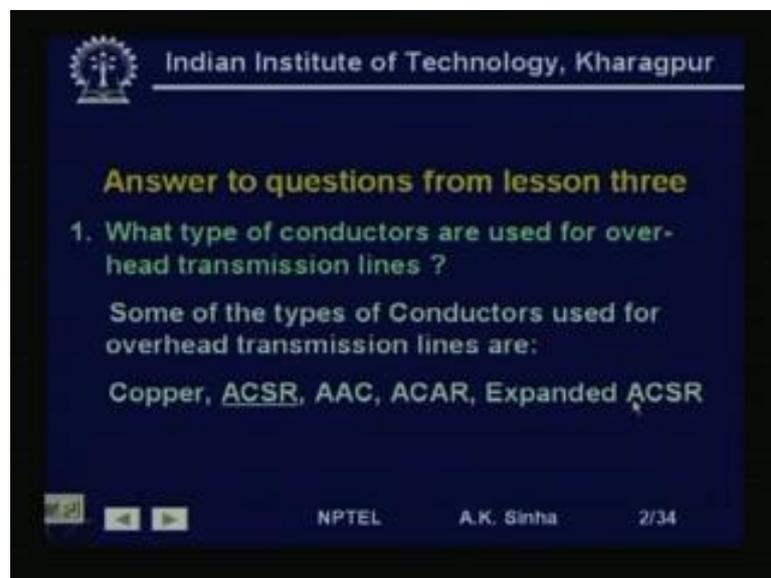


Power System Analysis
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Lecture - 4
Inductance Calculation

Welcome to lesson 4 on Power System Analysis. Before we start this lesson 4, first I would like to take up the questions that we asked in lessons 3.

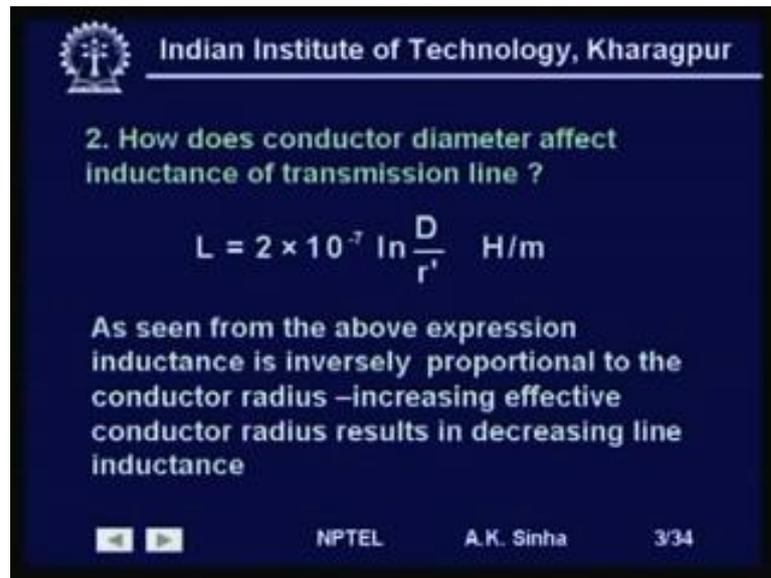
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First question was, what are the types of conductors used for over head transmission line? Well, some of the types of conductors used for over head transmission lines are copper conductors, which is very rarely use nowadays. ACSR, that is Aluminum Conductors Steel Reinforced conductors. Then, all aluminum conductors are ACAR conductors or expanded ACSR conductors, which are used.

As I have already said in lesson 3. Sometimes, we want to increase the effective radius of the conductor. And for that, we used expanded ACSR conductor in ESB transmission lines.

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2. How does conductor diameter affect inductance of transmission line ?

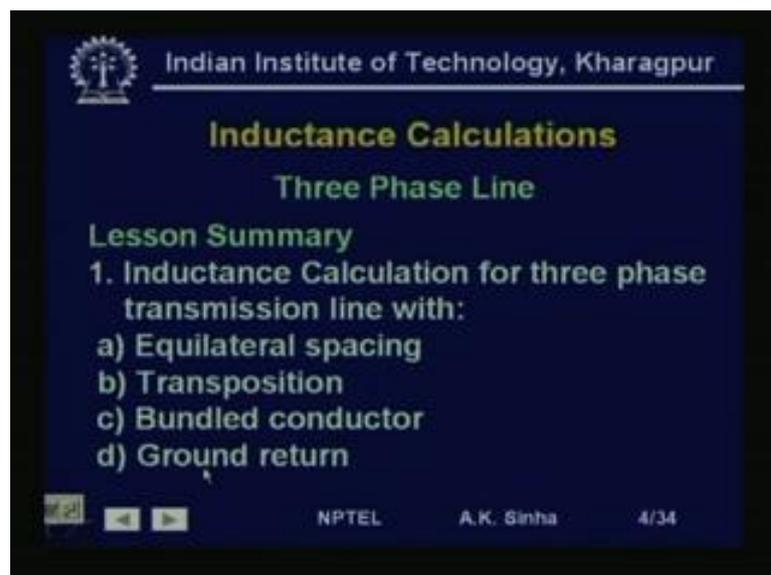
$$L = 2 \times 10^{-7} \ln \frac{D}{r'} \text{ H/m}$$

As seen from the above expression inductance is inversely proportional to the conductor radius –increasing effective conductor radius results in decreasing line inductance

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Next question was, how does conductor diameter affect inductance of a transmission line? Well, if you remember the equation for inductance, which was L , is equal to $2 \times 10^{-7} \ln \frac{D}{r'}$ Henry's per meter. Now, from this expression it is very clear. That inductance is inversely proportional to r' or the effective radius of the conductor. So, increasing effective conductor radius will result in decreasing line inductance. And that is why, as I said earlier, sometime, we use expanded ACSR conductors for transmission lines, specially in extra high voltage transmission lines.

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Inductance Calculations

Three Phase Line

Lesson Summary

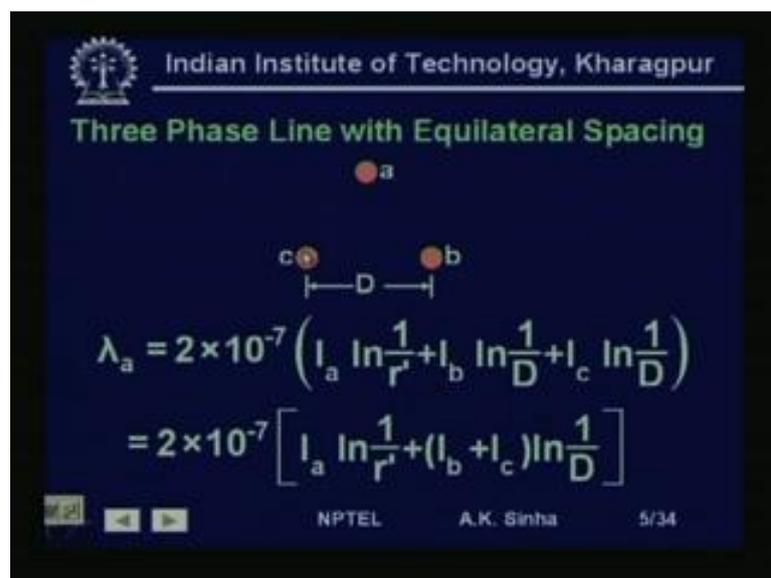
1. Inductance Calculation for three phase transmission line with:

- a) Equilateral spacing
- b) Transposition
- c) Bundled conductor
- d) Ground return

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Now, we will start our lesson 4, which will be on inductance calculations for three phase transmission lines. Well, in this lesson, we will discuss inductance calculation for three phase transmission lines with equilateral spacing, then with transposition. Again, with, when we are using bundled conductor lines and finally, lines with ground return. Well, first, we will take the calculation of inductance for a 3 phase line with equilateral spacing or equal spacing.

