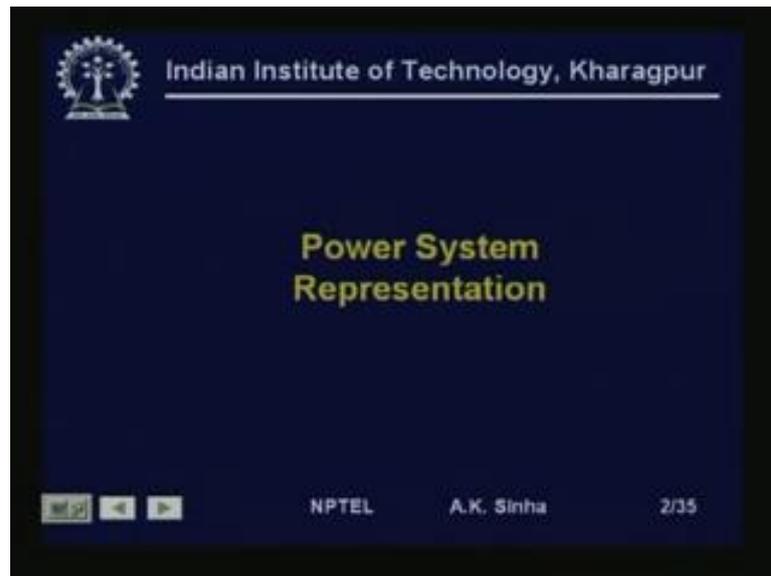


**Power System Analysis**  
**Prof. A. K. Sinha**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kharagpur**

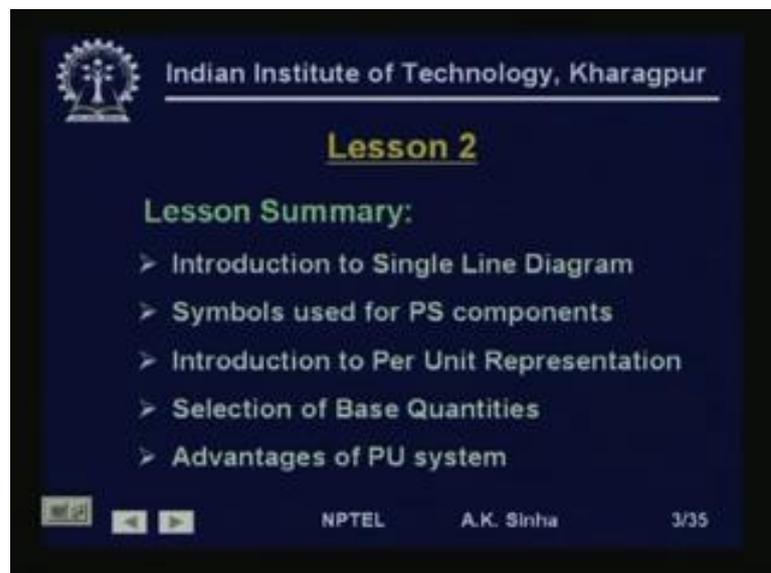
**Lecture – 2**

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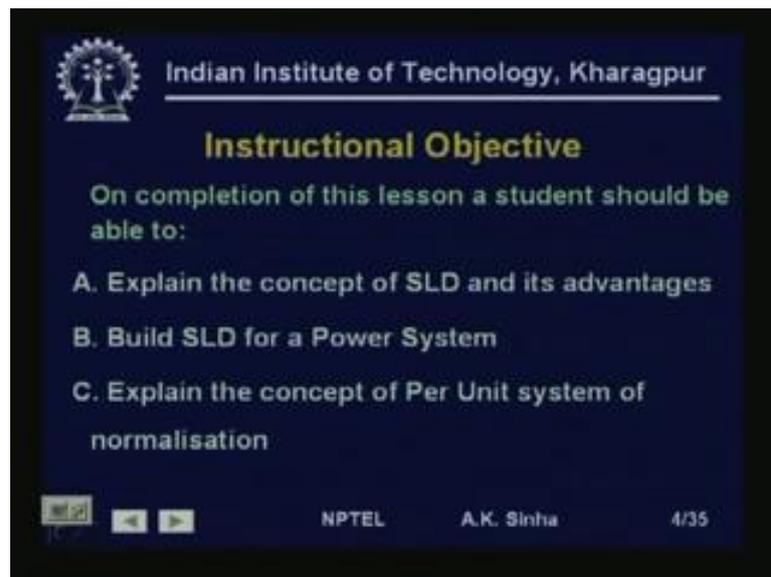
Welcome to lesson 2 on Power System Analysis. In this lesson, we will talk about power system representation. Well, in power system since it is a very complicated system. We use to for modeling, we represent the system in a simpler and easier to follow forms.

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And for that we will talk about how we represent this power system in. In this lesson 2, we will start with introduction to single line diagram. Then, we will talk about the symbols which are used for various power system components, in single line diagram. Then we will go on building single line diagram for power system. After that we will go to the per unit representation of power system. We will talk about how to select the base quantities. And finally, we will talk about the advantages of a per unit system.

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The slide is a dark blue presentation slide with white and yellow text. At the top left is the IIT Kharagpur logo. To its right, the text 'Indian Institute of Technology, Kharagpur' is written in white. Below this, the title 'Instructional Objective' is written in yellow. The main body of the slide contains the text 'On completion of this lesson a student should be able to:' followed by three bullet points: 'A. Explain the concept of SLD and its advantages', 'B. Build SLD for a Power System', and 'C. Explain the concept of Per Unit system of normalisation'. At the bottom, there are navigation icons (back, forward, search) on the left, and the text 'NPTEL A.K. Sinha 4/35' on the right.

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**Instructional Objective**

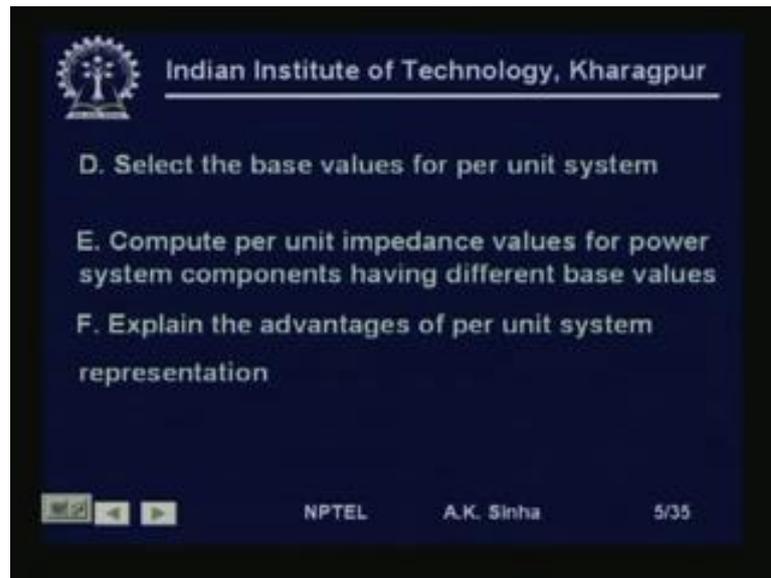
On completion of this lesson a student should be able to:

- A. Explain the concept of SLD and its advantages
- B. Build SLD for a Power System
- C. Explain the concept of Per Unit system of normalisation

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Well, once this you have gone through this lesson, you would be able to explain the concepts of single line diagram. And what are it is advantages. You will be able to build a single line diagram for a power system. You will be also able to explain the concept of per unit system of normalization.

