Kinematics of Machines

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Module – I Part – 1

Kinematics of Machines

The title of this course is Kinematics of Machines. So, let us start our discussion with the definition of these two words, namely machines and kinematics.

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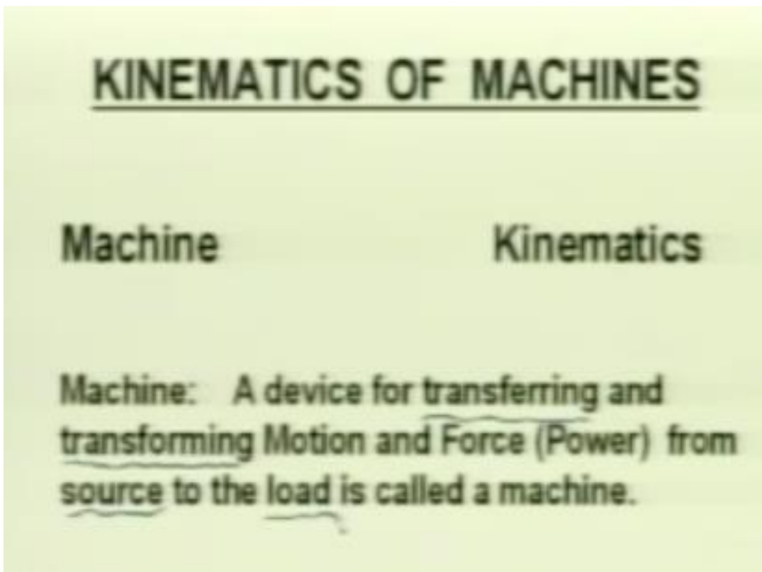

Broadly speaking, we can define machine as a device for transferring and transforming motion and force or power from the input that is the source to the output that is the load. Let me repeat, machine is the device for transferring and transforming motion and force from source to the load. The motion needs to be transformed as it is being transferred from the source to the load. The type of transformation that is needed is decided by the nature of the input motion that is available and the type of output motion that is desired. Some typical examples of the available input motion and desired output motion, we are listing now.

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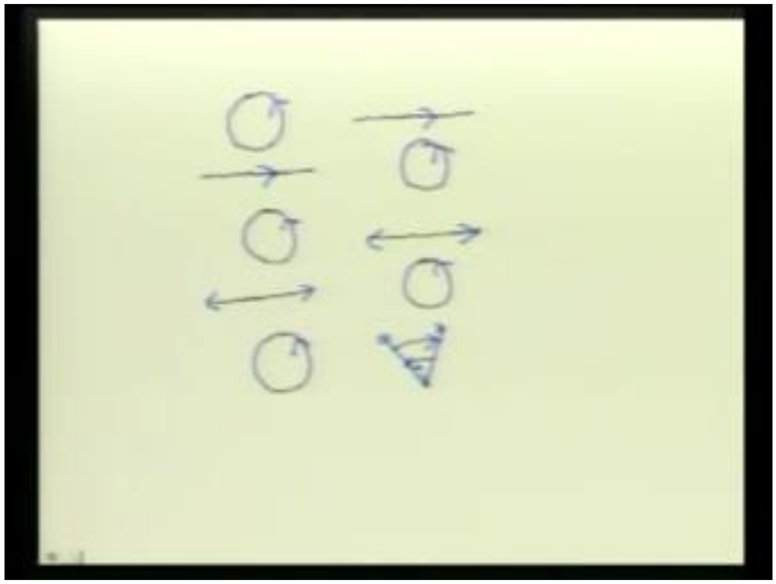

It may be the input motion available using the form of continuous rotation, whereas a desired output motion is a rectilinear motion or the vice versa. The input motion is a rectilinear motion and we need the output motion in the form of continuous rotation. Sometimes, the input motion may be continuous rotation, whereas the desired output motion is in the form of to and fro oscillation or again the vice versa. That is, the available input motion may be to and fro oscillation along a straight line, however, the desired output motion may be continuous rotation. Sometimes, both input and output motion may be continuous rotation but they may have to take place at different speed.

Sometimes, the input motion may be continuous rotation however, the output motion desired is that of an angular oscillation. It also needs from continuous input rotation to be transformed into intermittent rotary motion. At this stage, let us talk of a real-life machine that is, the most common machine tool which we call a lid. Here, the input motion is available at the shaft of the driving motor, which is the input motor, normally located at bottom left end of the machine.

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This motion has to be transferred to two different locations, that is to the head stock, to provide motion to the job and also to the tool post, that is to provide motion to the cutting tool. Let us see, during these two transfer processes, what kind of transformation is taking place? As we know, the job has to rotate continuously. So, the head stock has to be rotated at different speed than for the input motor, depending on the size of the job, whereas the tool post has to move along a straight line starting from the same input motion. Thus, we need to define stands of devices to transfer and transform from the input to the output.

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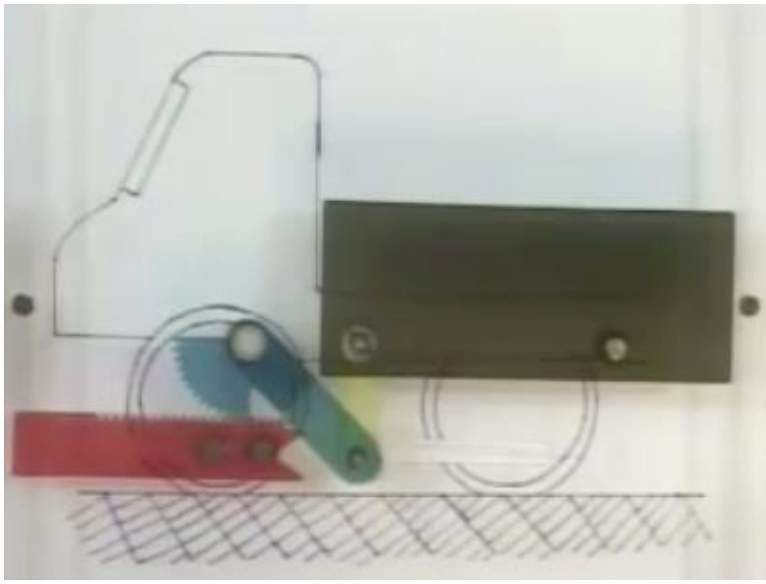

As the second example, let us look at the model of this tipper dump. Here, the input motion is linear provided by a hydraulic actuator and this linear motion has to be transformed into a rotary motion of this dumping bin. Thus, the linear motion at the hydraulic actuator from the input side is transferred to the bin and is also transformed into the rotary motion. One more point that one need to watch in this machine is that during the dumping position, the dumper is locked and cannot be disturbed by any accidental disturbance. Normally, in most such machines, various components undergo very little deformation. Consequently, these components can be assumed to be rigid. We didn’t the paradigm of the rigid body. The relative motion between various components can be studied from the view point of geometric constraints without any reference to force.

