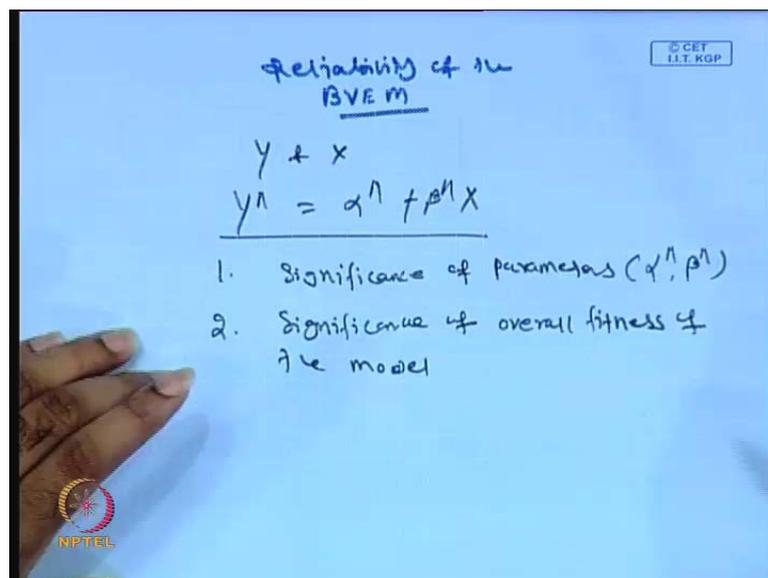


**Econometric Modelling**  
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**Lecture No. # 10**  
**Reliability BEM (Contd.)**

Good afternoon; this is Doctor Pradhan here; welcome to NPTEL project on econometric modeling. So, today we will continue the reliability part of bivariate econometric modeling. In the last class we have discussed the near depth reliability and the structure of reliability for the bivariate estimated econometric model. Now, we like to highlight this same issue again here because, some of the things we have not discussed last class.

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So, the thing is for 2 variables Y and X, our fitted model is like this: Y hat equal to alpha hat plus beta hat X. Now, the essential point is here we have 2 specific objectives; the first objective is to know the significance of the parameters and second objective is to know the overall fitness of the models.

In this particular... suppose reliability is concerned, we have 2 specific objectives. First objective is to know the significance of parameters and that is with respect to alpha hat and beta hats and second, the significance of the **significance of** overall fitness of the

model overall fitness of the model. So, we have 2 specific objectives so far as reliability is concerned.

So, first objective is to know the significance of the parameters that is the weightage of you know, each parameters when we fit the you know, regression equations with respect to X and Y. Then, obviously the impact can be negative or the impact can be positive which is just through the slope of the, you know, X coefficient or X variables. This which we have to just through slope of the X variable that is nothing but, beta coefficient and alpha coefficient is just to know the significance of the, you know, supporting factors.

Now, you know, just to put in a straight line equation, this is you know intercept and this is what we call it a slope. Now, we like to know whether this you know, intercept or the supporting component is significant one for influencing Y and whether X component is significant one. Again, for influencing Y now to know this one, we have standard procedure; so, that part the discussion of, this part particularly is known as the reliability of this estimated model.

So, before we... first we start this first objective; that is the significance of the parameters. So, the significance of parameter is that we have to represent the estimated models in a typical tabular form so that we can understand the exact structure of the reliability.

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$$Y^h = \alpha^h + \beta^h X$$

Estimated Parameters	Estimated Values	Variance of EV	SEY	t	P
$\alpha^h$	$\bar{Y} - \beta^h \bar{X}$	$\text{Var}(\alpha^h)$	$\sqrt{\text{Var}(\alpha^h)}$	$\frac{\alpha^h}{\text{SEY}}$	-
$\beta^h$	$\frac{\sum xy}{\sum x^2}$	$\text{Var}(\beta^h)$	$\sqrt{\text{Var}(\beta^h)}$	$\frac{\beta^h}{\text{SEY}}$	-

$$\text{Var}(\alpha^h) = \sigma_u^2 \frac{\sum x^2}{n \sum x^2} - X$$

$$\sigma_u^2 = \text{Error Variance} = \frac{\sum e^2}{(n-2)}$$

$$\sum e^2 = \sum y^h - \sum y^h L = \sum y^2 - \sum y^h L$$

Now, when we have estimated models, we had  $\hat{Y}$  equal to  $\hat{\alpha}$  plus  $\hat{\beta}X$  so then the standard table we have to design is here. We have estimated parameters; then second, the estimated values; then, third column represents variance - variance of estimated values. Then, standard error then t statistics, then you know probability level of significance, these are the structure of this particular you know, significance of the parameters; that means with respect to the first objective.

So, what are the estimated parameters for this particular you know, bivariate setup? The estimated parameter, first parameter is related to  $\hat{\alpha}$  and second parameter is  $\hat{\beta}$ ; this is what that means. Now, this table is altogether complete one so, this is what we have to design this entire table alright.

Now, these particular structures, what is estimated value  $\hat{\alpha}$ ? We will get it you know this is nothing but,  $\bar{Y} - \hat{\beta}\bar{X}$  and  $\hat{\beta}$  is equal to  $\frac{\sum XY}{\sum X^2}$  which we have discussed long back. Now, variance of estimated  $\hat{\alpha}$  that is nothing but variance of  $\hat{\alpha}$  and this is nothing but variance of  $\hat{\beta}$ . Then, standard error of  $\hat{\beta}$  that is nothing but, variance of  $\hat{\alpha}$  and this is square root of variance of  $\hat{\beta}$ . So, this is standard error; when we design T T statistic for this is  $t_{\hat{\alpha}}$  and this is  $t_{\hat{\beta}}$  and we like to know what is the significance levels.

Now, this model is you know theoretically is but, you know, technically or practically so far as the significance of the parameter is concerned, we have to evaluate in a proper sequence and that has to be compared with the tabulated value which we have discussed details in the last class.