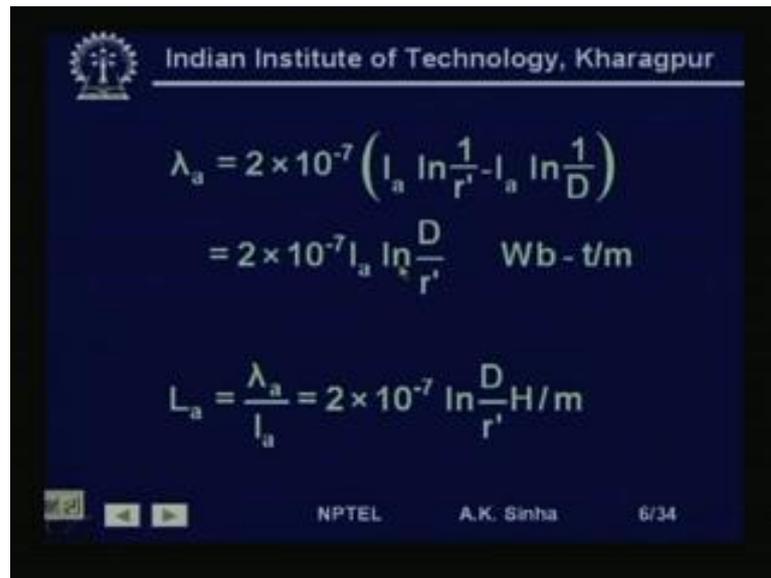
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Now, here in this diagram, you can see a, b, c are the three conductors, which are placed at the vertices of an equilateral triangle. That is the distance, among all three conductors are same. That is distance D. Now, if we calculate the flux linkage of conductor a, due to currents flowing in conductor a, b and c. Then, it will be equal to 2 into 10 to power minus 7 into I a log n 1 by r dash, where r dash is the effective radius of conductor a.

As we have seen, the effective radius of conductor a, is generally lower. It is about 0.7788 times the radius of the conductor, plus I b into log n 1 by D. That is the distance of conductor b from conductor a plus I c, the current flowing in conductor c into log n 1 by D, again the distance between conductor c and conductor a. This is equal to 2 into 10 to power minus 7, I a log n 1 by r dash plus I b plus I c log n 1 by D. That is, we have combined these two terms, we have got this.

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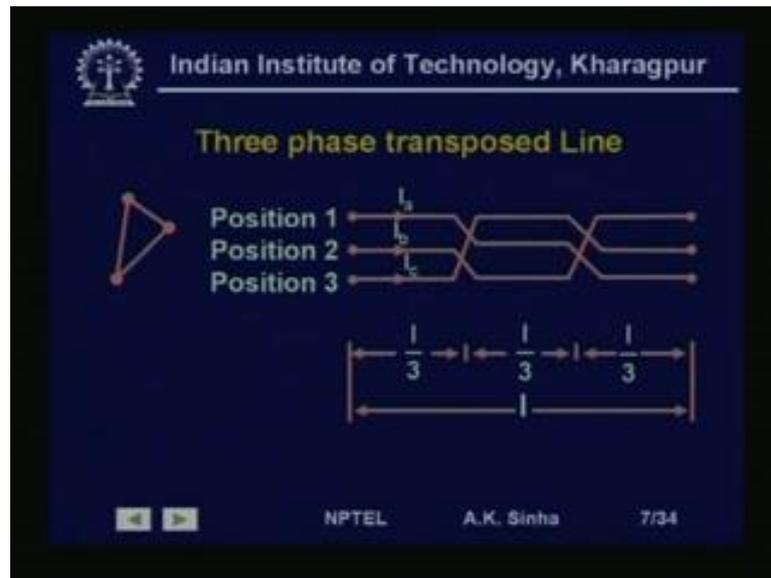
$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} \right)$$
$$= 2 \times 10^{-7} I_a \ln \frac{D}{r'} \quad \text{Wb-t/m}$$
$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{D}{r'} \text{H/m}$$

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Now, we know that, for a 3 phase three conductor line $I_a + I_b + I_c$ is equal to 0. Therefore, $I_b + I_c$ is equal to minus I_a . Therefore, we can write λ_a is equal to $2 \times 10^{-7} I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D}$, which results into $2 \times 10^{-7} I_a \ln \frac{D}{r'}$ Weber turn per meter.

Now, once we have got the flux linkage. We can calculate the inductance very easily, L_a inductance of conductor a is equal to λ_a / I_a , which comes out to be $2 \times 10^{-7} \ln \frac{D}{r'}$ Henry's per meter. Since, all the three conductors are equally spaced. Therefore, inductance of phase b and phase c conductors will also be same.

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Now, we take the inductance calculation for a 3 phase transposed line. As we have said earlier, because of physical limitations, it is not always possible to have an equilateral configuration for the conductors. So, by transposition, what we do is, we try to make the flux linkage of all the three phase conductance's more or less same. Now, what we do here is, if we have transmission line, say from this point to this point is three phase line.

What we do is, we change the position of the three phase conductance over one-third of length of the line. Like, if we see here for first one-third length of the line. The position of conductor of phase a is in position 1, phase b is in position 2, phase c is in position 3. Now, for the next one-third length, what we do is, the phase a conductor goes to position 2, phase b conductor goes to position 3 and phase c conductor goes to position 1.

And similarly, for the next one-third length of the line, that is the last one-third length of the line. The phase a conductor, now moves to position 3, phase b conductor moves to position 1 and phase c conductor move to position 2. In this way, the each phase conductor has gone through all the three positions for one-third length of the line.

And therefore, the total flux linkage for each of the phase conductors will be almost same. And this, because of this, the inductance will also be on the average being same for the three phase conductors. This makes the transmission line more balanced.

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$$\lambda_{a1} = 2 \times 10^{-7} \left[I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{31}} \right] \text{ Wb-t/m}$$

$$\lambda_{a2} = 2 \times 10^{-7} \left[I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{D_{12}} \right] \text{ Wb-t/m}$$

$$\lambda_{a3} = 2 \times 10^{-7} \left[I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{31}} + I_c \ln \frac{1}{D_{23}} \right] \text{ Wb-t/m}$$

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So, now, let us find out the flux linkage of phase a conductor in first one-third position of the conductor. This is, we write as λ_{a1} , this is equal to 2×10^{-7} into $I_a \log n 1$ by D_s . Here, I am writing instead of r dash, because sometimes, we use, instead of one conductor more conductors. So, we are taking about the self distance, which is r dash for a single solid conductor line.

So, it $I_a \log n 1$ by D_s plus $I_b \log n 1$ by D_{12} , because in this first one-third position current I_b is following in the conductor 2. And here, we have the distance D_{12} . Similarly, this plus $I_c \log n 1$ by D_{31} , which is again I_c is flowing in this conductor and the distance between them is D_{31} . Similarly, for the phase a conductor in the second one-third length of the line will be 2×10^{-7} into $I_a \log n 1$ by D_s . That is it is own distance plus $I_b \log n 1$ by D_{23} .