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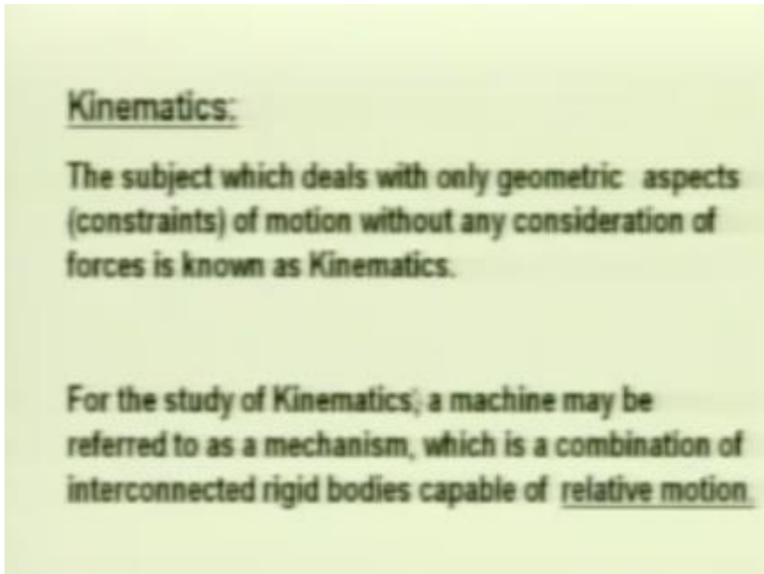

The subject matter which deals with this geometric constraint of relative motion, without any reference to the cause of the motion that is the force is called kinematics. We repeat, the subject which deals with only geometric aspects of motion without any consideration of force is known as kinematics. For the study of kinematics, a machine may be referred to as a mechanism, which is a combination of interconnected rigid bodies capable of relative motion. During the study of kinematics that is of a mechanism the idea of motion or relative motion predominates and the idea of force takes a back seat.

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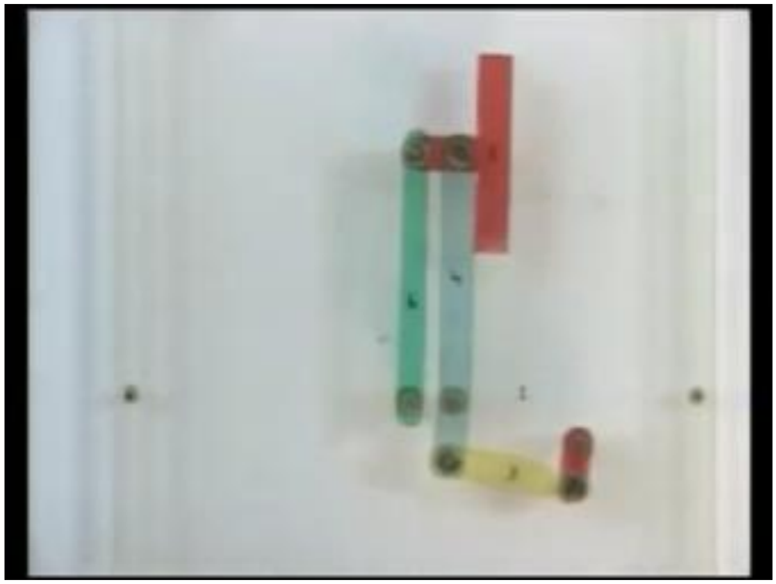

As an example of a mechanism, let us look at this one, which is a wind shield wiper mechanism for bigger vehicles like a bus. Here, the continuous rotation of the drive motor is converted into to and fro oscillation of the wiper lid. Of course, the power available at the motor shaft is used to overcome the friction force between the blade and the wind screen. However, designer is primarily interested to ensure a proper range of oscillation of the wiper blade, to generate the desired field of wiping. Consequently, we do not quality wiper machine, but we quality wiper mechanism.

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As a second example, let us look at this common pair of pliers. Here, if we want to study the relative movement between the two jaws, we can hold the lower jaw fixed and move only the upper jaw. As we see, these two jaws are simply hinged and the axis of the hinge is passing through this point. Consequently, all the points of this upper jaw are moving in circular arcs, centered at this hinge axis. Consequently, it will not provide a very good grip on a flat object and specially show if the object is long.

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However, we can design parallel job players by using a suitable mechanism. Using this mechanism, as we see the upper jaw is moving parallel to the lower jaw and consequently it will provide a very good grip on a flat object. So in such mechanisms, the generation of this specific motion characteristics is a primary interest rather than the forces on top invert.

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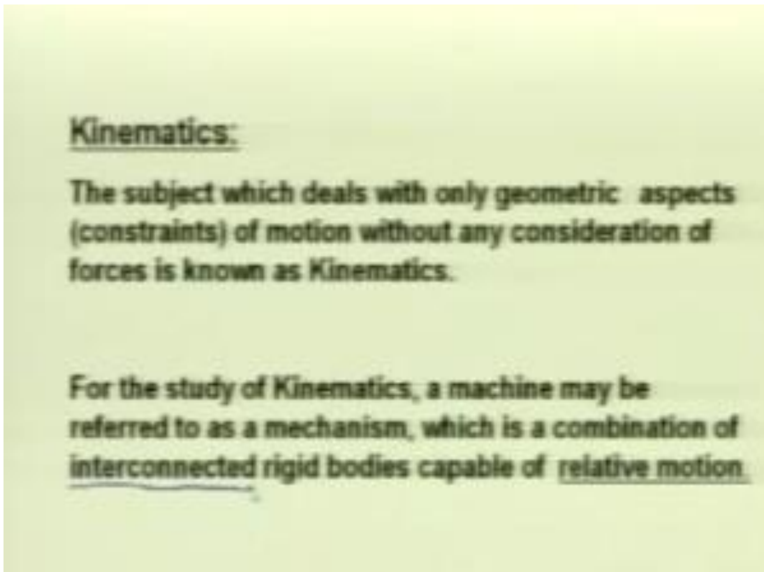

A clue to the behavior of a mechanism lies in the interconnection. These interconnections are technically called a kinematic pair. For the understanding of kinematics, a thorough understanding of the interconnections of the kinematics pair is essential. That is why, we shall now discuss in detail various kinematic pairs and the kind of relative motion that are permitted at these interconnections which are kinematic pairs. Before we go into the discussion of different types of kinematic pair, let me define certain technical terms. The number of coordinates that are needed to describe the relative movement permitted in a kinematic pair is called the degree of freedom.

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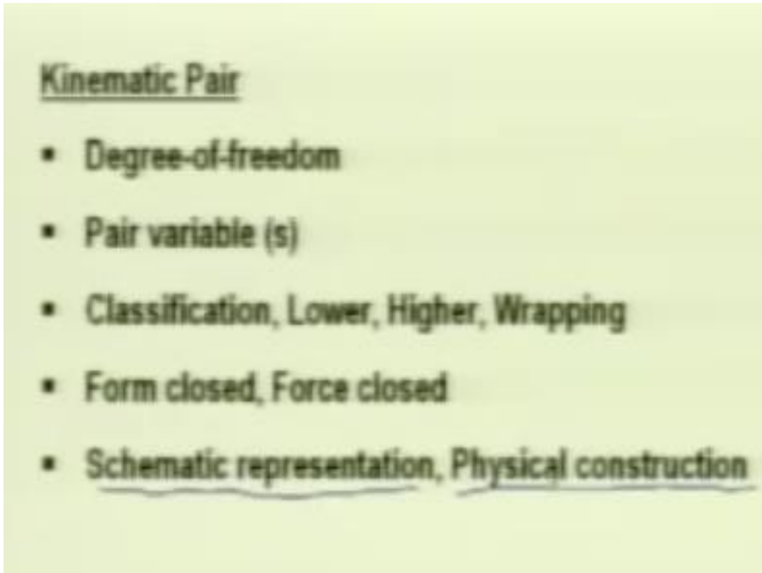

As we said, in every kinematic pair, there is some relative movement and to describe the relative movement, we need some coordinates. The number of such independent coordinates necessary to completely specify the relative movement is called the degree of freedom. The coordinates that are used to describe this relative movement is called pair variable. At this stage, let us recall that, a free an unconnected rigid body which does not have any kinematic pair to some other rigid body. A completely free unconnected rigid body has 6 degrees of freedom in a three-dimensional space. Three of these degrees of freedom are translational, say along three mutually perpendicular directions x, y, z. So there are three translations, one along x-axis, another along y-axis and along z-axis. The rest of three are rotational. As soon as, this free rigid body is connected to another rigid body through a kinematic pair, one or more of the 6 degrees of freedom are cluttered and only few are returned.