Now, what is all about this variance of  $\hat{\alpha}$ ? so basically the variance of  $\hat{\alpha}$  is derived there are you know technical procedure how you have to get the variance of  $\hat{\alpha}$  but, in the mean times variance of  $\hat{\alpha}$  is nothing but,  $\frac{\sigma_u^2}{\sum X^2}$ . So, here this is you know this particular item is a capital X and this particular x is a small x. This is nothing but, deviation format this is we can represent in  $X - \bar{X}$  alright.

Now, altogether there are 4 items,  $\sigma_u^2$   $\sum X^2$  then a n into  $\sum x^2$ . So, this is nothing but, variance of you know, variance of X so the question is, what is  $\sigma_u^2$  here? So,  $\sigma_u^2$  is  $\sigma_u^2$  is

called as an error variance here. This is otherwise called as error variance here we are calculating the variance of a particular variance  $x$  or particular variable  $y$ . We have to also calculate the variance of you can say  $u$  or  $e$  because, in a bivariate setup we start with 2 variables  $Y$  and  $X$ . But, ultimately we with the help of you know estimated model we get to know the or you have to create another variable called as  $u$  or otherwise called as error term.

Now, altogether when we have a fitted model then the entire system consists of you know 4 important columns. So, first column is related to  $Y$  column; it gives the information about  $y$  structure and we can get to know what is the variation of  $Y$  or you can say, standard deviation of  $Y$  or mean of  $Y$ . so, these are the statistics we have to draw from the  $Y$  column.

Similarly, in the  $X$  column we have series of  $X$  information corresponding to or  $Y$  component. So, we can also get to know the entire descriptive statistics of  $x$  variable. Now, next to  $X$  we start with a variable called  $\hat{Y}$ .  $\hat{Y}$  is nothing but,  $\hat{\alpha}$  plus  $\hat{\beta}X$ . now, with the help of  $\hat{\alpha}$  value and  $\hat{\beta}$  value and with the help of  $X$  information then we can create the  $\hat{y}$  columns. So,  $\hat{Y}$  column also we can get the descriptive statistic  $\hat{y}$  because,  $\hat{y}$  altogether here another variable which is designed through the help of  $Y$  and  $X$  and the estimated parameter  $\hat{\alpha}$  and  $\hat{\beta}$ .

Now, with respect to  $\hat{Y}$  and  $Y$ , we have to create another column called as a error columns; so that is represented as  $u$  columns or you can say  $e$  column. Now, corresponding to every figure of  $\hat{Y}$  and  $Y$ , we have to find out the error component for instance,  $u_1$  is equal to  $y_1$  minus  $\hat{y}_1$ . Similarly,  $e_2$  equal to  $y_2$  minus  $\hat{y}_2$  so the difference between the estimated  $Y$  and you know actual  $Y$ ; so this will give the error representation.

Now, once you have error series starting from  $u_1$  to  $e_n$  provided the system is  $n$  th observation then, you have to calculate the error variance. So, these error variance you know it is called as a  $\sigma^2_u$ ; so  $\sigma^2_u$ . Sometimes, you know this error variance we will represent here summation  $e^2$  by  $n - 2$  this summation  $\sigma^2_u$  equal to summation  $e^2$   $n - 2$ .

Here, you know basically this particular summation  $e^2$  by  $n - 2$ . We can put it in a other way: summation is  $e^2$  by  $n - k$  actually  $k$  is  $k$  is the number of

variables in this particular system or number of parameters, in this particular systems. Now, since this particular model is a bivariate one obviously, there are 2 variables and there are 2 parameters right. That is, alpha parameter and beta parameters; so as a result k is represented as here 2.

So, there is no point to write summation e square by n minus k because, it is already known to us that k represents the total number of variables in the system. That is, you can say y and x or number of parameter in the systems; that is alpha hat and beta hat. Now, but, when there is you know, multivariate system then these particular terms can be represented as a summation e square by n minus k. For instance, if we have trivariate models then obviously, summation is e square by n minus 3 because there are 3 variables in the system. Similarly, we have to extend one after another then obviously the n minus k component will be start, you can say expanding.

Now, sigma square e equal to summation e square by n minus 2 where summation e square is equal to summation y hat square plus summation y hat square; this is summation u y square minus summation y hat square. So, that means in other words it is nothing but, summation y square minus summation y hat square.

Let me explain how it has happened here. This is you know, usually derived in a technical procedures. So, the details you know calculating procedure of this particular terms we can analyze here.

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The image shows a handwritten derivation on a blue background. At the top right, there is a small logo that says "© IIT, ROO". The derivation starts with the equation  $y = y^{\wedge} + e$ . Below it,  $(y - \bar{y}) = (y^{\wedge} - \bar{y}) + e$  is written. To the right of this is a small graph with a vertical axis labeled 'y' and a horizontal axis labeled 'x'. A regression line is drawn, and a point on the line is labeled  $y^{\wedge} = \beta_0 + \beta_1 x$ . The mean of y is marked as  $\bar{y}$  and the mean of x is marked as  $\bar{x}$ . Below the graph, the equation  $(y - \bar{y}) = (y^{\wedge} - \bar{y}) + e$  is repeated. Then,  $y = y^{\wedge} + e$  is written again. The next step is the expansion of the sum of squares:  $\sum_{i=1}^n y^2 = \sum_{i=1}^n (y^{\wedge} + e)^2$ , where  $i = 1, 2, \dots, n$ . This is then expanded to  $\sum_{i=1}^n y^2 = \sum_{i=1}^n y^{\wedge 2} + \sum_{i=1}^n e^2 + 2 \sum_{i=1}^n y^{\wedge} e$ . The final result is  $\sum y^2 = \sum y^{\wedge 2} + \sum e^2$ .

