Dynamics of Physical System

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Lecture - 2
Newton's Method and Constraints

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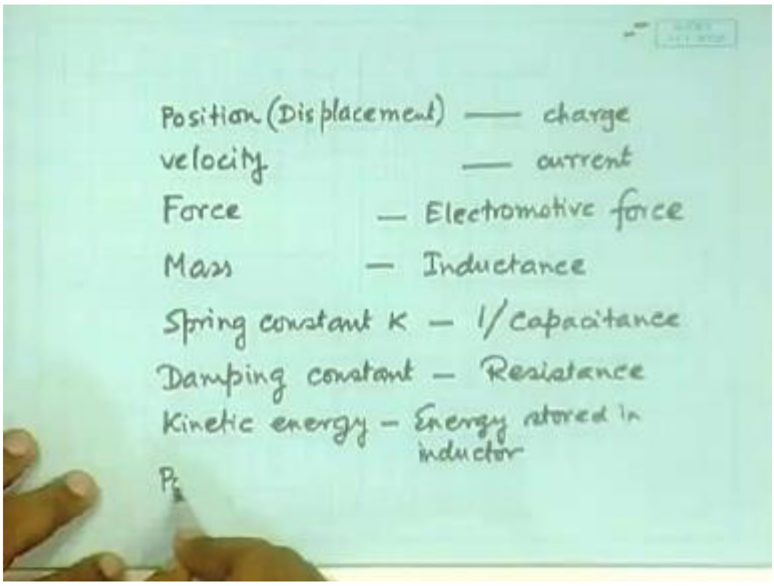

In the last class, we have seen that electrical quantities and mechanical quantities can be equivalanced, and let us start by just recapsulating where we were. We saw that in mechanical domain, the basic coordinate is position for displacement for from some the term position and it is electrical equivalent would be charge, the rate of change of position is velocity. So, velocity is equivalent to the rate of change of charge which is current, then acceleration is rate of change of current DIDT, force is equivalent to electromotive force. What else, mass is equivalent to inductance, in the sense that mass has the property of resisting change in it is inertial status, inductance has the property of resisting any change in current.

Then you have spring constant K, that is equivalent to 1 by c 1 upon capacitance good, they damping constant are frictional coefficient is equal to resistance. Finally, the two things I forgot to talk about in the last class, in mechanical domain there are two types of energy the kinetic energy and the potential energy. So, let see what they are equivalent to the kinetic energy is how much half m v square, half m, m is proportional to...

Student: m.

M is equivalent to l, v is proportional to…

Student: i.

I, so half l i square it becomes equivalent to what is half l i square, the energy stored in the inductor. So, kinetic energy is equivalent to the energy, so if in a circuit there are three inductors, the kinetic energy or equivalent of the kinetic energy, in that circuit would be the total energy stored in the three energies, potential energy then would be equivalent to the energy stored in the capacitors.

So, this sort of computes the story of the equivalences, it is not easy or common sense to see that the energy stored in the inductor is actually has the property of a kinetic energy, and the energy stored in the capacitor has a property of potential energy. But, it should you know that, that was essential content of the past class, last day's class. So let us start from there, we said that the Newton's method can be written simply as force is equal to mass into acceleration.

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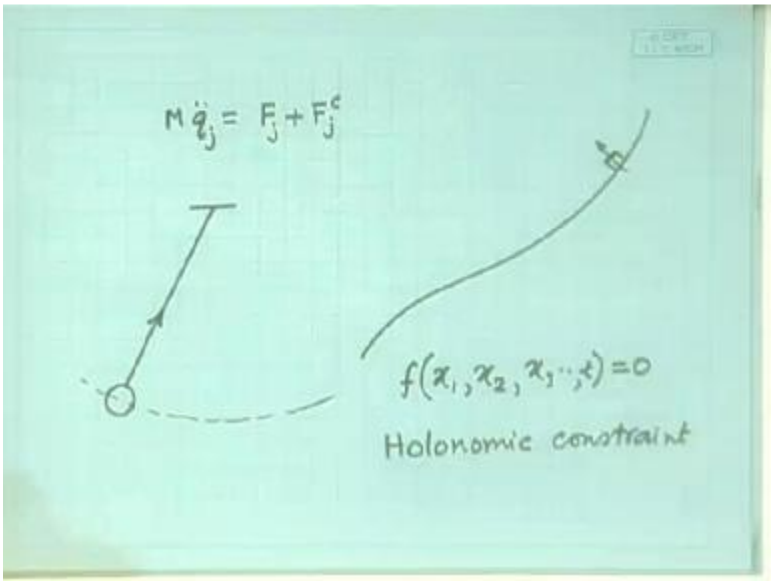

So, we will write it as the mass into acceleration in the left hand side, because that is the derivative part. So, M q double dot is mass into acceleration is equal to, equal to what whatever the force is, now this force we said has two components, one something that is impressed from outside, either it can be some externally applied force or it can be a force applied by the other elements of the system.

For example, if it is sliding against something two bodies are sliding against each other, there is one body experiences a force applied by the other body, which is also part of the system. For that body that this frictional force will have to be included, if this body is connected to another body by means of a spring, then because of the spring it the other body pulls with some force, so that is also included here.

