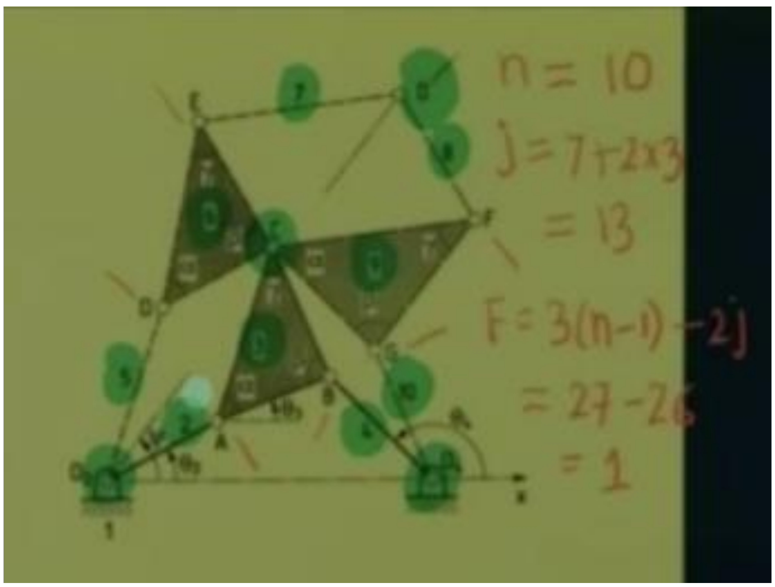
Kinematics of Machines

Prof A. K. Mallik
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Module-3 Lecture-4

In this lecture, we will continue our discussion on displacement analysis of planar linkages by analytical method. Today, we shall start our discussion with an example. Let us look at this figure which is a kinematic diagram of a 10-link mechanism.

(Refer Slide Time: 00:37)



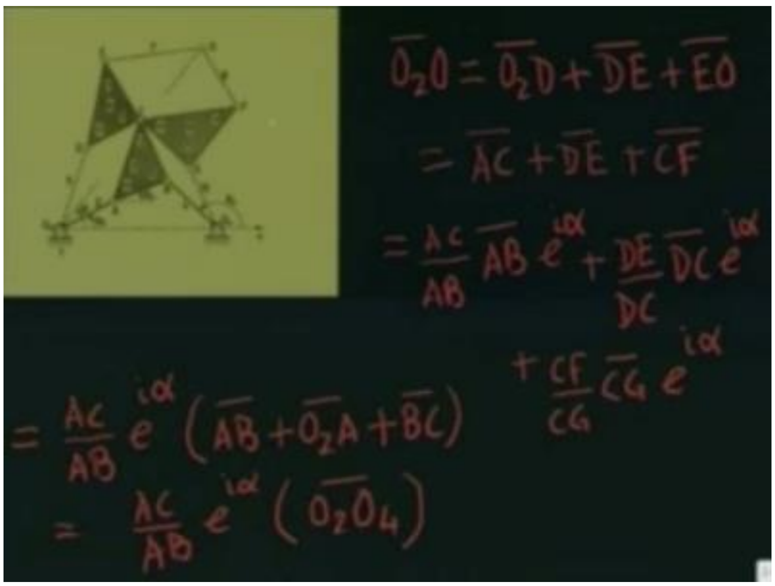
This is the fixed link 1, then O_2A is the link 2, ABC is this ternary link, link 3, link 4 is O_4B, link 5, link 6 is again a ternary link, link 7, link 8, link 9, which is again a ternary link, and link 10. So, we have ten links. Now let us look at the kinematic pairs.

As we see, at this point O_2, three links are connected namely 1, 2 and 5 so this is a second order hinge. Similarly, at O_4 three links are connected namely 1, 4 and 10 so this is again a second order hinge. And finally, at C three ternary links namely 3 6 and 9 are connected so this is again a second order hinge. Let us now calculate the degrees of freedom of this 10-link mechanism.

We have already seen the total number of links, n is 10. The number of kinematic pairs j is equal to, we have seven simple hinges namely at O, F, G, B, A, D, and E. So, j_1 is 7 and there are three second order hinges, one at O_2, one at O_4 and the other at C. So, j = 7 + 2∗3 = 13. So, the degree of freedom, F = 3(n – 1) – 2j = 3∗9 – 2∗13 = 1, i.e., F = 1.

So we see it is a single degree of freedom mechanism that means, whenever any link moves all other links move in a unique fashion. The question is, as this mechanism moves let us see how does the point O moves. To find that we will use the links as link lengths vectors and try to find the $\overline{O_2O}$.

(Refer Slide Time: 03:25)



Here as we see, the $\overline{O_2O}$ can write as $\overline{O_2D} + \overline{DE} + \overline{EO}$. One has to note that in this particular mechanism O_2ACD is a parallelogram. Similarly, O_4BCG is also a parallelogram and OECF is another parallelogram. That means, the link AC is same as the link O_2D; link length EO is same as the link length CF and so on. Not only that, three ternary links namely link 3, link 6 and link 9 consists of three similar triangles as indicated by these three angles namely α, β and γ in each of these triangles. These three triangles ABC, CGF and CED are three similar triangles.

Let me now write the $\overline{O_2D}$ as same as the $\overline{AC}$ because they always remain parallel and $\overline{EO}$ which is same as $\overline{CF}$ because $\overline{CF}$ and $\overline{EO}$ always remain parallel and of equal length.

Therefore, $\overline{O_2O} = \overline{AC} + \overline{DE} + \overline{CF}$. Now, the $\overline{AC}$ can be written as $\frac{\overline{AC}}{\overline{AB}}\ \overline{AB}e^{i\alpha}$ (this form takes care of the magnitude of $\overline{AC}$. and $\overline{AC}$ is at an angle α in the counter clockwise direction from the $\overline{AB}$. So, we have to multiply it by $e^{i\alpha}$). So, the first term is, $\overline{AC} = \frac{\overline{AC}}{\overline{AB}}\ \overline{AB}e^{i\alpha}$. Similarly, we can write $\overline{DE} = \frac{\overline{DE}}{\overline{DC}}\ \overline{DC}e^{i\alpha}$ and $\overline{CF} = \frac{\overline{CF}}{\overline{CG}}\ \overline{CG}e^{i\alpha}$

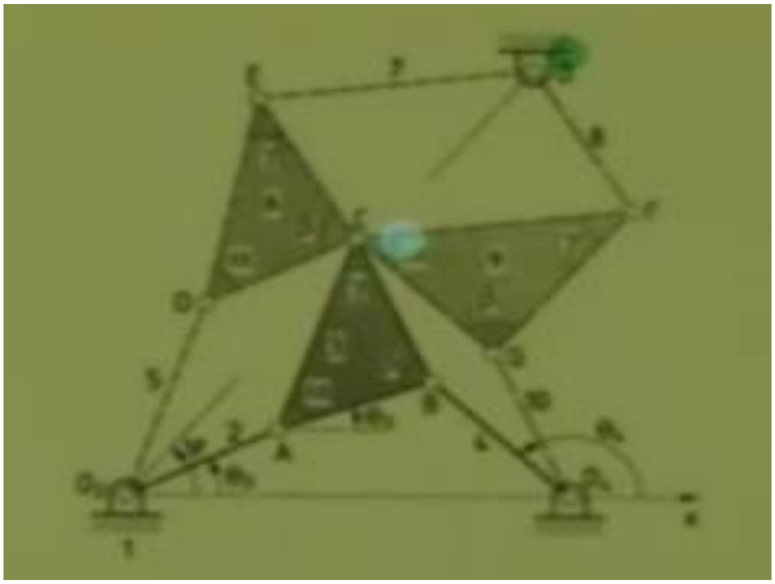
Now as we see, because these three triangles are similar triangles; $\frac{\overline{AC}}{\overline{AB}}$ is same as $\frac{\overline{DE}}{\overline{DC}}$ it is also same as $\frac{\overline{CF}}{\overline{CG}}$. As all these three ratios are same, so we can take any one of them out say, $\frac{\overline{AC}}{\overline{AB}}$, can take common from all these three expressions and $e^{i\alpha}$ can also take common. That leaves us with $\overline{AB}$ from the first term; then $\overline{DC}$ which is same as the $\overline{O_2A}$ because $\overline{DC}$ is same length as $\overline{O_2A}$ and they always remain parallel.

