Introduction to Nonlinear Dynamics
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Module - 01
Lecture - 01
A Brief Introduction to Modelling

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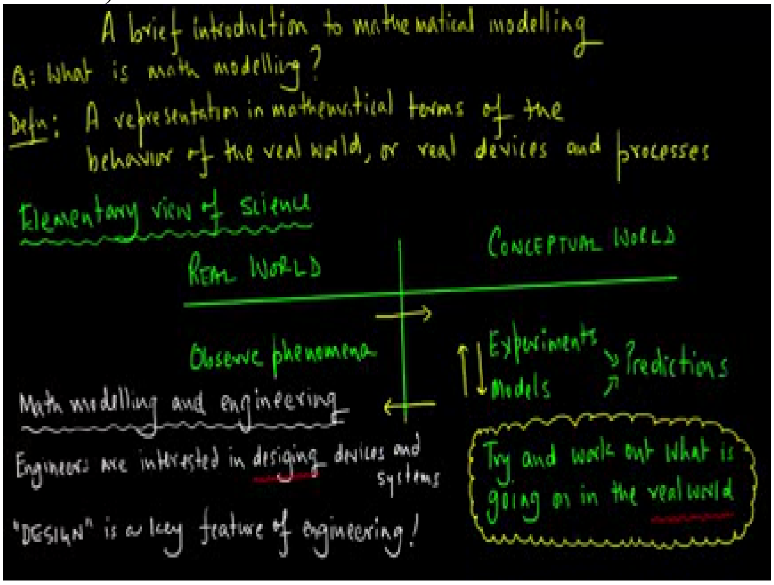

Now, this lecture is going to be very a brief introduction to mathematical modelling. Here we aim to explain what goes on in mathematical modelling and why we actually do it, and then finally we end with some examples to highlight the importance of math modelling in the real world. We start by asking, what really is mathematical modelling? We offer a rather simple minded definition; that is, representation in mathematical terms of the behaviour of a real world or of real devices and real processes.

We now take a rather elementary view of science. So what we do is we take the world and we break it up into two books, one is real world and other is the conceptual world and we go ahead and divide it into two different worlds. In the real world, is where we observe phenomena and in the conceptual world we conduct experiments and device models both of which leading to predictions.

The central idea is to try and work out, what are all is going out in real world? That is the central idea, so we highlight the real world. Now, as this is really central to the modelling process we go ahead and highlight it. In the modelling process, it is rather important, to go from real world to conceptual world and then back from conceptual world to the real world.

Now within the conceptual world, insights from experiments should inform models and inferences from models should guide experiments.

Now let us consider math modelling and engineering. Now what we find is that engineers are typically interested in designing devices and systems. And to that end design turns out to be rather important and key features of engineering.
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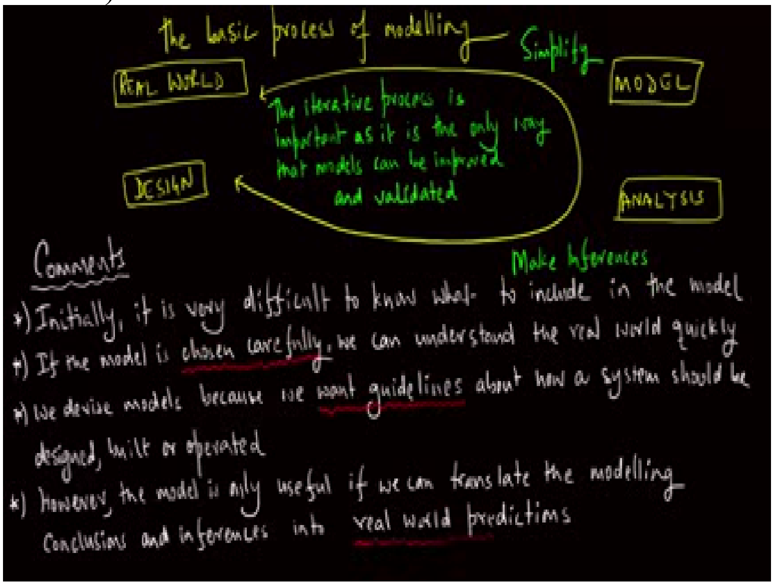