So, all these forces are to be included, so this has to include the externally applied force, it has to include the forces between elements which are connected by springs. Forces between elements, which are connected or interacting, which with the help of frictional elements, also and which is crucial the constraint forces. So, let us just recapsulate what we said about the constraint forces, most of the motions of bodies are constant in some way.

As we showed the they example, that if you have a pendulum like this then it is bob is constraint to move on the surface on a sphere and that happens, because of the constraint applied a force on the body. So, this force then has to been included, if you have a surface like this and you release some kind of a body here and it slides down, why does it follow this particular equation, because it is acted on by a force that is the constraint force.

So, when you write the Newton in equation then, this constraint forces have to be included clear, that is why the Newton in equation we will have to written as plus F c this is the constraint forces. And if you have n number of such bodies, for each body then it will have to be written with the subscript, for each body you will have the equation like this for all the n bodies you will have the equation like that clear. Now, we need to say if you more things about the constraints.

Constraints can be of a few different kinds see these constraints, let us how can we mathematically express the character of this constraints. What kind of constraint is this, here we can express the constraint as some kind of a function, some kind of a eligible equation this one can we not. So, it was originally a three dimensional space and we can then expressed a equation which will restricted it to a lower dimensional space clear.

So, that requires one equation to be written what is the form of that equation, it will normal be of this form some kind of a f of if there are many such variables. So, you will write x 1, x 2, x 3 and so on so forth is equal to 0. In this case the position is given by x 1, x 2, x 3 and a specific equation relating this x 1, x 2, x 3 equal to 0 gives the constraint equation. In general this might also with this function might also defined on time, imagine, imagine that this it is moved imagine this is being moved, so in general this might also contain time.

Now, any constraint that can be expressed in this form is note as holonomic constraint, remember this one is called holonomic constraint. Some holonomic constraints do not need this term t that is it is independent of time, like this and some holonomic constraints will need additionally this concept of time, the dependence on time, but both are holonomic constraints. If you easily see that the holonomic constraints actually reduce the dimensionality of the system.

For example, here the bob is constrained to move on the surface of the sphere and therefore, we can as we will see we can define a new coordinate system on the surface of the sphere and that is actually what we do. For a pendulum it will be stupid to write in terms of x, y, z, it will far more logical to write in terms of theta and phi, spherical coordinate system why, because what we are actually doing is that we are writing in terms of a lesser, smaller number of coordinates, constrained to the constraint surface remember that.

So, holonomic constraints are actually our advantage, holonomic constraints are advantage we will have to learn how to take advantage of that in a systematic way. In case of the pendulum just by looking at it by applying our common sense, we saw that no, no we do not need to use x, y, z we can better use theta phi. But, that is observation there has to be some kind of systematic way of going about it that is what we will learn.

But, let us a natural question is can there be a non holonomic constraint a constraint, but non holonomic nature yes, that there can be something that cannot be expressed in this form. Imagine, in your hostel do you play carrom the what is are constrained, because they have remain within that boundary, but can you express their equation by means of like this, know still it is a constraint.

That kind of a constraint, which is expressible as an inequality not a equality, here is a equality right. But, a body constrained move within the boundaries of the carrom board is given by a inequality, this point that point, this point let to be 0 and that point let it be l. So, x coordinate is constraint within 0 and l it is inequality, that is the non holonomic constraint, billiard is a non holonomic constraint.

If you have a container and molecules moving within that, they are constraint by a known non holonomic constraint. Because, they have to be within that and even very simple situation can give rise to non holonomic constraint let me imagine, very interesting example.

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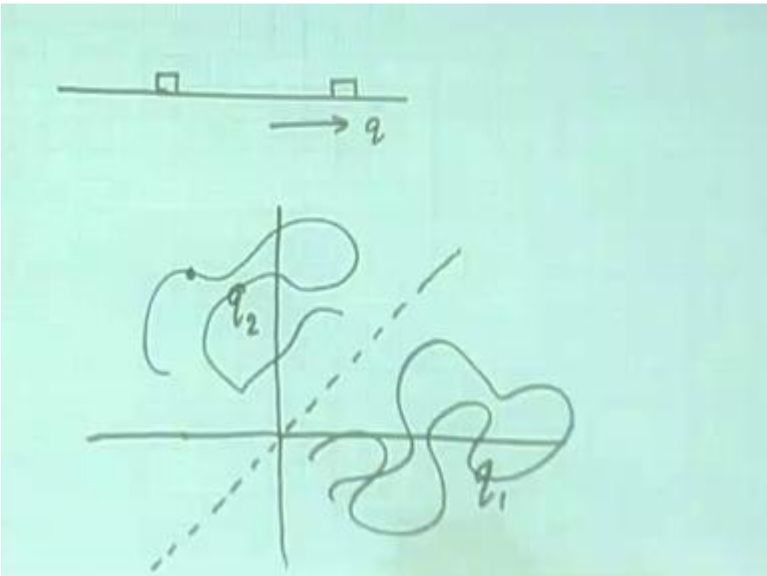

Suppose you have got a line and somebody that can move on this line, this is another body that can move on this line, one dimensional problem not a difficult thing at all. So, it is the q coordinate of this body and that body and they are free to move are their motion constraint by anything yes of course, they cannot occupy the same position, their body is they cannot occupy the same position.