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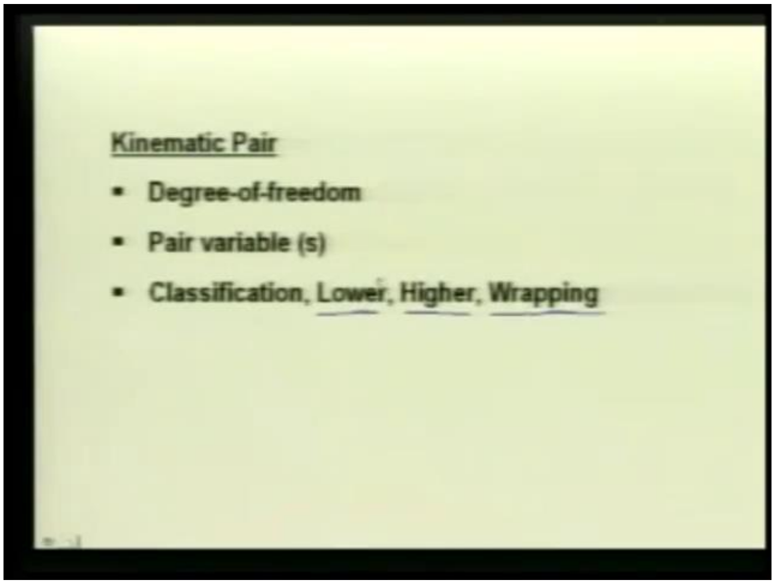

Kinematic pairs can be normally classified under three headings namely, lower pair, higher pair and wrapping pair. In a lower kinematic pair, the two bodies which are connected by this pair has area contact. Suppose, we talk of two bodies, namely 1 and 2, if they are connected by a lower pair, then at this connection, body 1 will have a surface contact with body 2. Similarly, in a higher pair the contact between the two bodies has only a line contact or a point contact. Whereas in a wrapping pair, one body completely wraps over the other. The typical example is of a belt and a pulley or a chain and a sprocket where the belt completely wraps around the pulley or the chain completely wraps around the sprocket.

There is another way of classifying the kinematic pair. Sometimes, it is called a form closed pair. Sometimes it is called a forced closed pair. Let me now explain, what do you mean by a form closed? If the contact between the two bodies at the kinematic pair is maintained by the geometric form, then we call it a form pair. Whereas, if the contact needs to be maintained by the application of an external force, then we will call it a force closed pair. All these concepts will become clear when we get into specific examples.

Next, we shall discuss all these various kinds of lower pair and higher pair through their schematic representation. I want to insist that the diagrams that will be shown are only for the schematic representation. Their physical construction can be very different as will be exemplified when we get into specific kinematic pair.

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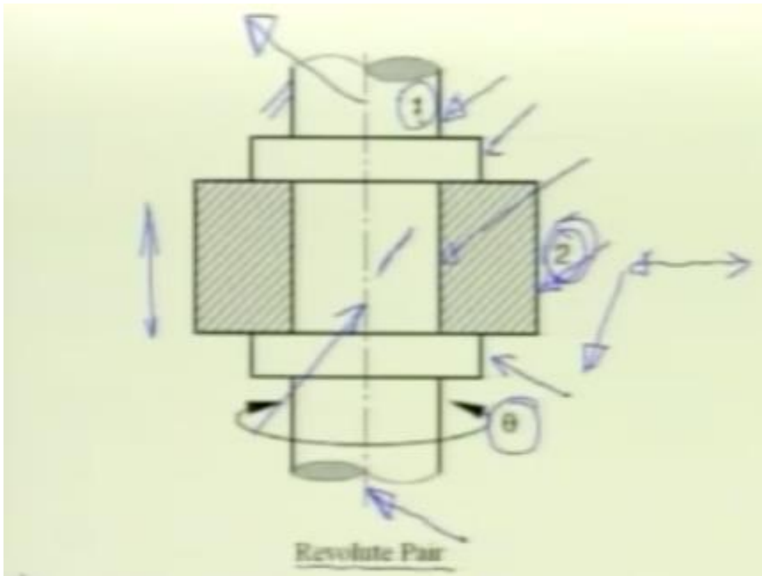

Let us now, discuss six different types of lower pair. As shown in the above slide, two bodies namely, body 1, body 2 are connected by this kinematic pair which is known as a revolute pair. This is a simple hinge joint. This portion of body 1 is a cylindrical pin which goes into a cylindrical hole in body 2. Also note there are two collars on body 1. To study the possible relative movements between these two bodies 1 and 2, let us consider body 1 to be fixed and let us see what type of motion we can give this to body 2. Obviously, the body 1 cannot translate along this vertical direction. The body 2 cannot translate in this horizontal direction. The body 2 also cannot translate perpendicular to these two directions. Consequently, all the three translational degrees of freedom of body 2 have been restrained by this revolute pair. The only relative motion that is permitted is rotation of body 2 about this vertical axis. The pair variable theta represents this relative rotation between body 1 and body 2 about this vertical axis. We should also note that this particular revolute pair is characterized by this axis, which is a fixed line in space.

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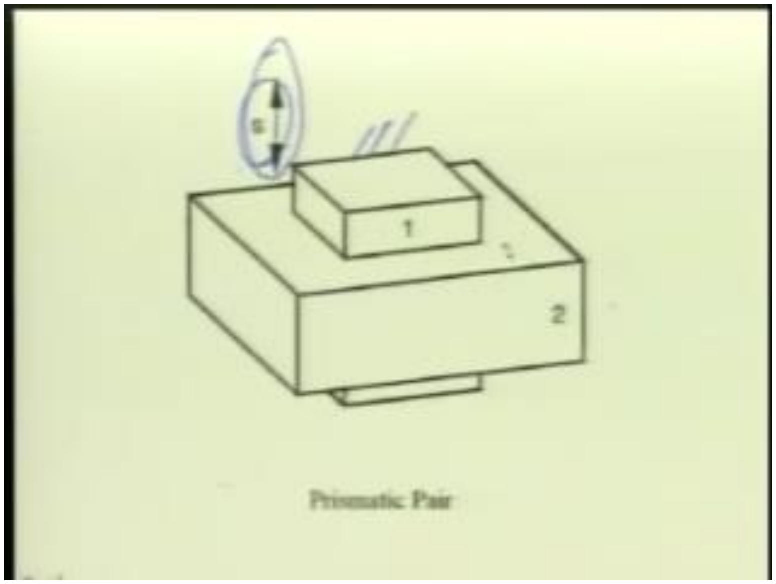

The above slide will show what is known as a prismatic pair. In this prismatic pair, again if we hold this body 1 fixed, then body 2 can only move along in vertical direction, no other rotation of body 2, no other translation of body 2 is possible. Consequently, this S representing the relative sliding between body 1 and 2 is the pair variable and this has single degree of freedom just like a revolute pair.