If we see here, in this I_b 's, now in this position I_a 's is in this position the distance between this two is D_{23} plus $I_c \log n D_{12}$. Again, if we see I_c is flowing in this and I_a is flowing in this. So, distance between them is D_{12} . Similarly, for the last one-third position portion of the line, we have a flux linkage with phase conductor equal to 2×10^{-7} into $I_a \log n 1$ by D_s plus $I_b \log n$ by D_{31} . Because, now I_b is occupying position 1 and I_a is occupying position 3 plus $I_c \log n 1$ by D_{23} . As we see here, I_c now is occupying this position and I_a is occupying this position, this is D_{23} .

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$$\lambda_a = \frac{\lambda_{a1}\left(\frac{1}{3}\right) + \lambda_{a2}\left(\frac{1}{3}\right) + \lambda_{a3}\left(\frac{1}{3}\right)}{3} = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

$$= \frac{2 \times 10^{-7}}{3} \left[3I_a \ln \frac{1}{D_s} + I_b \ln \frac{1}{D_{12} D_{23} D_{31}} + I_c \ln \frac{1}{D_{12} D_{23} D_{31}} \right]$$

$$\lambda_a = \frac{2 \times 10^{-7}}{3} \left[3I_a \ln \frac{1}{D_s} - I_a \ln \frac{1}{D_{12} D_{23} D_{31}} \right]$$

$$= 2 \times 10^{-7} I_a \ln \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s} \quad \text{Wb-t/m}$$

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Now, the total flux linkage of the conductor over the whole length, can be given by sum of these multiplied by their lengths. And if you want to take average per meter length, the flux linkage will be λ_{a1} into l by 3 plus λ_{a2} into l by 3 plus λ_{a3} into l by 3 divide by l . Or the average flux linkage of phase a conductor, which occupies all the three position is going to be the average of λ_{a1} plus λ_{a2} plus λ_{a3} , divided by 3. That is the average of the λ in all the three positions.

This, when we substitute the values of λ_{a1} , λ_{a2} and λ_{a3} , comes out to be 2×10^{-7} divided by 3 into 3. $I_a \log \frac{1}{D_s}$ plus $I_b \log \frac{1}{D_{12} D_{23} D_{31}}$ plus $I_c \log \frac{1}{D_{12} D_{23} D_{31}}$, which will finally, turn out to be, because we can now combine these two terms I_b and I_c terms. Because, the denominator, the two terms \log terms are same. Therefore, this we can write as λ_a is equal to 2×10^{-7} divided by 3 into 3 $I_a \log \frac{1}{D_s}$.

Now, I_b plus I_c is equal to minus I_a . So, we can write this as minus $I_a \log \frac{1}{D_{12} D_{23} D_{31}}$. Now, we can combine these two terms and then we will get 2×10^{-7} , $I_a \log \frac{\sqrt[3]{D_{12} D_{23} D_{31}}}{D_s}$, Weber turn per meter. So, the average flux linkage per meter, for a conductor a, comes out to be this much. And as we have seen, since all the three phase conductors, occupy all the three phases. So, the average flux linkage is for phase b and phase c conductors will also be same.

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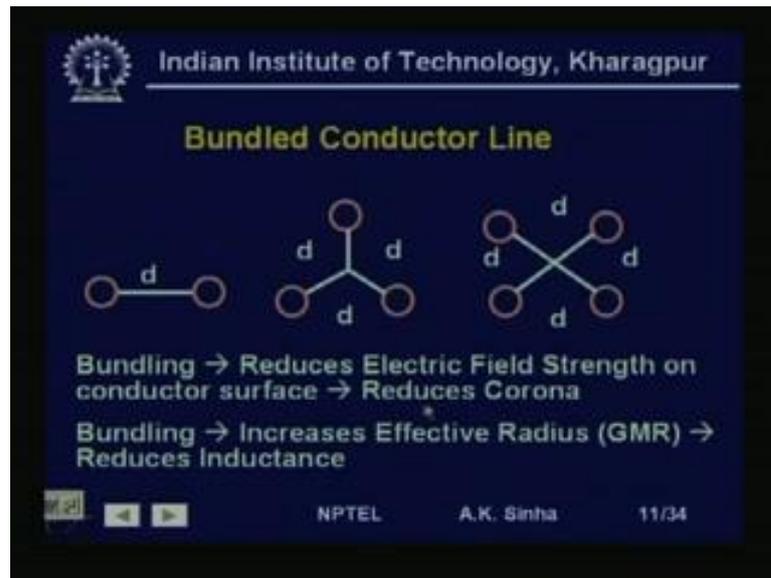
$$L_a = \frac{\lambda_a}{I_a} = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_s} \text{ H/m per phase}$$
$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}}$$
$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m}$$

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Now, once, we have got the flux linkage, average flux linkage, we can get the inductance, average inductances per meter. Length of the line as λ_a by I_a , which turns out to be $2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{12}D_{23}D_{31}}}{D_s}$. Where, this term cube root of $D_{12}D_{23}D_{31}$ is called the equivalent distance of the three phase system.

That is the equivalent distance between the three conductances; this is also seen as the GMD of the three phase conductances. Therefore, L_a is equal to or the inductance of phase a is equal to $2 \times 10^{-7} \ln \frac{D_{eq}}{D_s}$, where D_{eq} is the GMD of the 3. Now, as we have said earlier, that in order to reduce the inductances of the transmission system. We can do this by increasing the resistance, increasing the radius of the conductor. That is the effective radius of the conductor, should be increased. And this is can be d1 by means of bundle conductances. In fact, bundle conductances are used for two purposes.

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One is it reduces the electric field strength on conductor surface. And therefore, it reduces corona losses and radio interference and audible corona losses all the aspects which is associated with corona get reduced. Now, how does this happen. Now, if you see, if we use the conductor with the radius r , then its volume will be πr^2 . Now, if we take two conductors with the same, sorry, its surface area is πr^2 .

If we take the two conductors with the same cross-sectional area, then its radius is not going to be r by 2, its radius is going to be r by $\sqrt{2}$. Now, if we have this, then the surface area of one meter length of the conductor. In case, we are using two conductors instead of 1, will be now, π into r by $\sqrt{2}$ multiplied by 2. So, it will be $\sqrt{2}$ into r , whereas in the other case, it will be πr only.

So, here, what we see the surface area of the conductor increases considerably and because of this the electric stress reduces. Another effect, that we get, when we bundle the conductors is that, it increases the effective radius or the GMR of the self distance of the conductor considerably. And this reduces the inductance and this is one of the reasons, why we use this bundle conductors.

These bundled conductors are used by having more than 1 conductor, which are supported by a conducting frame at regular intervals, along the transmission line. Here, you see a bundle conductor with two conductors here. This is a bundle conductor with three conductors and bundle conductor with four conductors. In fact, for very high voltage line, sometimes we use more than four conductors also.

In India, normally we are using two conductors or four conductors for 400 kV line. And two conductors sometimes are used for 220 kV line.

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$$D_s = \sqrt[4]{(r' \times d)^2} = \sqrt{r'd}$$

$$D_s = \sqrt[9]{(r' \times d \times d)^3} = \sqrt[3]{r'd^2}$$

$$D_s = \sqrt[16]{(r' \times d \times d \times d \sqrt{2})^4} = 1.091 \sqrt[4]{r'd^3}$$

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \text{ H/m}$$

Diagrams illustrating conductor configurations:

- Two conductors at distance d .
- Three conductors in a triangle with side length d .
- Four conductors in a square with side length d .

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Now, if you look at the effective radius of the conductor. When, we are using two conductors, which are placed at a distance d , normally this distance d is approximately 10 times the diameter of the conductor. So, here, if we look at this two conductor consideration a bundle with two conductors, then D_s will be equal to, we are taking all the distances. The distance, it is own distance from itself will be r dash.