Now, imagine if I plot there, what will be the configuration space in this case, configuration space we introduce that concept in the last class. The minimum number of coordinates position coordinates, that we need in order to express the complete position of status of this system, this system contains two bodies. So, you need two position coordinates and the configuration space will be then q 1 and q 2, q 1 is the position of this one and q 2 is the position of that one from something.

And then in this q 1, q 2 space these bodies will just move around their motion; that means, in the q 1, q 2 space the motion of these two will be given by some kind of a trajectory. Notice, I am not talk about the trajectory of this one given by some orbital placed over that one given by another I am talking about then in the configuration space the total positional status of this system consisting on these two bodies given by a point a point.

Now, that point can move around like this which would then mean, suppose it in here which would then mean that the position of this fellow q 1 is here minus something and position of that fellow q 2 is plus something. So, this is a configuration point is it clear, that here we are not talking about individual positions, but their collective positions, the position coordinates of the whole thing represented graphically in the configuration space.

Now, notice this line then becomes a forbidden line, this line means what q 1 and q 2 are the same 45 degree line, q 1 and q 2 are the same means that is the forbidden state they cannot take the same position, which means that in the configuration space, this configuration point can either wonder in this side or in this side, but cannot cross. So, it can go like this no problem fine, but it cannot cross, because crossing would mean that the at some point of time, the two occupy the same position.

That means, within the configuration space then the configure is the point is then constrained what kind of constraint is that, it is a inequality constant it cannot take this value, either it is here or here. So, it is also a non holonomic constraint fine, so the non holonomic constraints as you can see that does not reduce the system dimension holonomic constraints do. Unfortunately, most of the engineering systems have some kind of holonomic constraints, so it will be more important for us to understand or to be utilize the advantages of holonomic constraints.

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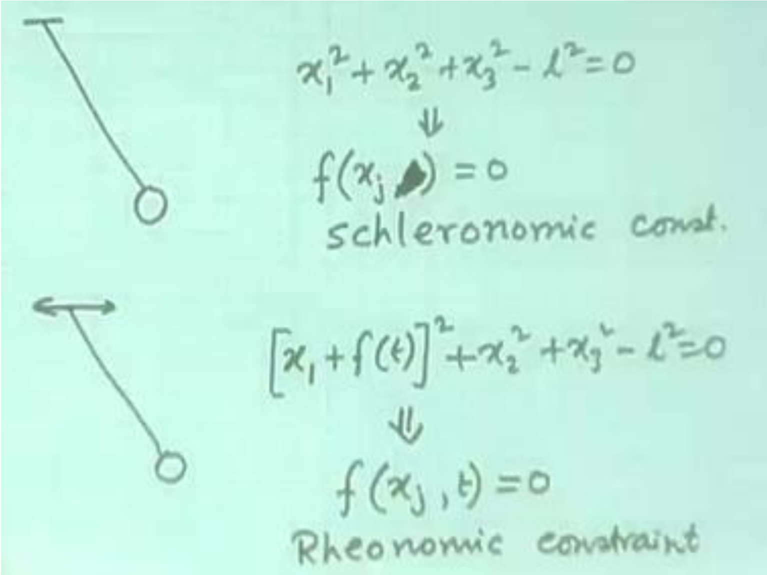

So, let us pay some more attention to that, first in case of the simple pendulum, if you express in terms of x, y, z, what is the holonomic constraint equation, it is constant to lie on the sphere how does we expressed as, it will be x 1 square plus x 2 square plus x 3 square is equal to x square assuming this to be the body origin, it will be trivial to move the origin to this point, because that would be a simple first step that anybody would do.

So, this minus l square is equal to 0 and we expressing as some function of x's equal to 0 in this point. Now, suppose this point is moved as a function of time then what, then also at every position suppose it is in here, then also from that position it will be a surface of a sphere again it moves here, again from that position it will be a ((Refer Time: 19:59)). So, essentially what is happening is that, this if it is moved like this, then also it is a constraint.

But, what kind of constraint can we express that, this is a x direction suppose in that case you will say that is x 1 plus that function of time, this square plus x 2 square plus x 3 square minus l square is equal to 0, that will be the constraint equation. Notice that, this is expressed as f x j comma t equal to 0, this is expressed not as this and this is expressed as f x j comma t is equal to 0. Actually they have some names you will find in textbooks that this has the name that has a name, this name is schleronomic constraint and this name is do not be scared by the names I will not shoot you if you forgot this names.

But, still why I am writing it because, in books if you come across this words you should know what they are, then not fruit's, hanging in the tree is that you can pluck and eat, they are specific names or specific type of constraints you should know that is all. So, if you come across these words you should be able to say that these are specific types of constraints. But, you will forget if this type what is the name, there is no problem about it because everybody cannot remember everything I know.

So, these terms were coined these are no coming from Latin and still these are the terms used. I will go by assuming that you will forget these names and I am not take that as a big thing seen, whatever it is the point is these are the names when come across them in the text books if you know what they are. So, we have seen that there are two types of constraints in the main, holonomic constraints and non holonomic constraints.

