r_R \cos \varphi$. So $y_C = y_D - r_R \cos \varphi$. So $\sqrt{(r_P)^2+e^2}+y(\theta) - r_R \cos \varphi$, where φ is the pressure angle and we already got an expression for $\tan \varphi = \frac{(y' - e)}{\sqrt{(r_P)^2+e^2}+y(\theta)}$ I get the x and y-coordinate of the contact point, in this XY coordinate system, because e is known to us. Given the function y, we can calculate this and get the value of φ for values of θ. Once we know φ, we can get x_C which is $e + r_R$ sine φ.

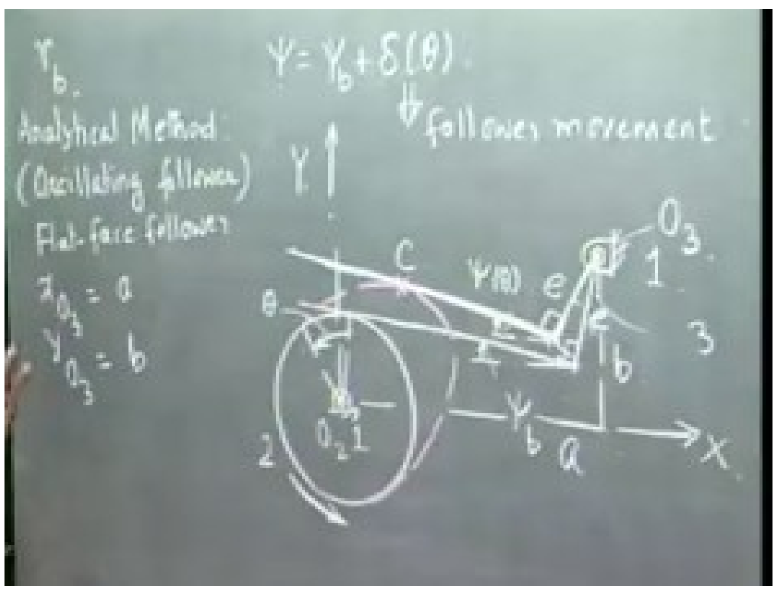
Similarly, given the quantities y_C, I can get because I know the value of φ from here, I can calculate cos φ. rp is nothing but $r_b + r_R$ that is known, $r_b + r_R$. So I get x_C and y_C. So the polar coordinates of this point C, $r_c = \sqrt{(x_C)^2+(y_C)^2}$ and $\psi_C = \tan^{-1}(x_C/y_C)$. Now this is the contact point polar coordinates in the fixed frame of reference but the cam profile is nothing but the locus of the contact point in the cam fixed system. So ψ is measured from this vertical line, but this vertical line which is the line belonging to the fixed link 1, after kinematic inversion, that is if we hold the cam fixed, we will rotate in the clockwise direction. So if this is the configuration at θ, so I draw a line at an angle θ from the vertical and this line I hold fixed. So as a consequence, this Y-axis rotates in the clockwise direction and that angle will be always theta.

So what will be the polar coordinate in this cam fixed system? If the angle is measured from this line, which I called as before $\psi_{C'}$. So $\psi_{C'}$ is nothing but $\theta + \psi_c$ and $r_c = \sqrt{(x_C)^2+(y_C)^2}$. So these are the polar coordinates representation of the contact point in a coordinate system fixed in the cam. This line, which I am holding the cam, when the cam has rotated by an angle θ, I draw this line at an angle θ counter clockwise direction from the vertical. I measure this $\psi_{C'}$, from this line in the clockwise direction.

I get the coordinates x_C, y_C because the pressure angle φ, I can always get once y, y', e and r_p, these quantities are known. So for various values of θ, I can get x_C and y_C. So for various values of θ, I can get r_c, and for various values of θ, I get first ψ_C and then add θ to get $\psi_{C'}$. And if I go on plotting these values r and ψ, r is the distance from O_2 and ψ is measured from this line, which has been drawn at an angle θ from the vertical line. Then plotting all those points and the locus of all these points will give me the cam profile. So this is how we get the analytical expression for the cam profile for a translating roller follower.