So here, instead of $\overline{DC}$, we write same vector $\overline{O_2A}$ and that leaves me with the $\overline{CG}$ which is same as $\overline{BC}$ because $\overline{CG}$ and $\overline{BC}$ are of equal length and they remain parallel. So, I write this as $\overline{BC}$, which means the summation of these three vectors namely $\overline{O_2A}$ + $\overline{AB}$ + $\overline{BC}$ is nothing but $\overline{O_2O_4}$. So, $\overline{O_2O}$ = $\overline{O_2O_4}\ \frac{\overline{AC}}{\overline{AB}} e^{i\alpha}$.

As the mechanism moves, all the vectors changes but $\overline{O_2O_4}$ never changes because O_2 is a fixed point; O_4 is a fixed point, so the $\overline{O_2O_4}$ is always on X-axis without changing its length. Neither the length AC nor length AB changes, because those are the rigid link lengths. Same is true for this angle α which is again the angle α of this ternary link. Consequently, as the linkage moves the $\overline{O_2O}$ never changes which means the point O never moves in this mechanism. Though this is single degree freedom mechanism all other points move as the mechanism moves but the point O never moves.

Just now we have seen that in the 10-link planar mechanism with specific dimensions there is one point which was not moving though the overall mechanism had a single degree of freedom.

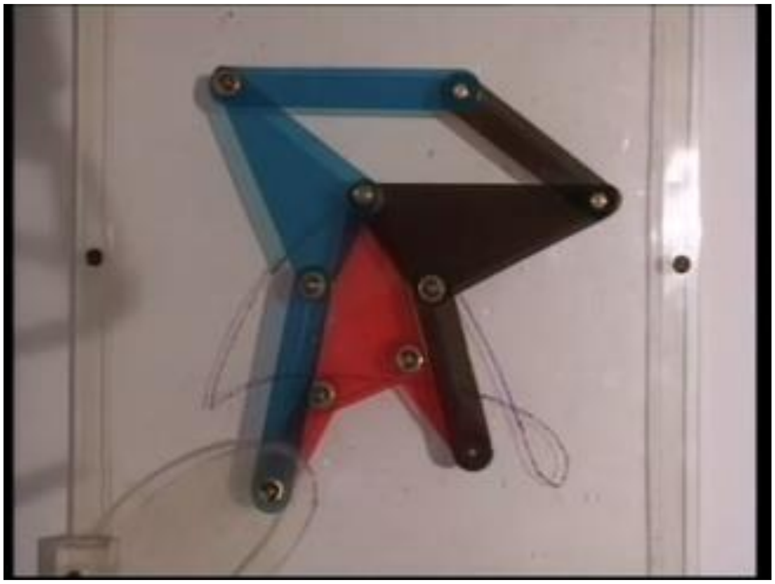
(Refer Slide Time: 09:17)



Consequently, if we fix point O which was not moving with the fixed link by putting a hinge, thus converting this hinge at O to a higher order hinge connecting three links namely 1, 7, and 8. As a result, now we have an assembly of links where there are three 4 bar links: one 4 bar link consisting of 1, 2, 3, and 4 with the coupler point at C; there is a second 4 bar linkage consisting of link 1, 8, 9, and 10 with the same coupler point C; and the third 4 bar linkage consisting of link 1 that is the fixed link, link 7, link 6 and link 5.

Now, all these three 4 bar linkages have the same coupler point C and this assembly moves in a unique fashion. In other words, it means there are three different 4 bar linkages we can generate the same coupler curve at the point C. As a result, this gives a wider choice to the designer to choose one of these three linkages to produce the same coupler curve. This is now what I will demonstrate with a model.

(Refer Slide Time: 10:32)



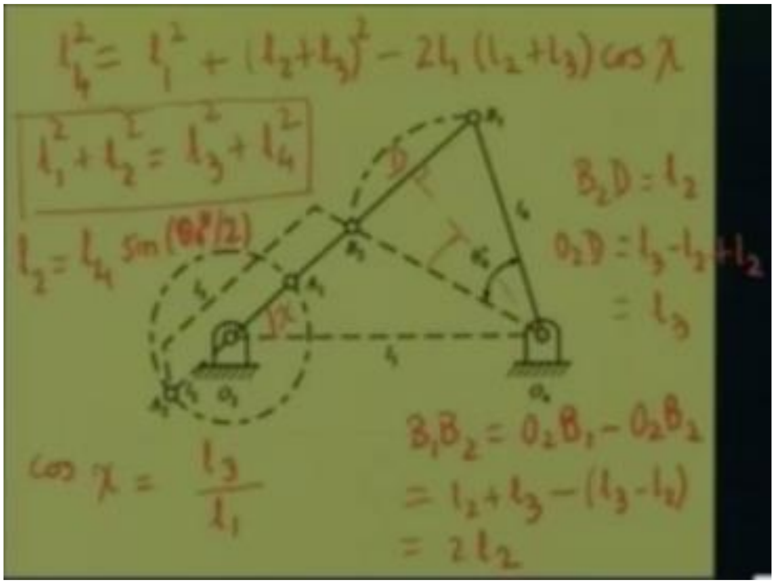
Let us now look at the model of the 10-link mechanism which we have just discussed. There is a fixed link and there are three second order hinges, all connected to the fixed link which we marked previously as O_2, O_4 and O. There are three grey moving links, there are three red moving links and there are three blue moving links. We also note that, this link length is equal to this link length and this link length is equal to this link length. Thus, this point O_2A and this was C and this was D probably forms a parallelogram.

Similarly, we have a parallelogram here and we have a parallelogram there. And these three ternary links, this red ternary link, the grey ternary link and the blue ternary links, they are similar triangles. That means, this angle is equal to this angle, this angle is equal to this angle, and this angle is equal to this angle.

We have seen as a consequence of these special dimensions, this 10-link mechanism move in a unique fashion because it has single degree of freedom. And this coupler point C can generate this coupler curve, whether I use only this 4-bar linkage or this 4-bar linkage or this 4-bar linkage. All these three 4-bar linkages generate the same coupler curve and this gives the designer a wider choice to choose a particular one which may be convenient for the purpose.

Now, we shall discuss some useful results for 4R-linkage which are most commonly used and these results will be obtained analytically.

(Refer Slide Time: 12:37)



For example, let us consider this 4R-linkage namely, O_2, A, B, and O_4. As shown in this diagram, at this configuration $O_2A_1B_1O_4$, the two links O_2A_1 and A_1B_1 are collinear.

Consequently, this link O_4B_1 has taken one of its extreme positions. It cannot go further to the left. This crank-rocker mechanism, rotates with crank O_2A. There is another configuration when O_2A_2 and A_2B_2 again become collinear and the corresponding configuration of O_4B that is, and this O_4B_2 is the other extreme position of this follower link, which is link 4.

So we are considering a crank-rocker mechanism and we see that, as the follower goes from O_4B_2 to O_4B_1, during this movement, the crank rotates from O_2A_2 to O_2A_1. That means, it moves through an angle θ_2^*. During the return from B_1 to B_2, the crank rotates from A_1 to A_2. So it rotates to an angle $(2\pi - \theta_2^*)$. If the crank rotates at constant speed, then the time taken for the follower motion during the B_2B_1 and B_1B_2 are not same.

Normally, we would like to have a quick return that is returning from B_1 to B_2, it rotates through an angle $(2\pi - \theta_2^*)$ and the follower motion that is B_2 to B_1, it rotates through an angle θ_2^*, which is more than π. The quick return ratio can be defined as $\theta_2^*/(2\pi - \theta_2^*)$.

Our objective is, to determine the relationship between the various link lengths namely, the fixed length l_1, the crank length l_2, the coupler length l_3, and the follower length l_4 so that we can determine whether there is any quick return effect or not. Towards this end, we consider this figure, which has been drawn for a mechanism without any quick return. One extreme position is O_2, A_1, B_1, O_4 and the other extreme position is O_2, A_2, B_2, O_4. During these extreme positions, that the crank and the coupler are always collinear. This is the outer dead center which is $O_2A_1B_1$ and this is called the inner dead center when O_2A_2 and A_2B_2 are opposite to each other. The angle between them, once I can say 0^0, in the other case it is 180^0.