Holonomic constraints offer the advantage that you can if you are clever enough, reduce the system dimension in case of non holonomic constraint you can do that. So, if you have non holonomic constraints you are force to use the whole set of coordinates for each body 3 coordinates x, y, z. So, normally you if there are 3 bodies, then how many coordinates will be there 9 coordinates, if it is constant by non holonomic constraint, you will have use all the 9 coordinates.

But, if there is one holonomic constraint, then the system dimension reduces by 1, if there are 2 holonomic constraints, the system dimension reduces by 2, if there are 3 holonomic constraints, so goes on reducing. Imagine here you have the equation as this that is one equation, that makes the dimension 1 less from 3 to 2, but imagine that if it is a planar pendulum, then there is one more constraint equation that is say y is equal to 0 or x 2 is equal to 0.

Say x 2 is equal to 0 is what is a constraint equation then, it is also in this form x 2 is equal to 0 geometrically what are you doing, here this equation one constraint equation defines some kind of a sub space. The second constraint equation give us another sub space and then the ultimate constraint is what, the intersection between the two. Since, it is the intersection between the two, then the system dimension reduces by the number of holonomic constraints.

So, the system dimension reduces by a number of holonomic constraints and that; obviously, is a major advantage. Now, the point is that in the classical Newton method there is no systematic way of utilizing this advantage, let me illustrate by doing it for this simple pendulum, if you write the equation for the simple pendulum by the Newtonian way x, y, z do it.

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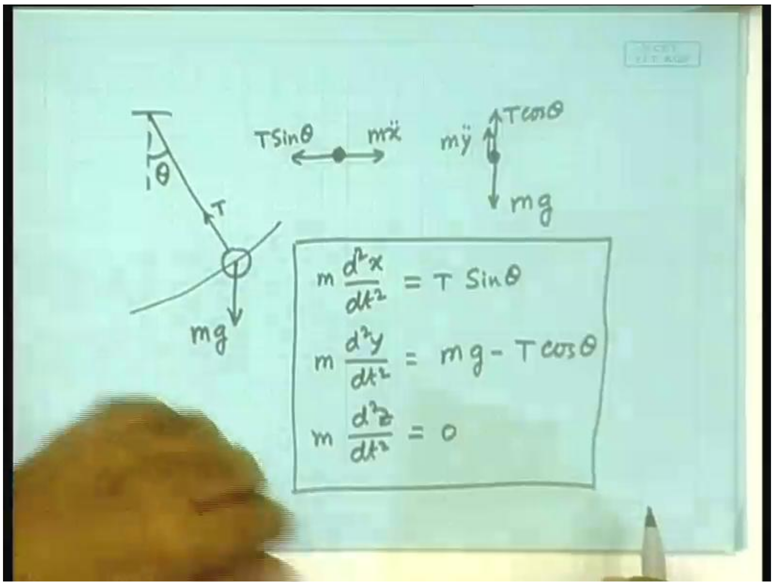

Write the equation you have the simple pendulum, here we will take the theta, here there would be a tension working t, here there would be m g working. So, x coordinate, y coordinate and z coordinate, if we draw the free body diagram these were as shown, the D' Alembert's way then we will have to do two free body diagrams along the x coordinate along the y coordinate.

Along the x coordinate what will be the equation, in this direction what is the pull theta is here T sin theta T sin theta, that will be counted balanced by mass into acceleration in that direction m x double dot. So, that is the free body diagram in the x direction, in the y direction here is a pull working that is T cos theta, so T cos theta, in this direction m g. So, you have the m g minus T cos theta working and naturally there will be a component of the acceleration mass into acceleration m y double dot, that will be the free body diagram simply equation, you have the equations given.

So, m d 2 x d t 2 is equal to T sin theta m d 2 y d t 2 is equal to m g minus T cos theta and z direction m d 2 z d t 2 is equal to 0, there is nothing working in the direction. So, the Newtonian state of equations are like this, how many equations did you need 3, how many coordinates did you need 3 and also T becomes included in these equations, do know how much is you do not know really and as it swings the T varies all time.

So, here is something that is very difficult to know, but it has gone into the formulation, so, that is the problem of the Newtonian method that the constraint force gets into the formulation. And in case of a situation like this will you able to write down what the constraint forces very difficult, so it was realized soon after Newton that something has to be done about the constraint forces, we cannot write down the equations properly or you can really solve the equations, if the constraint force is get it to the system equations that is one difficulty.

