Kinematics of Machines

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Module – 11 Lecture – 01

In this module we shall discuss synthesis of cam profile; that is determination of cam profile to generate the desired follower motion. We have already seen that the motion of the follower is described by the movement or displacement of the trace point.

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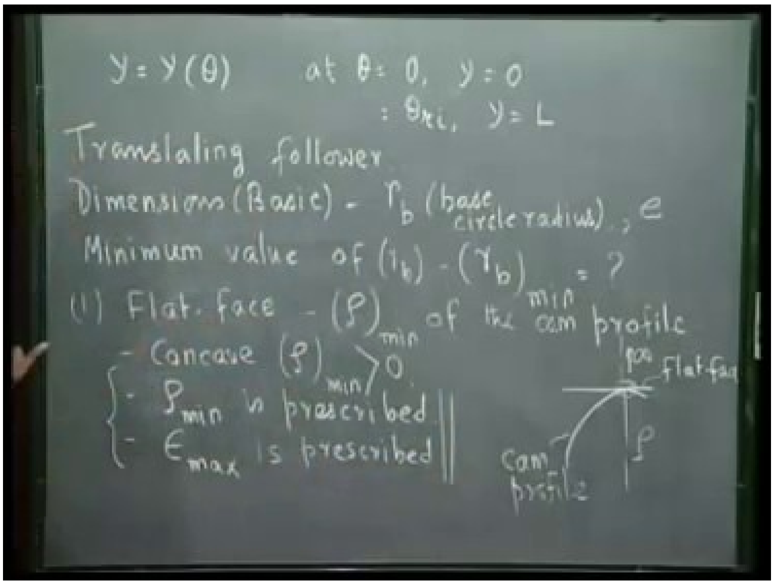

It can be described as a function of the cam rotation θ. y; the displacement of the trace point as a function of θ. This is given to us, that is the desired motion.

It can be given either in this analytical form or in the form of a displacement diagram if we go for graphical synthesis. It is known that this θ is measured from the beginning of the rise that is at θ = 0, displacement of the follower is also 0 and at $\theta = \theta_{ri}$, that is at the end of the rise, the displacement of the follower is the lift.

Now, we shall restrict our discussion for translating follower. However, before we get into the discussion of cam profile synthesis we shall see that to produce the desired motion of the follower, we cannot come out with a unique cam, unless we determine some basic dimensions of the cam. So first we determine the basic dimensions of the cam. As we will see, one is the base circle radius r_b, we have to first determine; what must be the base circle radius that is the size of the cam.

We have to come up with, what is the proper offset of the follower which we denote by e. This is the desired motion of the follower, we have accepted the follower to be a translating type, but before we come up with the cam curve that is the cam profile, we have to first determine; what should be the base circle radius of the cam and what should be the offset of the follower axis.

However, it is obvious that we should always go for minimum value of the base circle radius, because smaller the value of r_b, smaller is the cam size, with increasing r_b, the cam size also increases. Our objective will always be to use the minimum value of the r_b that we can use. However, we cannot choose this r_b minimum arbitrarily. Let us see minimum value of r_b, which we call $(r_b)_{min}$. The minimum value of r_b is decided by various considerations.

For example: one is: suppose we are talking of a translating flat-face follower. Now this minimum value of r_b of a flat-face follower will be governed by what is the minimum value of the radius of curvature of the cam profile. So we say $(\rho)_{min}$, where ρ is the radius of curvature of the cam profile. The cam curve will have different amount of radius of curvature at different points, but we cannot take the radius of curvature of the cam profile below this value of $(\rho)_{min}$.

For example, for a flat-face follower, it is obvious that the cam profile has to be concave which means ρ everywhere must be positive i.e., greater than zero, curvature is positive so $(\rho)_{min}$ must be greater than zero. That is not all, in fact, just zero is not sufficient, because as we will see, when two surfaces are in contact, suppose this is the flat-face and this is the cam surface.