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And how to select the base quantities for the per unit system, and a power system having different voltage levels and different power ratings of the equipment. Then, we will also be able to compute per unit impedance values for power system components, having different base values. And finally, you would be able to explain the advantages of per unit systems of representation.

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So, we will take up these two forms of representation of a power system. The first one is the graphical representation, in terms of single line diagram. And the second one will be

how to represent the various electrical quantities, in terms of per unit values. So, first we will talk about the single line diagram for a power system.

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### Single Line Diagram

- Power systems are extremely complicated electrical networks
- Three phase networks – all devices are installed in all three phases and each power circuit consists of three conductors.
- A complete conventional diagram showing all the connections is very complicated and impractical.

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Well, as we have said in lesson 1. Earlier power systems are extremely complicated electrical networks. In fact, they are most probably the most complicated man-made systems. Another problem that these power system network is that, we always deal with 3 phase networks. That is all the lines are 3 phases means, each power circuit consists of three conductors. All the devices are installed in all the 3 phases. A complete conventional diagram showing, all the connections is going to be very complicated.

And it will be impractical, because it will be very difficult to read this diagram. And understand the connections between various components, because of the complications. Which will be coming because of so many lines had to be shown on the diagram. Now, we will try to take advantage of some of the basic characteristics of the power system, in trying to simplify the system for representing it.

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We know that 3 phase systems are designed as balance system. And are generally operated as a balanced 3 phase system, which means that all the equipment which are the 3 phase equipment like generators, motors, transformers. All are designed to be balanced system means. All the 3 phases have equal impedances have the same voltage levels and so on.

And also wherever we use even single phase systems, like your residential loads. There also we try to put these loads, in such a way that, they are equally divided on all the 3 phases. And so the system is more or less a balanced system. Now, if we have a balanced system. Then what we have is all the 3 phases are having the same currents and voltages.

And therefore, whatever we have, whatever values of currents and voltages in one phase is there, is going to be in the other two phases. Except that they will be out of phase by 120 degree from each other. And therefore, it is possible to make a single phase, or a single line representation of this balanced 3 phase system. This is the property that we try to take make use of in building single line diagrams.

Now, single line diagrams are concise way of communicating the basic arrangement of power system components. That is what we need in case of representing graphically the power system network is; to see the basic interconnections of various components. So, that we know what is connected where and how? So, this is what we try to do using the single line diagram.

Now, a single line diagrams which are also called SLD's, use single line to represent all the 3 phases. As I said earlier, that 3 phases since they are all of same kind having same similar currents and voltages. Therefore, we use a single line to represent all the 3 phases.

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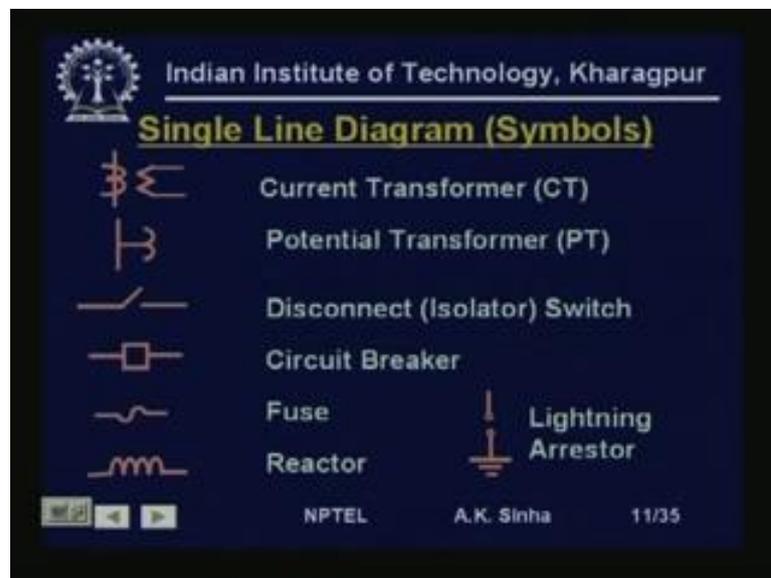
Since, we use only one line to represent all the 3 phases. Single diagrams are also called one line diagrams. And these diagrams show the relative electrical interconnections of various components. Such as generators, transformers, transmission and distribution lines, loads, circuit breakers, and all these components which go in to make the power system network. Now, single line diagram when we are using the graphical representation, we need to use symbols for the various power system components, although there is no universally accepted set of symbols.

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But, we use certain symbols which are more or less. Universal in the sense, that most of the people use these kinds of symbols. Generators is represented like this. Similarly, transformers 2-winding transformers are represented with 2-windings, shown 3-winding transformers. With 3-windings like this and auto transformers are shown in this fashion.

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We have symbols for current transformers, potential transformers, disconnect, isolators, which is circuit breakers, fuses, reactors, lighting arresters. All these components, different symbols are used for these components. And by combining these symbols, we build the power system network diagram.

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### Single Line Diagram (Bus-bars)

**Busbar (bus) – Node in electrical circuit**  
(one bus for each phase)

**Buses -** aluminum or copper bars or pipes  
and can be several meters long

**Buses in SLDs -** short straight lines  
perpendicular to transmission  
lines and to lines connecting  
equipment to the buses.

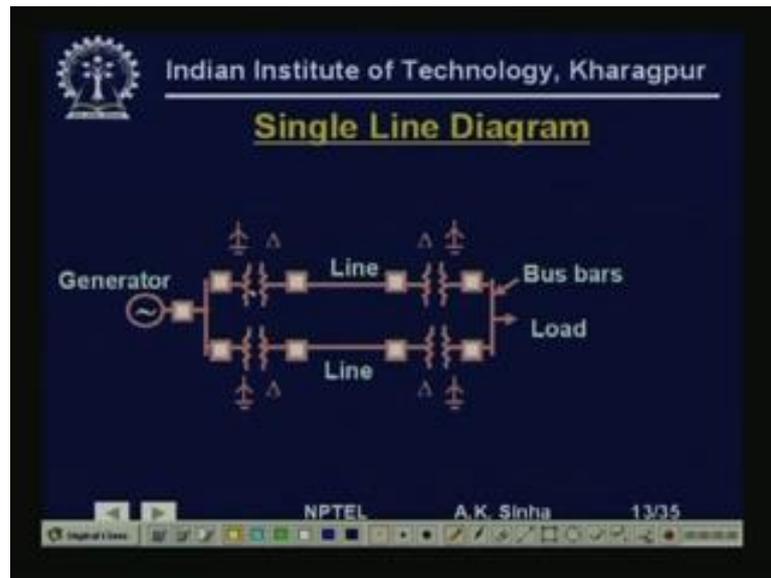
NPTEL A.K. Sinha 12/35

Now, in power system we have a concept of a bus-bar. Bus-bars are nothing but, nodes in electrical circuit. So, basically in power system, we call these nodes of electrical circuit as the bus-bars. This is required in power system, because you cannot join various lines, which may be very thick lines just by connecting at one point. So, what is used is a long bar is used. And these various lines and components are connected to this bar by means of nuts and bolts.