Now, this particular system **this particular system say** let us say we have a system; say  $y$  equal to  $\hat{y}$  plus  $e$ . So, this is how we start the process. What we will do? Let us we call it equation number one then we you can say subtract  $\bar{y}$  on both the sides so this is nothing but,  $\hat{y}$  minus  $\bar{y}$  so plus  $e$ .

Now, you see here so the actual the actual representation is like this now this is what we call as the  $x$  series and this is what we will call it  $y$  series. So, then we have a estimated you know corresponding to  $y$ ; so we can get the **you can say we can get the**  $\bar{y}$  you know  $\bar{y}$  and corresponding to  $x$  we have to get the  $\bar{x}$ . So, this is what we call is a mean of  $y$  and this is called as a mean of  $x$ .

Now, with respect to  $y$  and  $x$  information, our objective is to get the estimated line; that is called as a best fitted line. Now, let us assume that this best fitted line can be represented like this. So, this is  $y$  here which is equal to  $\hat{\alpha}$  plus  $\hat{\beta}x$   $\hat{\beta}x$  now this particular point is very relevant because, in this particular point where  $\hat{y}$   $\bar{y}$  or exactly equal to  $\bar{y}$  so, this is a... we are representing here that means this entire representation we can write it here like this  $y$  minus  $\bar{y}$  is equal to  $\hat{y}$  minus  $\bar{y}$  plus  $e$  because  $y$   $\bar{y}$  and  $\hat{y}$   $\bar{y}$  is equal at that point of you know equilibrium; so it is not a issue.

So, what we have to do instead of writing this one? We call it this is small  $y$  in deviation format and this is what we will call it  $\hat{y}$  in a deviation format and this is  $e$ . This is also as usual error terms. Now, this is what we have derived from here; now put it in a proper way. So, it is  $y$  equal to  $\hat{y}$  plus  $e$ ; now, what we have to do? This is original equation we have  $y$  minus  $\bar{y}$  equal to  $\hat{y}$  plus  $e$  this  $y$  and this  $\hat{y}$  is in a capital format and this  $y$  and this  $\hat{y}$  is you know deviation format; there is a huge difference between this deviation and actual.

Now, we have transferred the actual to deviation format. For this simplicity is concerned, now what we have to do? We have to apply summation. We first apply square in both the sides and then we have to apply the summation to get the entire structures. Now, what we have to do? If we do that then the entire structure becomes summation  $y$  square equal to summation  $\hat{y}$  plus  $e$  whole square. Obviously,  $i$  equal to 1 to  $n$  here this is  $i$  equal to 1 to  $n$  because  $i$  represents the sample units; it will start from 1 to  $n$  because, we are in the process of cross sectional modeling and our sample unit represent here  $i$ .

Now, obviously  $i$  equal to 1 to up to  $n$  **alright**; now, what you have to do? This particular component  $y$  hat  $y$  hat bar, this is what we can write in the format like this;  $y$  hat squares  $i$  equal to 1 to  $n$  plus summation  $e$  square  $i$  equal to 1 plus 2 summation  $y$  hat into  $e$ . So, this if you, if we expand this particular you know, right hand side of this equation then, we will get summation  $y$  square equal to summation  $y$  hat square plus summation  $e$  square plus 2 summation  $y$  hat into  $e$ . But, this particular term is exactly equal to 0 this particular term is exactly equal to 0. Now, the question is how it becomes 0; so let me explain here.

(Refer Slide Time: 15:32)

The image shows a hand pointing to a blue surface with handwritten mathematical equations. The equations are as follows:

$$\sum y^{\wedge} e = 0$$

$$y^{\wedge} = y^{\wedge} - \bar{y}^{\wedge} = y^{\wedge} - \bar{y}$$

$$= \alpha^{\wedge} + \beta^{\wedge} x - \alpha^{\wedge} - \beta^{\wedge} \bar{x}$$

$$= \beta^{\wedge} \frac{(x - \bar{x})}{x}$$

$$e = y - y^{\wedge}$$

$$= y - \beta^{\wedge} x_i$$

$$\sum_{i=1}^n y^{\wedge} e = \sum_{i=1}^n \beta^{\wedge} x_i (y_i - \beta^{\wedge} x_i)$$

$$= \beta^{\wedge} (\sum x_i y_i - \beta^{\wedge} \sum x_i^2)$$

$$= \beta^{\wedge} (\sum x_i y_i - \frac{\sum x_i y_i \cdot \sum x_i^2}{\sum x_i^2})$$

$$= \beta^{\wedge} (\sum x_i y_i - \sum x_i y_i) = \beta^{\wedge} \cdot 0 = 0$$

The structure is here; our point is here to prove that summation  $y$  hat  $e$  equal to 0 so first of all what is  $y$  hat  $y$  hat is equal to  $y$  hat minus  $y$  hat bar so this is nothing but,  $y$  hat minus  $y$  bar **alright**. Now, if we will simplify then it is nothing but,  $\alpha$  hat plus  $\beta$  hat  $x$  minus  $\alpha$  hat minus  $\beta$  hat  $x$  bar. Again, if you simplify then it is nothing but,  $\alpha$ ,  $\alpha$  hat cancels so,  $\beta$  hat into  $x$  minus  $x$  bar which is equal to  $\beta$  hat into small  $x$ . That is what we call it deviation; that means, this particular item is small  $x$ .

So, this is one part of the problem then  $e$  equal to  $y$  minus  $y$  hat so that means that means  $e$  equal to  $y$  minus  $y$  hat which is nothing but,  $y$  minus  $\beta$  hat  $x$   $\beta$  hat  $x$   $i$  now this is  $y$   $e$  and this is  $y$  hat now we have to integrate now summation  $y$  hat  $e$  is equal to summation because this is summation here so this  $\beta$  hat  $x$  into  $y$  minus  $\beta$  hat  $x$  so obviously this is  $x$   $i$  and this is  $x$   $i$  this is  $y$   $i$  so like this so of course,  $i$  equal to 1 to  $n$  and

this side  $i$  equal to also one to  $n$  actually the term is 2 into summation  $\hat{y}$  into  $e$  but, if we prove that summation  $\hat{y}$  equal to 0 then obviously, 2 into 0 equal to 0.