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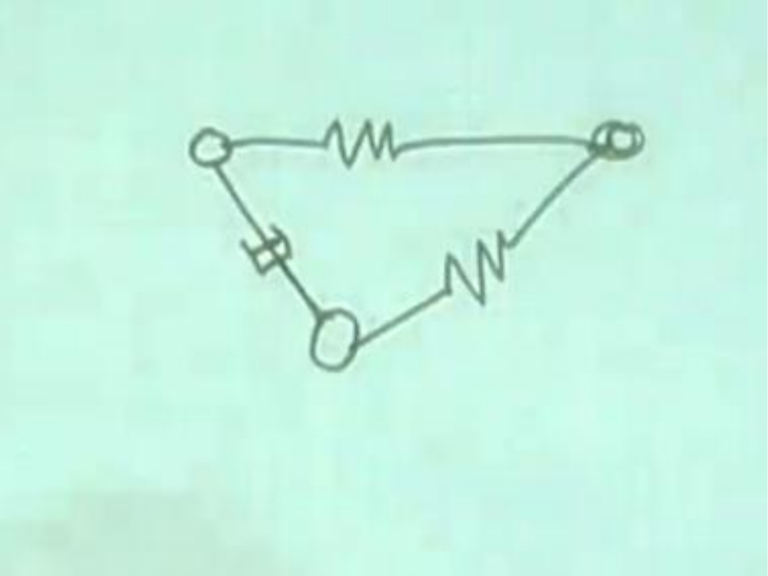

Second difficulty is that if you have say many bodies interacting by means of something like. So, this is a frictional element or the another spring element, so like this, then all these mutual interactions should be consider, on this body not only the force due to gravity. But, also all the forces to be consider and they are vectors, can you see even if the system is, so simple only three bodies with this write it down the differential equation would be mesh.

Because, all these have to be written in vectorial terms, all these will have different directions. And then the Newton's equation itself will be a vectorial equation will be tough and I am sure if I give you this problem in your exam you will fail that is why, we need to do something about these forces. What are these forces, ((Refer Time: 30:25)) out of these there was a problem with this one, out of this also a problem with this one, because these are all vector forces.

Something needs to be done do that something needs to simplify that and thirdly the Newtonian technique does not offer any direct way of reducing the system dimensions. These three, where recognized as a major practical difficulties with direct application of the Newton’s method true, Newton gave the basic idea of writing the differential equation.

And after we had the Newtonian approach that is when we started being able to solve the equation for the mass for the example, it was possible to write down the differential equation for the motion of the marsh, calculate the initial condition solve it and then predict where it will be it actually was that. So, all these major advantages came because of the Newton’s law, but these were the practical difficulties, we ultimately need to write down the differential equations.

So, the within a 100 years after Newton the solution of this practical problems came and they are very valuable when we try to write down the differential equations for engineering systems, so let us try to learn how this practical difficult to where the work, that will need a bit of oriented to mathematics. So, do not gets ((Refer Time: 31:53)) scared about that most of you are more or less conversant with math's, in fact at home with math's I will say, so do not get scare about that.

So, let us recognize the first problem, the first problem was that there were constraint forces that were troublesome. Here, we have to write down the equations using the constraint forces and that is a mesh, if there are many constraint forces then it will be even more mesh. About a century after Newton people like Lagrange, D' Alembert, Hamilton they proved this technique.

Then noticed, that there is a specialty of the constraint forces, they do more work have you noticed. ((Refer Time: 32:57)) The constraint force is in this direction and this fellow is moving in this direction perpendicular direction and the constraint force is doing no work. So, instead of writing the equations in terms of forces, if I write the equation in terms of the work done, we are true we can straight away get read out the constraint forces can be...

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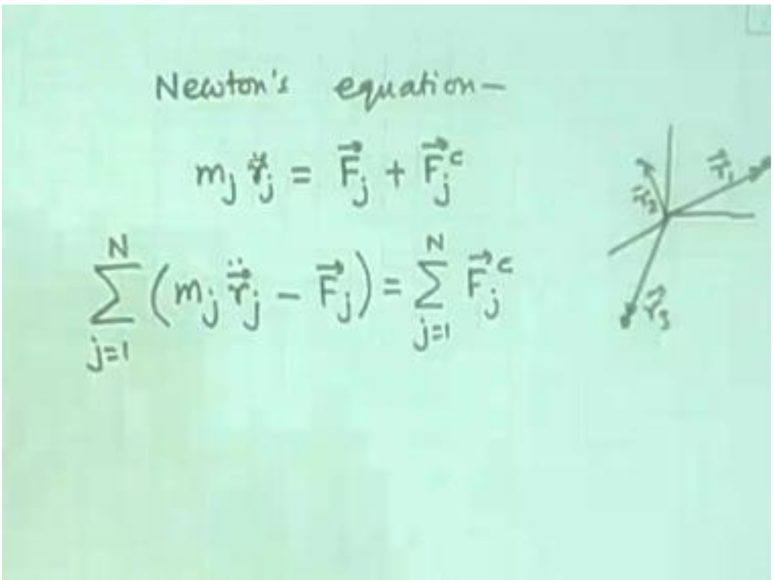

So, the next stage was to write down the equations in terms of the work, work done by the constraint forces will be 0. Now, that give rise to a very solid way of write it down the equations, but let us first start with the Newton's laws by rewriting the Newton's laws. The Newton's equation would be m for each mass point from a coordinate system, every mass point will have a r, a vector another vector and r, so r 1, r 2 third mass point r 3 and all that.

So, for each one m j, r j double dot this r j is the vector how to I normally in print we mark that by a bold space. But, let us put a small arrow over right to mean that it is a vector is equal to F j which is the vector plus F c j, for each mass point we will have a equation like this. And then, we can add them up to obtain equations of this form I will put this one here to keep the right hand side only with the constraint force, so that we can eliminate that later we will write it as add them up all the equations.