This was the system which was used from very beginning. And it is still continuing though if you go for go and see a high voltage substation. You will find bus-bars are very similar to transmission lines. They are thick aluminum conductors put on overhead towers and the various lines and other components are joined to this. So, basically electrically if you see a bus-bar is nothing but, a node.

Now, in single line diagrams bus-bars are used or represented as short lines, short thick lines which is used perpendicular to the transmission lines. And is connecting these lines. So, when we show different lines connected at a bus-bar. What we have is the lines normally will be shown as the horizontal lines. Then the bus-bar will be a vertical line, somewhat thicker than the transmission lines which are being shown.

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Now, here for an example, I am showing you a single line diagram of a power system. Now, in this diagram, you see we have a generator, we have circuit breakers. Then we have a bus-bar which is connecting this generator to various transformers. We have transformers connected to the bus-bar through circuit breakers. We have another transformer here for the other line. And the lines the two parallel lines are connected to the transformers through circuit breakers shown here.

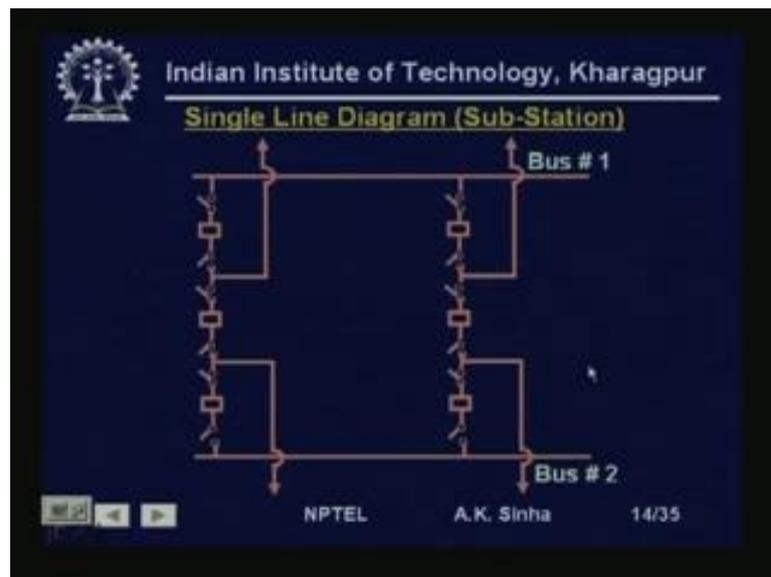
Then, we have another transformer on the other side. And this is again connected through circuit breakers to a bus-bar to which a load may be connected. So, here what we see is how we represent a power system network, which is consisting of a generator connected through two step up transformers; connecting high voltage transmission line. And then we have step down transformers to reduce the voltage, connecting to a low voltage bus-bar to which load is connected.

So, in this way we can build a single line diagram. And now I will show you how we build a single line diagram using the symbols. Here, we have taken an example of a generator 3 phase generator. Connected by means of step, two step up transformers to a double circuit transmission line. Then through two step down transformers to a low voltage bus-bar to which load is connected.

So, what we have is, we have used the generator symbol here. Then, we have a circuit breaker with for protection of the generator, connects it to a bus-bar shown here. Then, we have again circuit breakers to circuit breakers on the primary side of the transformer.

Or the low voltage side for the two transformers. And then on the secondary side or the high voltage side again this is transformer is connected, by means for these circuit breakers to the two high voltage transmission lines, which again at the other end, through the circuit breakers is connected to the high voltage side of the step down transformers. And the low voltage side of the step down transformers is connected to a bus-bar by means of these two circuit breakers. And on this low voltage bus, we have the load.

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Next we have a substation with 2 bus-bars and 4 transmission lines. We have these transmission line 1, 2, 3 and 4 connected to these two bus-bars. By means of circuit breakers shown here, and the isolator switches shown here. So, this is how we represent a power system network, using single line diagram. This is much simpler, much easier to understand, because it is showing you the relative interconnection of the various components. And it is much more simplified and easy to see and draw.

Next, we will talk about representing the power system components with it is electrical quantities. Most of the time, we represent these electrical quantities, in terms of power system per unit system. One of the reason for this is, because power system consists of equipments which are very wide varying ratings, and if we use all these different voltage levels and the power levels. Then, there is always a chance of making mistakes, because we are talking in terms of somewhere kilowatts, somewhere megawatts, somewhere kilovolts, somewhere just volts. So, chances of making mistakes is much higher. But, if

we represent these values in terms of per unit, we will see what advantages we get out of this.

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### Per Unit Normalization

- In Power Systems electrical quantities such as POWER, VOLTAGE, CURRENT, IMPEDANCE etc. are very often expressed as per unit of a base or reference value.
- Per Unit value is expressed by equation:

$$\text{Per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

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So, in power system electrical quantities such as power, voltage, current, impedance etcetera, are very often expressed as per unit of a base or reference value. That is what we do is we represent these values on a reference, with respect to a reference or a base value, per unit value is expressed by the equation. Per unit quantity is equal to actual quantity divided by the base value of that quantity.

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### Choice of Base Quantity

Two Independent Base Values are selected

- $V_{\text{base}}$  – Voltage Base Value
- $S_{\text{base}}$  – Complex Power Base Value

#### Single Phase System

$$P_{b1\phi} = Q_{b1\phi} = S_{b1\phi} \quad I_b = \frac{S_{b1\phi}}{V_{bLN}}$$

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Now, we have for interest 4 values. That is power, voltage, current and impedance, the out of these 4 values, if we choose any two. The other two can always be represented in terms of these two. That is we need to choose only 2 values independently, the other 2 values will come out to be dependent on these. The two independent base values which are selected for per unit representation in power system, is generally a base voltage value and the complex power base value.

So, we use V base and S base as our reference values. And the values for Z base, that is the impedance base. And the current base I base will get derived from V base and S base. Now, let us start with a single phase system. For a single phase system we have P base single phase is equal to Q base single phase is equal to S base single phase. That means, we choose the complex power base. And that base is going to be that value is also the base value for the real, as well as reactive power. Now, the base value for the current, I base will come out to be S base by V base. That is S base single phase by V base lying to neutral or a single phase value.

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$$Z_b = R_b = X_b = \frac{V_{bLN}}{I_b} = \frac{V_{bLN}^2}{S_{b1\phi}}$$

$$Y_b = G_b = B_b = \frac{1}{Z_b}$$

**Convention for per unit system:**

1. Value of  $S_b$  is same for the entire system
2. Ratio of  $V_b$  on either side of a transformer is selected to be same as the ratio of transformer voltage ratings

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The impedance base value Z base; Z b is equal to R b the resistance base or the reactance base. All of them will have the same base value. And this is equal to V base lying to neutral by I base. This can also be written as V base lying to neutral square divided by S base single phase. Of course, if we are talking about admittance, admittance will be just the inverse of the impedance. So, Y base is equal to G base the conductance base or the susceptance base B base. And that will be equal to 1 by Z base.