Now, what we have to do? Here, just we take  $\hat{\beta}$  common then, summation  $x_i y_i$  minus  $\hat{\beta}$  is equal to common here. So,  $\hat{\beta}$  summation  $x_i^2$  then this  $\hat{\beta}$  is you can say, we have taken common  $\hat{\beta}$  so then it is nothing but,  $\hat{\beta}$  into summation  $x_i y_i$  minus what is  $\hat{\beta}$ ?  $\hat{\beta}$  is nothing but, summation  $x_i y_i$  divided by summation  $x_i^2$ . We have again summation  $x_i^2$ ; so this summation  $x_i^2$ , this summation  $x_i^2$  cancels. That means, equal to  $\hat{\beta}$  into summation  $x_i y_i$  minus summation  $x_i y_i$ . so, summation  $x_i y_i$ ; so this and this is canceled. That means, it is nothing but,  $\hat{\beta}$  into 0 which is nothing but, equal to 0 alright. Now, so that means the entire structure is like this; so  $\hat{\beta}$  equal to summation  $x_i y_i$  / summation  $x_i^2$ . We are just expanding the  $\hat{\beta}$  value here; so obviously, summation  $x_i^2$  / summation  $x_i^2$  cancels. The left out term is summation  $x_i y_i$ . Obviously, summation  $x_i y_i$  is here so this is summation  $x_i y_i$  so that means it is equal to 0.

Now, we have proved that summation  $\hat{y}$  equal to 0. Now, you come to this stage here so summation that means summation  $y^2$  / summation  $y^2$  is equal to summation  $\hat{y}^2$  plus summation  $e^2$ . Now, we will start our process here; so what is exactly this particular [ ]? so this particular [ ] is like this.

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Handwritten mathematical derivation on a blue background:

$$y = y^{\wedge} + e$$

$$y = y^{\wedge} + e$$

$$\sum_{i=1}^n y^2 = \sum_{i=1}^n y^{\wedge 2} + \sum_{i=1}^n e^2$$

Labels and arrows:

- TSS (Total Sum Squares) points to  $\sum_{i=1}^n y^2$
- ESS (Explained Sum Squared) points to  $\sum_{i=1}^n y^{\wedge 2}$
- RSS (Residual Sum Squared) points to  $\sum_{i=1}^n e^2$

Definitions:

- Total Sum Squares:  $\sum_{i=1}^n (y - \bar{y})^2$
- Explained Sum Squared:  $\sum_{i=1}^n (y^{\wedge} - \bar{y})^2$
- Residual Sum Squared (Unexplained Sum Squared):  $\sum_{i=1}^n (y_i - y^{\wedge})^2$

Equation:  $e = y - y^{\wedge}$

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We start with you can say,  $y$  equal to  $\hat{y}$  plus  $e$  then you know we transfer into  $y$  equal to  $\hat{y}$  plus  $e$  then after you know doing so you know process so we get to we have received summation  $y$  square equal to summation  $\hat{y}$  square plus summation  $e$  square so this is  $i$  equal to 1 to  $n$  and this is  $i$  equal to 1 to  $n$  and this is also  $i$  equal to 1 to  $n$ . This is how the entire structure is all about; that means, our point is here to justify the significance of the alpha parameter and beta parameters and just this particular task. So, we need to have variance of alpha hat and to have variance of alpha hat and to have variance of beta hat we need to integrate with again with error variance because this particular variance of alpha hat depends upon the variance of error variance and again for you can say a variance of beta hat we need **also** error variance.

So, we like to know, what is the exact component of error variance; explaining by this process, we are in the stage that summation  $y$  square equal to summation  $\hat{y}$  square plus summation  $e$  square. This particular term is called as a TSS and this particular term is called as a ESS and this particular term is called as a RSS; this particular term is called as a RSS. What is exactly this particular term? That means, this is called as a total sum square; this is explained sum square and this is called as a residual sum square. So, that means, this is what we call as a total sum square **total sum squares**. Then, this is explained sum square, explained sum squares and this particular term is called as a this particular term is called as a residual - residual sum squares. Sum square it is otherwise known as unexplained sum square unexplained sum squares; this is otherwise called as a unexplained sum square.

Now, what is exactly a this particular term so this is nothing but, summation  $y$  minus  $\bar{y}$  whole squares  $i$  equal to 1 to  $n$  and this particular term is nothing but,  $\hat{y}$  square this is nothing but, summation  $\hat{y}$  minus  $\bar{y}$  whole squares  $i$  equal to 1 to  $n$  then this is nothing but, summation you can say  $y_i$  minus  $\hat{y}_i$  whole square  $i$  equal to 1 to  $n$ . In fact, the entire process is started from here only because, our entire model is nothing but,  $e$  equal to  $y$  minus  $\hat{y}$  and the way we are minus minimizing the error sum we have received the alpha hat component and beta hat component.

Now, to justify the significance of this particular parameter alpha hat and the parameter beta hat we again come down this particular process. Now, we have to explain how this means there is lots of interesting facts behind this particular structure. So, let us see how is this particular structures alright.