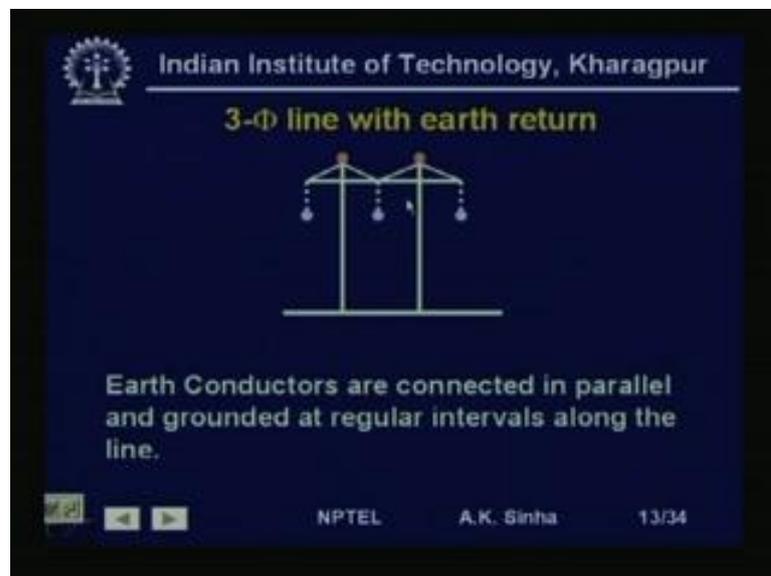
Then, it is distance from this conductor will be d , then distance of this conductance with itself will be r dash and distance of this conductor with this will be d . So, there are four distances involved and it will be 4th root of r dash into d whole square. That is r dash square into d square, which is equal to square root of r dash d , which is certainly much higher than r dash, because d is much larger than r dash.

Same thing, for three conductors we can get, line distances. That is, three distances for each conductor, one is self distance and two for the other two conductors and same thing, for each one of the three conductors. Therefore, we get self distance D_s as 9th root of r dash into d into d whole cube. This is equal to cube root of r dash into d square. Similarly, when we are using four conductors, we are got 16 distances, one for itself and three more for the other three conductors distance, from this conductor to other three conductors.

Therefore, we get the self distance D_s is equal 16th root of r dash into d into d into d root 2. This is the distance between this conductor and this conductor. Same thing will happen for this and this conductor and so on. So, in 2 to the power to 4, which is comes out to be 1.091 into 4th root of r dash d cube. And as we have seen, earlier an inductance, we can get for these bundle conductances as l is equal to 2 into 10 to the power minus 7, $\log n D e q$ by D_s .

Where, D_s , is what, is the GMR for the bundle conductance. Since, this has increase considerable as compared to single solid conductor, having the cross sectional area. Therefore, the inductance, get reduced considerably.

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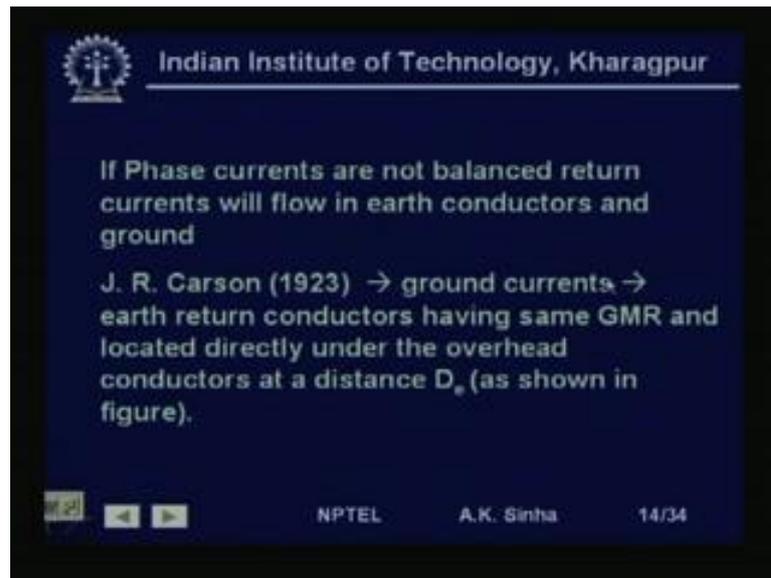


Now, we will take up three phase line with earth return. Now, this is a normal situation. Those three conductors may be horizontally placed and we have at the top of the transmission tower earth wires or the ground wires. These ground wires as I have said earlier in lessons 2. That these ground wires are used to protect the phase conductors from direct lightening stroke.

Now, these ground conductors are normally conducted or connected to the tower and at each tower footing, they are grounded. So, earth conductors are connected in parallel and grounded at regular intervals along the transmission line. If they are not grounded at each tower, may be, they will be grounded at every alternate tower or so. So, how do, we find out the inductances for such a system?

Normally, when the system is working as a balance system, they would not be any return current flowing. Because, the sum of the three currents will be 0 all the time, but in case of unbalance current flowing in the system. We have some currents flowing through these return conductors, which will again go to the ground and will flow through the earth.

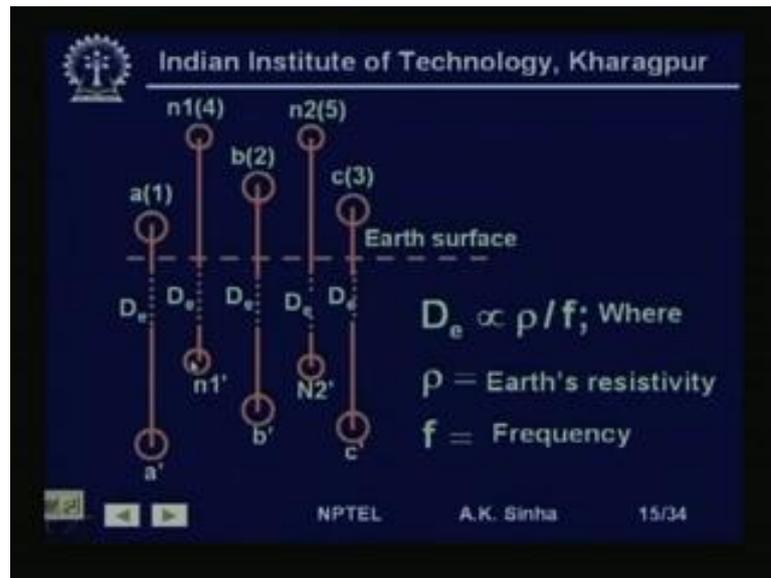
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So, if phase currents are not balanced, return currents will flow in earth conductors and ground. Because, since these earth conductors are connected to the tower and again, there are grounded by the footing, ground tower footing. So, the current flowing in these earth conductors will go into the ground. And will this disperse and flow in the ground also.

Now, it was J.R. Carson in 1923, who effectively or who proposed, how to take care of these ground currents in the system, in the transmission system. So, which was later modified, he was the first person to model this ground currents into a system. What he did was, he said that, earth return conductors are basically or the earth current can be basically, represented by earth return conductors, which are having the same GMR as the overhead conductors.

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And are located directly below the overhead conductors, at a distance D_e , as shown in this figure, that is here we have three phase conductors a, b and c. And we have two earth conductors are the ground conductors, which are n 1 and n 2. Now, what we he proposed as for such a system. He said that, the ground current flowing can be represented by conductors a dash, which is directly below a into the ground at a distance D_e , from that conductor.