Now, the stress here at this contact zone which we call contact stress is decided by the radius of curvature of these two mating surfaces. Like here, if this is the ρ of the cam profile and this ρ of the flat-face follower is infinite because it is a flat surface. So, the contact stress will be decided by this radius of curvature. So, the allowable contact stress level for the given force decides what must be the minimum radius of curvature.

We will not get into the detailed discussion of how to determine this contact stress and how does it depend on ρ; we shall assume that $(\rho)_{min}$ is prescribed from this contact stress point of view. For a flat-face follower, the $(\rho)_{min}$ of the cam profile is prescribed. Another consideration we know that the eccentricity of the driving effort should not be too large, because then there is a chance of the follower to cock inside its bearing guide and jam the follower. To prevent this jamming, we should also say what is the maximum allowable eccentricity of the driving effort which we denoted by ε.

So we also say ε_{max} is also prescribed. So, right now we discuss a translating flat-face follower. To determine the minimum value of the radius of the base circle and the offset that must be used so that these two conditions namely; minimum value of the radius of curvature of the cam profile and maximum allowable value of the eccentricity of the driving effort are not violated.

In our subsequent discussion, we shall assume $(\rho)_{min}$, ε_{max} and y(θ) are all given to us and find what is the minimum value of r_b and that of the offset for a flat-face follower.

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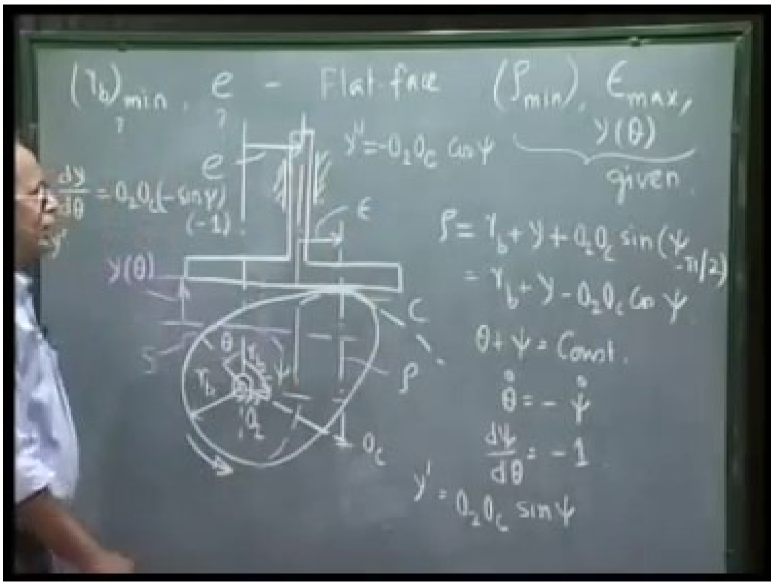

So, our next task is to determine the $(r_b)_{min}$. That is the minimum value of the base circle radius and the offset that we can use for a flat-face translating follower, when minimum radius of curvature of the cam profile and maximum eccentricity of the driving effort and the desired follower motion are all prescribed.

So, these are to be determined and these are given. As we said that this is a flat-face follower, this is a cam which is rotating in the counterclockwise direction, this is the base circle, and this is the base circle radius. The cam center is here, the camshaft axis, let me call it O_2, this is the follower axis along which the follower translates and this distance is called the offset in the positive direction for this rotation of the cam and that is e and this is the contact point.

The distance of the contact point from the guide axis, this is the eccentricity of the driving effort ε. The lowest position of the follower when the follower is in contact with the base circle is where the cam profile is touching the base circle. So, if we draw a horizontal tangent to the base circle, this is the lowest position of the follower when this point S was vertical.

This is the level from which the follower starts rising. At the beginning of the rise, contact was here when that contact was along the vertical line O_2S; O_2S was vertical, the cam has rotated by an angle θ and consequently the movement of the follower is given by this y which is this function of θ.

Now, let us see this contact point if we call it C, the center of curvature of the cam profile at C, let it be here which is O_C; O_C is the center of curvature of the cam profile at this contact point C. Now what do you mean by centre of curvature or the radius of curvature that for three infinitesimally separated time instants the circle with centre at O_C and O_CC has radius which is same as that of the cam profile.