This is very simple. But, in power system what we have is, we use large number of transformers for voltage transformation, from low voltage to high voltage, and then from high voltage to low voltage at various points. Therefore, what we need to do is, we need to choose these base values very properly. One of the convention that we follow for choosing the base values, for per unit system is that value of S base  $S_b$  is same for the entire system. That is the base value for the complex power is same for the whole system.

Now, the ratio of the base value for the voltage. On either side of a transformer is selected to be same, as the ratio of the transformer voltage ratings. This is very important, because on the two sides of the transformer you have two different voltage levels. And we can choose different voltage bases on the two sides. It is very advantageous; if we use these bases on the two sides of the transformer. The voltage bases on the two sides of the transformer to be in the same ratio, as the turns ratio of the transformer. Generally, what we try to do is, we choose the base voltage values on the two sides of the transformer, as the nominal or the rated voltage of the transformer on the two sides. This we will see later has lots of advantages.

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Diagram of a transformer equivalent circuit:

- Primary side: Current  $I_{1pu}$ , voltage  $V_{1pu}$ , and leakage impedance  $R'_{pu} + jX'_{pu}$ .
- Secondary side: Current  $I_{2pu}$ , voltage  $V_{2pu}$ .
- Core: Labeled "Ideal Trans".

Equations:

$$V_{1pu} = \frac{V_1}{V_{1b}} = \frac{N_1}{N_2} \times \frac{V_2}{V_{1b}}$$

Using

$$V_{1b}/V_{2b} = V_{rated1}/V_{rated2} = N_1/N_2$$

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Now, let us see we have a transformer. Now, this transformer can be represented as a leakage impedance. That is R in per unit and j in per unit referred to the primary side. And an ideal transformer and then we have the voltage  $V_2$  per unit. On the secondary side and  $I_2$  per unit current on the secondary side. We have neglected the magnetizing

part of the transformer here. Most of the time in power system analysis, unless we are interested in finding out the losses in the core losses as such.

We most of the time will be neglecting that. The core loss or the magnetizing part of the transformer winding. Because, its effect is very, very small and neglecting this does not cause much errors. And also it simplifies our diagram considerably. Now, if you look at this circuit diagram. Then we can write,  $V_1$  per unit is equal to  $V_1$  by  $V_1$  base, which is equal to  $N_1$  by  $N_2$  in to  $V_2$ . Because  $V_1$  is equal to  $N_1$  by  $N_2$  in to  $V_2$  and this divided by  $V_1$  base.

Now, if we are choosing our two base values, on the two sides of the transformers base voltage values.  $V_1$  base divided by  $V_2$  base is equal to  $V_{\text{rated } 1}$  divided by  $V_{\text{rated } 2}$ , this is equal to  $N_1$  by  $N_2$ . This is what as I said if we follow these conventions. That base values for the voltage at the two sides of the transformer is in the ratio of the rated voltages. Which will be also the turns ratio of the transformer.

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$$V_{1pu} = \frac{N_1}{N_2} \frac{V_2}{(N_1/N_2)V_{2b}} = \frac{V_2}{V_{2b}} = V_{2pu}$$

$$I_{1pu} = \frac{I_1}{I_{1b}} = \frac{N_2}{N_1} \frac{I_2}{I_{2b}}$$

Using

$$I_{1b} = S_b/V_{1b} = S_b / \{(N_1/N_2)V_{2b}\} = (N_2/N_1)I_{2b}$$

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Then, we have  $V_1$  per unit is equal to  $N_1$  by  $N_2$  in to  $V_2$  divided by  $V_1$  base. Now,  $V_1$  base will become  $N_1$  by  $N_2$  in to  $V_2$  base. Because,  $V_1$  base by  $V_2$  base is equal to  $N_1$  by  $N_2$ . Therefore, this is equal to  $V_2$  by  $V_2$  base which is equal to  $V_2$  per unit. That is what we find is the voltage at the two sides of the transformer in a per unit values, are going to be same, though the actual values of the voltages will be quite different. Because, one side for a 11 KV to 132 KV transformer. The low voltage side value will be somewhere around 11 KV whereas, the high voltage side value will be

around 132 KV. But, in case of per unit system if we have chosen the base value of voltage, on the low voltage side as 11 KV and on 132 KV side. As 132 KV, then on both sides we have the value of the voltage equal to 1 per unit.

So, this in per unit system, the transformer voltages are going to be same. Similarly, we can write  $I_1$  per unit is equal to  $I_1$  by  $I_1$  base, which is equal to  $N_2$  by  $N_1$  into  $I_2$  that is  $I_1$  divided by  $I_1$  base. Now, using  $I_1$  base is equal to  $S$  base by  $V_1$  base. This will be equal to  $S$  base divided by  $N_1$  by  $N_2$  into  $V_2$  base, this finally comes to  $N_2$  by  $N_1$  into  $I_2$  base. That is we are seeing that  $I_1$  base and  $I_2$  base are related by a relationship  $I_1$  base is equal to  $N_2$  by  $N_1$  into  $I_2$  base. That is it is in the inverse ratio of the base values. That we had chosen for the voltage. Where we had  $V_1$  base by  $V_2$  base is equal to  $N_1$  by  $N_2$ . Here we have  $I_1$  base by  $I_2$  base is equal to  $N_2$  by  $N_1$ .

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$$I_{1pu} = \frac{N_2}{N_1} \frac{I_2}{\left(\frac{N_2}{N_1}\right) I_{2b}} = \frac{I_2}{I_{2b}} = I_{2pu}$$

Transformer impedance referred to primary side

$$Z_{1pu} = Z_1 \frac{I_{1b}}{V_{1b}}$$

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Now, if you look at the per unit quantities I can write  $I_1$  per unit is equal to  $N_2$  by  $N_1$  into  $I_2$  divided by  $I_1$  base. Now,  $I_1$  base is equal to  $N_2$  by  $N_1$  into  $I_2$  base. Therefore, it comes out to be  $I_2$  by  $I_2$  base which is nothing but,  $I_2$  per unit. That is the current in per unit values for  $I_1$  and  $I_2$  is going to be same, which shows since the voltage in per unit is also same. And the current is also same which shows this ideal transformer is simply a 1 is to 1 transformer in per unit system. So, it can as well be neglected.

So, what we find that, if we have used a per unit system with proper voltage base selection. Then this auto transformer or the turns transformation, the voltage

transformation ratio, on current transformation ratio which comes for the transformers are no longer required in per unit system. So, transformers can be simply eliminated. And it is leakage impedance can be put in per unit values. Now, the same thing we can see for the impedance. The transformer impedance on the primary referred to the primary side is  $Z_1$  per unit will be equal to  $Z_1$  into  $I_1$  base divided by  $V_1$  base.