Same thing for n 1, we have a conductor n 1 dash, which is in the ground or which is directly below this in the ground at a distance D_e and so on. Where, he said D_e , the distance at which these conductors are from the overhead conductors is proportional to ρ , the earth resistivity. And inversely proportional to frequency of the system or the frequency of the current, which is flowing in these conductors.

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$$D_{k'k} = D_{kk} \text{ (m); } D_{kk'} = D_e$$

$$D_e = 658.5 \sqrt{\rho/f} \text{ m } (\rho = 100 \Omega - \text{m})$$

$$R_{k'} = 9.869 \times 10^{-7} f \text{ } (\Omega/\text{m})$$

$$\sum_{k=1}^{2(3+2)} I_k = 0$$

$$\lambda_k = 2 \times 10^{-7} \sum_{m=1}^{(3+2)} I_m \ln \frac{D_{km'}}{D_{km}} \text{ Wb-t/m}$$

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So, now we can see that, D for any conductor k , we have the ground conductor $D_{k'k}$. The distance of it is own, that is same as D_{kk} , which is the GMR of that conductor. And the distance $D_{kk'}$ for any conductor k to k' is D_e . And the $D_{kk'}$, which is the distance of this conductor to itself is same, as the GMR of this conductor. And he said that, D_e is equal to 658.5 into square root of ρ by f .

Now, this is an empirical formula, which was found after lots of experimentation. Where, ρ is as resistivity and if one does not know the earth resistivity properly. Then, one can always choose 100 ohm meter as the earth resistivity, which is the resistivity for that earth. He also found out, that the resistance of these image conductors are the conductors, which are representing the ground current is given by $R_{k'}$ is equal to 9.869 into 10 to power minus 7 f , where f is the frequency per ohms per meter.

So, these D_e and $R_{k'}$ are empirical values and these have been found out after lots of experimentation. Now, for this system, that we have, we have five conductors here and five conductors as images, below these conductors at a distance D_e , from the overhead conductors. Now, this makes a total of 10 conductors. Now, some of the currents in all the 10 conductors will have to be equal to 0.

So, in this system, sum of the current I_k is equal to 0, for k is equal to 1 to 2 times into three phase conductors plus 2 neutral conductors. That is sum of all the 10 conductor currents is equal to 0. Now, for such a system, we can find out the flux linkage, for any conductor very easily. And λ_k , in this case is given by 2 into 10 power minus 7

λ is equal to 1, 2, 3 plus 2. That is three phase conductors plus 2 ground conductors, $\frac{1}{D_{km}} \ln \frac{D_{km'}}{D_{km}}$.

That is distance from the conductor to the conductors, which are the image conductors divided by D_{km} , the distance between 2 overhead conductors. And these include ground conductor as we are seeing. So, λ_k is $2 \times 10^{-7} \ln \frac{D_{km'}}{D_{km}}$, λ is equal to 1 to 5, $\frac{1}{D_{km}} \ln \frac{D_{km'}}{D_{km}}$ by D_{km} Weber turn per meter.

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$$\lambda = LI$$

Where,

λ is a (3+2) vector

I is a (3+2) vector

L is a (3+2) × (3+2) matrix whose elements are :

$$L_{km} = 2 \times 10^{-7} \ln \frac{D_{km'}}{D_{km}} \text{ H/m}$$

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Since, λ is equal to LI , where L is the inductance and I is the current. Now, in this case, since there are five conductors, we have λ for all the five conductors. That is $\lambda_a, \lambda_b, \lambda_c$ and $\lambda_{n1}, \lambda_{n2}$. So, λ is a 3 plus 2 vector, I again, since the current is following in all these five conductors. So, I is a 3 plus 2 vector.

Now, L which is the inductance matrix is also a 3 plus 2 into 3 plus 2 matrix. Whose elements L_{km} is equal to $2 \times 10^{-7} \ln \frac{D_{km'}}{D_{km}}$. This comes out from the previous relation of λ_k . So, λ_k , we know. So, if we sum this up and then we find out the inductance, sorry, the conductors, then L_{km} will come out to be this much.

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$$\begin{bmatrix} E_{Aa} \\ E_{Bb} \\ E_{Cc} \\ 0 \\ 0 \end{bmatrix} = (R + j\omega L) \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_{n1} \\ I_{n2} \end{bmatrix}$$

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And therefore, we can write this as matrix equation, where we are saying E_{Aa} is the voltage drop in phase A conductor. E_{Bb} is the voltage drop in phase B conductor. E_{Cc} is voltage in phase C conductor. For the neutral conductors if there are grounded, their potential is 0.

So, voltage drops are going to be 0, for the two ground conductors, this is equal to R plus $j\omega L$, which will be again a 5 by 5 matrix into I_a current in phase a. I_b current in phase b. I_c current in phase c, I_{n1} current in neutral 1 and I_{n2} current in neutral 2 or ground 2.

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$$R = \begin{bmatrix} (R_a + R_k) & R_k & R_k & R_k & R_k \\ R_k & (R_b + R_k) & R_k & R_k & R_k \\ R_k & R_k & (R_c + R_k) & R_k & R_k \\ R_k & R_k & R_k & (R_{n1} + R_k) & R_k \\ R_k & R_k & R_k & R_k & (R_{n2} + R_k) \end{bmatrix} \Omega/m$$

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Here, the matrix R is a 5 by 5 matrix and this is represented by for R_A , what we have is, R_A plus R_k dash. That is this is for R_A , the resistances for conductor A. Now, R_k dash is the resistances for the image conductor, which is below A. Then, we have R_k dash for the other 4. Similarly, for R will be R_{12} will be R_k dash R_2 with 2 will be R_B plus R_k dash R_{23} will be R_k dash and so on. So, this is a 5 by 5 matrix, which is a resistance in ohms per meter for this five conductor system.

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$$\begin{bmatrix} E_{Aa} \\ E_{Bb} \\ E_{Cc} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_{n1} \\ I_{n2} \end{bmatrix}$$

Now, if you write the complete system equation for the voltages across the line conductors. Then, we will have the voltage as $E_A a$, $E_B b$, $E_C c$. That is the voltage across the two points of the phase A conductor, phase B conductor and phase C conductors. Similarly, for the neutral conductors, since the neutral conductors are grounded, this voltage will be 0.

This voltage is going to be equal to the impedance, multiplied by the current which gives the voltage drop across the two points. Now, here the impedances will be Z_{11} , Z_{12} , Z_{13} , Z_{14} , Z_{15} . All these impedances, this will give a 5 by 5 matrix. And the currents will be the, three phase currents I_a , I_b , I_c , I_{n1} , the current in neutral conductor 1 and I_{n2} in the conductor 2.

So, this for the five conductor system, we have five set of equations and this impedance matrix is a 5 by 5 matrix, which we can divided into four different sub matrices. As shown here, Z_A which is 3 by 3 matrix, which indicates the Z_{11} , Z_{12} Z_{21} , Z_{22} , Z_{23} ,

Z 31, Z 32,, Z 33. These impedances are relating the voltages across the phase conductors with the currents following in the phase conductor.

Similarly, the voltages across the phase conductors, due to currents following in the neutrals, can be given by the impedances as shown in the Z B. That is Z 14, Z 15, Z 24, Z 34, Z 35. And Z C indicates the voltage across the neutral conductors, due to currents in the phase conductors. These impedances are Z 41, Z 42, Z 43, Z 51, Z 52, Z 53. Similarly, for the voltage n th across the neutral conductors, due to currents in the neutral conductors will be given by this matrix Z D, which is Z 44, Z 45, Z 54, Z 55. So, we have three sub matrices Z A, which is a 3 by 3, Z B which is 3 by 2 matrixes, Z C, which is 2 by 3 matrixes and Z D, which is a 2 by 2 matrix. This is the complete system equation that we have, where the impedance terms are given as for the diagonal elements.