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$$\sum y^2 = \sum \hat{y}^2 + \sum e^2 \dots (1)$$

$$TSS = ESS + RSS$$
 Dividing  $\sum y^2$  on both the sides of eqn 1

$$\frac{\sum y^2}{\sum y^2} = \frac{\sum \hat{y}^2}{\sum y^2} + \frac{\sum e^2}{\sum y^2}$$

$$1 = \frac{\sum_{i=1}^n \hat{y}_i^2}{\sum_{i=1}^n y_i^2} + \frac{\sum_{i=1}^n e_i^2}{\sum_{i=1}^n y_i^2}$$

$\beta^1 = \frac{\sum xy}{\sum x^2}$

$$\frac{\sum \hat{y}^2}{\sum y^2} = \frac{\sum_{i=1}^n (\beta^1 x_i)^2}{\sum_{i=1}^n y_i^2} = \frac{\beta^{12} \sum x^2}{\sum y^2}$$

Now, we have the component summation y square equal to summation y hat square plus summation e square. That means, what we can conclude total sum square is equal to explained sum square plus residual sum square **alright**. So, we are now in a position to say that total sum square is equal to explained sum square and residual sum square i have highlighted earlier. That, you know when you have y series and we have x series that is our you can say a beginning; so we have y information and we have x information and through the process we have received the error component. That is how we can say it is all about you can say statistics or econometric; so that means, we like to verify that whether x is totally influencing the y component or x is partly influencing y and some of the other part can be explained in other way.

For instance, if x is not 100 percent influencing y then obviously there is some point of lacking; so that lacking part, we have to discuss and that is nothing but, it is called as a residuals. So, that means when we have y series; we like to know what is the total sum square; that is nothing but, sum of y i minus y bar the deviation and its squares. That means the variation from all these points to the, you can say, from the arithmetic mean now total sum square is equal to explained sum square. That is nothing but, summation y hat minus y hat bar squares and rest is summation e squares that is residual sum square.

Now, put it technically. What I will do? Let us assume that this is equation number 1; so what I will do? I will divide summation y square both the sides; so dividing summation y

square on both the sides of equation 1 then, what do you have? You see here so summation y square divided by summation y square is equal to summation y hat square by summation y square plus summation e square by summation y square alright.

Now, this particular term is exactly equal to to 1; this is equal to 1. Now, this is one component and this is another component. So, that means 1 equal to summation y hat square by summation y square plus summation e square by summation y square **alright**. This **is how** means we are in a position to draw like this. obviously, i equal to 1 to n here i equal to 1 to n here; so this is i equal to 1 up to n here alright.

Now, we have 2 parts; so we call it this is part A and this is we call it a part B. Let us first explain what is this part; a component. So, part of a component is like this summation y hat square by summation y squares what is y hat exactly. So, y hat is nothing but, summation small beta hat means beta hat and small x this is whole square i equal to 1 to n divided by summation y square obviously i equal to 1 to n alright.

Now, what is beta? That means, if we simplify further then it is nothing but, beta hat square then summation x square divided by summation y squares **alright**; what is beta hat? Beta hat actually, beta hat is equal to summation x y by summation x square beta hat by beta hat equal to summation x i by summation x square. Now, you see here if you simplify further then what we can do? Summation put t here; so summation I will write it here again.

(Refer Slide Time: 27:56)

The image shows a handwritten derivation on a blue background. At the top right, there is a small logo for '© CET I.I.T. KGP'. The main derivation starts with the equation for the coefficient of determination,  $R^2$ , which is the ratio of the explained variation to the total variation. It is written as  $A: \frac{\sum \hat{y}_i^2}{\sum y_i^2} = \frac{(\sum \hat{y}_i)^2}{(\sum y_i)^2} \cdot \frac{\sum x_i^2}{\sum y_i^2}$ . This is then simplified to  $\frac{(\sum \hat{y}_i)^2}{\sum x_i^2 \cdot \sum y_i^2} = R^2$ . A note next to this equation says 'Coefficient of determination' and 'Square of Correlation Coefficient'. Below this, the correlation coefficient  $r$  is defined as  $r = \frac{\text{Cov}(X, Y)}{\sigma_x \cdot \sigma_y} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y}) / n}{\sqrt{\sum x_i^2 / n} \sqrt{\sum y_i^2 / n}}$ . This is further simplified to  $r = \frac{\sum x_i y_i}{\sqrt{\sum x_i^2} \sqrt{\sum y_i^2}}$ . A curved arrow points from the  $R^2$  equation to the  $r$  equation, indicating that  $R^2 = r^2$ . In the bottom left corner, there is a logo for 'NPTEL'.

Summation  $\hat{y}$  square by summation  $y$  square is equal to summation  $x y$  whole square divided by summation  $x$  square whole square into summation  $x$  square divided by summation  $y$  square; this is what  $\hat{y}$  square by summation  $y$  square.

Now, you see here this is summation  $x$  square and this is summation  $x x$  square to the power again to... so this is how it is cancelled so that means it is nothing but, summation  $x y$  whole square by summation  $x$  square into summation  $y$  square **alright**. This is the left out term from this you know component; That means, this is what we have received from the part a. so, part A it will expand this part A that is the variance of explained ratio between explained sum square to total sum square so means the exact term is summation  $y$  square equal to summation  $\hat{y}$  square plus summation  $e$  square; that means, the total sum square equal to explained sum square plus residual sum square.

Now, what we have done? We divide the total sum square both the side so then the left side of this problem is equal to 1. Then, right part of the first part is the explained sum square divided by total sum square. This is how it is called; as a you know ratio between the explained sum square to total square then, the ratio between residual sum square to total sum square. Now, we like to know if we have a component explained sum square to total sum square, what is that issue and if you know the ratio component is residual sum square divided by total sum square, what is that component? So, then we have to now you know, interpret accordingly.

Now, by this process we are in the, we are you know coming to a position that summation  $\hat{y}$  square by summation  $y$  square that is nothing but, ESS by TSS is nothing but, summation  $x y$  square by summation  $x$  square into summation  $y$  square. This is what we call it is just like  $r$  square this is what we call it a  $r$  square; that is what is  $r$  square  $r$  square is nothing but, square of square of correlation coefficient this is what is called as a correlation coefficient.