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Transformer impedance referred to secondary side

$$Z_2 = Z_1 \left( \frac{N_2}{N_1} \right)^2$$

$$Z_{2pu} = Z_1 \left( \frac{N_2}{N_1} \right)^2 \left( \frac{I_{2b}}{V_{2b}} \right)$$

$$= Z_1 \left( \frac{N_2}{N_1} \right)^2 \cdot I_{1b} \left( \frac{N_1}{N_2} \right) / V_{1b} \left( \frac{N_2}{N_1} \right)$$

$$= Z_1 \left( \frac{I_{1b}}{V_{1b}} \right) = Z_{1pu}$$

NPTEL A.K. Sinha 21/35

And if it is referred to the other side, secondary side, then  $Z_2$  is equal to  $Z_1$  into  $N_2$  by  $N_1$  square. Therefore,  $Z_2$  per unit will be equal to  $Z_1$  into  $N_2$  by  $N_1$  square which is nothing but,  $Z_2$  into  $I_2$  base by  $V_2$  base, this well again substituting the values we have  $Z_1$  in to  $N_2$  by  $N_1$  square into  $I_1$  base into  $N_1$  by  $N_2$ . For  $I_2$  base we can write and divide by  $V_2$  base can be written as  $V_1$  base into  $N_2$  by  $N_1$ .

Which finally, comes out to be  $Z_1$  into  $I_1$  base divided by  $V_1$  base which is equal to  $Z_1$  per unit. That is what we are seeing is that, even the impedance referred to on the two sides in per unit system are going to be same. And therefore, the transformation ratios for referring the impedance from one side to the other side of the transformer is no longer required. So, whether the impedance is referred to on the primary side. Or on the secondary side of a transformer, it does not make any difference in per unit system, because it is going to be same on both side. And that is why we can very easily eliminate all these transformation ratios, which we have to do in actual system. This is a very great advantage of per unit system of representation, because in a power network we have very

large number of transformers. And we have, if we are working with actual quantities, we will have to take care of all these turns ratios all the time.

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**Change of Base**

$$Z_{p.u.new} = \frac{Z_{actual}}{Z_{bnew}} = \frac{Z_{p.u.old} Z_{bold}}{Z_{bnew}}$$

$$Z_{p.u.new} = Z_{p.u.old} \left( \frac{V_{bold}}{V_{bnew}} \right)^2 \left( \frac{S_{bnew}}{S_{bold}} \right)$$

NPTEL A.K. Sinha 22/35

In power system, we also use equipments at various with various ratings, different ratings. And therefore, we need to convert the impedance values from one rating to another. Because as I said earlier the equipment manufacturers will generally specify the value of the impedance of any equipment, in terms of percentage, or per unit values based on the name plate rating of that equipment.

We might choose a different base for power, for the whole power system. We may have a difference base value for the voltage of the system in that part. Therefore, we need to convert the impedance from one base value to another base value. So, suppose we have the value of impedance given at one base, then how do we convert it to another base. So, we have this relationship Z per unit new value is equal to Z actual value by Z base new, which is equal to Z per unit old into Z base old which is the actual value. Because, if we multiply the per unit value by the base value, we get the actual value. So, this is Z actual is Z per unit old in to Z base of old divided by Z base new. This can be written as Z per unit old into V base old by V base new square into S base new by S base old. When we substitute for Z base old as V base old square by S base and Z base new as V base new square by S base new.

If we substitute that, this is the relationship that we will get Z base Z per unit new is equal to Z per unit old into V base old divided by V base new square into Z base new by

Z base old. So, if we know the old values or the equipment value at particular base. And we want to convert it to other base, we can find out the value of the impedance at this new base values.

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**Three Phase System**

$$S_{b1\phi} = \frac{S_{b3\phi}}{3}$$

$$S_{b3\phi} = P_{b3\phi} = Q_{b3\phi}$$

$$V_{bLN} = \frac{V_{bLL}}{\sqrt{3}}$$

NPTEL A.K. Sinha 23/35

Till now we talked about single phase system. The per unit system representation can be extended. Or used in 3 phase system also in a similar way. What we have is for a single phase system, if we have chosen a base  $S$  base for single phase, this is equal to  $S$  base for 3 phase system divided by 3. We know that for any equipment rating. If we have the 3 phase rating, then per phase rating is going to be 1/3rd of that.

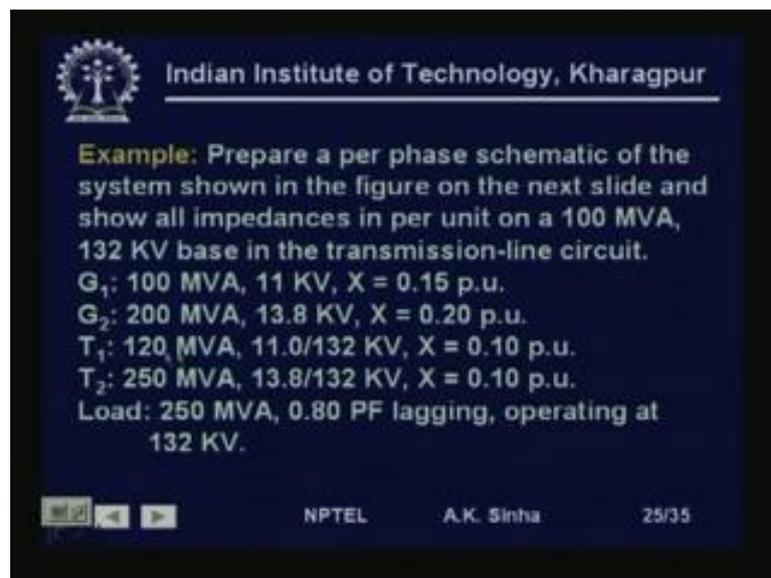
So, similarly we can choose the 3 phase base for complex power, which will again be valid or will be same for the real power, as well as the reactive power. So,  $S$  base 3 phase is equal to  $P$  base 3 phase is equal to  $Q$  base 3 phase. In case of 3 phase system instead of using line to neutral voltage, which is the per phase voltage in the system. We use line to line voltage and the relationship is  $V$  base line to neutral is equal to  $V$  base line to line divided by root 3. Or  $V$  base line to line is equal to root 3 times  $V$  base line to neutral.

Therefore, we get  $I$  base which is equal to  $S$  base single phase divided by  $V$  base line to neutral as equal to  $S$  base 3 phase divided by root 3  $V$  base line to line. So, if we have  $I$  base known, then we can write  $Z$  base is equal to  $V$  base line to neutral by  $I$  base, which is same as  $V$  base line to neutral square divided by  $S$  base single phase. And if we are using 3 phase system, then this multiplied by 3 and this multiplied by 3 will give me  $V$  base line to line square.

Because,  $V$  base line to line is  $\sqrt{3}$  times  $V$  base line to neutral divided by  $S$  base 3 phase. Therefore,  $Z$  base for a single phase system, as well as 3 phase system is going to be same. Except that the bases for 3 phase system has to be chosen as  $S$  base for 3 phase is three times the  $S$  base for single phase. And  $V$  base for 3 phase system should be the line to line voltage phase. Again  $Z$  base is same as the base for resistance, reactance or the impedance. And if we take the reciprocal of it, we get the base value for the admittance.

So, we can use this per unit system both in single phase. Or 3 phase system normally power systems are 3 phase system. So, we will be using the 3 phase system, where we will be using the power base as 3 phase power base. And we will be using the voltage base as the line to line voltage.

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**Example:** Prepare a per phase schematic of the system shown in the figure on the next slide and show all impedances in per unit on a 100 MVA, 132 KV base in the transmission-line circuit.