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Diagonal elements (k = m)

$$Z_{kk} = R_k + R_{k'} + j\omega 2 \times 10^{-7} \ln \frac{D_{kk'}}{D_{kk}} \Omega/m$$

Off - Diagonal elements (k ≠ m)

$$Z_{km} = R_{k'} + j\omega 2 \times 10^{-7} \ln \frac{D_{km'}}{D_{km}} \Omega/m$$

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That is Z k k, which means Z 11, Z 22 and so on are given by R k plus R k dash plus j omega 2 into 10 to power minus 7, log n D k k dash by D k k ohms per meters. Whereas, these are the distances, as we have seen in the system in the system diagram earlier. Off diagonal elements that is Z k m, which is basically Z 12, Z 13 or Z 54, all these off diagonal elements.

That is, when k is not equal to m is given by R k dash plus j omega into 2 into 10 to power minus 7 log n D k m dash by D k m ohms per meter. So, we can compute all these

diagonal and off diagonal elements, that is the elements of this imperial matrix using these relationships.

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The slide displays the following equations:

$$\begin{bmatrix} E_p \\ 0 \end{bmatrix} = \begin{bmatrix} Z_A & Z_B \\ Z_C & Z_D \end{bmatrix} \begin{bmatrix} I_p \\ I_n \end{bmatrix}$$

$$E_p = \begin{bmatrix} E_{Aa} \\ E_{Bb} \\ E_{Cc} \end{bmatrix} \quad I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad I_n = \begin{bmatrix} I_{n1} \\ I_{n2} \end{bmatrix}$$

At the bottom of the slide, there are navigation icons, the text "NPTEL", "A.K. Sinha", and "22/34".

Now, what we can do, since we have already divided this system into four sub matrices. Now, we can write this whole system of equation in a short form like E_p , where E_p is the voltage drop across the face conductors, 0 is the voltage drop across the neutral conductor, this is equal to Z_A, Z_B, Z_C, Z_D into I_p . Where I_p is the current flowing through the face conductors and I_n is the vector of current flowing through to neutral conductors.

As shown here, E_p is equal to E_{Aa}, E_{Bb} and E_{Cc} are three phase conductor voltage drops, I_p is equal to the phase currents I_a, I_b, I_c . And I_n is the current following through the two neutrals I_{n1}, I_{n2} . Now, from this set of equations, we can write this into two separates sets of equation as E_p is equal to Z_A, I_P plus $Z_B I_n$. E_p is equal to Z_A, I_P plus Z_B into I_n .

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$$E_P = Z_A I_P + Z_B I_n$$
$$0 = Z_C I_P + Z_D I_n$$
$$I_n = -Z_D^{-1} Z_C I_P$$
$$E_P = [Z_A - Z_B Z_D^{-1} Z_C] I_P$$
$$E_P = Z_P I_P$$
$$Z_P = Z_A - Z_B Z_D^{-1} Z_C$$

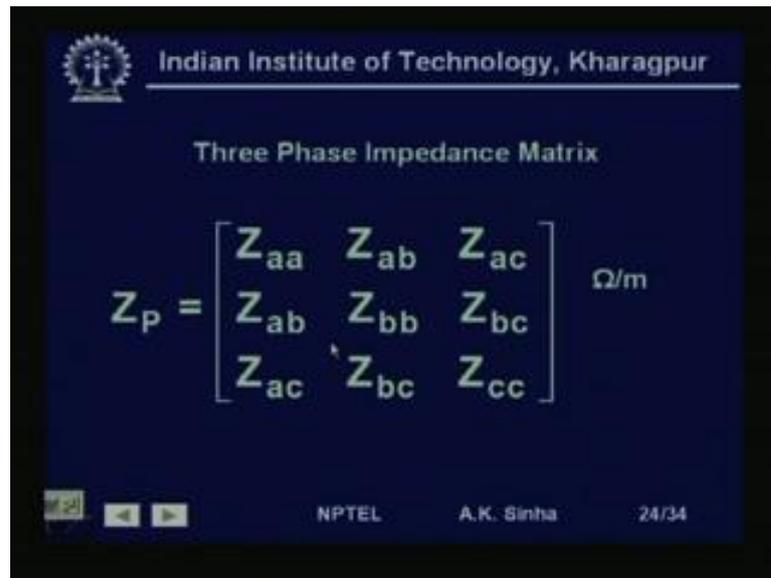
NPTEL A.K. Sinha 23/34

Similarly, 0 is equal to $Z_C I_P + Z_D I_n$. 0 is equal to $Z_C I_P + Z_D I_n$. Now, from this second equation, we can take this term. This term on the other side then we have I_n is equal to minus $Z_D^{-1} Z_C I_P$. That is I_n is here and we have taken this on this side. So, it is minus $Z_C I_P$. Now, we pre multiply both sides by Z_D^{-1} .

So, we will get I_n , I_n is equal to minus $Z_D^{-1} Z_C I_P$. And therefore, putting for this I_n here in this first equation, we will get E_P is equal to $Z_A I_P$ minus because this minus term is coming $Z_B I_n$. So, I_n is given by this relationship, so minus Z_B into Z_C into Z_D^{-1} , sorry, Z_B into Z_D^{-1} into $Z_C I_P$. So, we have substituted for I_n from here and then we are writing this expression like this.

Now, we can write this whole as E_P is equal to $Z_P I_P$, where Z_P is this matrix. Here, this will be a 3 by 3 matrix Z_P is equal to $Z_A - Z_B Z_D^{-1} Z_C$. So, this is what we will get, that is we have eliminated in this equation, the current I_n and we are writing the all the equations in terms of the phase variable. Only thing is, this is taken care by using Z_D^{-1} and Z_C from this expression.

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The slide displays the following content:

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Three Phase Impedance Matrix

$$Z_P = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \Omega/m$$

NPTEL A.K. Sinha 24/34

Therefore, we will get now Z_p , which will be a 3 by 3 matrix as Z_{aa} , Z_{ab} , Z_{ac} , Z_{ba} or Z_{ab} will be same Z_{bb} , Z_{bc} and Z_{ca} , Z_{bc} , Z_{cc} . So, this is now a 3 by 3 matrix, from where we have eliminated the currents in the neutral conductors or the ground conductors. So, here we are getting a relationship, only for the phase conductors, E_p is equal to Z_p into I_p .

So, the current following in the ground conductors or the ground is now eliminated. And we can now get a relationship, only for the conductor's currents. In the phase conductors relating the voltage drop in the phase conductors. Now, for a fully transpose line, what we will have is since these phase conductors will be occupying all the three positions for one-third length of the line. Therefore, we will get the average values for all the three flux linkages and the inductances. And therefore, we will have the same impedance for all the three phased.

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For Fully Transposed Line

$$\hat{Z}_p = \begin{bmatrix} \hat{Z}_{aa} & \hat{Z}_{ab} & \hat{Z}_{ab} \\ \hat{Z}_{ab} & \hat{Z}_{aa} & \hat{Z}_{ab} \\ \hat{Z}_{ab} & \hat{Z}_{ab} & \hat{Z}_{aa} \end{bmatrix} \Omega/m$$

NPTEL A.K. Sinha 25/34

So, self impedance Z_{aa} , will be same as Z_{bb} , will be same as Z_{cc} . That is all the three phase will have the same self impedances. And the mutual impedances will also be equal for all the three phases. So, Z_{ab} and Z_{ac} will be equal which will be also equal to Z_{bc} and Z_{ca} .