You see, what is correlation? Then, correlation is simply nothing but, covariance of  $X Y$  divided by sigma  $x$  into sigma  $y$ . If we will simplify further then it is nothing but, summation  $x$  minus  $\bar{x}$  into  $y$  minus  $\bar{y}$  divided by  $n$  or divided by summation  $x$  square by  $n$  square root; then summation  $y$  square by  $n$  square root; so this  $n$  this  $n$  this  $n$  cancelled, alright.

Now, if this is R component, this particular component is nothing but, summation x y this is summation x y divided by summation x square into summation y square **alright**. Now, if we will make it square then obviously r square equal to summation x y whole square divided by summation x square into summation y square so what we have is received from here only so that means this particular ratio explains some square to total sum square is nothing but, the r square component that means what is r square here r square represent the square of correlation coefficient but, you know this particular component is very much true when we are in the bivariate process but, when there is multivariate process then this uh you know ratio between explained sum square to total sum square cannot be represented as a simple correlation coefficient that is something different.

What is this difference? The difference is actually, this particular r square component is represented as a coefficient of determination so this particular component this r square component is represented as a coefficient of determination; this particular item is represented as a coefficient of determination; so, what is this coefficient of determination?

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The slide shows the following derivation:

$$\Sigma y^2 = \Sigma \hat{y}^2 + \Sigma e^2$$

$$1 = \frac{\Sigma \hat{y}^2}{\Sigma y^2} + \frac{\Sigma e^2}{\Sigma y^2}$$

$$= \frac{ESS}{TSS} + \frac{RSS}{TSS}$$

$$1 = R^2 + \frac{RSS}{TSS}$$

Correlation:  $-1 \leq r \leq 1$

$$R^2 + \frac{\Sigma e^2}{\Sigma y^2} = 1$$

$$R^2 = 1 - \frac{\Sigma e^2}{\Sigma y^2}$$

Case 1:  $R^2 = 1$ , Perfectly fitted model:  $\Sigma y^2 = \Sigma \hat{y}^2$

Case 2:  $R^2 = 0$ , Unfit:  $\Sigma y^2 = \Sigma e^2$

Logos for NPTEL and I.I.T. KGP are visible in the bottom left and top right corners of the slide respectively.

Now, coefficient of determination that means, you see here we had, we have here y square is equal to summation y square equal to summation y hat square plus summation e square. So, that is what we have received; 1 equal to summation y hat square by

summation  $e^2$  plus summation  $y^2$  by summation  $y^2$ . This is what we have received and by the process this is otherwise known as ESS by TSS and this is what we had RSS by TSS.

Now, this particular component by the you know, by the process of derivations, what we have received it is nothing but, simply you can say R square. Usually, when we will represent the coefficient of that determination then it is nothing but, represented as a capital R square. So, what we have written earlier, it is called as a small R square; that means, small and capital R square; both have same in the case of in the case of bivariate models. So, bivariate model **in the** that means in the case of bivariate model, the coefficient determination and the square of correlation coefficient are similar so that means they are same but, the interpretation is somewhat different in the correlation coefficient. What we have to study? You know, association between the 2 variable degree of association between 2 variables.

Now, here R square capital R square we have judged ratio between explained sum square by total sum square and explained some square is nothing but, total sum of the x component that is explained items and divided by total sum of y component which is nothing but, dependent component. Now, we like to know what is the percentage influence of independent variable to dependent variable or you know explanatory variable to explained variable that is what we are now in the process so that means R square is the ratio between explained sum square to total sum square by default it is equal to 1 here plus RSS by TSS here RSS by TSS here.

Now, there are you know beautifully interpretation here; so, what is this beautiful interpretation? **you know** You know, fortunately this particular item can be again turned into this one. Now, we know correlation coefficient is usually in between minus 1 less than equal to 1 so this is how the correlation coefficient range this is correlation coefficient range correlation coefficient range **alright** now correlation coefficient range R square R square is always in between 0 to 1 so this is the range of coefficient of determination so what is the coefficient of determination it is the ratio between explained sum square to total sum square means technically or you can say it would go by physical interpretation it is the variation of you know total variation of explained items to you can say total variation on y.

So, this is how it is called as a R square or coefficient determinations. Coefficient determination, coefficient of determination is nothing but, percentage of proportion variation of y which is explained by the you know, proportion variation of x. This particular term is called as a proportion variation of y which is explained by proportion variation of x and this particular component is represented as proportion variation of y which is explained by proportion variation of y that means this is total sum square is nothing but, y square; so this is our total component.

We like to know what is the x inflation y, what is e inflation y, so that is why it is known as a proportion variation of y. This is proportion variation of y which is explained by this you know, proportion variation of x because ESS is you know, the entire component of ESS depends upon the x component only then this is nothing but, proportion variation of y which is x means which is not explained properly that is what we called as a RSS. That means, that will taken care by u component so  $1 = R^2 + \frac{RSS}{TSS}$ .

Now, so we have the range 0 R square and 1; so, this will give you the model signal. This will give the reliability of the model signal. So far as the second objective is concerned, now you see, we start with the first objective and by default we are now going to explain the second objective. So, that is the overall fitness of the model. Now, the moment will get R square that is the proper structure, how you have to you know receive this R square and how you have to go for its statistical level of significant because, suppose a first objectivity is concerned with respect to alpha hat and beta hat, so we are applying the t statistic. Now, when we are going for you know, over all fitness of the model then, we have to use the f statistics.

Now, we are just explaining how we are receiving the error variance and how it is connected to total variance of y and total variance of x. now, by this process we like to explain how is the structure of this significance of the individual parameters that to alpha hat and beta hat. And, in the other side by means by using all these you know TSS ESS and RSS, we like to explain how the overall fitness of the module will be statistically significant. So, that means we have 2 clear cut objectives in our mind first is the significance of the parameter and the significance of the overall fitness of the module. So, before i means, before i highlight the entire structure of the R square significance label and the typical parameters significance variable. We like to highlight here the

influence of R square because the value of R square always in between 0 to 1; so if it is 0 how is this structure and if it is 1 how is the structure? Let us see here.