So, for this configuration as we see, the angle between O_2A_1 and O_2A_2 is π. That means there is no quick return. It takes π amount of rotation of the crank for the forward motion and again another π amount of rotation for the return motion. This θ_4^* gives you the swing angle of the rocker. Let us now derive what is the relationship between various link lengths. If we consider the triangle $O_2B_1O_4$, what do we see? O_2O_4 is of length l_1 and O_2B_2 is of length $(l_2 + l_3)$ and O_4B_1 is l_4.

And let us say, this angle is let me denote it by χ. So, considering the triangle $O_2O_4B_1$, I can write, $(l_4)^2 = (l_1)^2 + (l_2 + l_3)^2 - 2l_1(l_2 + l_3)cos\chi$ .Now, to determine the angle cosine χ, let me draw a perpendicular from O_4 to the line B_1B_2. $O_2B_1B_2$ is an isosceles triangle because this length O_4B_1 is always equal to O_4B_2. So this perpendicular bisector meets B_1B_2 at the mid point. Now, let me call this point say, D.

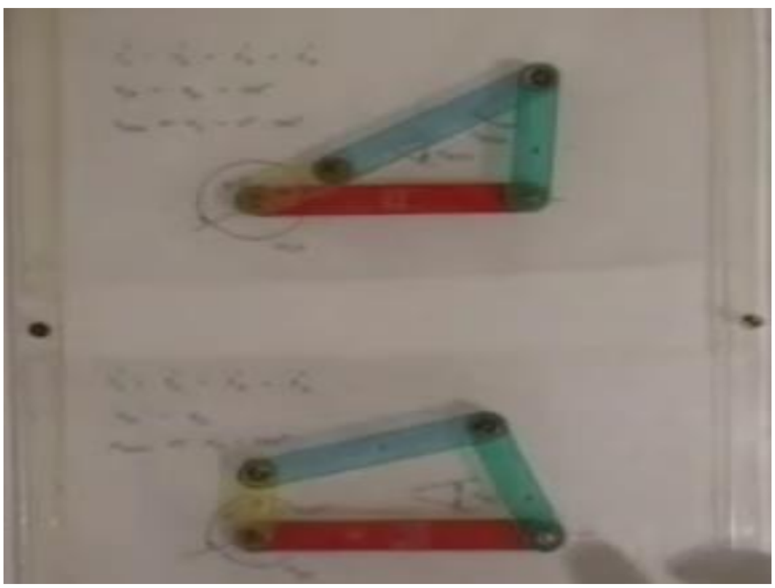
We can easily see that, $B_1B_2 = O_2B_1 – O_2B_2$. Now, link length $O_2B_1 = l_2 + l_3$ and O_2B_2 is A_2B_2 is l_3 and O_2A_2 is l_2, so this $O_2B_2 = (l_3 – l_2)$. Therefore, $B_1B_2 = l_2 + l_3 – (l_3 – l_2) = 2l_2$ i.e., $B_1B_2 = 2l_2$ and half of that is $B_2D = l_2$. So, we can write $B_2D = l_2$. So $O_2D = O_2B_2 + B_2D = l_3 – l_2 + l_2 = l_3$. So cosine of this angle chi is O_2D divided by O_2O_4, so $\cos\chi = O_2D/O_2O_4$. O_2D is l_3 and O_2O_4 is l_1, by substituting these values we get, $\cos\chi = l_3/l_1$. If we substitute this in above equation, we can easily show that we will get,

$$(l_1)^2 + (l_2)^2 = (l_3)^2 + (l_4)^2$$

Thus, for a 4R-linkage, to have no quick return effect that is, without any quick return effect, the link lengths of a 4R-linkage must satisfy this relationship between its link lengths. l_1 is the fixed link length, l_2 is the crank length, l_3 is the coupler length and l_4 is the follower length. Not only that, we can also see that, B_2D which we have got as $l_2 = l_4 \sin(\frac{\theta_4^*}{2})$. So, there is another relationship for such a linkage without quick return that is, {crank length = follower length $* \sin(\theta_4^*/2)$}.

So, these are the two very important relationships which can be used while designing a mechanism. Let us now look at the model of this crank rocker linkage without quick return that we have just discussed.

(Refer Slide Time: 21:12)



This is a crank rocker linkage where $(l_1)^2 + (l_2)^2 = (l_3)^2 + (l_4)^2$. This is the one extreme position, where the crank and the coupler has fallen in one line and this is one extreme position of the follower. Now, as it rotates to 180^0, again the crank and the coupler becomes collinear giving rise to the other extreme position of the follower. As a result, if the crank rotates at uniform speed, the forward and return motion of the follower takes equal time and there is no quick return.

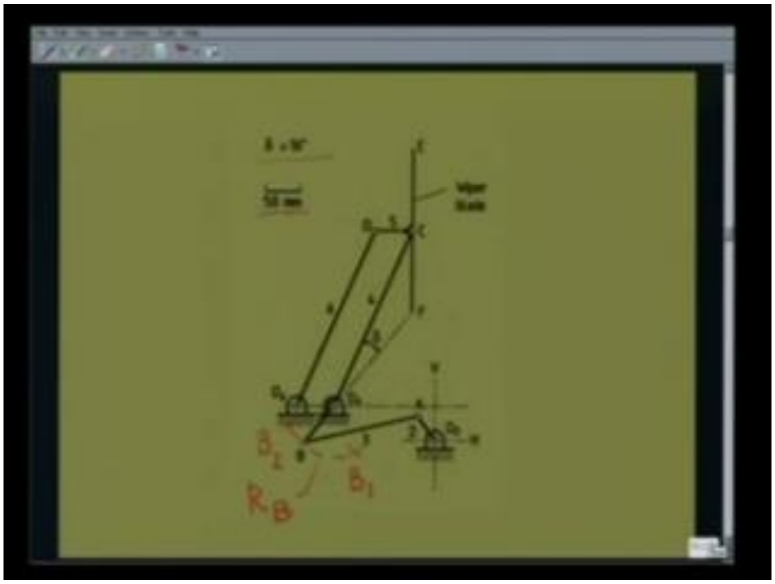
Now, let us look at the model of this crank rocker linkage, where $(l_1)^2 + (l_2)^2 \neq (l_3)^2 + (l_4)^2$. As a result, the extreme position of the follower that it takes is due to the unequal rotation of the crank. One extreme position is here, when the crank and the coupler has fallen in one line. Now from here, it rotates through this angle, again the crank and the coupler fall in one line, giving rise to extreme position of the follower.

So here, as we see the follower doesn't take equal time during its forward and return motion. There is some quick return effect depending on of course, whether I am rotating clockwise or counter-clockwise. Here, θ_2 from here to there is more than here to there, so if we rotate it clockwise, you can see the return is quicker.

Let us now look at another model where again it is a crank rocker linkage but $(l_1)^2 + (l_2)^2 \ll (l_3)^2 + (l_4)^2$. Consequently, the quick return effect will be much more predominant. For example, this is one extreme position and the other extreme position is taken here. So, the angle that the crank rotates is more than π and the angle through which it returns is much less than π. So if we move the crank at uniform speed, the quick return effect is much more predominant. The follower is taking much longer to come from right to left extreme and much less time to go back.

Now that we have discussed both graphical and analytical methods of displacement analysis, let me show you through an example, how both these methods can even be combined while designing a particular mechanism.