$G_1$ : 100 MVA, 11 KV,  $X = 0.15$  p.u.  
 $G_2$ : 200 MVA, 13.8 KV,  $X = 0.20$  p.u.  
 $T_1$ : 120 MVA, 11.0/132 KV,  $X = 0.10$  p.u.  
 $T_2$ : 250 MVA, 13.8/132 KV,  $X = 0.10$  p.u.  
Load: 250 MVA, 0.80 PF lagging, operating at 132 KV.

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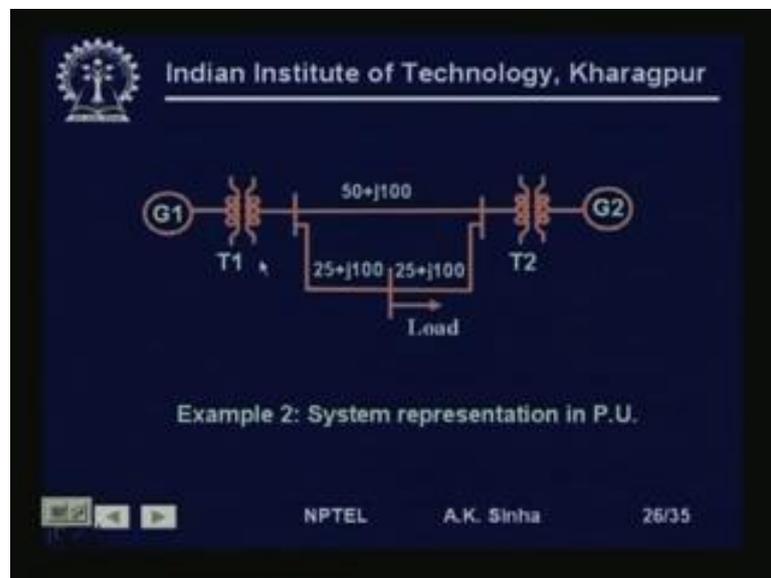
Now, let us take a simple example for this per unit representation. Here we have taken a problem where we say that prepare, a per phase schematic of the system shown in the figure, which is shown in the next slide. On the next slide, and show all the impedances in per unit on a 100 MVA base. 132 KV voltage base in the transmission line of the circuit. That is the impedance the power base for the whole system is chosen as 100 MVA.

Whereas, the voltage base in the transmission line circuit is chosen as 132 KV. The voltage base in the other parts of the circuit, as we will see will depend on the voltage transformation ratio on the two sides of the transformers. Now, for this system we are

given that we have a generator G 1 with 100 MVA rating at a 11 KV and impedance of 0.15 per unit, another generator G 2 with a 200 MVA rating. And voltage rating of 13.8 KV and reactance X is equal to 0.2 per unit.

We have a transformer T 1 with a rating of 120 MVA. And voltage ratio of 1 KV to 132 KV and the reactance of 0.1 per unit, another transformer T 2 with a power rating of 250 MVA. And a voltage transformation ratio of 13.8 KV to 132 KV. And a reactance of 0.1 per unit on its base, that is on it is rated value base. We have a load connected to the system which has a value of 250 MVA at 0.8 per factor lagging, and operating at 132 KV.

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So, this is our system we have a generator G 1, connected through a transformer T 1 to this bus. And a generator G 2, connected through transformer T 2 through this bus. This is, these two transformers are step up transformers. And we have three transmission lines, working at 132 KV and a load of 200 MVA at 0.8 power factor lagging connected at this bus. Now, let us see how we convert this in per unit system. The impedance values, here for the transmission lines are given in ohms.

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Determine the per unit impedance of the load for the following cases:

- Load modeled as a series combination of resistance and inductance.
- Load modeled as a parallel combination of resistance and inductance.

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So, what we have to do is determine the per unit impedance of the load for the following cases. Load model has series combination of resistance and inductance. That is we can put this load as a series combination. Or we can put this load as a parallel combination of resistance and inductance.

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**Solution:** All the data presented are in per unit on a base specified for each component. In our analysis, we will convert all these quantities to a common system base specified in the problem.

Base KV in the transmission line = 132 KV

Base KV in the generator circuit  $G_1$   
 $= 132 \times 11/132 = 11$  KV

Base KV in the generator circuit  $G_2$   
 $= 132 \times 13.8/161 = 11.31$  KV

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So, what we do in solving this problem. First all the data represented are in per unit on a base specified for each component. So, the base values for the reactance's are generators transformers, all of them are given on their own rating. So, in our analysis we will

convert all these quantities to a common system base specified in the problem, which is 100 MVA. Now, base KV in the transmission line part is 132 KV.

Base KV in the generator circuit G 1 will be 132 in to 11 by 132, which is the voltage transformation ratio for the transformer. So, it will be 11 KV on the generator side. Similarly, base KV in the generator circuit G 2 is going to be 132 into 13.8 divided by 161, this is equal to 11.31 KV. So, base values on two sides of the transformers are to be chosen based on the voltage ratio of the transformer or the nominal ratio of the transformer or the turns ratio of the transformer.

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We now convert all the parameter values to per unit on the common base specified.

$G_1: X = 0.15 \times 100/100 \times (11/11)^2 = 0.15 \text{ p.u.}$   
 $G_2: X = 0.20 \times 100/200 \times (13.8/11.31)^2 = 0.1489 \text{ p.u.}$   
 $T_1: X = 0.1 \times 100/120 \times (11/11)^2 = 0.0833 \text{ p.u.}$   
 $T_2: X = 0.1 \times 100/250 \times (13.8/11.31)^2 = 0.05955 \text{ p.u.}$

The base impedance in the transmission-line circuit is

$(132)^2/100 = 174.24 \text{ ohms}$

NPTEL A.K. Sinha 29/35

Now, we convert all the parameter values to per unit on the common base specified. So, G 1 the value given is X is equal to 0.15. Now, we are converting it to our new base. So, S base old by S base new is both are 100. Similarly, the voltage also base old and base new voltage are also same. So, the value of the impedance remains same. So, using the same relationship, we find out the value of reactance for G 2 which comes out to be 0.1489 instead of 0.2

Similarly, when we convert it for the transformers we get 41 the value on 100 MVA base, as 0.0833 per unit. And for T 2 we get 0.05955 per unit. The base impedance of the transmission line circuit is going to be 132 KV square by 100, because the voltage base is 132 KV for the transmission line. And the MVA base for the whole system is chosen as 100. Therefore, the base impedance is KV square, base KV square divided by base

MVA, which is equal to 174.24 ohms. So, we know the Z base for the transmission line circuit as 174.24 ohms.

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$$Z_{\text{trans-line}} = (50 + j200) / 174.24$$
$$= 0.2870 + j 1.1478 \text{ p.u.}$$

The per unit impedance of the transmission lines connecting the load bus to the high voltage buses is given as

$$Z = (25 + j 100) / 174.24 \text{ p.u.}$$
$$= 0.1435 + j 0.5739 \text{ p.u.}$$

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Therefore, we can find out for each transmission line, the per unit value of the impedance. So, for the transmission line having impedance 50 plus j 200, the per unit value comes out to be 0.2870 plus j 1.1478 per unit. Similarly, for the transmission line having impedance value of 25 plus j 10, the value comes out to be 0.1435 plus j 0.5739 per unit.