So, if we draw the circle with O_C as centre and O_CC as radius, then this circle and the cam profile three infinitesimally separated positions are same on the cam profile and on this circle. So, we can say for three infinitesimally separate positions this O_CC which is the radius of curvature ρ does not change. So what is the expression for this ρ? That is this distance + this distance + this distance, this distance is y and this distance is nothing but r_b.

If we join these two points O_C and C and if we call this angle as ψ, then we can write the radius of curvature ρ is nothing but $r_b + y +$ this vertical height. That is, $\rho = r_b + y + O_2O_C \sin(\psi - \pi/2)$.

Let me repeat the radius of curvature at this particular instant when the cam has rotated through an angle θ, is this distance which is O_2O_C sin of this angle and this angle is $(\psi - \pi/2) + r_b + y$. Now this expression is true, ρ does not change or O_2O_C does not change for three infinitesimally separated time instants which means we can differentiate this expression up to twice assuming ρ and O_2O_C to be constant, because these do not change for three infinitesimally separated time instants.

If we simplify it, we can write $\rho = r_b + y - O_2O_C \cos \psi$. Similarly, if we replace this cam by this circle and this equivalence is true only for three infinitesimally separated position that is very important as we will learn during the equivalent higher pair mechanisms for lower pair linkage, that radius of curvature is same for this circle and this cam profile.

So, we can replace this cam by the circle up to three instants. Now we see this O_2S and O_C they are not moving at least for three infinitesimally separated instances. So, this angle $\theta + \psi$ we can write constant and this relationship is not valid for all t, but definitely valid for three infinitesimally separated time instants. So we can differentiate these expressions up to twice holding this as constant and ρ and O_2O_C as constant.

This gives us, $\dot{\theta}$ ($d\theta/dt$) = $-\dot{\psi}$ or we can write $d\psi/d\theta = -1$. Since $d\theta/dt = -d\psi/dt$, so $d\psi/d\theta = -1$. Now, here if we differentiate it once as I said earlier ρ and O_2O_C as can be treated as constant and if we differentiate with respect to θ which is nothing but differentiating with respect to t because θ is ωt, we get $dy/d\theta = O_2O_C \sin \psi$.

This is 0, if we differentiate this is 0. So, $dy/d\theta$ bring to this side, we get $O_2O_C (-\sin \psi)*$ $d\psi/d\theta$ and $d\psi/d\theta$ is -1. So, that gives us y' which is $dy/d\theta$, here prime denotes differentiation with respect to θ. So, $y' = O_2O_C \sin \psi$. As I said, this we can differentiate up to second time. So, if we differentiate again, we get, $y'' = O_2O_C \cos \psi * d\psi/d\theta$ and $d\psi/d\theta$ is -1, so y'' we can write $-O_2O_C \cos \psi$.

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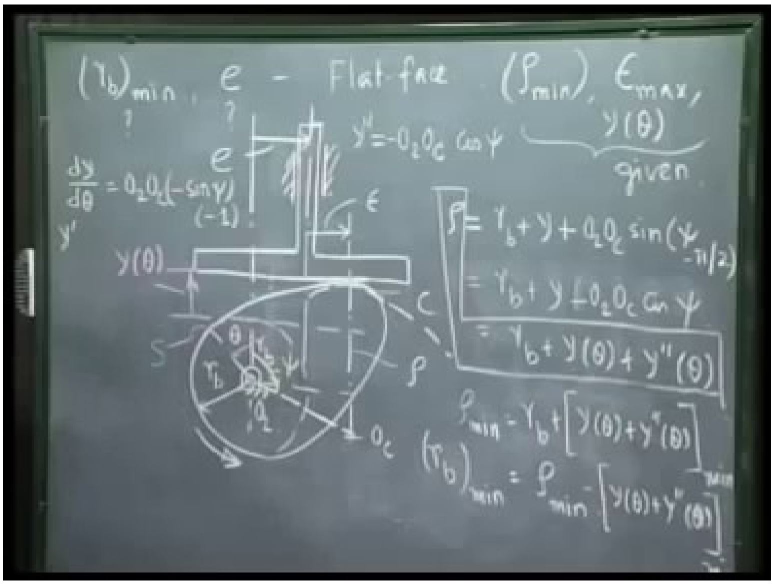

In expression of ρ, $-$ O_2O_C cos ψ is nothing but $d^2y/d\theta^2$; the second derivative of y with respect to θ. So $\rho = r_b + y(\theta) + y''(\theta)$.