Let us now look at the basic process of modelling. We always start with a real world problem and construct a model, which is then subjected to analysis and finally it all feeds into design. So we got to and fro between the real world and final design. In getting to the model we need to simplify the real world and the objective of the analysis is to allow us to make inferences. The iterative process between the real world and final design is important.

As it is the only way that models can be improved and validated. A few comments are in order. Now initially it is very difficult to know, what really to include in the model. So, if you happen to be a research student and struggling in formulating a good initial model rest assured there are lots of others who will be in the very same boat. But, if the model is chosen very carefully, we can then understand the real world rather quickly.

Now the reason, we device models is because we want guidelines about how system should be designed, built or operated. However, the model is only really useful, if we can translate the modelling conclusions and the inferences into real world predictions.
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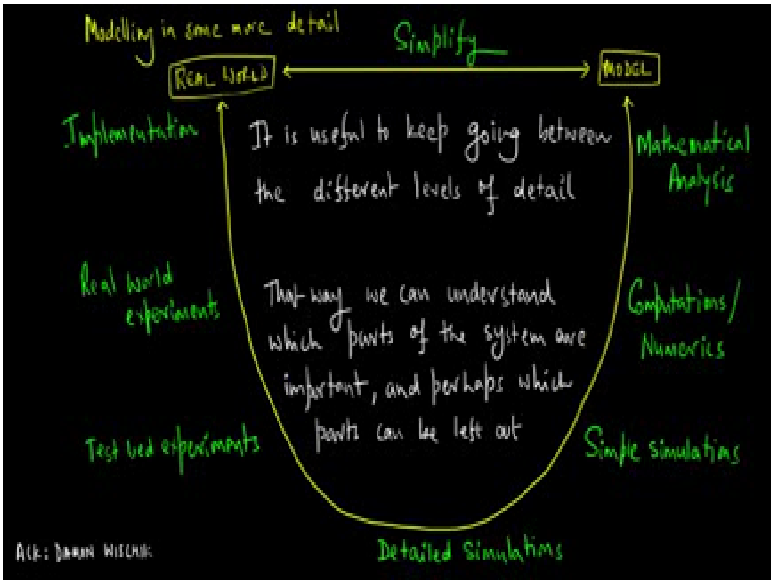

Now let us look at the process the modelling in some more detail. We would actually start with the real world at one end and with the mathematical model at the other end. And of course, while we connect with real world to a model, we would need to make some simplifications to the real world along the way. And of course, the model has to be connected all the way back into the real world.

We will start with mathematical analysis followed by computational/numerical analysis. Conduct simple simulations and then move on to more detailed simulations. Then try and conduct some very simple test bed experiments before moving on to the real-world experiments and finally implementation in the real world. It is really useful to keep going between the different levels of detail.

That way we can understand which parts of the system are important and perhaps which parts can be left out. The above diagram was adopted from performance modelling notes of Damon Wischik