(Refer Slide Time: 24:31)

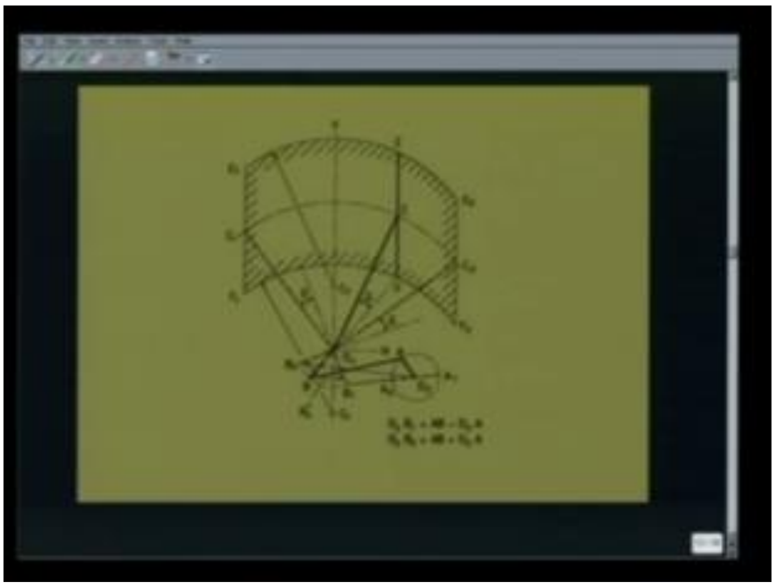


As an example, let us consider this wind shield wiper mechanism, which is a 6-link mechanism. This figure shows the mechanism to a particular scale, for example, this distance is 50 mm, that is the scale of this diagram. Let us now see this mechanism. This has O_2ABO_4, this part is a crank-rocker mechanism and then this link 4 is extended beyond O_4 and we have another 4R mechanism namely, O_4, C, D and O_6 and the second 4R mechanism that is O_4CDO_6 is in the form of a parallelogram. O_4C is same as length in O_6D and length CD is same as the length O_6O_4. And this wiper blade is an integral part of the coupler of this parallelogram linkage that is link 5. That means the wiper blade is same as link 5. The first part of the question is what is the wiping field? That means, as this crank O_2A is driven by a motor, what is the field that is wiped by this wiper blade?

Now to solve this problem, what we do? First, we use a little bit of graphical method. We study only this 4-bar linkage namely, O_2ABO_4 and determine the extreme positions of this link O_4B that is, link 4. For that, B is going along a circle with O_4B as radius. This is the circle with O_4 as center and O_4B as radius, that we call the path of B say k_B. The extreme positions of B will be taken up when the links O_2A and AB become collinear. So, the farthest point B can go is, when the distance of B from O_2 is (O_2A + AB). So, I take O_2 as center and draw a circular arc with ($l_3 + l_2$) as radius and let that intersect k_B at this point, and I call it as B_2.

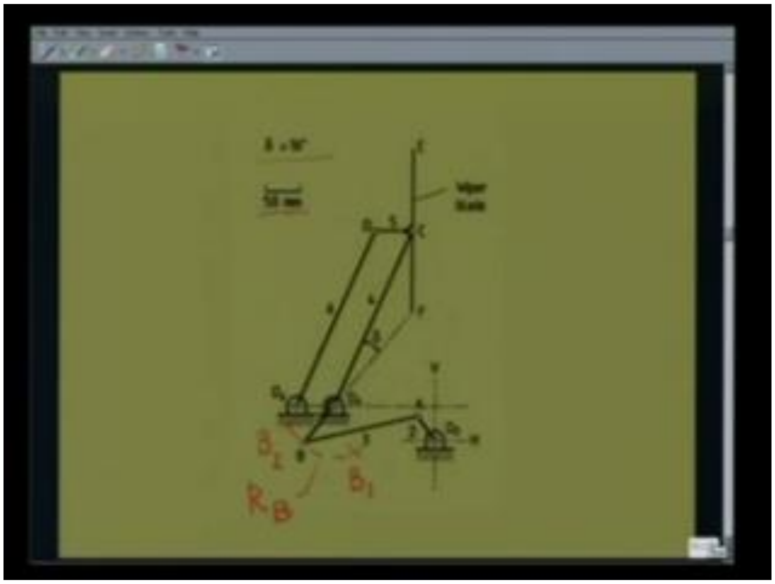
And other extreme position of B will be taken up again, when AB and O_2A will be collinear and the distance of B from O_2 will be equal to $(l_3 – l_2)$. So, we draw a circular arc with O_2 as center and $(l_3 – l_2)$ as radius and let that intersect k_B at this point B_1. So O_4B_2 is one extreme position of link 4 and O_4B_1 is the other extreme position of link 4. Now as we see the link 4, this extension O_4C is not in line with O_4B. There is an angle δ which has been prescribed as 16^0.

(Refer Slide Time: 27:56)



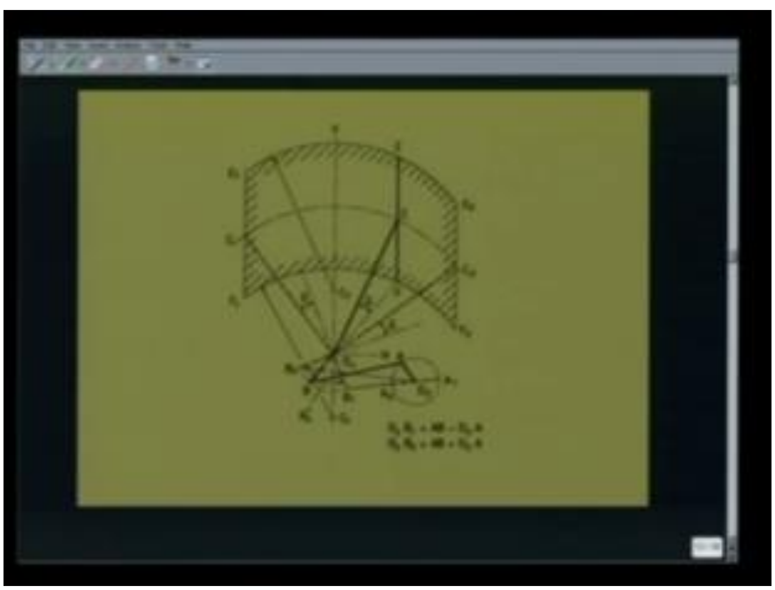
So, we have determined B_1 and B_2 as explained earlier. When corresponding to B_2, I draw this line O_4C_2 at an angle δ which was 16^0 and corresponding to B_1. Again, I draw at an angle δ 16^0 to get C_1. So, O_4C_2 and O_4C_1 are the two extreme positions of the link 4.

(Refer Slide Time: 28:31)



Now the question is, as we know, because the second part of the mechanism was a parallelogram, so this line CD always remains horizontal and the wiper blades always remain vertical, because there is no rotation of the coupler of this parallelogram linkage.

(Refer Slide Time: 28:45)

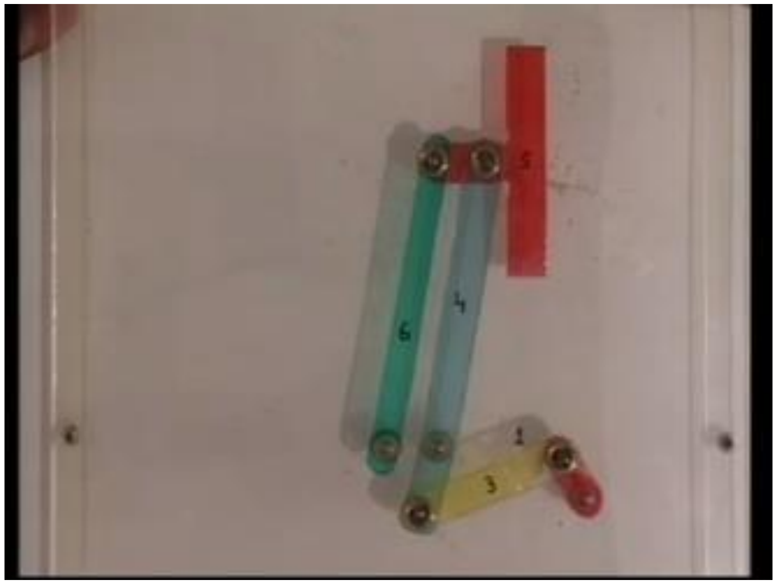


We have already determined the extreme position C_1 and C_2, so I can draw the wiper blades as E_2F_2 and other extreme positions when C is at C_1 as E_1F_1. So these are the two extreme positions E_2F_2 and E_1F_1 for the wiper blade. To determine the wiping field, we see that, because it is a parallelogram linkage, the point C which goes in a circle with O_4 as center and O_4C as radius, all the points of the coupler move in identical curves.

That means, the curve generated by the points E or F that is, the end of the wiper blades also will be similar circles, exactly of same radius as O_4C. Only thing the center of the circle will be shifted from O_4 to CE for the point E and O_4 to CF for the point F. That is, CE, CF is same as E_2F_2 and E_1F_1. So, with center as CE and radius as O_4C which is same as CEE_1, I draw this circular arc. Similarly, with center as CF, I draw this circular arc, and these are the two extreme positions and the wiping field is what has been shown by these hatched lines. So we have obtained the field of wiping for this particular mechanism.