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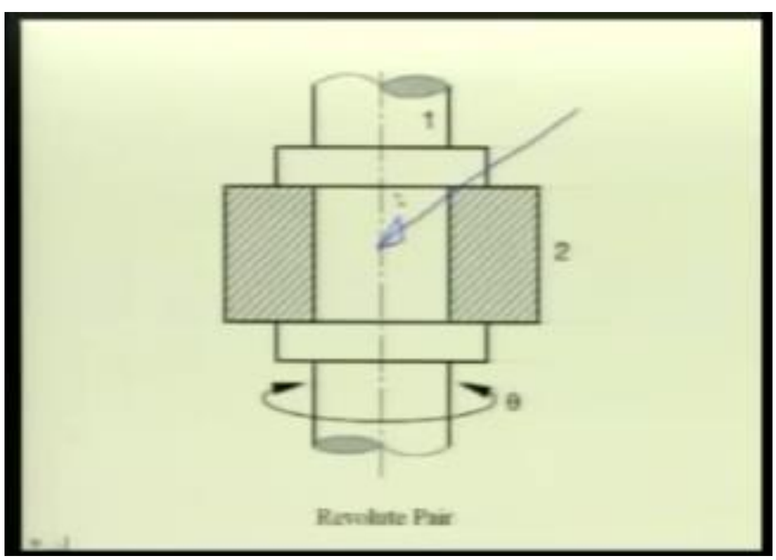

However, I would like to emphasize, one very critical difference between a revolute pair and a prismatic pair. If we go back to the revolute pair, we saw that, revolute pair is characterized by this axis, which is a line is fixed in space.

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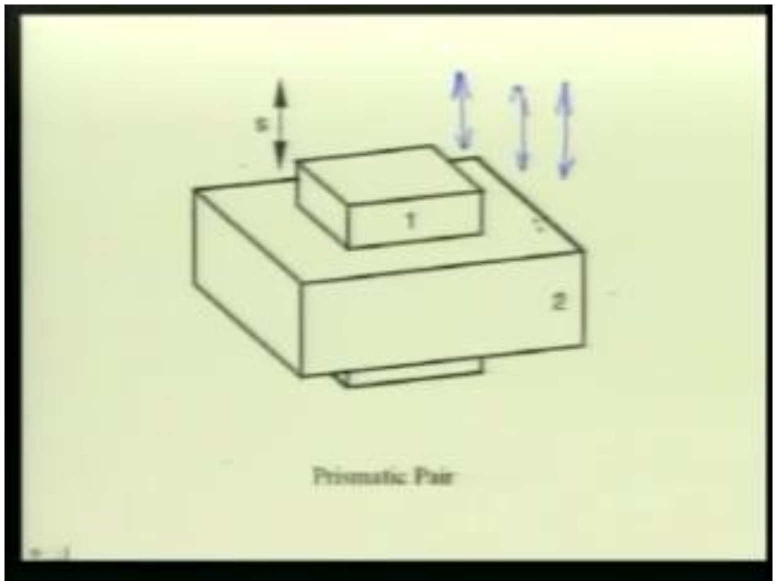

Whereas, in the prismatic pair, the translation is along vertical direction and that can be represented by all vertical lines, we do not need any fixed line in space. So, this is only a direction not in axis. At this stage, I would like to emphasize that these two diagrams are only schematic.

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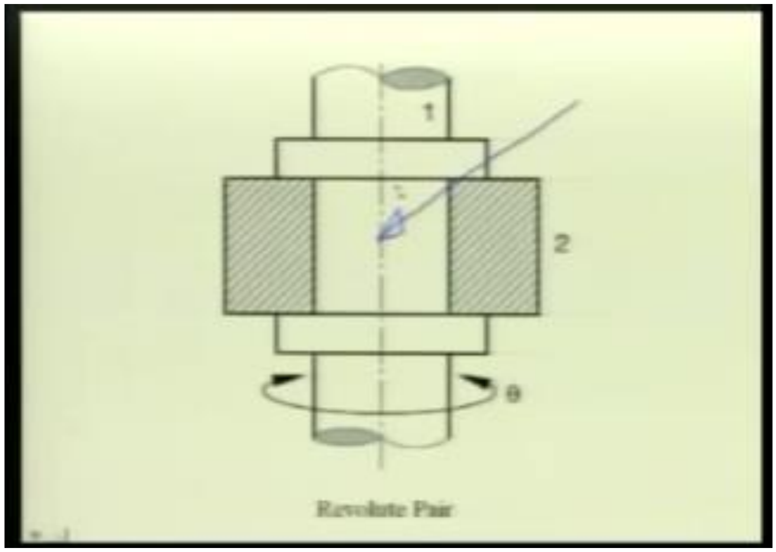

For example: a revolute pair can be constructed by a bearing between the shaft and the foundation. So, the physical construction of a revolute pair can be very different because that bearing also allows a rotation about a fixed axis.

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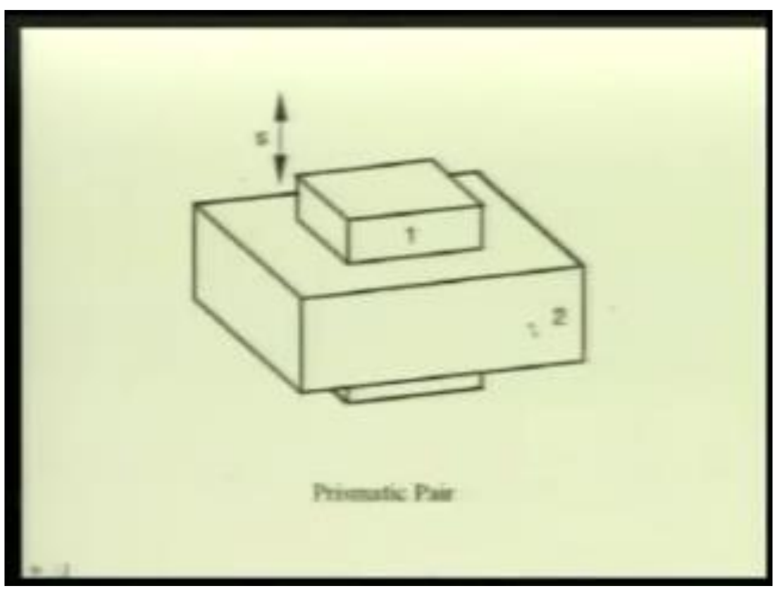

Similarly, a piston within a cylinder in an IC engine, for example forms a prismatic pair because the only relative motion between the piston and the cylinder is possible along the length of the cylinder. The next slide shows what the screw pair is.