So far we have discussed in great details, the determination of the basic dimensions and the cam profile synthesis with reference to translating follower.

(Refer Slide Time: 41:11)



For the oscillating follower is concerned, determination of basic dimension is a little more involved. So what we will do, we will assume that the basic dimensions like base circle radius is determined. It is already given to us and the desired follower motion is prescribed and how to synthesize the cam profile or how to determine the cam profile by analytical method? So we continue our discussion with analytical method: but with reference to oscillating follower. Here again, I will restrict our discussion, only to a flat face follower. Exactly same methodology can be applied to roller follower, but the algebra involved is a little different. But the methodology is just the same what we follow for flat face follower is equally applicable for a roller follower.

Now first let me discuss, how do we describe or how do we express the follower motion incase of an oscillating follower? As we know, the size of the cam is described by this base circle radius. Let's say this is the base circle radius of a cam, this is cam shaft axis. Now when the oscillating flat face follower is in contact with the base circle that is the extreme position from where the follower motion is measured or the rise of the follower starts. Let us assume that the follower face which is in contact with the base circle radius is represented by this line and this follower is hinged at this point. This is an oscillating follower. So it is hinged at this point. There is a revolute pair with the fixed link and the whole follower will oscillate about that point. So I draw a perpendicular to this follower face. This is 90^0. I represent the follower by these two straight lines and these gives me another kinematic dimension which is necessary. So here as we know the fixed link is 1 and cam is link 2. This is the base circle of the cam and this is the follower which is link 3. So let me call this revolute pair at O_3.

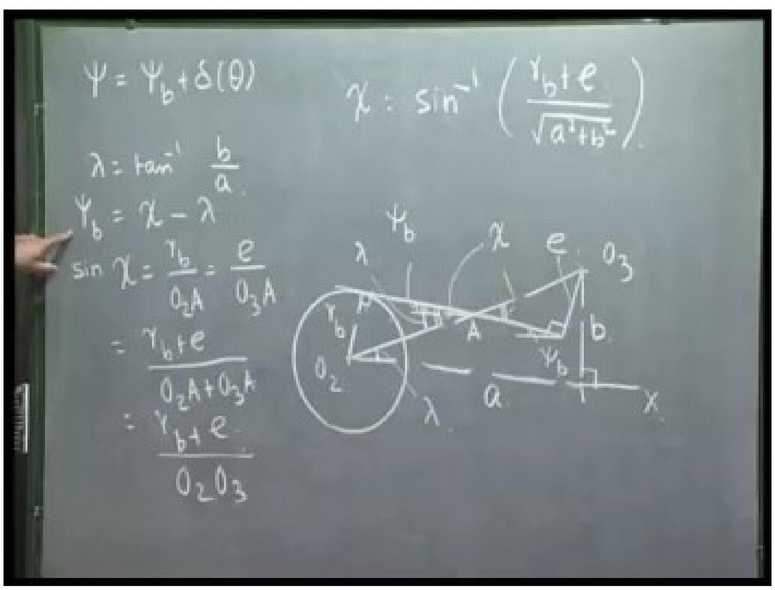
Now due to the shape of the cam, as the cam rotates this point was here and the follower was at this position and now follower will be in this position and if I drop a perpendicular to this line this dimension remains e. This is moving in a circle. So this rigid body rotates in the clockwise direction as the cam rotates in the anticlockwise direction. This is the basic motion geometry of the flat face follower which is oscillating and as we see, this distance changes because the contact point on the follower face moves.

Now how do I express this particular follower motion? Suppose I draw a horizontal line, this is the lowest position of the follower which I express by this angle ψ_b and when the a follower has rotated in the counterclockwise direction, this I called $\psi(\theta)$. So $\psi(\theta) = \psi_b + \delta(\theta)$, some angle which is a function of θ. It is this $\delta(\theta)$ which represents the follower movement. Instead of a trace point, I am talking of this follower movement in terms of this angle directly. That is more convenient. We should also note that we have already got another kinematic dimension namely this e which is perpendicular drop from O_3 to this follower face which remains same because this is the same rigid body. This angle is always 90^0.