This is the most important relationship between the radius of curvature of the cam profile in terms of the base circle radius and the displacement function, $y(\theta)$. This will tell us that ρ is minimum when this quantity is minimum, they are changing with θ, both $y(\theta)$ and $y''(\theta)$. When this whole quantity is minimum that is when for a given r_b, ρ is minimum. So that clearly tells us that minimum value of r_b has to be given value of $\rho_{min} - y(\theta) + y''(\theta)$, we have to calculate the minimum value of this expression. For a given value of $y(\theta)$, we first find for which value of θ, $y(\theta) + y''(\theta)$ is minimum and what is that minimum value, then $(r_b)_{min}$ for a given value of ρ_{min} we can get from this expression.

So, you can see, but any lesser value of $(r_b)_{min}$ when you add this the value of ρ will turn out to be less than ρ_{min}. From this figure, we should be also being able to determine what is the offset? We have already determined $(r_b)_{min}$ when those are given, now given the ε_{max} we should be able to find out what is the e? that is the offset that is desired. Towards that end, we already had an expression for $dy/d\theta$, that was y' which was O_2O_C sin ψ. This we already had.

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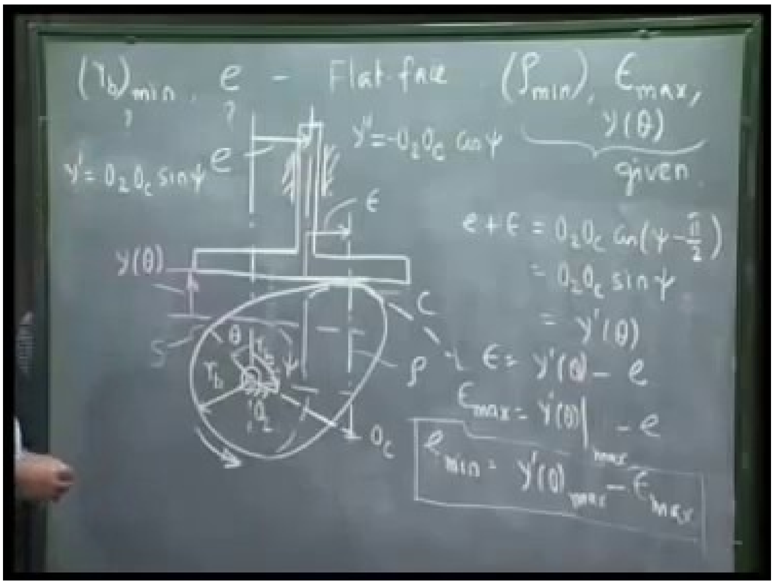

Let us calculate this horizontal distance which is e + ε; where ε is the eccentricity of the driving effort; e is the offset. It is the distance of the follower axis from the camshaft, that is O_2O_C cosine of this angle which is $O_2O_C \cos(\psi - \pi/2)$ which is same as $O_2O_C \sin\psi$.

Here we just now proved that dy/dθ is nothing but $O_2O_C \sin\psi$. So, this we can write, y′(θ). So ε = y′(θ) – e, the driving eccentricity of driving effort is governed by this y′(θ) and the offset e.

Now, maximum value of this ε takes place when y′(θ) is maximum – e. So, the minimum value of the offset is $y'(\theta)_{max} - \varepsilon_{max}$. This equation determines the minimum value of the offset that has to be used, so that ε never goes beyond this ε_{max} which is prescribed. We have to only see; what is the maximum value of y′(θ).