So, we get finally, for a fully transpose line, we get the values or thus impedance series impedance for the three phase system with ground return. As Z_p is equal to Z_{aa} , Z_{ab} , Z_{ab} , Z_{ab} , Z_{aa} , Z_{ab} , Z_{ab} , Z_{ab} , Z_{aa} . This matrix will give me the impedance, series impedance of the transmission line, with ground return.

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Where,

$$\hat{Z}_{aa} = \frac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc})$$
$$\hat{Z}_{ab} = \frac{1}{3}(Z_{ab} + Z_{ac} + Z_{bc})$$

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So, Z_{aa} is equal to as I said the average value and Z_{ab} will be also equal to the average value the mutual and the self, will be given by these two relationship.

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Example: The conductor configuration of a completely transposed three-phase overhead transmission line with bundled conductors is shown in the next slide. All the conductors have a radius of 0.74cm with a 30cm bundle spacing.

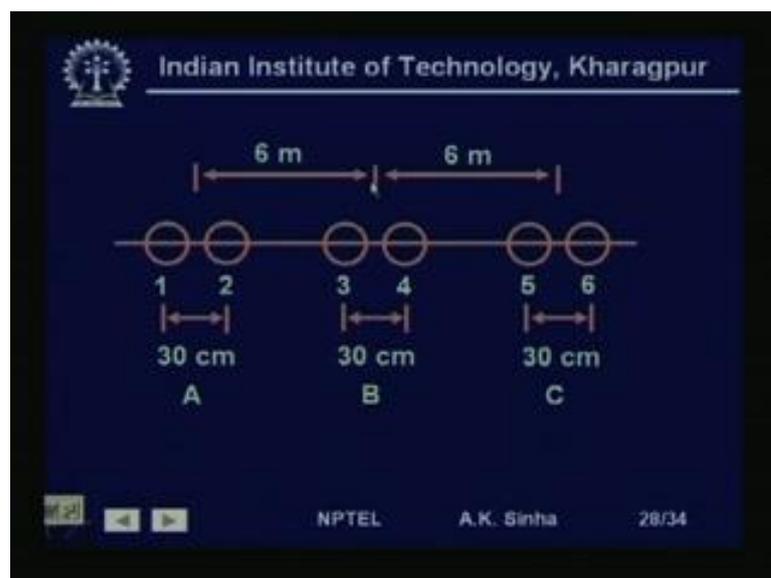
(a) Determine the inductance per phase in mH/km and in mH/m.

(b) Find the inductive line reactance per phase in ohms/m at 50Hz.

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Now, let us take an example, for finding out the inductance of a three phase transmission system. The example, that we are taking is for the conductor consideration of a completely transpose three phase over a transmission line, with bundle conductor is shown. All the conductors have a radius of 0.74 centimeter with is 30 centimeter bundle spacing.

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That is, we have a 30 centimeter bundle spacing for the conductors and each conductor as a radius of 0.74 centimeter. The distance between the two phase conductors is 6 meters. That is from center to center. And this is a horizontally spaced configuration. The line is fully transposed. For this line, determine the inductance per phase in milli Henry per kilometer and in milli Henry per meter. Find the inductive line reactance per phase in ohms per meter at 50 hertz.

So, this is the question, that we will have to find out the inductances per phase of the line in milli Henry per kilometer or a milli Henry per meter. We will also need to find out the inductive line reactance per phase in ohms per meter at 50 hertz.

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Solution: For the given configuration as shown in the figure on the last slide, we have the mutual GMD between the different phases given by

$$D_{ab} = (r_{13} \cdot r_{14} \cdot r_{23} \cdot r_{24})^{1/4}$$

$$= (6 \times 6.3 \times 5.7 \times 6)^{1/4}$$

$$= 5.9962 \text{ m}$$

Similarly,

$$D_{bc} = 5.9962 \text{ m}$$

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So, for this system, what we have to do is, we have to find out the equivalent distances. So, for the given configuration as shown in figure, we have the mutual GMD between the different phases given by D_{ab} is equal to $r_{13}, r_{14}, r_{23}, r_{24}$. That is distance from here to this conductor, distance from this to this conductor, distance from this to this conductor, distance from this to this conductor.

And 4th root of that, because again if you take the distances from this to this and this to this, this to this and this to this, that will also be same. So, it will 4th root of D_{ab} is equal to the 4th root of distance between 13, 14, 23, 24. So, this is equal to 6 into 6.3 into 5.7 into 6. That is 13 is 6, 23 is 5.7, 14 is 6 plus 30 centimeters. So, 6.3 and 24 is 6. So, we take the 4th root, it comes out to be 5.9962, it is same as almost 6 meters.

Similarly, for between B and C, if we do this, we will get that same distance or the same GMD. But, for D_{ca} , if we look at this between C and A, then we have the distance as A 1 to 5, 1 to 6, 2 to 5 and 2 to 6.

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$$D_{ca} = (r_{15} \cdot r_{16} \cdot r_{25} \cdot r_{26})^{1/4}$$

$$= (12 \times 12.3 \times 11.7 \times 12)^{1/4}$$

$$= 11.9981 \text{ m}$$

The equivalent equilateral spacing between the phases is given by D_{eq} defined as

$$D_{eq} = (D_{ab} \times D_{bc} \times D_{ca})^{1/3}$$

$$= (5.9962 \times 5.9962 \times 11.9981)^{1/3}$$

$$= 7.5559 \text{ m}$$

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Therefore, D_{ca} is equal to r_{15} into r_{16} into r_{25} into r_{26} , the 4th root of this. That is equal to 12 into 12.3 into 11.7 into 12, 4th root of that is 11.9981, which is very near to 12 meter. Now, the equilateral spacing between the phases is given by the D_{eq} defined as D_{eq} is equal to D_{ab} into D_{bc} into D_{ca} and cube root of that. So, if we find out the equilateral spacing. The D_{eq} , that comes out to be 5.9962 into 5.9962 into 11.9981 cube root of that, that comes out to be 7.5559 meter.

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Self GMD of all the three phases, owing to symmetry, are equal and hence the equivalent self GMD for the system can be given by

$$D_s = (.7788 \times r \times 30)^{1/2}$$
$$= 4.1580 \text{ cm.}$$

Inductance per phase for the given system is

$$L = 2 \times 10^{-7} \times \ln(D_{eq} / D_s) \text{ H/m/phase}$$
$$= 1.04049 \times 10^{-6} \text{ H/m/phase.}$$
$$= 1.04049 \times 10^{-3} \text{ mH/m/phase.}$$
$$= 1.04049 \text{ mH/km/phase.}$$

NPTEL A.K. Sinha 31/34

Now, the self GMD of all the three phases point to symmetry are equal. That is, the self distance each conductor has the same radius and the distance between the two conductors of a bundle is same. That is 30 centimeter. So, therefore, the D_s for each phase conductor is going to be equal to r dash. That is 0.7788 into r for the conductor into 30 centimeter, which is the distance between the two conductors of the same bundle.

The square root of this comes out to be 4.1580 centimeter. In fact, we could have taken in all the four distances as we have seen earlier D_s is r dash into D square root of that. So, this is equal to 4.1580 centimeter. Now, you see this is much larger than 2 times of 0.7788 into r which is 0.74 centimeter. Therefore, bundling has increased the effective radius considerably.