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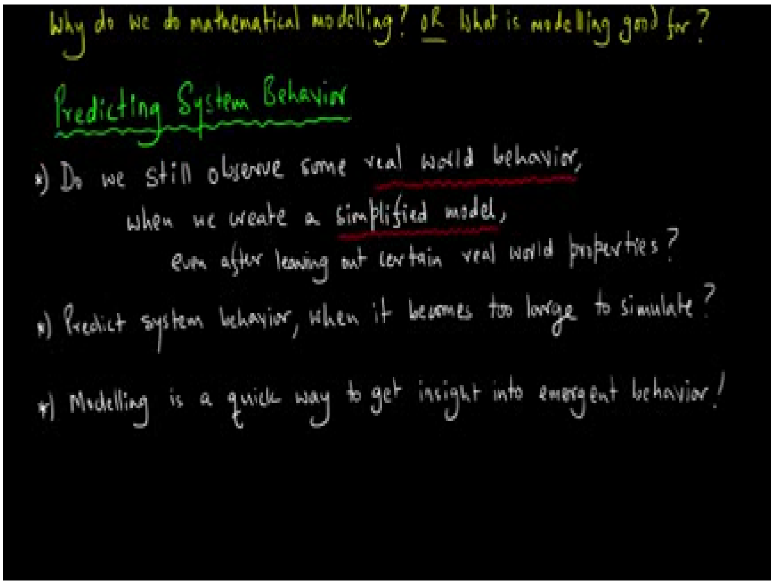

Now we get on to the question, on why we actually do mathematical modelling. Or in other words, what really is mathematical modelling good for? Well, one key reason is predicting system behaviour. Now we would like to know whether we can still observe some real world behaviour.

When we create a simplified model of the system even after leaving out certain real world problems, we like to know where, where we would predict the system behaviour, even when it becomes too large to simulate. Modelling turns out to be a quick way to get insight into emergent behaviour.

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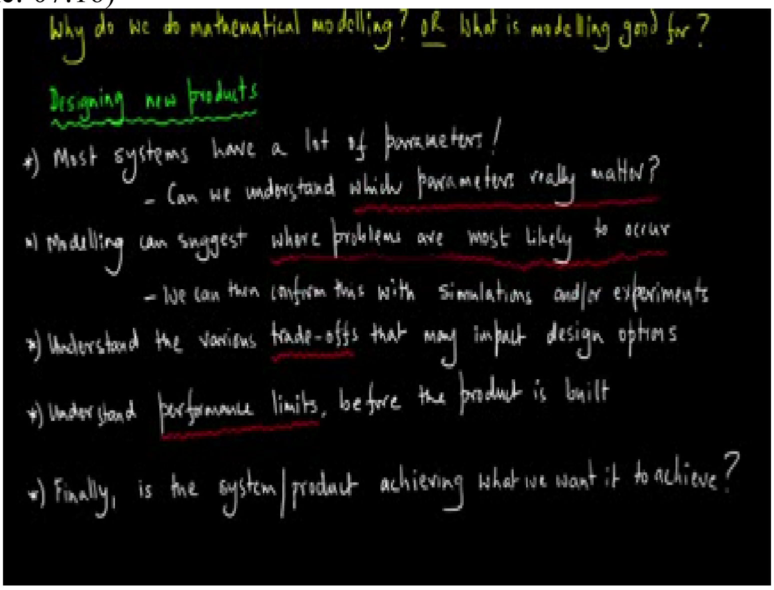

Another important reason is the design of new products. For example, most systems have lot of parameters. Can we understand which of these parameters really matter in the system? Modelling can also suggest where problems in fact are most likely to occur. We can then actually conform this with simulations and or experiments. In fact, we can also avoid or fix these problems with the help of modelling.

We can also understand various trade-offs that may impact design options. We can also understand performance limits before the product is actually built. And finally, we can understand whether the system or the product is achieving what we really wanted to achieve.