As you see, if any lower value of ε is used then ε_{max} will be more because of this – e. So, this is the minimum value of offset. So, we have got both the relations minimum value of r_b and minimum value of the offset. From here, we can also get what is the face width of the follower that is necessary because as you see the contact point is changing, right now it is here, in some other instance it may be there, so what is the minimum width of the follower?

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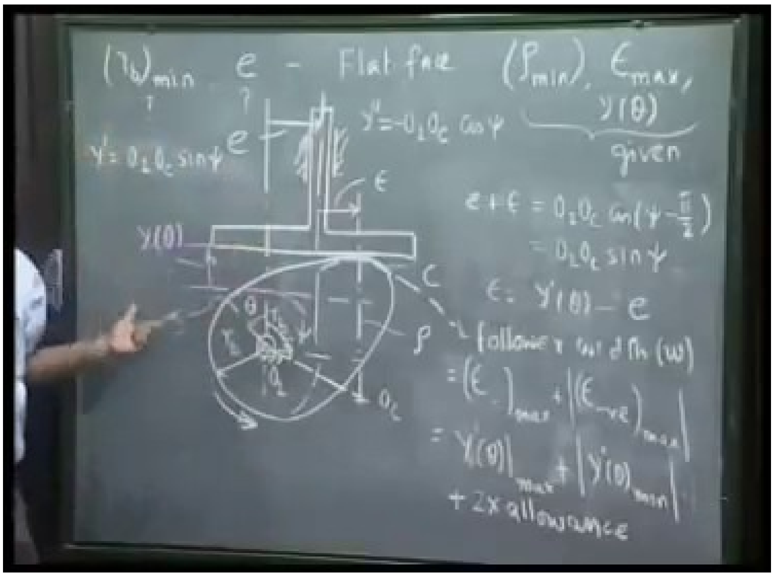

To do that as we see this distance eccentricity of driving effort ε is given by $y'(\theta) - e$. So, when $y'(\theta)$ is positive, ε is positive, this will be more than this and the contact is on the right of this follower axis and when $y'(\theta)$ is negative $y'(\theta) - e$ which will be negative; that means the contact will be on the left of this. So, total width of the follower that is necessary is maximum positive ε + maximum negative ε, we have to take the magnitude of this value plus the magnitude of the positive value.

So, if we call that minimum follower width, if I call it 'w' that is $(\varepsilon_{+ve})_{max} + (\varepsilon_{-ve})_{max}$ and then we take the absolute value, which is clearly seen that e cancels and this will come out to be $y'(\theta)|_{max} + |y'(\theta)_{min}|$ which is negative. So, we take the absolute value. So, the total follower width is given by $y'(\theta)|_{max} + |y'(\theta)_{min}|$ and normally we keep some allowance on either side, if the contact point comes up to here we should keep an allowance here similarly if the contact point comes up to here we keep on same amount of allowance here.

So, 2 times some allowance which the designer can choose. Given this, ρ_{min}, ε_{max} and $y(\theta)$, we have determined $(r_b)_{min}$ and the minimum amount of offset and also the minimum amount of follower which that is necessary for this cam follower system to function smoothly.

Now we shall discuss how to determine the optimum values of minimum values of the base circle radius and the offset for a roller follower.

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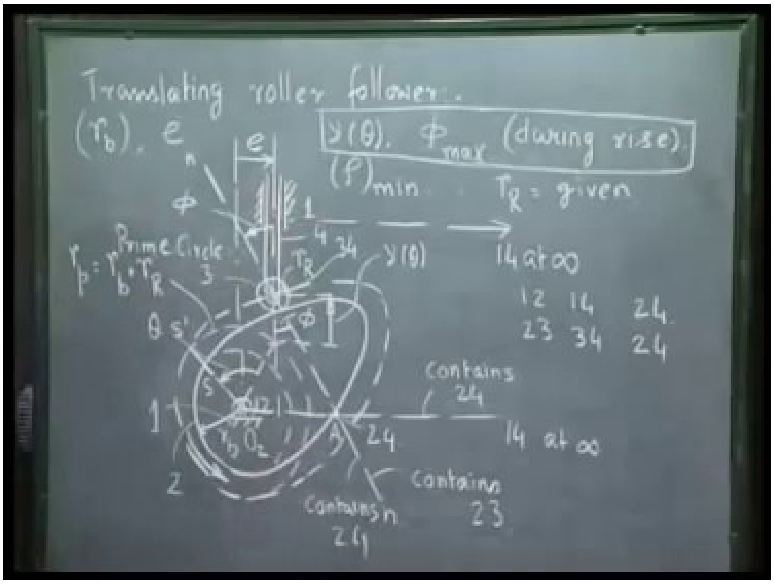