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The base impedance in the load circuit is same as the base impedance in the transmission line circuit.

Also, the load is specified as

$$250 \times (0.8 + j 0.6) = 200 + j 150$$

We now look for the two different types of combination for the load resistance and reactance.

NPTEL A.K. Sinha 31/35

Also we have the load is specified as 250 MVA at 0.8 per factor lagging. So, that can be converted in to 200 megawatt plus j 150 means, 150 megawatts. We now look for different types of combinations of load resistance and reactance. That is we are asked to find out the series combination as well shunt combination, for the impedance of this load.

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i. Series connection:

$$Z_{load}^* = \frac{(132)^2}{(200 + j 150)}$$

$$= 55.7568 - j 41.8176$$

$$Z_{load} = 55.7568 + j 41.8176 \text{ ohms}$$

$$Z_{load,pu} = \frac{(55.7568 + j 41.8176)}{174.24}$$

$$= 0.32 + j 0.24 \text{ p.u.}$$

NPTEL A.K. Sinha 32/35

So, for series combination what we have is, we find out the impedance we know that  $S$  is equal to  $V I$  conjugate. So,  $I$  conjugate if we take, therefore  $I$  conjugate will give you  $V$  by  $S$ . And therefore, if we take  $V$  square by  $S$  we get  $Z$  conjugate.  $Z$  load conjugate comes out to be, because 132 KV is the voltage level at that. And this is the load, therefore the load impedance turns out to be 55.7568 minus  $j$  41.8176. This is  $Z$  load conjugate the star is indicating, this is conjugate value.

Therefore, the actual value of the load is going to be 55.7568 plus  $j$  41.8176 ohms. This is going to be there, because this has to be a reactive load or a inductive load. So, this plus must be there, this cannot be a negative, because we have load which is having a lagging power factor. So,  $Z$  load per unit will again we just dividing it by  $Z$  base comes out to be 0.32 plus  $j$  0.24 per unit.

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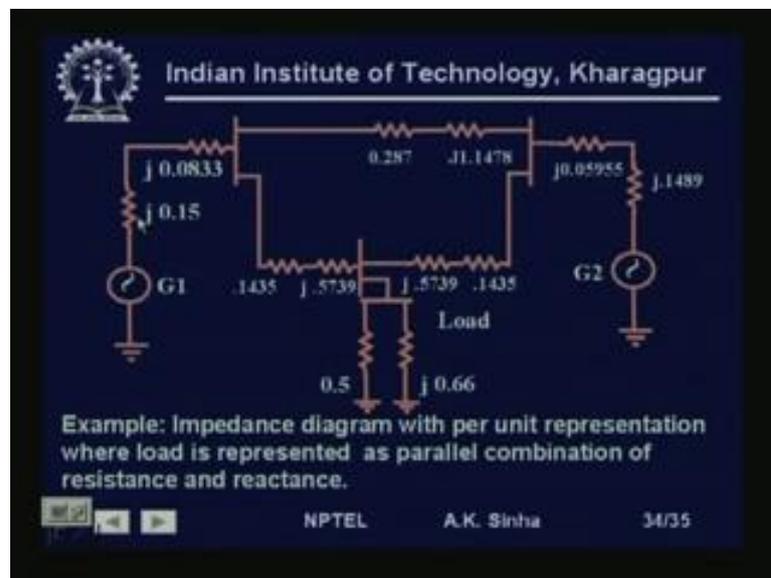
ii. Parallel connection:

$$R_{\text{load}} = (132)^2/200 = 87.12 \text{ ohms}$$
$$= 87.12/174.24 = 0.5 \text{ p.u.}$$
$$X_{\text{load}} = (132)^2/150 = 116.16 \text{ ohms}$$
$$= 116.16/174.24 = 0.66 \text{ p.u.}$$

NPTEL A.K. Sinha 33/35

If we take the parallel combination, again we can find out the value of resistance part, which is KV square divided by P. That comes out to be 87.12 ohms which in per unit comes out to be 0.5 per unit. The reactive part can also be found in the same way V square by Q, which is coming out to be 116.16 ohms which comes out to be 0.66 unit.

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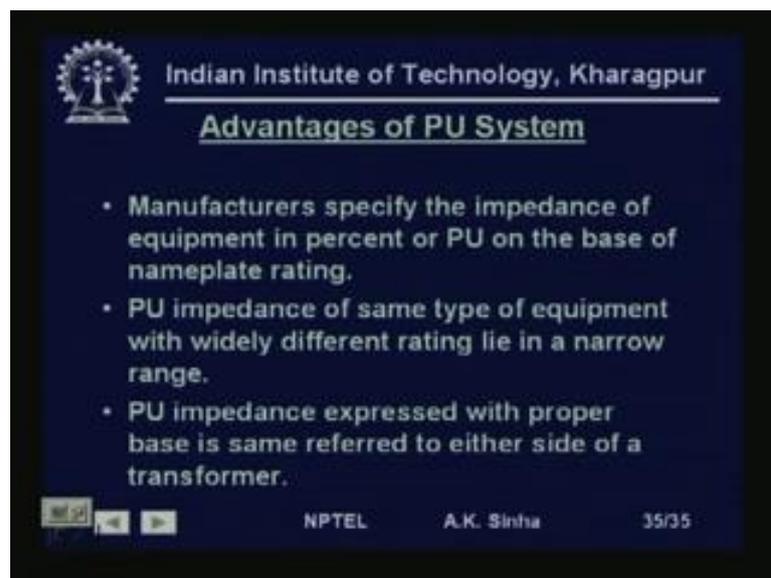


And therefore, if we put all these impedance values on the single line diagram. Then we get this network diagram. The generator with its impedance at j is equal to 0.15 generator G 1. Then we have the transformer impedance, then we have the transmission line

impedance. This is the other transmission line, this is the other transmission line. And this is the transformer impedance, this is the generator impedance.

All these are in per unit on 100 MVA base, and the voltage bases are respective bases on the two sides of the transform. So, using this all these values, this transformers are no longer existing in the system except for the leakage impedance part. The load here is shown as the parallel combination, similarly a series combination can also be shown.

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Now, we will see what are the advantages? After we have solved this problem. We will can see what are the advantages of this per unit system? One is manufacturers specify the impedance of equipment, in percent or per unit on the base of the nameplate rating. That is what we were given is for a generator at 100 MVA base and it is rated KV the reactance was given. Similarly, for generator G 2 at 200 MVA and it is rated voltage the reactance was given.

We had to convert them to a single base for our system. But, most of the manufacturers will provide you the impedance of the equipment in per unit, or percent value. If you multiply per unit by hundred you get the percent value. So, this is one thing which we get. So, we know the manufacturers will provide in this form. This per unit impedance of the same type of equipment with widely different rating, lie in a narrow range.

That means, for generators, if you see for a thermal generator. The value of the impedance or the synchronous impedance will be of the order of 0.8, or 0.9 per unit. For most of the generators, whatever may be their rating, it may be a 10 megawatt generator,

it may be a 500 megawatt generator. But, the value is going to lie between say 0.5 to 1 per unit or 1.2 per unit in that small range, whereas the actual value will vary considerably from a few ohms to milliohms.