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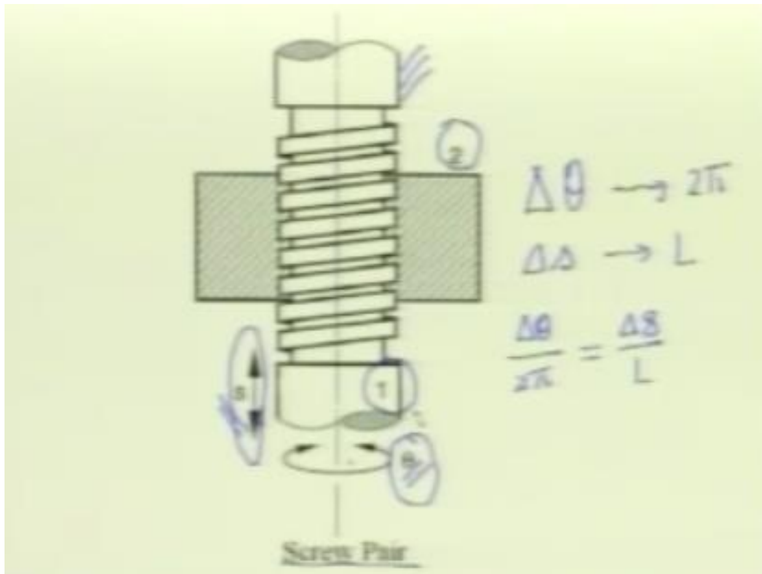

In the screw pair, the body 1 is the screw and the body 2 is a nut. So, if we fix body 1, then the only relative motion of body 2 that is possible is rotation ‘θ’ about the axis of the screw, but as the nut rotates it also translates along the axis of the screw. So, S can also represent the relative motion. It may appear that it has 2 degrees of freedom because two pair variables, namely: θ and S are required. But a little thought would convince us, that this screw pair also has single degree of freedom because as rotation θ goes to 2π, if I say it is the rotation by representing by change in theta. If Δθ goes to 2π, then change in S, ΔS goes to L, where L is the lead of the screw. Thus, θ and S are not independent, but they are related by this expression {Δθ/2π = ΔS/L}. Consequently, only one of these, either θ or S needed to completely specify the relative movement. The next figure shows what is known as the cylindric pair.

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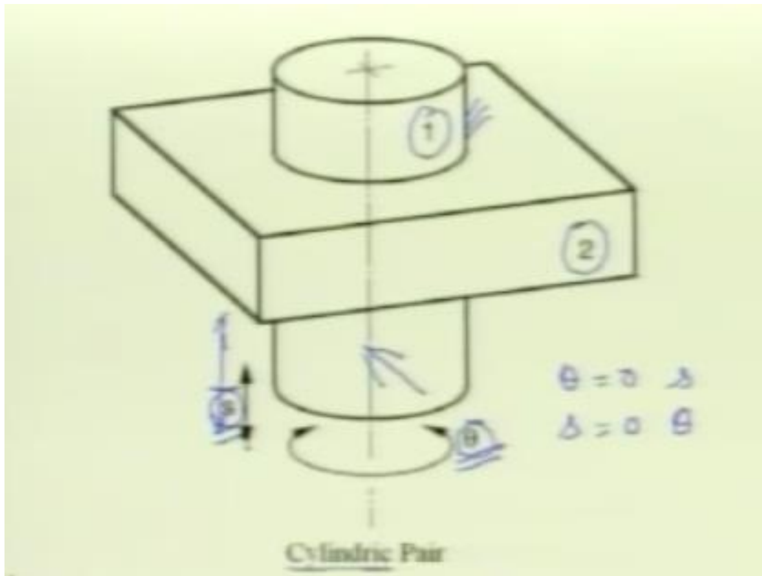

Here, in a cylinder body 1 passes through a cylindrical hole in body 2. If we hold body 1 fixed then body 2 can rotate about this vertical axis and we can also translate body 2 with respect to body 1 along this vertical direction, but here we see the pair variable θ and S are independent. I can keep θ equal to zero and give any amount of S or I can hold S equal to zero and give any amount of θ. Thus, two pair variables θ and S are independent. Consequently, this cylindrical pair has 2 degrees of freedom.

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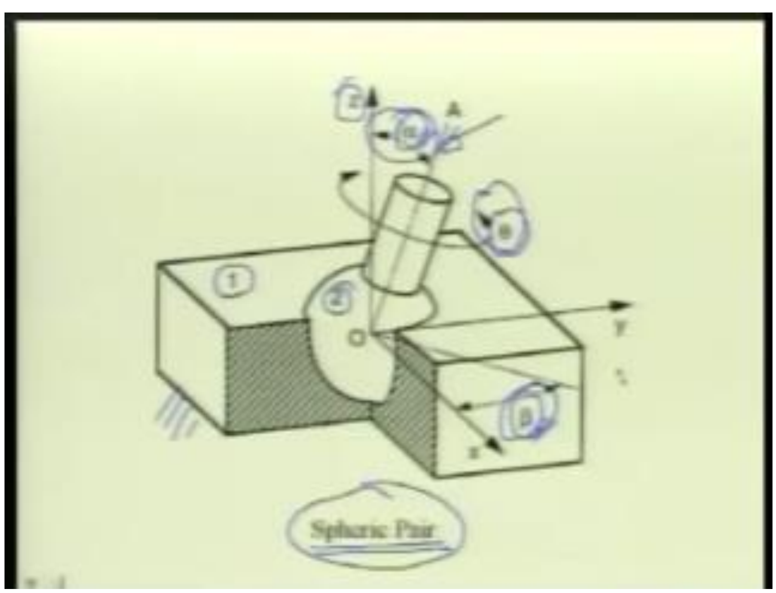

The above slide shows, what the ball and socket joint is? There is a spherical ball in body 2 and there is a spherical cavity in socket 1. This pair is known as a spheric pair. It is obvious from the schematic representation, if we hold body 1 fixed and then body 2 loses all its translated degrees of freedom. It cannot translate in x, y or z direction. However, it retains all its three rotational degrees of freedom. These three rotational degrees of freedom can be represented by three independent angles, namely: alpha and beta. These two angles alpha and beta locates the line OA in this three-dimensional space x, y, z and this alpha and beta are completely independent. I can give any amount of alpha and any amount of beta and take OA somewhere else in this three-dimensional space. But even after locating this axis OA, there is another degree of freedom theta, which denotes the spin of body 2 about its own axis OA. Thus, we have 3 degrees of freedom in this spheric pair and the three pair variables are alpha, beta and gamma. Again it is needless to say that, this is only a schematic representation. Nobody would be able to make such a ball and socket joint. So obvious question is: how the ball entered into the socket?

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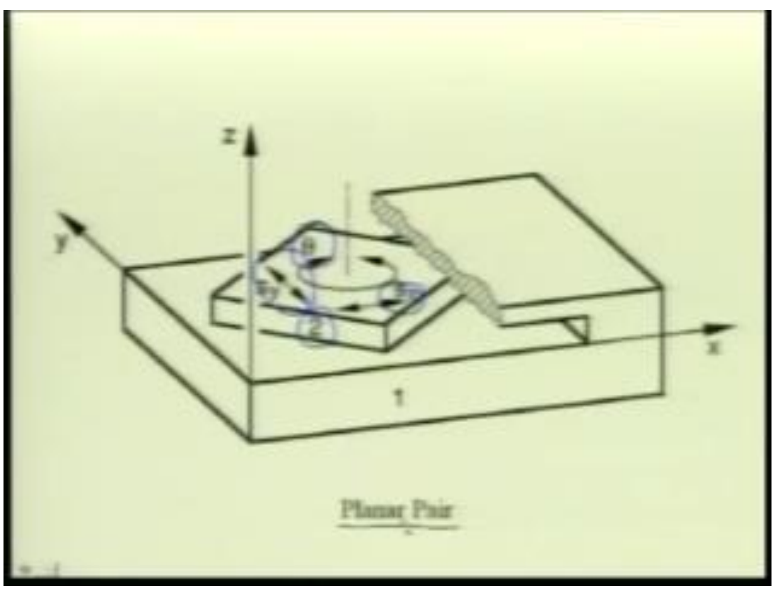

The next kinematic pair, as shown in this slide is called a planar pair. Here as we see, body 2 can have translation along x-direction given by the pair variable S, along the y-direction, given by the pair variable S_y and also rotate about the z axis given by the pair variable theta and this S_x, S_y and θ are three independent coordinates, consequently planar pair has 3 degrees of freedom. The next figure shows, what we called a spheric pair, but we also know note that this is force closed.