So we are discussing a translating roller follower and we have to determine; what are the optimum values of the base circle radius and the offset that is needed. What are given to us is the desired motion of the follower y(θ) and the maximum value of the pressure angle which can be allowed.

The maximum possible value of the φ that is the pressure angle which is allowed. As we know during the rise of the follower this angle is critical and it should not be allowed to go beyond a prescribed maximum. So φ_{max}; let me say during rise, this is also prescribed and ρ_{min} of the cam profile, in case of a flat-face follower is dictated by the contact stress point of view, but what we will do here, we will satisfy this maximum pressure angle condition for a given y(θ) and we will determine r_b and e.

Once I get r_b and for a given y(θ), the cam profile becomes unique and we can calculate what are the ρ of that cam profile at various points for various values of θ and we should only check that this condition is not violated. So, what we will do, for given y(θ) and φ_{max}, how to determine the values of r_b and e for a translating roller follower.

Let's say this is the base circle radius, this is the camshaft axis. And as I said the follower axis is offset with this be a follower axis, the roller center is here, this is the roller which is hinged to the follower and as we see here an extra dimension is necessary that is this given roller radius, this is decided not from kinematics, but from the force transmission point of view what is the minimum size of the pin that decides the roller radius. So, we also assume that r_R

is also given. This is the amount of offset, the cam is rotating in the counter-clockwise direction.

So, if this is the base circle radius, then we know if we draw the pitch curve, which is a curve parallel to the cam profile and the distance between them everywhere is the roller radius. This is the prime circle, this is the base circle radius r_b and this is the prime circle whose radius r_p is $r_b + r_R$. If this is the contact point then joining the contact point with the roller centre that is the common normal between the cam and the roller surface and it is this angle that this common normal makes with the direction of the follower movement is called the pressure angle.

So first of all, let us try to get an expression for this pressure angle in terms of r_b and other quantities. Now what is the lowest point of the follower when the trace point comes here meets the prime circle. This is the lowest point of the follower. So, the rise of the follower at this instant is so much; this is the lowest point of the roller centre when θ is zero and when the cam is rotated by an angle θ, this is the displacement of the follower which we call $y(\theta)$.

Now let us see, what is θ? When the contact point was here that is when the roller was at its lowest position and now the contact point between the prime circle and the pitch curve is here. So, this is the cam rotation θ. Let me call this O_2, this is S, this is S′. When this line O_2SS' was here, then the roller was at its lowest position.

Now, to determine the pressure angle in terms of these geometrical quantities we take advantage of the Aronhold-Kennedy Theorem. This is the mechanism as we see of 4 rigid bodies. The fixed link is link 1, the cam is link 2, the roller is link 3 and the follower is link 4. So, it is a 4-link mechanism consisting of 1, 2 and 3, 4. Between 1 and 2, there is a revolute pair at O_2, so this is also the relative instantaneous center 12. 3 and 4 are connected by a revolute pair here at this roller centre.

So, this is also the relative instantaneous center 34. 4 is in vertical translation with respect to 1. So, 14 is that infinity in the horizontal direction. So, we can say 14 is in the horizontal direction at infinity. We can draw any horizontal line and 14 lies at infinity because all parallel lines meet at infinity. So we can say, if we draw a horizontal line through 12, 14 at infinity.

So we have determined 12, 34, 14. Now we also know 23 that is between the cam and the roller, 23 must lie on this common normal. We do not know where but definitely it lies on this common normal, this contains 23, where exactly it will be located in this line nn, that will depend on the amount of slip that is taking place.