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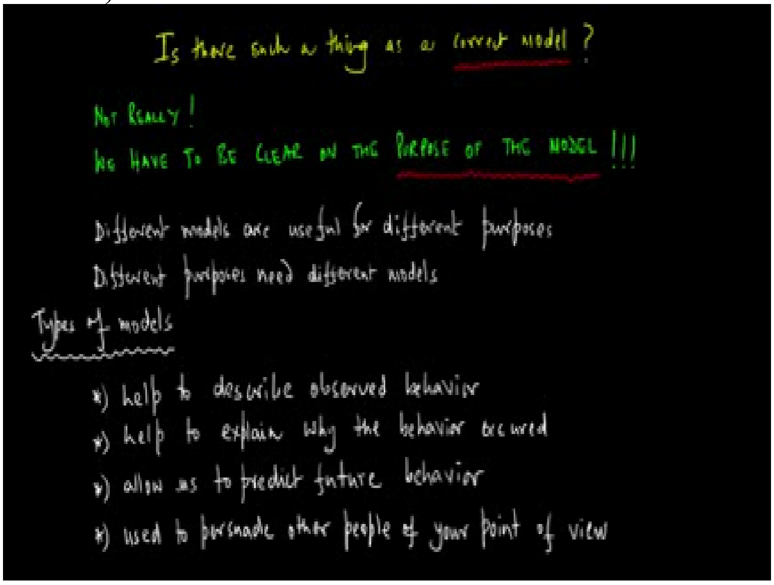

With all the stock of modelling we should also be asking, is there such a thing as a correct model? The short answer is; not really. Now we have to be crystal clear on the purpose of a model. Different models are useful for different purposes and different purposes in fact, need different models.

There can be different types of models. Some merely help to describe observed behaviour. Others help to explain, why the behaviour occurred in the first place? We have models that would allow us to predict future behaviour. And of course, we could also have models which are used to persuade other people of your point of view.

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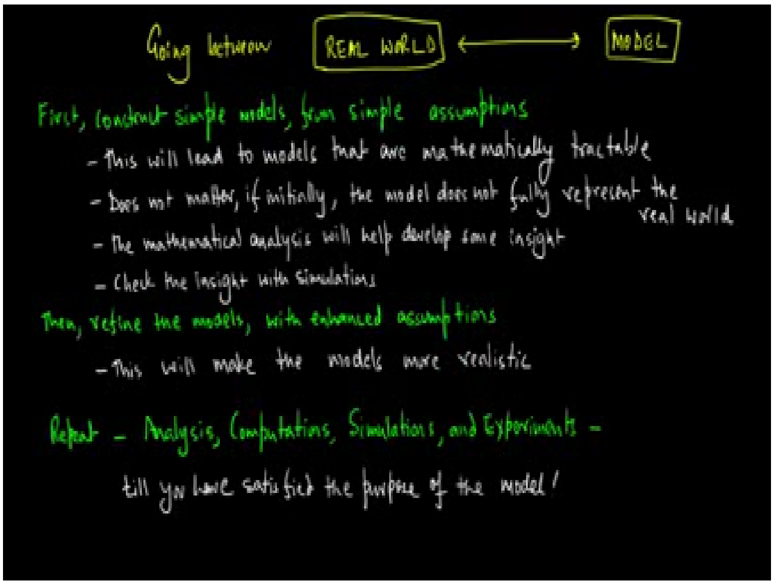

I remember I said that going between real world and the model can actually be quite hard. So we now offer some pointers on how to cope with this part of the modelling process. First always construct simple models from simple assumptions. This will lead to models that are mathematically tractable. And it does not really matter if initially the model does not fully represent the real world.

The mathematical analysis will in fact, help develop some insight. And one should always check that insight with simulations. After this part, we should refine the models with enhanced assumptions. Of course, this will make models more realistic. And one should repeat the analysis, the computations, simulations and the experiments till we have satisfied the purpose of the model.