So, sigma j is equal to 1 to say N, N number of bodies m j r double dot j this term minus this minus F j this is equal to the total number of constraint forces add it together. So, that is the Newton's law and we are now trying to get read of this diagram, we have just mention that the constraint force now does not work is it always true I will come back to this.

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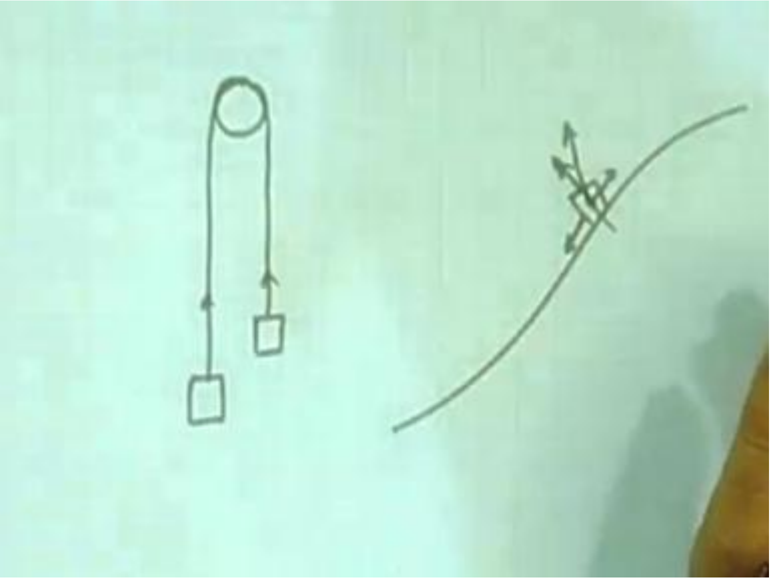

Imagine this situation, where is the constraint force, the tension and the pulling does it do know what know it does. For this one if it is moves by q amount, then it does work and this fellow also it work, but then overall taking the system into concentration we cancel off can you see that. So, it is not always true that the constraint force does not do any work, but overall it will sum up to 0 and that is why we needed it to sum up, we need the sum, because of that we get need to get read of the constraint forces.

So, the constraint force in some situations may do some work, but ultimately when summed over all the bodies in the system that gives it just one situation. Suppose, a body is sliding down the surface, the constraint force is acting like that it is always sliding down in a direction that is orthogonal to the constraint surface, constraint force and therefore, you are happy that there is no work done by it or suppose there is a friction. Then, the constraint force will not act in the orthogonal direction, but whether it will act in a that kind of a direction.

In that case what, you might argue that now the constraint force is doing work, yes and that is why in such situations we will break up the constraint force into a component orthogonal and a component in the direction of the motion. And the one in the direction of motion is caused by the friction as we will include it in this term, because it caused by friction it is not really a constraint force, the constraint force is the one that is orthogonal to it and it does work clear.

So, even if this body is applying a force on this body which is in that direction, we will break it up into the constraint force part and the non constraint force part and the non constraint force part was the one that is due to friction will be included as in the given forces ((Refer Time: 38:42)). So, we are still happy constraint forces it will do no work, there are situations for example, where is that like this it is by moved ((Refer Time: 38:58)) is it now true that the constraint force is not doing any work.

Imagine carefully, here the point of suspension is being moved like this and the fellow is, is it always true that the fellow is moving in a direction that is orthogonal to the string, why not be. Then, what then that is the mathematical nice city that Lagrange thought off, he is saying that at any specific position of my such point of suspension, at any specific position this bob has some admissible motion. That means, if it is like this it can only move in that direction which is orthogonal, that is the admissible motion, admissible displacement.

So, in a given position this bob always knows always has a specific admissible direction, admissible motion. So, even if it actually motion moves in a direction that is not always orthogonal, but it has a specific admissible motion and that admissible motion always is orthogonal at every point of time is orthogonal to the constant force. So, he said take a camera and freeze it, if you freeze it at every point of time you will see that individual at every point of time the admissible motion is orthogonal to the constraint.

So, take freeze shot at every moment of time is that it is happening, so when we talk of this kind of admissible motion. That means, at every point what is the direction it could move, that is admissible motion whether it does move or not that is the different issue, but it could move in that direction and that is something that is called admissible displacement, in some books you will find the word for virtual displacement, but virtual often give rise to some confusion among this rule, so I prepared to use the term admissible displacement

So, what is the technical meaning that at any specific point of time given a specific constraint what is the possible direction of it is motion, so that is the admissible motion admissible displacement.