But let me apply the Aronhold-Kennedy Theorem. We know that 12, 14 and 24 must be collinear; this is 12 and 14 lies on this line at infinity. So, this line must contain 24. This is 34 and 23 is contained on this line, so we can write 23, 34 and 24, these 3 relative instantaneous centers are also collinear out of which we know 23 is on this line, 34 is here, so this line also must contain 24.

So 24 is contain on this line and this line also contains 24; so obviously this point of intersection that decides the location of 24. This angle same as this angle which is φ. So let me name this point as A. A is the location of 24, that is the relative instantaneous center between the cam and the follower because cam is link 2 and follower is link 4.

Once we have located this 24 at A, we can easily get the expression of this pressure angle φ. Let this vertical line intersects this horizontal line so O_2 at this point which we call B, and this roller center let me call it C.

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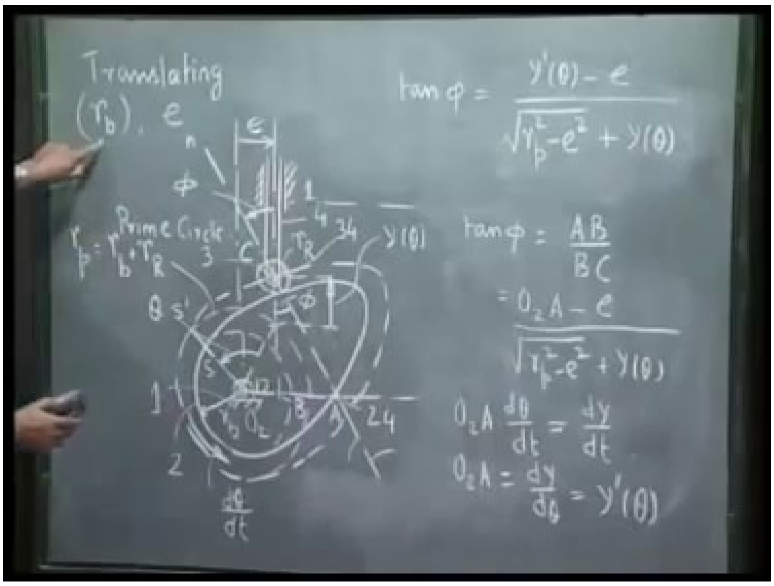

So, what we see that tan φ is AB/BC. If φ is the pressure angle, then tan φ is AB/BC, tangent of this angle is AB/BC, where A is at 24 and B is the point of intersection of the guide of the follower with the horizontal line through O_2, that is line joining O_2 and 24.

Now let me see what is AB? AB is nothing but O_2A – O_2B and O_2B is nothing but e. So we can write this as (O_2A – e)/BC and what is BC? BC is this vertical distance out of which this is y and this is $\sqrt{r_p^2 - e^2}$, I see a right-angle triangle here, hypotenuse is r_p that is a prime circle radius and this distance is e. So, this vertical side is $\sqrt{r_p^2 - e^2} + y(\theta)$ that gives me BC.

Now what is O_2A? We know if we consider this point 24 as a point on body 2, whatever velocity we get; if we consider it to be point on body 4; we will get the same velocity because it is 24, 24 means these two coincident points on body 2 and body 4 have the same velocity at this configuration.

So, if we consider this to be a point on body 2 that is the cam which is rotating with constant ω = dθ/dt, then the velocity of this point is upward and the magnitude is O_2A ∗ dθ/dt. So we can write O_2A ∗ dθ/dt that is the velocity of this point 24 in the vertically upward direction.

Now, if we consider it to be point on body 4 that is the follower, as we know the follower is a translating member so all points on the follower have the same velocity. So, this 24 considering it to be point on body 4, it has a velocity which is nothing but the velocity of the follower which is dy/dt. That clearly tells me O_2A is nothing but dy/dθ, (since (dy/dt)/(dθ/dt) = dy/dθ) which we write as y′(θ) where prime denotes the first derivative with respect to θ.