Let me repeat; First we said, determine the path of B which is the circle with O_4 as center and O_4B as radius. On this circular path, I locate B_1 and B_2 using the relation $O_2B_1 = AB – O_2A$, $O_2B_2 = AB + O_2A$. Once I got the extreme positions of link 4, I draw O_4C_2 and O_4C_1 corresponding to O_4B_2 and O_4B_1, because link 4 is a rigid link, the same angle δ is maintained between the line O_4B and O_4C. Once we get the extreme positions of the point C, I draw the wiper blades which always remain vertical because of the parallelogram linkage as E_2F_2 and $E_1F_{1.}$ Because of the parallelogram linkage, all the coupler points generate same circular arc as O_4C. Only thing, the center of the circle is shifted in a symmetric fashion from C_2 to E_2 that is O_4 to CE; C_2 to F_2 that is O_4 to CF; these are all on the same vertical line. As a result, we get the complete field of wiping as generated by this particular wiping mechanism.

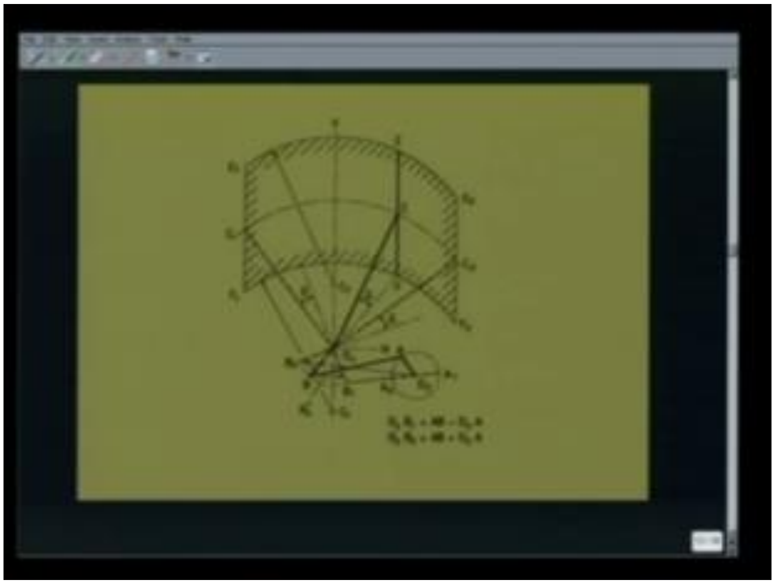
(Refer Slide Time: 31:42)



We demonstrate the same wiper mechanism that we have just now studied. This is the same wiper mechanism consisting of a crank rocker, consisting of link 2, link 3, and link 4. O_2 and O_4 are the two fixed hinges. And there is another parallelogram linkage starting from here link 4, link 5, and link 6. This link length is same as this link length and this link length is same as this link length. So, it is a parallelogram, and because it is a parallelogram linkage, the coupler always remains parallel to itself, it never changes its orientation.

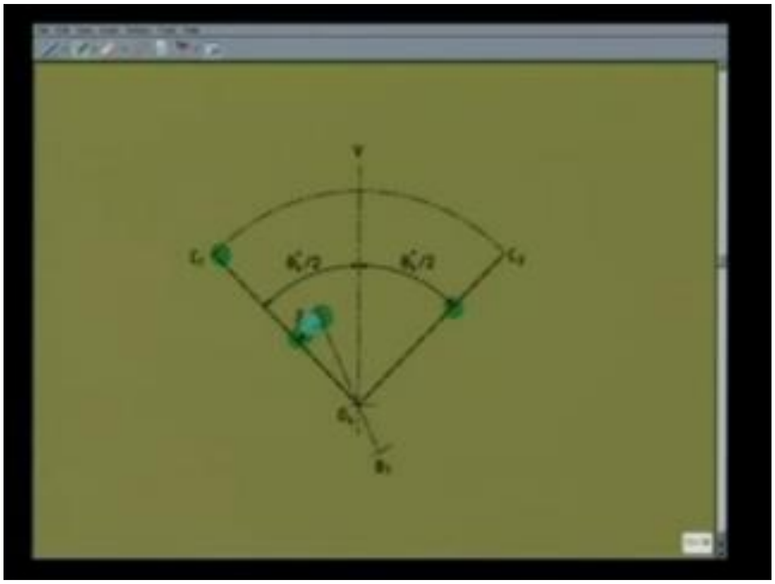
So, as the motor rotates, the wiper blade goes from left to right generating a field of wiping, but the coupler blade always remains vertical.

(Refer Slide Time: 32:49)



Now that we have obtained this field of wiping for this particular given mechanism, let us observe that this wiping field that is $E_1E_2F_2F_1E_1$. This field of wiping is not symmetrical about the vertical line passing through O_4. This O_4V is the vertical line passing through O_4. But the field of wiping is more on the right and less on the left. So, as a designer, maybe we can make a very little change to make this field of wiping symmetrical about this vertical line. So, the second part of the problem is retaining all other link parameters same, change only the angle δ which was given a 16^0. Change only this angle δ to make the field of wiping symmetrical about the line O_4V. To solve this problem, what do we do? We find, what is the angle of oscillation of this rocker link O_4B? That is, from O_4B_1 to O_4B_2 that is the θ_4^* which we call the angle of swing. This θ_4^* is not symmetrical about the vertical line. As a result, the field of wiping is also not symmetrical about the vertical line.

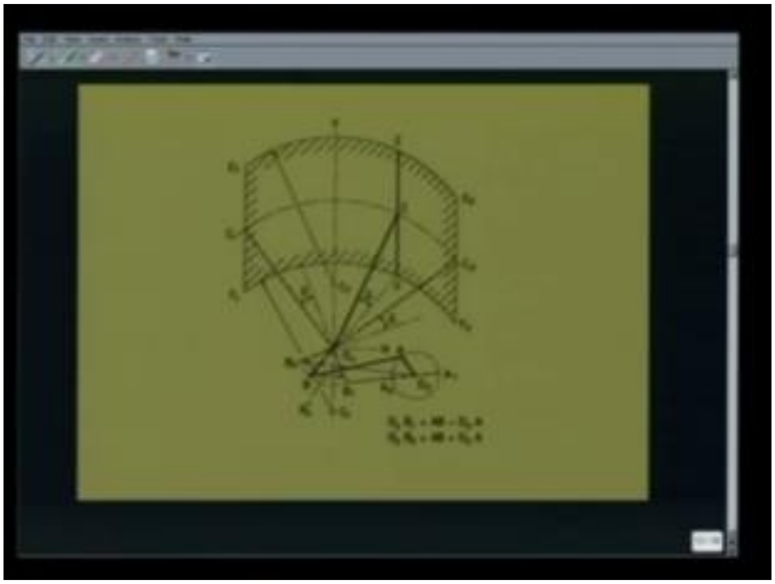
(Refer Slide Time: 34:16)



This θ_4^* we measure. And then, $\theta_4^*/2$ is symmetrically about the vertical line O_4. We have already seen that B_1 which is the extreme position of the link O_4B which we have obtained earlier. And now O_4C_1 must be like this to make the field of wiping symmetrical because now we have made O_4C_1 and O_4C_2 symmetrically placed about the vertical line just at an angle $\theta_4^*/2$ and $\theta_4^*/2$ because θ_4^* is entirely decided by all the link lengths that cannot change. But now, this is O_4B_1 and this is O_4C_1. So, the extension of O_4B_1 and this line O_4C_1, the angle is δ. This is the angle δ, which should be provided rather than what we had earlier as 16^0.