Now the relative position of O_2 and O_3 and these two revolute pair is also two important kinematic dimensions. Suppose I draw X-axis here and Y-axis here. So the x and y-coordinate of O_3 are two more important dimensions, which I call it a and b, x-coordinate of O_3 is a, y-coordinate of O_3 is b, where O_3 is the hinge point for the oscillating follower and O_2 is the cam shaft, e is another kinematic dimension which has been obtained by dropping a perpendicular from O_3 to the follower face and then the follower is treated as this rigid body consisting of two straight lines at right angle. This is the lowest position when the follower was in contact with the base circle that is when this point was here. Now the cam has rotated through an angle θ. As the cam has rotated through an angle θ, the follower has rotated through an angle δ(θ) and the angle that this follower face makes with the X-axis or horizontal line that I write as ψ.

At θ = 0, δ(θ) = 0, and $\psi = \psi_b$. So δ(θ) is the follower movement which is the function of θ, that is how it prescribed. And our objective will be to find the polar coordinates of this contact point C which will define the cam profile when I express this coordinate in the cam fixed coordinate system. That is basically the method. But to do this, we need to do a little bit of elaborate geometry as we will see just now.

(Refer Slide Time: 49:11)



For an oscillating flat face follower, the angle that the follower face makes with the horizontal was expressed as $\psi = \psi_b + \delta(\theta)$. So before we try to get the contact point coordinates, first let me get the expression of the angle ψ_b, which the follower face makes at its extreme position when it is contact with the base circle. So let us say this is the base circle, with O_2 as center and the follower face is tangent to the base circle. This is the point O_3, where the follower is hinged and this angle is 90^0. The angle that this follower face makes with the horizontal, that we call ψ_b.

If we draw a line which is horizontal, this angle is ψ_b. Now the line O_2O_3 is a horizontal line this distance we call 'a' and this distance we call 'b'. This is 90^0. This is the X-axis. Now this angle is clearly seen to be $\tan^{-1}$ (b/a) and this angle is nothing but this angle. So this angle is $\tan^{-1} \lambda$. Lambda (λ) is the angle which this line is making with the horizontal. So this angle is same as this angle. This is λ.

So $\lambda = \tan^{-1}$ (b/a) and ψ_b is this angle. I can get ψ_b, if I find this angle and then subtract λ from there. So to find this angle, let me join this line this is at 90^0, this is at 90^0 and this angle is same as this. Let this angle be χ, then $\psi_b = \chi - \lambda$ and I can easily get χ, if I write this as the base circle radius (r_b), this dimension is e and this point let me name A, the point of intersection of the flat face at the lowest position with O_2AO_3. So this angle is equal to this angle. This angle is 90^0. This angle is 90^0. So from here, I can write $\sin \chi = r_b/O_2A$ and from here I can write $\sin \chi = e/O_3A$. So both are same.

Using this relation, we can write, $\sin \chi = (r_b + e)/(O_2A + O_3A) = (r_b + e)/O_2O_3$ and O_2O_3 is nothing but $\sqrt{a^2+b^2}$, because this angle is 90^0. So $\chi = \sin^{-1}\frac{r_b+e}{\sqrt{a^2+b^2}}$ and $\psi_b = \chi - \lambda$, where λ is this angle. The angle O_2O_3 makes with X axis that is $\tan^{-1}$ (b/a). So once these geometric dimensions like a, b, e and r_b are given, we are in a position to find the value of ψ_b.

In our next lecture I will take off from this point and we will assume this value of ψ_b which we can obtain given the values of a, r_b, e, b, etc. Then I will try to get the coordinates of the contact point between the flat face and the cam profile and that will give me the cam profile in polar coordinates as we did in case of translating follower.