Now, R square the entire component is R square R square plus summation e square y summation y square is exactly equal to 1. Now, this is how we have observed; now since this is our target, so what we will do? We will take R square equal to 1 minus summation e square by summation y square, this is what we have received form this you know simplification.

So, what we have to do here now? Let us say, if case 1: **case** 1 if R square is equal to 1 then, what will happen? If R square equal to 1 this particular item is equal to 0; this particular item exactly equal to 0 so R square equal to 1 means this is equal to 1 and this particular item is equal to 0. So, that means, the model is the absolutely fit for this you can say problem.

So, when R square is 1 then it is the best fitted models. Now, when R square exactly equal to 1 then, the unexplained component the percentage of unexplained component is exactly equal to 0. That means, there is no way you has a impact on you can say y variables. So, that means, the 100 percent the percentage influence of x on y; so this is how, this is the case where R square exactly equal to 1. But, in real life situation or real life problem, it is very difficult to get a situation when R square exactly equal to 1 **alright**.

In the other side, when R square equal to 1 then it is called as a complete fitted or perfectly fit model. This is what we will call as a perfectly fitted model - perfectly fitted models but, this is not the sufficient condition. this is the necessary conditions the when R square equal to 1 the overall fitness of the model is very high or very high means it is excellent one so that means it is completely fitted model estimated model so it can be used for forecasting and for but, the sufficient condition is that when R square is exactly equal to 1. Then, corresponding to the first objective with respect to significance of the alpha hat and beta hat; it has to be significant - highly significant. Then, the model we can say that it is best fitted model otherwise of R square is exactly 1 and model is you know the significance of the model is explicitly high and other side the parameters are not statistical significant or few parameter are statistical significant and other parameters are not even significant at a very lower. Then, the model cannot be used as a forecasting.

Even if R square equal to 1 because we are just in the beginning of this process and we have R square equal to 1 and parameters are not all parameters, are not statistically highly significant then there is a serious problem in the modeling. So, there will be some you know complex problem in between. So, that complex problems we have not highlighted; we will highlight details when we will proceed you know, when we will proceed accordingly. So, we like to know in later stage, not now. So, what we can now, you can explain that when R square equal to 1, just we interpret that it is perfectly fitted the models keeping other things it remains constant.

Now, case 2: when R square is equal to 0, **R square equal to 0** then, the model is completely unfit. That means the entire variations will be receive from u only. That means, this particular item is equal to 1 and this is equal to 0. Now, when R square equal to R square equal to 1; so that means, this is equal to 0 when R square equal to 1 then summation y square is equal to summation y hat squares; this is summation y square equal to summation y hat square.

When it is unfit then summation y square is equal to summation e square **alright** but, this is rare and this is rare. Why it is rare? It may, it may not be rare but, this is you know very extreme situation. The reality is that when we will when we are in the process of you know fitting a model then, obviously we must have some theoretical knowledge. So, when we have a theoretical knowledge then, obviously means most of the instances R square cannot be equal to 0. It may be very low level but, it cannot be 0. If your R square value is coming 0 that means, your theory is not absolutely that means identification of problem with relate to all variables are not systematically.

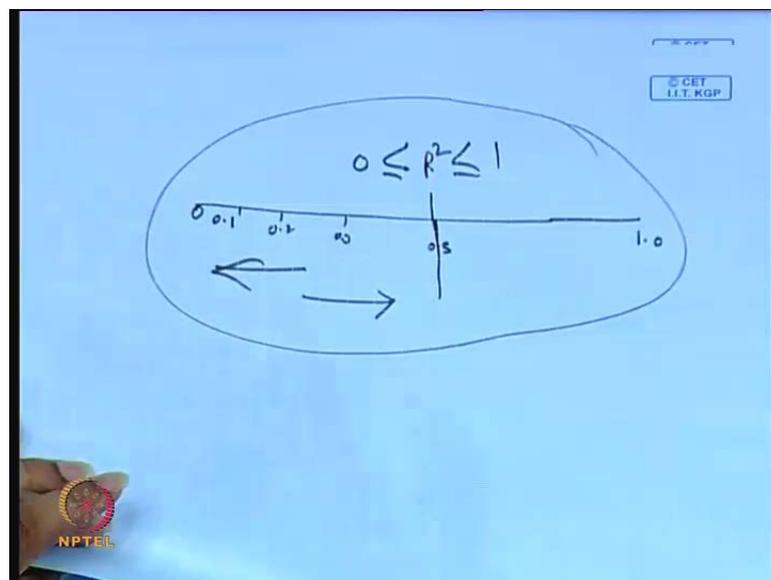
So, there is some kind of problems; that is why before going to fit this particular model that is your theoretical knowledge may be very perfect and you must be in a position to to identify exactly the structural variables. If your initial you know, initial homework is very tough then, obviously later stage of modeling will not face problems. Otherwise, it is just like a continuous process until you get the best fitted models. If you do not go stepwise you process then, obviously every time we will go back to again original position till we get the better fitted model.

So, that is why each and every stage should be perfectly before we going to next stage so in reality we have R square one extreme equal to 1 and another extreme R square equal

to 0 but, it is very essential and it is also very essential. So, what is the actual is, when R square variable is close to 1 then, it is called as a best you know means a better fitted model. We cannot say best fitted model when we will call is a best fitted model then, obviously R square equal to 1.

Now, when R square R square is close to 1 then it is called as a that means, the fitness of the model will start increasing that means, you start from R square equal to point 0 point 0 1 point 0 2 point 0 3 point 0 4, 0 5, like this; so we will go up to point 0 0.

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Now, we have 3 different ranges; in fact, see here; the range is like this. So, take a case here so this is what we called 0 and point one 0.1 then 0.2 then 0.3 like this then this is 0.5. Then of course, this is 1.0; this is how the R square ranges this is the R square range. so when we will call it as R square 0 less than 1. So, the range will be like this so this is you know middle; now if you are in this stage the model fitness or model accuracy will start declining when we are moving this side then, the model accuracy will be start increasing.