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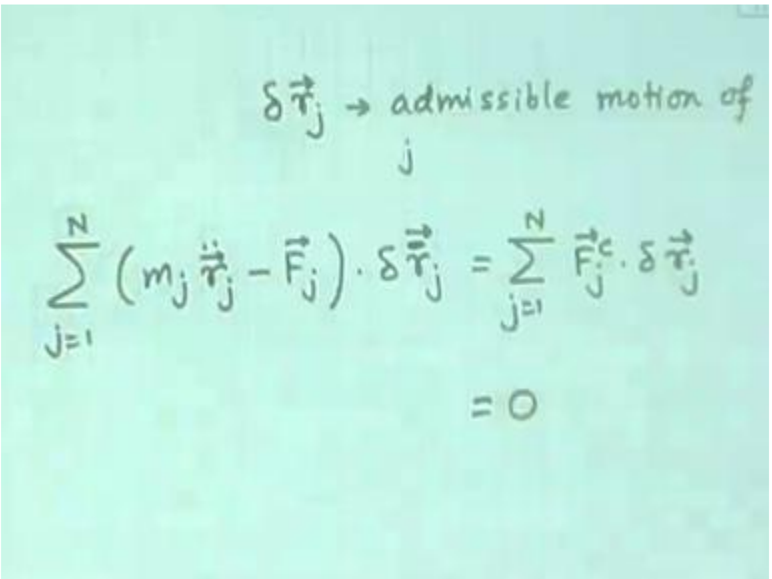

And that is given by the symbol delta, so if r is the position coordinate then delta r is it is admissible motion for the j body delta r j. So, this is...

Student: ((Refer Time: 42:18))

No, yes, but admissible motion is related to instantaneous position, so if this fellow is moving the ((Refer Time: 42:34)) also moving that is, but then fix it at any point of time here is a string and therefore, this is the direction of the constraint force. In this position in which direction can the bob move, bob can always move in orthogonal direction.

Student: ((Refer Time: 42:53))

Yes, yes that is not the actual case, but every point of time that is true, every point of time if you freeze it, it can only move in that direction. Again at some other point of time let it move again freeze it, it is here, but it can only move in the direction that is orthogonal to the direction of the string.

Student: ((Refer Time: 43:21))

Yes it is moved.

Student: Yes.

Yes it is move true that is why it is a conceptual nice city it is not immediately visible, he says that at every point of time take time instance separately fix the shot. Then, look at the bob, look at the string and say which direction could it move, then you will find that at the moment it can only move in the direction that is orthogonal. So, even though it is not moving in the orthogonal direction from this concept of nice city you see that at every moment of time, individually take a move by moment it is always moving in the direction that is orthogonal.

So; that means, this delta r j has to be taken at every instant of time and that is not same for this instant and that instant, it is different where was it here. ((Refer Time: 44:21)) So, we where is here from the Newton's equation in order to talk in terms of the work done, what you have to do, you have to this is the force, that is the force just multiplied by the admissible displacement that is what.

So, multiply both sides by the admissible displacement what you get is times this is the admissible displacement is equal to by delta this delta r j. So, you are multiplying it by just the delta r j which is the admissible displacement, here is the force, here is the displacement and therefore, this term is what, this term is also work. But, the right hand side is 0, because here we have multiplied the for the rth body, the constraint force times the admissible displacement that is always 0.

So, this in the right hand side is 0, we have completely got read of the constraint forces from the formulation. You might argue with that we still have a troublesome quantity, here we have got read of this, but we have brought in this admissible displacement let us we have do something about it, yes we will do something this about it sure. But, then we have we have been able to get read of the more troubles some fellow and then we will do something about this.

Now, do you have time yes another 10 minutes, let us get to the next conceptual stage, keep this equation written in your copy. This left hand side equal to 0 that is what we are derived.

Student: ((Refer Time: 46:46))

No we have added up for each one, it is 0 n number of 0's added up gives 0.

Student: ((Refer Time: 47:02))

Yes, so for each one it is 0 and for n number of 0s added together you get 0, so in the right hand side you have 0. But, then remember it was a conceptual quantity it is something that is not actual motion, remember this is not actual motion, this is the virtual or in books you will find virtual displacement I prefer to call it admissible displacement, you will find in some books also admissible displacement, so at every point of time this is the admissible displacement.

Now, what was the next problem, the next problem was that holonomic constraints offered you an advantage that can reduce the number of equations, but there was no systematic direct way of doing that when it comes to the Newton's law. So, we needed to do something about it, now if say normally the configuration space will be some trice n dimensional, but in order to facilitate our you know visual concept.

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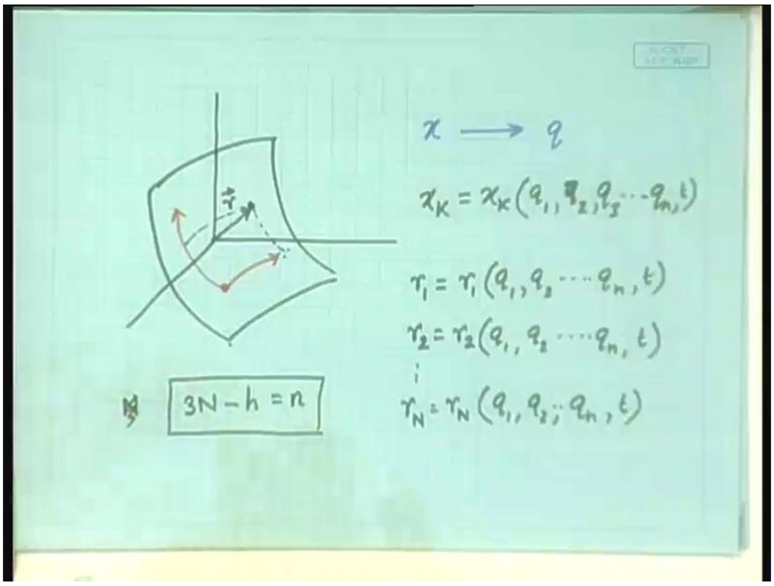


Let us draw a three dimensional configurational space and the constraint equation gives what a surface in this three dimensional space clear it will gives a surface. For example, the motion of the pendulum gave a spherical surface on which it have move, similarly for a different situations it will be different, but always it will be a surface. Because, a surface is what has the n minus 1 dimension, so it will be a surface suppose it has a surface like this can you see a surface like this, there is no reason to for it to be a straight surface or something like that.