So thus, we arrive at an expression for tan φ, and it is this expression from where we will be able to determine the required values of r_b and e as I shall show you a little later. So, let me write the expression $\tan\varphi = \frac{y'(\theta) - e}{\sqrt{r_p^2 - e^2} + y(\theta)}$, where r_p is prime circle radius, e is offset.

Thing to note that because we provided this offset in the correct direction for positive values of y′ that is positive value of dy/dt when the follower is rising the value of φ is reduced due to the presence of this offset. Without this offset whatever value we would have got by giving this offset, I am reducing the pressure angle, that is the objective of providing this offset.

Now this expression we can use to get the proper values of r_p and of course if we get r_p, we can get r_b and also the value of e. To do this we will take help of a geometric representation of this particular equation $\tan\varphi = \frac{y'(\theta) - e}{\sqrt{r_p^2 - e^2} + y(\theta)}$ and you should also note that when $y'(\theta)$ is negative the φ value will increase. Of course, the value of φ is negative, but the magnitude of the pressure angle will increase due to the presence of this offset while the follower is coming down that is during the return stroke of the follower.

But as I said earlier that does not matter, it is only during rise there is a chance of jamming of the follower in the guide and we reduce the pressure angle by providing this offset.

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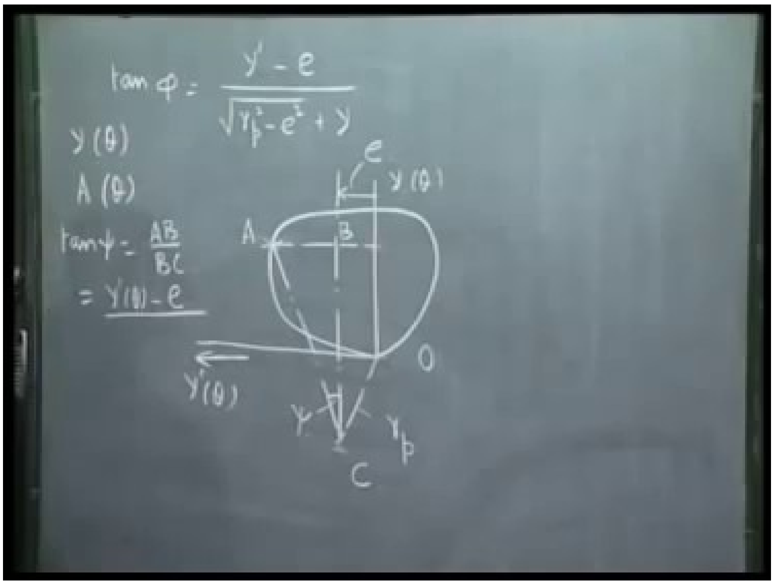

We have just now obtained this expression for the pressure angle φ as $\tan\varphi = \frac{y'(\theta) - e}{\sqrt{r_p^2 - e^2} + y(\theta)}$. As we know we have been given the desired displacement function y(θ) which is a periodic function because it repeats after every 2π rotation of the cam. If it is a periodic function, then if we plot y(θ) along this vertical line and $y'(\theta)$ along this horizontal line, then that will be a closed curve. I mean the shape of the curve is unimportant that will depend on the value of function y. Let us say, this is the curve and we get a closed curve when we plot y vs y′.

This is the origin let's say O. Now if we draw a line at a distance e, the offset and take a point on this line which is at a distance r_p from this origin O and if we call say this point C, C is located at this vertical line is at a distance e towards positive y′ from this y axis and OC is equal to r_p that locates this point C. Now we see if we take any point on this curve, any point means a particular point, this point corresponds to some value of θ.

Now let us see, that the angle that this line, say CA, A represents a particular value of θ. The angle that this line CA makes with the vertical is nothing but the pressure angle for that value of θ. If we drop a perpendicular, if we call this point B and if we call this angle ψ, then tan ψ = AB/BC and what is AB? AB = y′(θ) – e.

So, y′(θ) – e and what is BC? BC is this vertical distance which is nothing but y for the corresponding to point A, that is y(θ). And what is this vertical height? That is $\sqrt{r_p^2 - e^2}$. This is again a right-angled triangle; this hypotenuse is r_p, this horizontal side is e.