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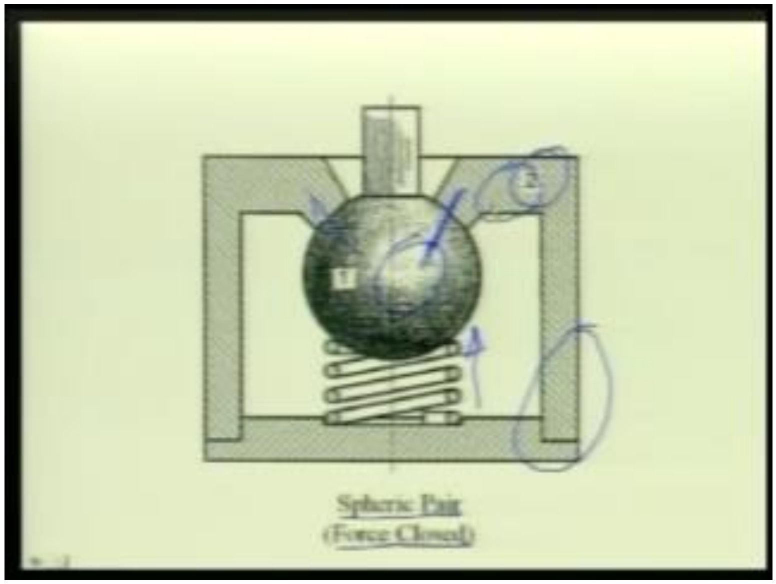

Here, the spherical surfaces of body 1 and body 2 are maintained in contact in these two zones by the application of this spring force. We also note, that this is how in real life a spheric pair will be constructed where this body 2 is assembled, rather than making in one piece as shown earlier. Let us now, recapitulate all these six types of lower pairs. As we said a lower pair should have surface contact and we must have noted that in all this schematic representation, the bodies 1 and 2 at contact over a finite surface.

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Type	Symbol	DOF	Pair Variable
Revolute (Hinge)	R	1	θ
Prismatic (Slide)	P	1	s
Screw (Helix)	H	1	θ or s, $\frac{\Delta\theta}{2\pi} = \frac{\Delta s}{L}$
Cylindric	C	2	θ and s
Spheric (Globular)	G	3	α, β and θ
Planar	E	3	s_x, s_y and θ_z

At first we discussed, what you called a revolute pair. The symbol that is used for a revolute pair is R, that is the first letter of this word revolute. This revolute pair is nothing but, in a non-technical language called a hinge joint. The degree of freedom of this revolute pair is 1 and the pair variable is θ which denotes the rotation. Next, we discussed, what we called the prismatic pair which is nothing but a sliding joint. Symbol for prismatic pair is P, which is again the first letter of the word prismatic. Here also the degree of freedom was 1 and the pair variable was S. Then, we discussed a screw pair. The symbol for screw pair is H. To remember this H, it will be nice to remember that screw is in the form of a helix and it is the first letter H of helix, we are using to represent the symbol of a screw pair. The degrees of freedom in screw pair was also 1 and the pair variable would be either θ or S, which are related by this relation $\{\Delta\theta/2\pi = \Delta S/L\}$.

Then we called discussed the cylindric pair. The symbol is C. Here, the degree of freedom is 2 because we need two independent variables, θ and S to represent the relative rotation and relative translation. Then we discussed spheric pair. The symbol for spheric pair we are using is G. To remember this G, the spheric pair can also called a globular pair and G is the first letter which is used to represent the symbol of a spheric pair. Here the degree of freedom is 3 and the pair variables as shown in that sketch where alpha, beta and theta. Lastly, we had we called a planar pair. We cannot use the symbol P which has already been used for the prismatic pair. So, we give it a new symbol called E. In German language, plane is called Ebony. Starting with the letter E and we are using that letter E to represent a planar pair. The degree of freedom in a planar pair is also 3 and the three pair variables will be S_x, S_y, representing translations in two mutually orthogonal directions and θ_z that was the rotation about the perpendicular axis which was represented as z-axis.

Now that we have discussed all kinds of lower pairs. Let us get into higher pair. In a higher pair, as told earlier, the contact between the two bodies takes place only along a line or at a point. As we see, between the pair of a gear teeth or between a cam and follower mechanism. The next slide schematically shows a higher pair between body 1 and body 2.

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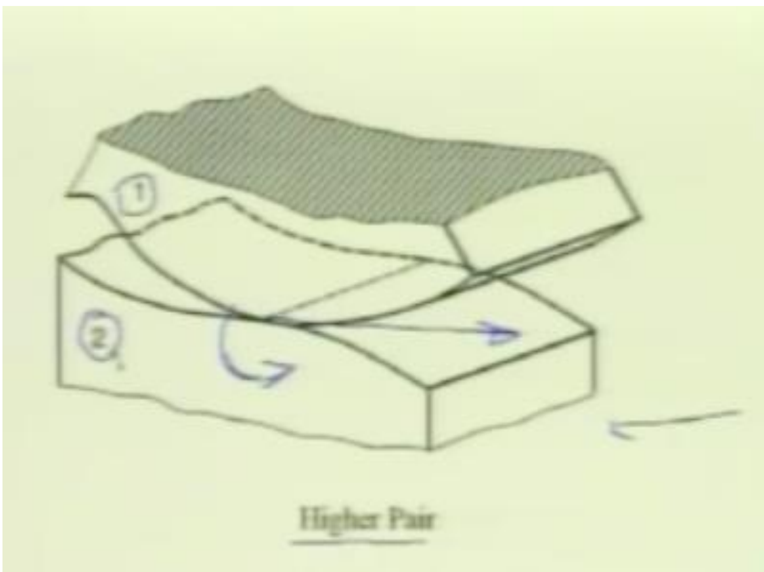

As we see, if we restrict the relative motion of body 2 with respect to body 1 in one plane, then there can be only relative translation in this direction and rotation about this axis and this is the line contact between body 1 and body 2. So, this is a higher pair.

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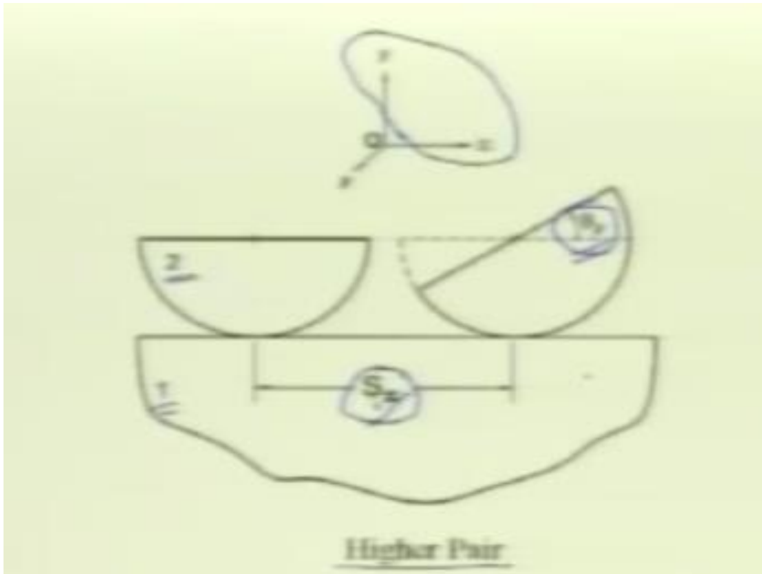

The above diagram again shows a higher pair between body 1 and body 2. As the body two moves relative to body 1 and assuming that the motion is restricted in this xy plane, then to describe this relative motion completely, we need two coordinates namely S_x and θ_z. S_x representing the relative sliding and the θ_z representing the rotation about the z-axis are completely independent. Thus, if a higher pair is used in a planar mechanism, then a higher pair will have 2 degrees of freedom and two pair variables namely S_x and θ_z.