But, then the motion in the configuration space is always constant to the surface, therefore the next conceptual step is to define a new coordinates system in this smaller surface, smaller dimensional surface what will you look like, it will look like we will say on this let this be origin and let this be the one coordinate and let that be the other coordinate. And then, suppose in the actual position is somewhere here, actual position somewhere here, we will say fine since this actual position is always on the constraint surface, we can resolve it into components in this directions.

And therefore, it has this much of this new coordinate and that much of this new coordinate, that defines the position clear. So, we are now taking the next conceptual step of going from one set off coordinates to another setup off coordinates in a smaller number of coordinates clear. So, initially you have the x 1, x 2, x 3 coordinates, but now we are going into a smaller number of coordinates let that we call q 1, q 2, q 3, fine.

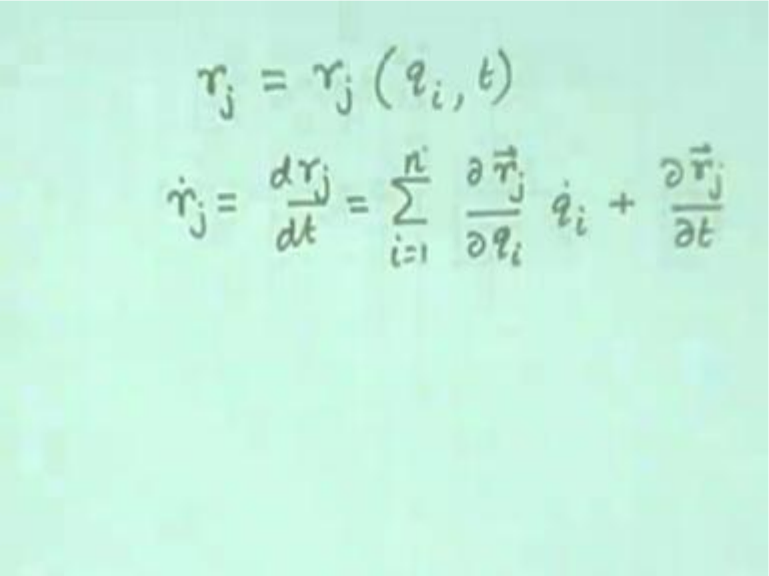
So, the number of q's would be that many less than the number of x's as the number of holonomic constraints, we have established that. So, we are going from the x coordinate system to the q coordinate system fine and that would be given by some kind of a transformation equation, where the x k can be expressed as functions of q 1, q 2 q 3 to q n and possibly also time. So, that is the coordinate transformation equation, imagine that in case of a equation ((Refer Time: 51:56)).

Like this in this case the new set off coordinates q's are the theta and phi, because that uniquely specifies the position on the constraint surface, original set off coordinates was x 1, x 2, x 3 x, y, z. Now, here we are trying to express x 1 in terms of theta and phi, x 2 in terms of theta and phi, x 3 in terms of theta and phi and that is what have written here, in general it will be as possible in this form. And since, each of the bodies are given by some kind of r coordinate r we are writing in terms of r the radial vector.

So, this will be thrice end time, but we can simplify it by writing an equations r, r 1 will be then a similar function of q 1, q 2, q n, t. Similarly, r 2 the position of the second body will be and to r N, Nth body I want to understand what we are doing, so each body was given in the old coordinate system by an r now, this r is expressed in terms of the new coordinate system q 1, q 2, q 3 and may be it can also be function of time, if it is a holonomic constraint there were the surface is also time variable, then it will be a function of time.

So, in general I have written like that, but that might be difficult to conceptualized, so you might say that r 1 is expressed as a function of the new coordinates q 1, q 2, q 3, and q n.

(Refer Slide Time: 54:33)



Then, we have this equation in general in short we can write r j is equal to r j q i comma t simple, short expression of what we have written ((Refer Time: 54:47)). The jth radial vector from the origin to the jth mass point is expressed in terms of the new coordinate system and time. If it that is, so then how do you express r dot j, because r dot j was representing the velocity we need to convert the velocities also.

So, r dot j will be d r j d t, then it has to be written as a chain rule it would be sum of i is equal to 1 to the number of n is the let me write somewhere, ((Refer Time:)) the number of bodies were capital N. So, number of configuration coordinates was thrice N, number of holonomic constraints where h this is small n, the number of new coordinates that we need. And that is what we are writing in terms of that 1 to small n and then it will be partially derivative of r j delta q i q dot i plus delta r j chain rule fine, we will stop here and then we will continue with this the next class.