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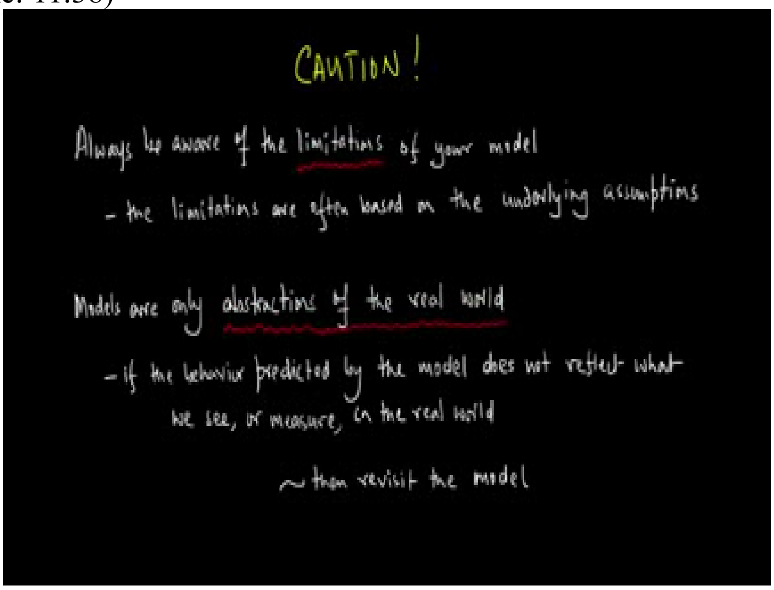

Now for some words of caution always be aware of the limitations of your model. The limitations are often based on the underline assumptions that you make, and always remember that models are only abstractions of the real world and if the behaviour that is predicted by the model does not reflect, what we see or measure in the real world; then it is time for you to revisit the model.

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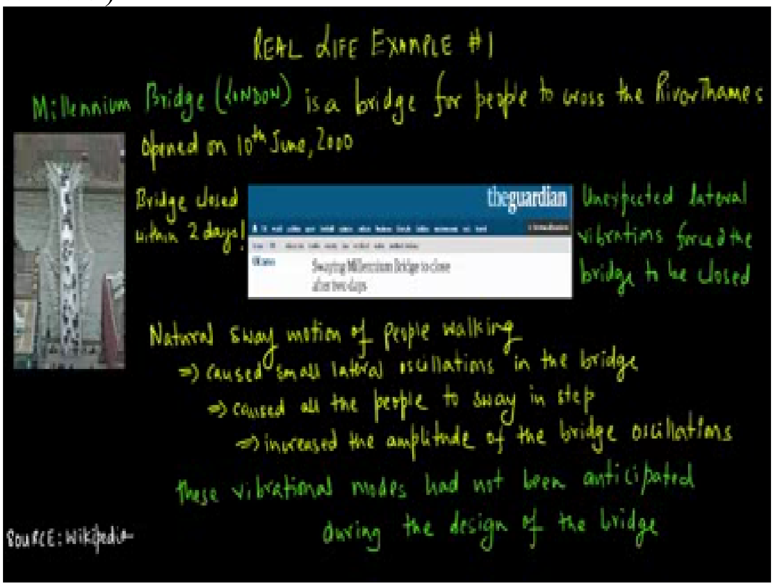

Now, let us talk about some real life examples and live in this lecture. The first example will be beautiful Millennium bridge, which is in London. It is the bridge for people to cross the river Thames. That is the picture of the bridge, so nice beautiful bridge, which is opened on 10th of June 2000. And then something rather dramatic happened the bridge closed within two days of it opening.

Essentially what happened was that unexpected lateral vibrations forced the bridge to be closed. Now, what really happened was that the natural sway of motion of people walking caused small lateral oscillations in the bridge. This in turn caused all the people on the bridge to sway in step. And which in turn increased the amplitude of the bridge oscillations. Now these vibrational modes had in fact not been anticipated during the design of the bridge.