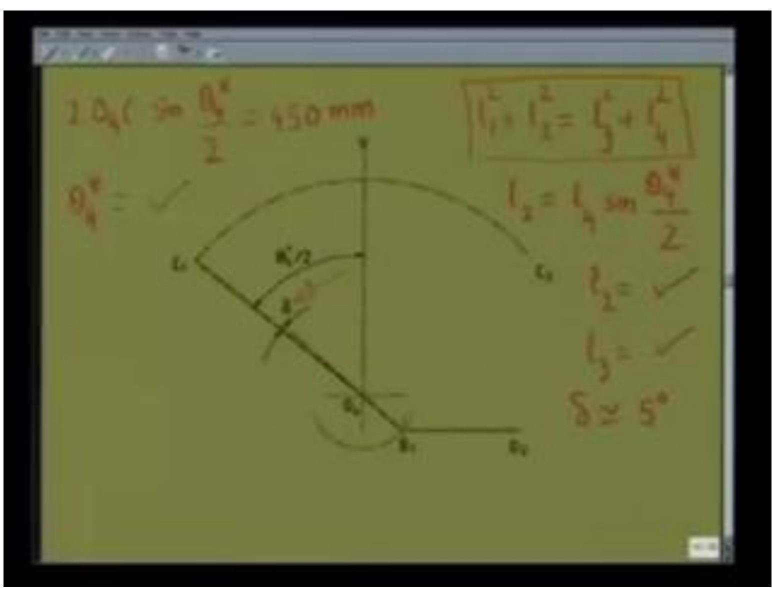
Now, the third part of the problem we can have some more specifications. For example, we define what is the width of this wiping field? That is, this horizontal distance C_1C_2. So, let me pose a problem that modifies this design.

(Refer Slide Time: 35:39)



But you are allowed to change only the crank length O_2A, this coupler length AB and the angle δ to satisfy three requirements. Namely, the wiping field should be symmetrical about the vertical line O_4B, the width of the wiping field that is, the horizontal distance between these two extreme positions E_1F_1 and E_2F_2 is say 450 mm to the same scale and there should be no quick return. That means, O_2A_1 and O_2A_2 corresponding to these two extreme positions of the follower the crank angle should be 180^0.

(Refer Slide Time: 36:25)



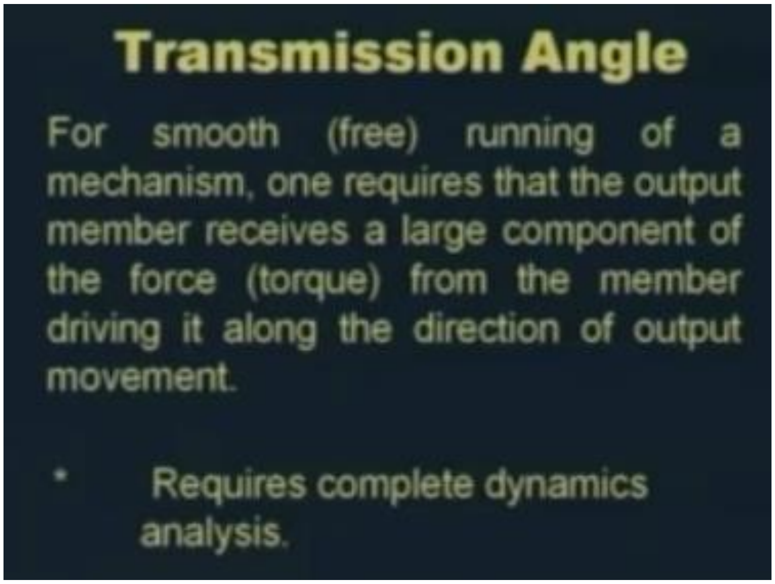
So, we have no quick return for which we need $(l_1)^2 + (l_2)^2 = (l_3)^2 + (l_4)^2$. Now, the width C_1C_2 we can easily see, it is $2O_4C * \sin(\theta_4^*/2)$. Because this angle is $\theta_4^*/2$, so this horizontal distance is $O_4C \sin(\theta_4^*/2)$, twice of that is the width of the field of wiping. This has been specified as 450 mm. The length O_4C has not been changed, so you had it already given in the design. So, substituting that value of O_4C, I can find what is the θ_4^* value. So, we determine θ_4^*, from this equation with the given value of O_4C. Now that we know θ_4^*, if you remember for no quick return, we also had a relationship that $l_2 = l_4 \sin(\theta_4^*/2)$.

Now l_4 that was the link O_4B was not allowed to be changed. So now that l_4 is given, θ_4^* we have obtained, so I can obtain the crank length O_2A as l_2. So I have obtained l_2. l_1 has not been changed, l_4 has not been changed and l_2 we have obtained, so using this relationship for no quick return, I can get the only remaining unknown that is l_3. So, we have designed the 4R-linkage O_2ABO_4. To obtain the required value of δ, we just with the new lengths l_2, l_3, l_1, l_4 were unchanged, I again obtain what is the path of B, which is this circle. And O_2B_1 as we know was $l_3 - l_2$. From O_2, I draw a circular arc O_2B_1 as $(l_3 - l_2)$. So I get the extreme position O_4B_1.

And the extreme position O_4C_1 is already known so the angle between the extension of O_4B_1 and O_4C_1 determines the required value of δ, which as we see is much less than the original value which was 16^0. This δ, if you draw it correctly comes out around 5^0. So we have modified the design to satisfy three requirements namely the field of wiping has to be symmetrical has to be symmetrical about the vertical line O_4B, has to be of a particular width and also there should not be any quick return effect such that the wiper blade takes equal time in the forward motion and return motion. It should not go very fast in one direction and very slowly in the other direction which will definitely disturb the driver.

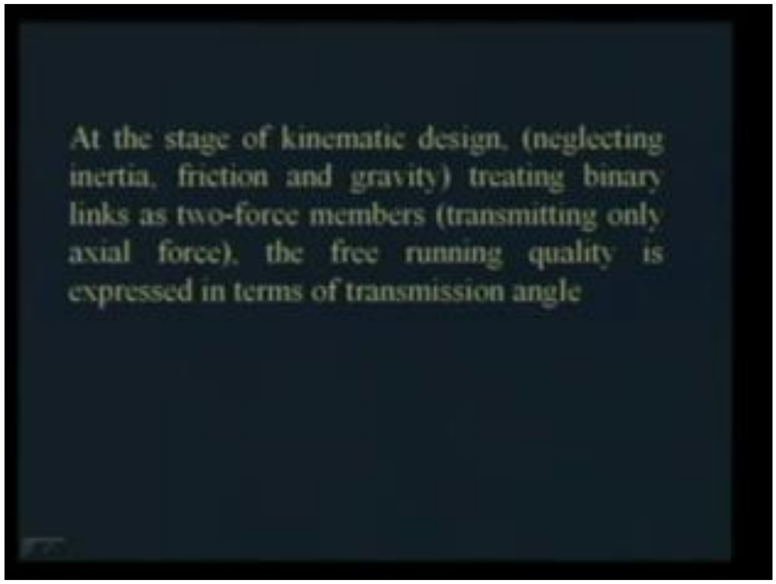
Now that we have completed our discussion on displacement analysis both by graphical and analytical method let me start with a very important index of a good mechanism. As you know, the mechanism has to satisfy the geometric requirements, but satisfying the geometric requirement is not all. For a real-life mechanism, it must move freely and this free running quality of a mechanism is quantified by a parameter, which is called transmission angle. Let me now discuss the concept of transmission angle and show how to calculate the transmission angle at least for 4 link mechanisms.

(Refer Slide Time: 41:05)



Let me repeat, for smooth running of a mechanism, one requires that the output member receives a large component of the force or torque from the member driving it along the direction of output movement. This will ensure that the mechanism runs freely. Not only satisfies the geometric requirements or kinematic requirements, it must have this quality of smooth or free running. However to ensure this smooth and free running, one needs to have a complete dynamic analysis. That will be discussed much later.

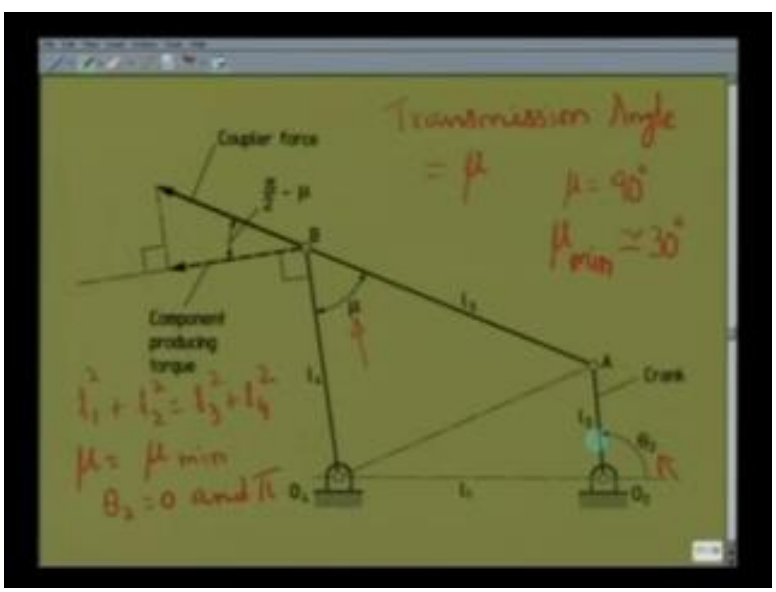
(Refer Slide Time: 41:53)



However, even at this stage of kinematic design, what we do?