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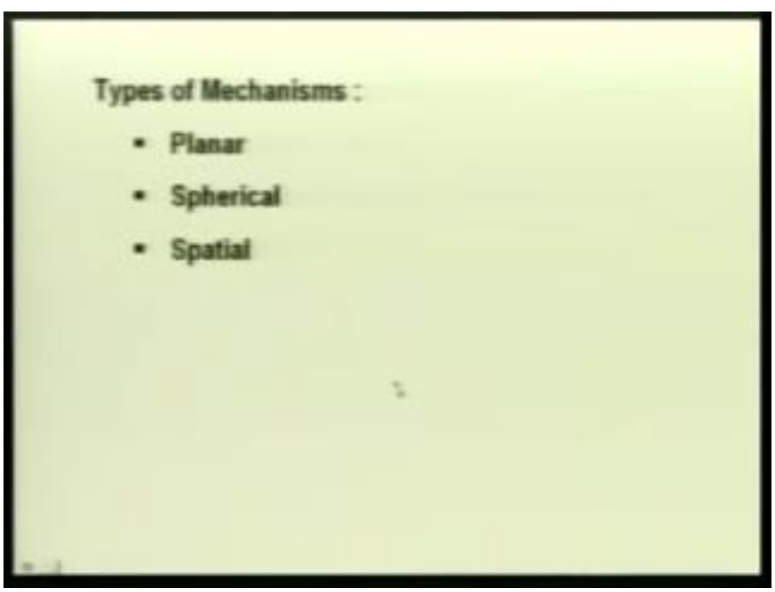

Now, we have defined all types of kinematic pairs. Let us talk about the classification of different types of mechanism. Broadly speaking, we can classify mechanism under three headings namely: planar, spherical and spatial. In a planar mechanism, all the points of the mechanism move in parallel planes and all these parallel planes can be represented by a single plane, which is called the plane of the motion and single view perpendicular to this plane of motion reveals the crew motion of all the points of a mechanism.

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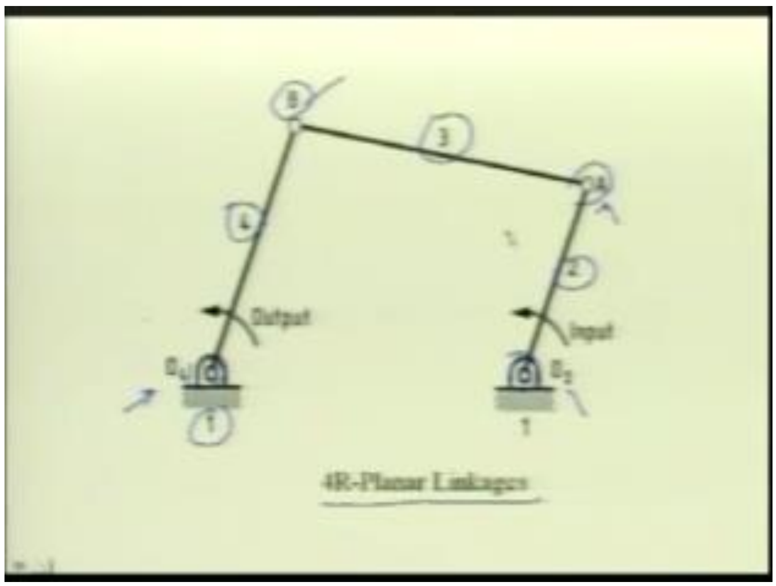

As an example, let us look at this planar mechanism. This is called 4R-planar linkages. Here, we have four rigid bodies, namely 1, 2, 3 and 4 which are connected by four revolute pairs at O_2, A, B and O_4. As the linkage moves, the input motion is transferred to the output motion through the rigid body 3. The input shaft at O_2 and the output shaft at O_4 are parallel. Not only that, what we can see all the four revolute joints have parallel axis at A, O_2, O_4 and B. There are four revolute joints but the axis of all these four revolute joints is parallel and perpendicular to this plane of motion. The next figure shows 4R-spherical linkage.

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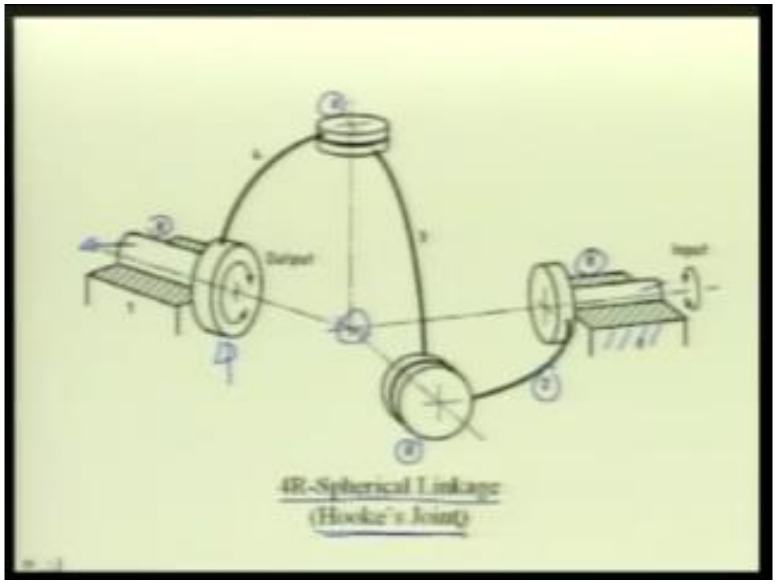

Here again, we have four rigid bodies connected by four revolute pairs. But the axis of all these four revolute pairs intersect at one point. This linkage can be used to transmit input motion from shaft 1 to output link number 4. The input is link 2 and link 1 is fixed. There is a revolute pair. If I rotate this input shaft, the motions are transmitted through all these revolute joins and ultimately the output link 4 rotates. So, this is a mechanism to connect two shafts whose axes are not parallel but intersecting. Such a joint is known as a Hooke's joint.

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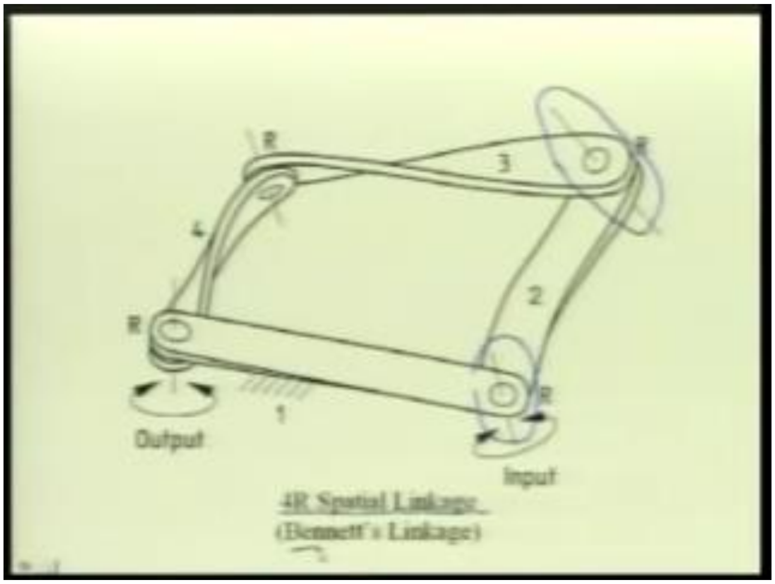

This figure shows again a mechanism with four revolute joints, but here, we call it a 4R-spatial linkage, because the points of all these links move neither in parallel planes nor on the surfaces of spheres. They move in a three-dimensional space and as can be clearly seen that the axis of this revolute pairs is skewed in space. In the sense, they are neither intersecting nor parallel. This linkage has a name, it is called a Bennett's linkage. It should be mentioned that unlike in a 4R-planar linkage or in a 4R spherical linkage or in this 4R-Spatial linkage, there can be relative motion only for specific dimensions.