So, this is a great advantage that even if we do not know the exact value while designing a system. We can choose an average value of per unit impedance, for the given type of equipment. And do all our design calculations based on that, before we really procure the equipment. Another great advantage of per unit system is, if we choose the voltage bases on two sides of the transformers as the turns ratio of the transformer or the voltage ratio of the transformers on the two sides. Then the transformation ratio is no longer there, that is the transformer except for its leakage impedance does not exist.

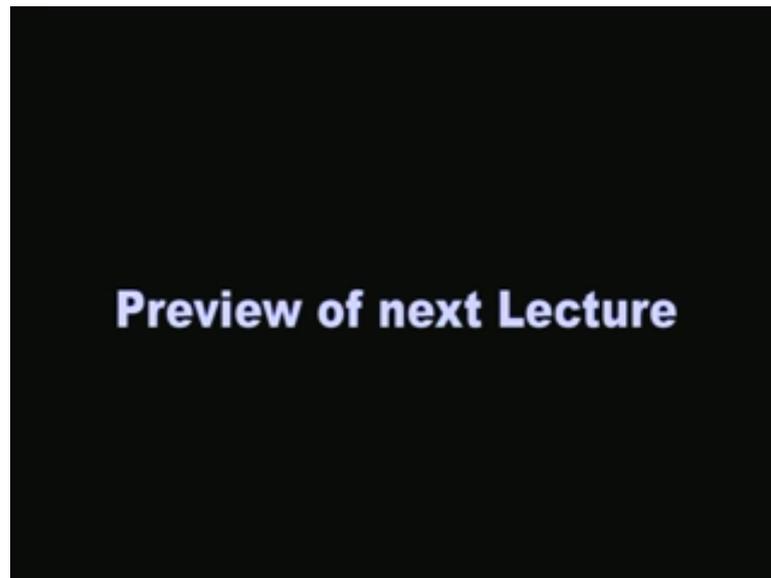
That is transformers or it is voltage transformation or the impedance transformation of the transformer is completely eliminated, when we work with per unit system. And this greatly enhances the computational simplicity for large networks. That is our computational effort reduces consequently. Anyway when you work with this per unit system and you get all the values in per unit. Finally, you need to use values which are the actual values.

And therefore, what you need to do is, the final values after doing all the calculations. And getting all the values of voltages currents, impedances in per unit, you will finally, have to convert them to actual values by multiplying them, by the respective base lines. So, once you do that you will get the final values. So, when we are doing this analysis for power system, what we do is? First convert the actual values to the per unit values.

And then do all the calculations on per unit values. And final results, we again convert them to actual values to see the actual values of currents and voltages and then impedances. So, this is how we work for power system analysis using the per unit system.

Thank you.

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Welcome to lecture 3 in Power System Analysis. In this lesson, we will talk about transmission line parameters.

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Questions from Lesson 2

1. What is a Single Line Diagram ?

- Single Line Diagrams (SLD) are concise way of communicating the basic arrangement of power system components.
- SLDs use a single line to represent all three phases.
- They show the relative electrical connections of generators, transformers, transmission and distribution lines, loads, circuit breakers, etc., used in assembling the power system.

NPTEL A.K. Sinha 2/37

Before I go in to the transmission line parameters itself. We will first talk about the questions that I asked at the end of lesson 2. Well, the first question was what is a single line diagram? Well, the answer to that question is single line diagrams are concise way for communicating, the basic arrangement of power system components. Single line diagrams use a single line to represent all the 3 phases. And they show the relative electrical connections of various electrical components, which are used in assembling the power system.

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2. How base values of voltages are chosen on two sides of a transformer ?

- Ratio of  $V_b$  on either side of a transformer is selected to be same as the ratio of transformer voltage ratings

NPTEL A.K. Sinha 3/37

The second question was, how base values of voltages are chosen on two sides of a transformer? Well, the answer to this question is, ratio of base values on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings. Well, this is necessary because, when we use this voltage bases on the two sides as the voltage ratio of the transformer. Then, we find that the per unit impedance on the two sides of the transformer are same. And therefore, it eliminates the use of the ideal transformer in the power systems. This helps considerably in reducing the calculations for the power system circuits.

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3. For a 500 Mw, 22 kV generator the per unit impedance is 0.8 pu on its own base. What is its pu impedance at 100 Mw and 33 kV base ?

$$Z_{p.u.new} = Z_{p.u.old} \left( \frac{V_{bold}}{V_{bnew}} \right)^2 \left( \frac{S_{bnew}}{S_{bold}} \right)$$

$$Z_{p.u.new} = 0.8 \left( \frac{22}{33} \right)^2 \left( \frac{100}{500} \right) = 0.0711$$

NPTEL A.K. Sinha 4/37

And the third question was for a 500 megawatt 22 KV generator, the per unit impedance is 0.8 per unit on it is own base. What is it is per unit impedance at 100 megawatt and 33 KV base? Well, as we had seen in lesson 2, the Z per unit on the new base is given as Z per unit under old base multiplied by old V base divided by new V base square multiplied by new MVA base divided by old MVA base. So, using this relationship, once we substitute these values we will get Z per unit at the new base of 100 megawatt. And 33 KV as 0.8 into 22 by 33 whole square multiplied by 100 by 500, which will come out to be equal to 0.0711 per unit.

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Next we will talk about the electrical parameters. Now, we are using conductors, overhead conductors. And these conductors has certain amount of resistance to the current flowing through them. So, depending on this resistance, there is going to be power loss in these conductors. Therefore, we must find out what is the resistance of the conductor

Then, we will also talk about inductance this comes. Because, once the current flows in the conductor it sets up magnetic field because of which there is certain amount of inductance. Since, these conductors are at high voltage. There is a voltage difference between the two phase conductors. As well between phase conductors and ground. Therefore, there is capacitance involved between them. So, we will also talk about capacitance. So, these three parameters are the most important parameters for a transmission line, electrical modeling.

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### Resistance

The dc resistance of a conductor at a specified temperature T is:

$$R_{dc,T} = \frac{\rho^T l}{A} \Omega$$

Where  $\rho^T$  = conductor resistivity at temperature T  
l = conductor length (m)  
A = conductor cross-sectional area (m<sup>2</sup>)

NPTEL A.K. Sinha 13/37

Well, as far as resistance is concerned, the dc resistance of a conductor at any temperature is given by  $R_{dc}$  is equal to  $\rho l$  by  $A$ , where  $\rho$  is a function of temperature. And that is the resistivity of the conductor changes with temperature. If the temperature goes up resistivity will also increase.  $L$  is the length of the conductor and  $A$  is the conductor cross sectional area. I would like you to answer the following questions.

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### Answer the following questions

- What type of conductors are used for overhead transmission lines ?
- How does conductor diameter affect inductance of transmission line ?
- Why bundled Conductors are used in Extra High Voltage lines ?

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First is what type of conductors are used for overhead transmission lines? Second question is how does conductor diameter affect inductance of transmission line? And the

third question is why bundle conductors are used in extra high voltage lines? So, I hope after going through this lesson, you should be able to answer these questions.

Thank you.