We neglect inertia, friction, and gravity and treat all the binary links as two-force members that is, transmitting only axial force. With this assumption, the free running quality of a mechanism can be expressed in terms of what is called transmission angle. Let me explain this concept of transmission angle for a 4R crank-rocker linkage.

(Refer Slide Time: 42:21)



This diagram shows a crank rocker linkage namely, O_2, A, B and O_4 and let O_2A be the crank. As we said, if we assume that coupler AB is a two-force member, then the entire force that AB exerts on the output member O_4B is along the line AB. So this is the direction of the coupler force. However, it is only this component which is perpendicular to the follower, produces torque to drive the coupler. So, you have to ensure that, this angle is as small as possible.

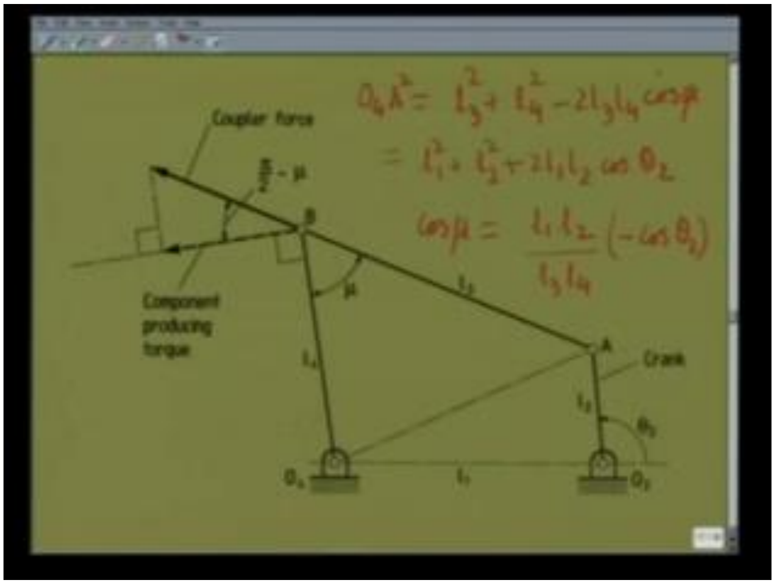
However for defining the transmission angle, it is defined as the acute angle between the coupler that is AB and the follower O_4B. This is what is shown as the angle μ. So, we define the transmission angle is equal to μ, which is the acute angle between the coupler and the follower.

Obviously, the base possible value of μ is 90^0. Then the entire coupler force is used to produce torque about O_4, to drive the follower. However, as the mechanism moves, this angle μ changes, but one is to ensure that μ does not fall below a particular minimum value and normally, minimum value of μ prescribed around 30^0.

Next, we will show, because we are only interested in ensuring the minimum value of μ, can you find out for what crank position that is, for what value of θ_2 the minimum transmission angle offers? Because this angle μ keeps on changing with the crank position, it depends on θ_2. It can be easily shown that, if it is a crank rocker linkage without any quick return that is $(l_1)^2 + (l_2)^2 = (l_3)^2 + (l_4)^2$, then μ attains it minimum value, that is μ_{min}, when this angle θ_2 is either 0 or π. That is the crank is along the line of frame O_4O_2. That is θ_2 is either 0 or π. To get this result, that μ attains its minimum value for θ_2 equal to 0 and π, if there is no quick return, that is this $(l_1)^2 + (l_2)^2 = (l_3)^2 + (l_4)^2$, this relationship holds good.

It is very easy to show, if we consider the length O_4A. O_4A, I can write in terms of l_3, l_4 and μ. To obtain an expression for the transmission angle μ, let us consider the triangle O_4AB, we can write $(O_4A)^2 = (l_3)^2 + (l_4)^2 - 2l_3l_4cos\mu$. Same way, I consider again the triangle O_4O_2A and we can write, $(O_4A)^2 = (l_1)^2 + (l_2)^2 - 2l_1l_2\cos(\pi - \theta_2)$, that is $(O_4A)^2 = (l_1)^2 + (l_2)^2 + 2l_1l_2\cos\theta_2$.

(Refer Slide Time: 46:29)

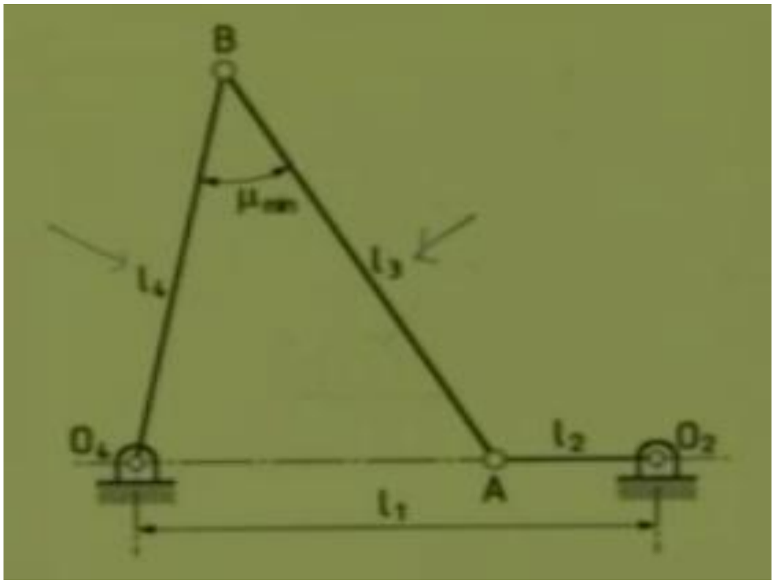


If this crank rocker linkage has no quick return effect that means $(l_1)^2 + (l_2)^2 = (l_3)^2 + (l_4)^2$, then I can get expression for cosμ as,

$$\cos\mu = \frac{l_1 l_2}{l_3 l_4}\,(-cos\theta_2)$$

For μ to be maximum, we see that the values of θ_2 can be either 0 or π. Because we have to remember, if this angle is more than 90^0, then I will take π – μ (this angle between the coupler and the follower as my transmission angle). Transmission angle is defined as the acute angle between the coupler and the follower.

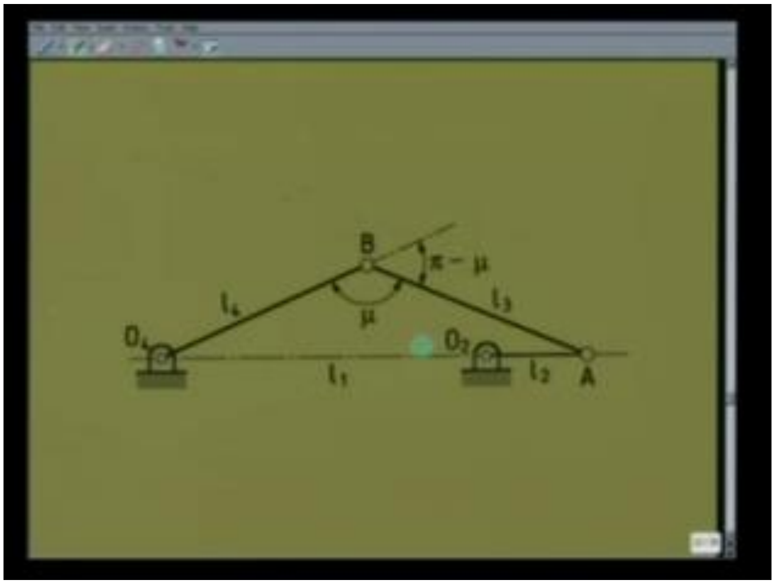
(Refer Slide Time: 49:04)



Here in this diagram, angle happens to be acute so it is μ. But if when this angle becomes obtuse, then I have to take $180^0 - \mu$ as my transmission angle. So, transmission angle is minimized, when θ_2 is either 0 or π for this particular situation. That means, the crank falls in line with the frame that is O_4O_2. This will show now through models. But if there is quick return effect then the minimum transmission angle occurs either at $\theta_2 = 0$ or at $\theta_2 = \pi$. Only when, there is no quick return effect then it will be at both locations $\theta_2 = 0$ and π.