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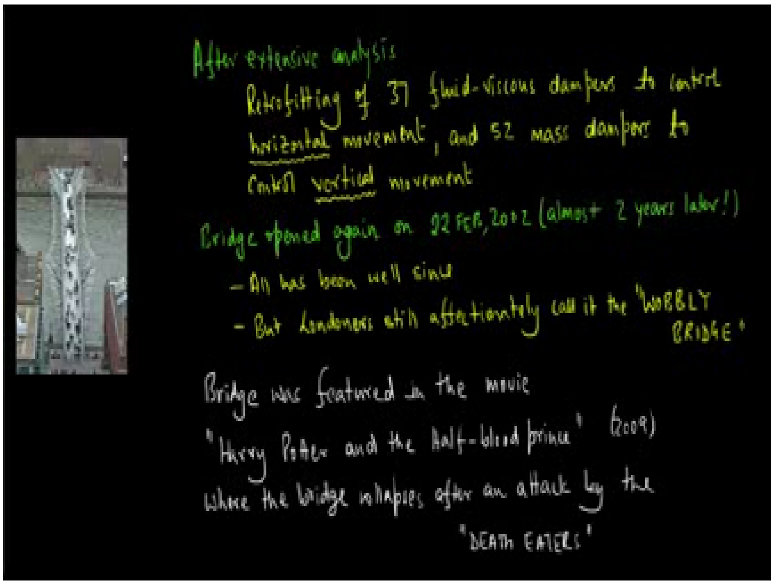

After extensive analysis, following solution was proposed, retrofitting of 37 fluid viscous dampers to control horizontal movement and 52 mass dampers to control vertical movement. The bridge opened again on the 22 of February 2002. Almost two years later, all has been well since with the bridge. But Londoners still rather affectionately call it the Wobbly bridge.

The bridge was also featured in the very famous movie "Harry Potter and the Half-blood prince" in 2009. Now in the movie the bridge actually famously collapses after an attack by the notorious Death Eaters.

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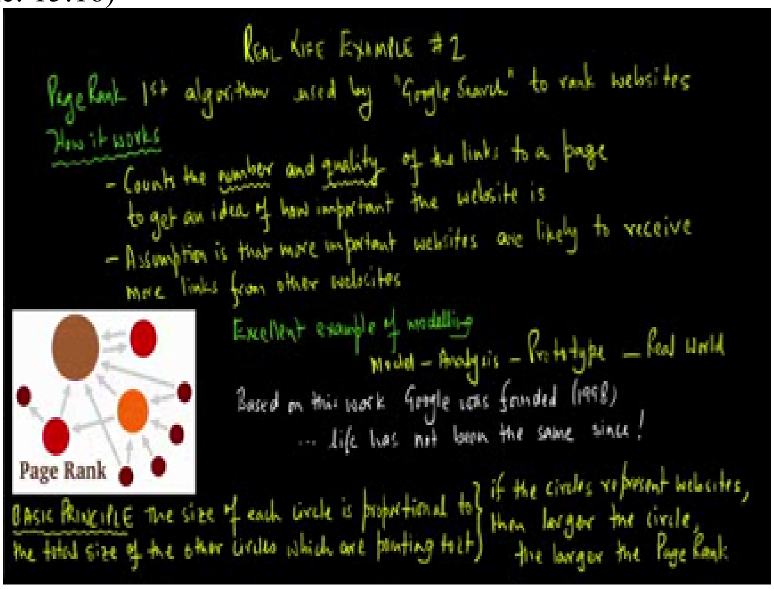

Now let us look at another real life example. This is an example that we use almost everyday of our life it is PageRank. The PageRank was the first algorithm that was used by Google search to rank websites. Now how it works is roughly as follows. The algorithm counts the number and quality of the links to a page, to get some idea of how important the website actually is.

The underlying assumption is that the more important websites are likely to receive more links from other websites. Now in the time frame above let us highlight underlying principle, the size of each circle is proportional to the total size of the other circles, which are pointing to it. So essentially if the circles represent websites, then larger the circle, the larger the PageRank. Now it turns out to be absolutely excellent example of modelling.

The models were constructed, analysis was done, prototypes developed and taken on to the real world. Based on this work Google was founded in 1998 and life has not really been same since. So hopefully this video has managed do to give you at least some insight into, what mathematical modelling is; why one should be really doing it at the first place and how it can truly have tremendous impact in our area's (17:25).