Now, always our objective is to go this side, not this side. So, that the model fitness or overall fitness of the model will be start increasing now when your R square value will be closed towards one then it is the signal of or it is just like a green signal it is the outcome of the best fitted model. So, when we are closing to 0 or close to 0 then, obviously it is a, it will give you the red signal. That means we are we are diverting from

the best fitted model so we should not go towards the red signal whether you have to go towards the green signal where the best fitted model or the model accuracy will be start increasing so this would be our main agenda before we will go to this process.

Now, you come back to the original position, what is this actually structure? Our objective is here to test the R square whether R square is statistical significant or not.

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$$\begin{cases} \text{Var}(\hat{\beta}) = \frac{\sigma_u^2}{S_{xx}} \\ \text{Ser}(\hat{\beta}) = \sqrt{\text{Var}(\hat{\beta})} \end{cases} \quad \text{Ser}(\hat{\beta}) = \sqrt{\frac{\sigma_u^2 S_{xx}}{n S_{xx}}}$$

$$t_{\hat{\alpha}} = \frac{\hat{\alpha}}{\text{Ser}(\hat{\alpha})} \quad t_{\hat{\beta}} = \frac{\hat{\beta}}{\text{Ser}(\hat{\beta})}$$

$$H_0: \alpha = 0 \quad H_1: \alpha \neq 0$$

$$t_{\hat{\alpha}} = \frac{\hat{\alpha}}{\text{Ser}(\hat{\alpha})} \quad t_{\hat{\beta}} = \frac{\hat{\beta}}{\text{Ser}(\hat{\beta})}$$

Table ←

Further, we have to prepare the ANOVA tables; so we have to prepare the ANOVA table just like a... in the first objective, the first objective we have explained here. **The first objective what we have explained here is this is what the first objective we have explained here the first objective here** the fitness of the model is like this. so, we like to know the target component is t alpha hat and component is t beta hat. Now, we have received here summation e square by n minus 2; so that is what sigma u now variance of alpha hat you have.

Similarly, you have to go for standard error of alpha hat standard error of alpha hat is nothing but, variance of alpha hat. Now similarly, what we have to do here now we have to get the variance of beta hat here variance of beta hat is nothing but, sigma square u by summation x square. Similarly, standard error of beta hat standard error beta hat is nothing but, square root of variance of beta hat so this is what the beta hat parameter structures and alpha hat alpha hat parameter structure is alpha hat parameter structure is you can say that means, standard error of alpha hat is nothing but, sigma square u

summation  $x^2$  by  $n$  summation  $x^2$ . So, this is what the structure this is standard error of this terms alright.

Now, standard error of this much; so what you have to do? Once you have alpha variance of alpha hat, you can get the standard error of alpha hat. So, what is the issue here? Now, our objective is to know whether alpha hat is significant or beta hat is alpha hat is significant beta hat is significant. So, we need to calculate t of alpha hat and you need to calculate t of beta hat so further to know the significance of this particular alpha hat and beta hat so we have to apply a statistic hypothesis or we have to use the statistical hypothesis.

Basically, the statistical hypothesis is divided into 2 parts called as a null hypothesis and alternative hypothesis. So, this is null hypothesis then in contemporary to null hypothesis we have alternative hypothesis so we start with the null hypothesis that the suppose, our target is to test alpha is significant alpha is significant means alpha must have some value if alpha has a some value. Then, we on the basis of that value you have to test the significance now let us we start with that alpha is equal to 0 so alpha 0 usually fit that alpha equal to 0 let us say alpha alpha equal to 0 and we have to test alpha naught equal to 0.

Once you you know reject this small hypothesis then we are in the right trac[k]- if you could not reject then that variable may not be statistical significant so that means so t of alpha hat is basically we will calculate technically is nothing but, alpha hat by standard error of alpha hat and p of beta hat is nothing but, you can say beta hat y standard error of beta hat now this is calculated statistic this is calculated statistic that has to be compare with the tabulated statistic so this is to be also compare with tabulated statistic.

Then we get to know whether this particular item is statistical significant or not and if it is significant at what level their significant so we have different structure of significance tailed 5 percent one tailed and 2 tailed and 10 percent 1 tailed and 2 tailed so starting procedure is we have to start with the 1 percent level. Then, if it is non significant then you have n to move to 5 percent. If it is not significant then, 5 percent; then you have to go to 10 percent but, if you will get significance at one percent then that means your model accurate is very very high and the reliability of the model is also that means if the reliability of the model is perfectly.

If we are getting significance at 10 percent level yes model is reliable one but, the degrees of reliability may be very less so when the variable is statistically significant in a close to one or at the level of one percent then obviously the model reliability or model accuracy is very high or absolutely.

Now, we will target or we have to reformulate or we have to design or redesign in such a way so that the parameters means involve in this particular systems, modeling systems should be highly significant highly statistically significant and mostly at it should be at the level of one percent only if it is so then the model reliability so far as the first order condition is...

Now, again for sufficient condition we have to go for R square that means, there are 2 problems here. So, your all parameters should be statistical significant at the higher level 1 percent level and same times your R square will be also statistically significant at the 1 percent level or that is at a higher levels. If it is so then, the model is absolutely fit for the forecasting but, the problem if parameters are significant and R square is not significant or R square is significant parameters are not significant; then the problem is very complicated.

So, that means there is some kind of fault or problem in between this process; so that process means has to be investigated further again there are certain problems in between; so that we are getting first part and we are not receiving the second part the systems. The system will be very much or perfectly when parameters are significant it should be R square should be statistically significant. If not then there is serious issue for this particular estimated model. We have to redesign or you have to rebuild till you get the best fitted models where both parameters are statistical significant and your R square will be statically significant. So, we will discuss details in the next class; thank you very much; have a nice day.