Therefore, now we can find out inductance per phase for the given system as L is equal to $2 \times 10^{-7} \times \ln(D_{eq} / D_s)$ Henry's per meter per phase, which is equal to 1.04049×10^{-6} Henry's per meter, per phase, which can be written as 1.04049×10^{-3} milli Henry per meter, per phase or 1.04049 milli Henry per kilometer, per phase.

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The inductive line reactance per phase at 50Hz is given by

$$X = 2\pi \times 50 \times 1.04049 \times 10^{-6} \text{ ohms/m/phase.}$$
$$= 3.270 \times 10^{-4} \text{ ohms/m/phase}$$

NPTEL A.K. Sinha 32/34

The inductive line reactance per phase at 50 hertz is given by x is equal to $2\pi f$ into the inductance. So, this is $2\pi \times 50$ into 1.04049×10^{-6} ohms per meter per phase. This comes out to be 3.270×10^{-4} ohms per meter per phase. So, this is, how we can calculate the inductance and the inductive reactance for any given three phase transmission system.

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Answer following questions

1. Why bundled conductors are used in EHV lines?
2. What is Transposition ?
3. How the effect of earth return current is taken into account in inductance calculation?

NPTEL A.K. Sinha 33/34

Now, before we finish I would like you to answer the following questions. First is, why bundled conductors are used in EHV lines. Second question is, what is transposition. And third question is, how the effect of earth return current is taken into account in

inductance calculation for a 3 phase line with ground return system. Specially, when this system is carrying unbalanced current, so with this we finish these lessons.

Thank you very much.

We will meet again for lesson 5. In which, we will talk about calculating the capacitance of the transmission line.

Thank you.

Preview of next lecture

Lecture - 05

Transmission Line Capacitance

Welcome to lesson 5, on power system analysis course. In this course, we will talk about the transmission line capacitance, before we get in to the calculation of transmission line capacitance.

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Questions from Lesson 4

1. Why bundled conductors are used in EHV lines?

Bundling → Reduces Electric Field Strength on conductor surface → Reduces Corona

Bundling → Increases Effective Radius (GMR) → Reduces Inductance

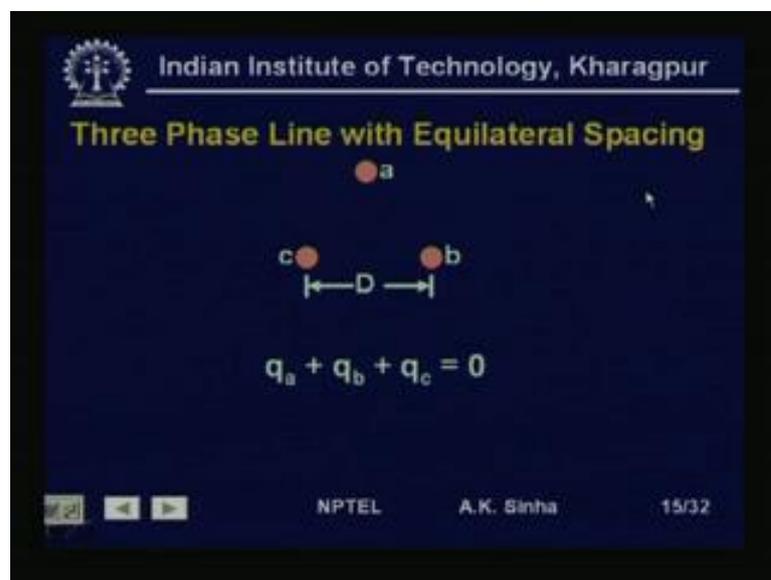
$$D_s = \sqrt[3]{(r' \times d \times d)^3} = \sqrt[3]{r' d^2}$$

NPTEL A.K. Sinha 2/32

I would like to answer those questions. That I ask in lesson 4. First question was, why bundle conductors are used in EHV lines? Well, the answer to this question is bundling of conductors. That is, instead of using one single conductor, use of a number of conductors connected by conducting frames, reduces electric fields strength on conductor surface, which in effect reduces the corona losses, which result in power loss.

As well as radio interference and audible noise in the system, bundling also increases the effective radius of the conductor. And there by, reduces the inductance of the transmission line. This in effect will improve the regulation of the transmission line. As seen from here, the effective radius for a 3 conductor bundle, which are spaced at a distance d . From the center d , from each other with a radius r is given by 9th root of r dash into d into d whole cube, which is equal to cube root of r dash d square. This is much larger than r dash, which is used, when a single conductor is used. And therefore, bundling helps in reducing the inductance as it increases the effective radius.

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Therefore, we will take the case of a three phase system. We will start with the three phase line with equilateral spacing. Because, as we have seen for inductance calculation, we can always covert, if the line is transpose. We can always convert any system into the equivalent three phase equilateral spacing of the conductors by finding out the equivalent distance D e q . So, here we have equilateral spacing conductor a , b and c , each with a distance d from each other. We also assume that this system consist of only three conductors. So, the sum of the total charges will be equal to 0. That is q_a plus q_b plus q_c is equal 0.

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Solution: For a fully transposed three phase line, we have the line voltage given by

$$V_{ab} = 1/(2\pi k) \{ q_a \times \ln(D_{eq}/r) + q_b \times \ln(r/D_{eq}) \}$$

Where $D_{eq} = (D_{12} \cdot D_{23} \cdot D_{31})^{1/3}$
 $= (12 \times 12 \times 24)^{1/3}$
 $= 15.119 \text{ m.}$

Similarly,

$$V_{ac} = 1/(2\pi k) \{ q_a \times \ln(D_{eq}/r) + q_c \times \ln(r/D_{eq}) \}$$

NPTEL A.K. Sinha 28/32

So, for solving this, we will take case of a fully transposed three phase line. So, we have the fully transposed three phase line. The voltage is given by the relationship V_{ab} is equal to $1/(2\pi k) \{ q_a \times \ln(D_{eq}/r) + q_b \times \ln(r/D_{eq}) \}$. Where D_{eq} is equal to $(D_{12} \times D_{23} \times D_{31})^{1/3}$. That is cube root of D_{12}, D_{23} into D_{31} .

Now, substituting these values, we get this as equal to $12 \text{ meters} \times 12 \text{ meters} \times 24 \text{ meters}$. That is the distance between the conductor a and c, the cube root of this, will give us equal to 15.119 meter . Similarly, we can write the relationship for V_{ac} and V_{bc} . V_{ac} will be equal to $1/(2\pi k) \{ q_a \times \ln(D_{eq}/r) + q_c \times \ln(r/D_{eq}) \}$. Calculate these quantities for a transmission system.

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Example:

A three phase, 400kV, 50Hz, 350km overhead transmission line has flat horizontal spacing with three identical conductors. The conductors have an outside diameter of 3.28 cm with 12 m between adjacent conductors.

Determine the capacitive reactance-to-neutral in ohms/m/phase and the capacitive reactance for the line in ohms/phase.

NPTEL A.K. Sinha 27/35

So, next, this example, we have a three phase 400 kV 50 hertz 350 kilometer overhead transmission line. That has a flat horizontal spacing, which three identical conductors. That is, we have three identical conductors, places in a plat horizontal spacing. The conductors have an outside diameter of 3.28 centimeter. And that is the diameter of the conductor is 3.28 centimeter. And the distance between the adjacent conductors is and the distance between the adjacent conductors is 12 meter. Now, for this system determine the capacitive reactance to neutral in ohms per meter per phase and the capacitive reactance, for the line in ohms per phase.

Thank you.