Now, we have discussed different kinds of kinematic pair and different types of linkage. Let me say, what the subject that we study in kinematics.

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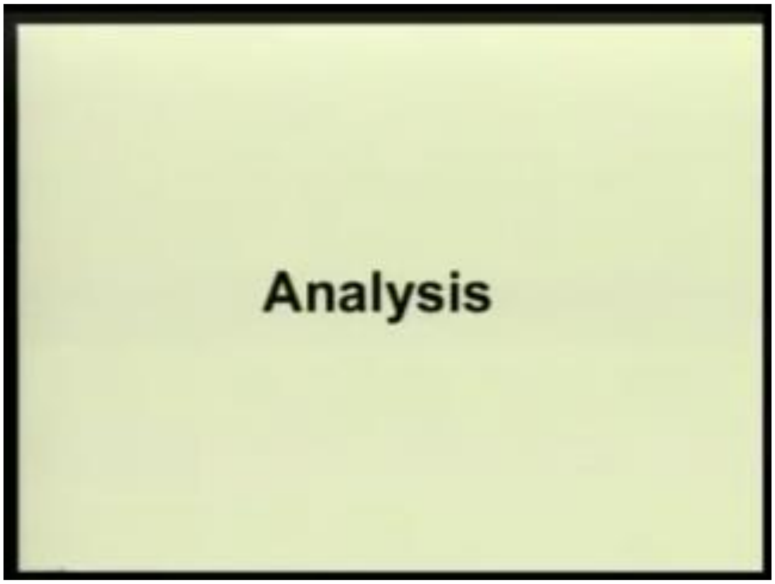

In kinematics, there are two types of problems, one is called analysis.

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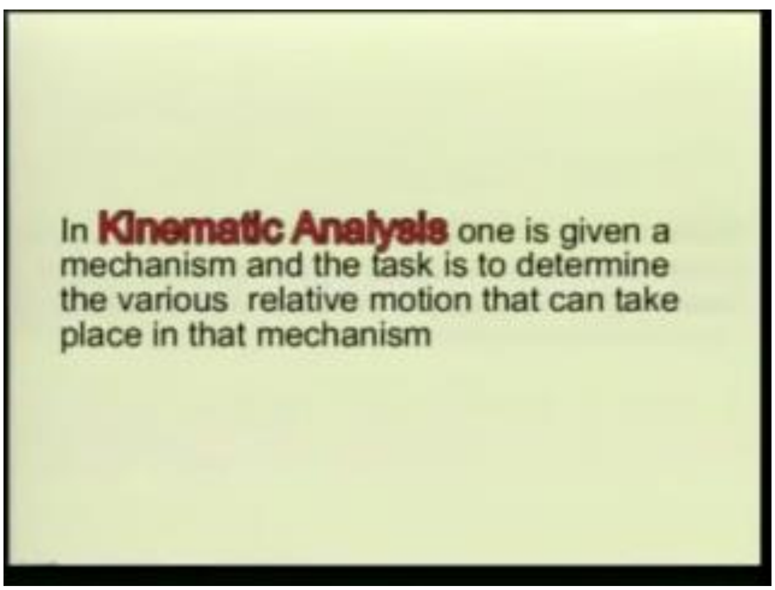

In kinematic analysis, one is given a mechanism and the task is to determine the various relative motion that can take place in that mechanism.

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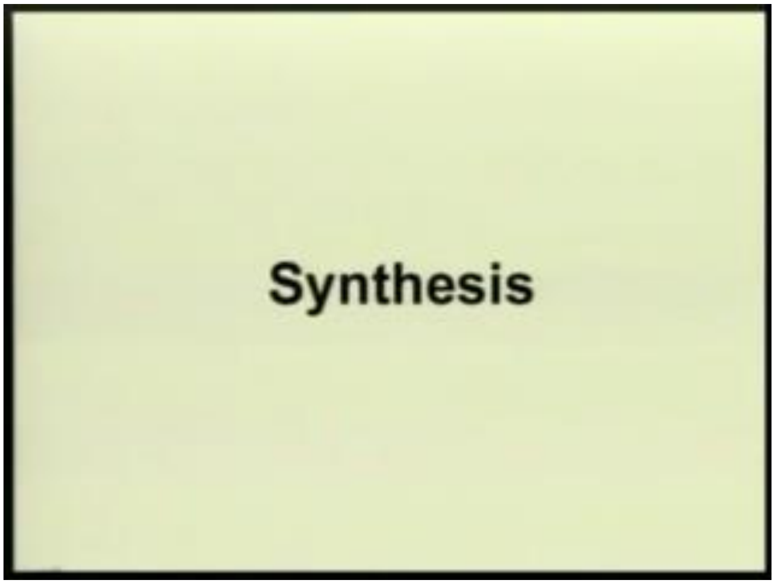

The other problem is, what is known as kinematic synthesis.

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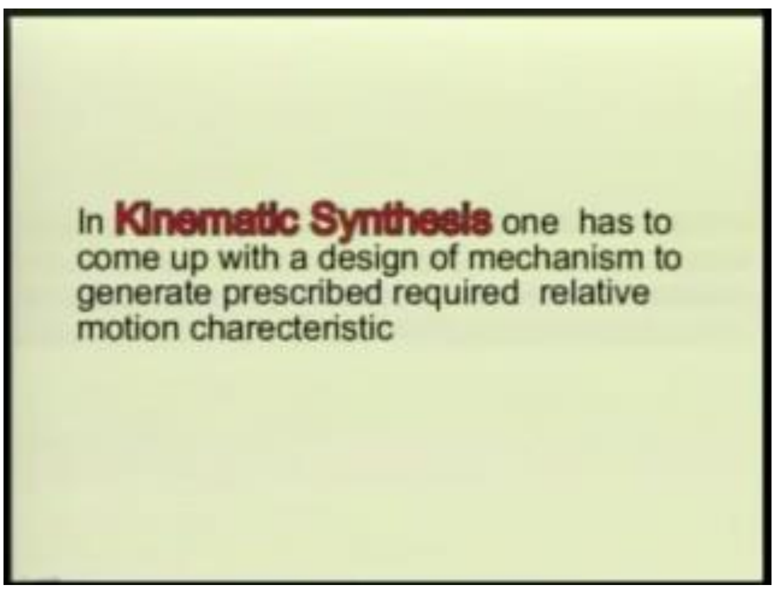

In synthesis one has to come up with a design of mechanism to generate prescribed required relative motion characteristics.

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This course will be addressing to both of these problems namely kinematic analysis and kinematic synthesis.