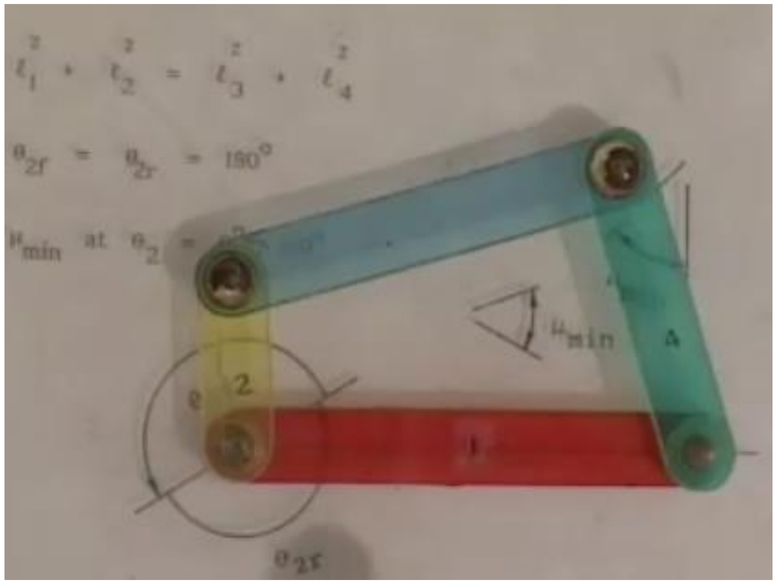
This figure clearly shows that, when the crank O_2A along the line of frame O_4O_2. The transmission angle that is the angle between the coupler and the follower is at its minimum value, this is what we call μ_{min}, because it is already acute.

(Refer Slide Time: 49:26)



In another situation, we see that the crank O_2A is again along the frame line O_4O_2 and the angle between the coupler and the follower that is this angle, is more than 90^0. So here, we will define $(\pi - \mu)$ as my transmission angle. It will always take the acute angle and this angle is minimized when O_2A is along the line of frame. So there are two situations: either O_2A is in this direction when this angle between the coupler and the follower is more than 90^0 and the transmission angle is $(\pi - \mu)$; the other situation is, when O_2A is again along the line of frame and the coupler and the follower makes an acute angle and that itself is the minimum transmission angle, μ_{min}. This now, we will demonstrate through models.

(Refer Slide Time: 50:32)



Consider this model of a crank rocker linkage without any quick return effect. That is, $(l_1)^2 + (l_2)^2 = (l_3)^2 + (l_4)^2$. For such a linkage, as I told you, the minimum transmission angle occurs when the crank is along the line of frame. This angle between the coupler and the follower reaches its minimum value. However, again when the crank falls along the line of frame, this angle is maximized, that is the transmission angle which is defined as the acute angle between the coupler and the follower that I can see by the extension of the coupler and the follower, that angle is minimized. So, the minimum transmission angle occurs at the two configurations: one is this and the other is this.

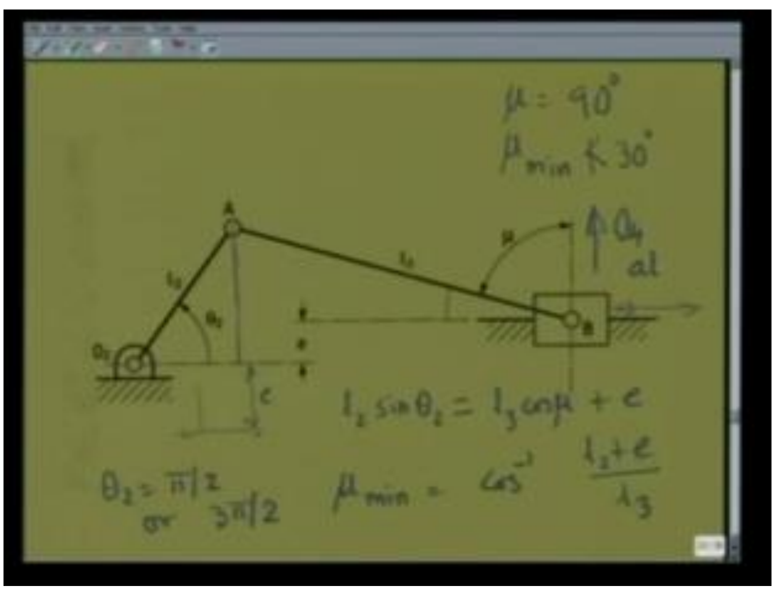
When the transmission characteristics are very bad, because most of the tool of the coupler is not going to drive the follower, this perpendicular component of this axial force is minimum. Here, it is very good. When it is 90^0, all the coupler force is trying to drag the follower. Here, the transmission is very good, here the transmission is worst and here the transmission is worst.

Now, let us look at this another model, where $(l_1)^2 + (l_2)^2 \neq (l_3)^2 + (l_4)^2$. Here, the minimum transmission angle occurs only for this configuration, when the crank and the frame are along the same line. And the angle between them is maximized so the transmission angle is minimum. Here, the transmission quality is poor and here the transmission quality is very good, when the angle is close to 90^0.

Let us look at this model again where, $(l_1)^2 + (l_2)^2 \neq (l_3)^2 + (l_4)^2$. So here, the minimum transmission angle occurs only for this configuration when the crank is along the line of frame and the angle between the follower and the coupler is very small giving rise to very poor transmission characteristics. However, in this configuration, when again the crank is along the line of frame, the angle is quite large and this is not the minimum transmission angle.

So, if there is no quick return, then the minimum transmission angle occurs either here or when this angle is 180^0, but if there is no quick return, then it happens both at this position and at the 180^0 position as shown earlier. We now explain the concept of transmission angle with reference to a slider-crank mechanism.

(Refer Slide Time: 53:28)



This figure shows a slider-crank mechanism, where the slider is at B undergoing horizontal translation and O_2A is the crank. It is obvious that, if the connecting rod AB is horizontal then the entire connecting rod force is in the direction of movement of the slider.

Consequently, we define the transmission angle as μ that is the angle between the connecting rod and a direction perpendicular to the line of movement. The most desired value of μ is 90^0, but to ensure smooth free running of the mechanism, μ_{min}, minimum value of μ should not fall below say, around 30^0. Now we can find out, for what value of crank angle, that is θ_2, the minimum transmission angle occurs. For that, we see this vertical distance is, $l_2 \sin\theta_2 = l_3 \cos(\mu) + e$. As θ_2 changes, l_3, l_2 and 'e' are constants, so μ changes. And the minimum value of μ will take place depending on whether 'e' is this way or suppose the offset was below this line, the direction of sliding is like this, then this distance would have been called 'e' depending on whether 'e' is upward or downward, one can easily find that,

$$\mu_{min} = cos^{-1}(\frac{l_2 + e}{l_3})$$

That occurs either at θ_2 equal to $\pi/2$ or $3\pi/2$, depending on the direction of 'e'.

This I leave for the students to decide for themselves and that will add to the understanding. One can also see this slider-crank mechanism had a 4Rlinkage with O_4B the hinge O_4 at infinity. Then you see, O_2ABO_4 is the equivalent 4R-linkage and μ is nothing but the angle between the coupler AB and the follower O_4B. This concept of transmission angle we have used is same for both the 4R-linkage, crank-rocker linkage, and this slider-crank mechanism.

Let me now summarize what we have learnt today. We continued our discussion on displacement analysis of planar mechanisms by analytical method. Then we obtained certain important results so far as 4R crank rocker linkages are concerned, with reference to its quick return effect and transmission angle and also where the minimum transmission angle occurs, which has to be ensured for a free running of a design mechanism. And we have also seen through an example, that how we can combine both graphical and analytical methods to improve the design or modify an existing design.