Digital Signal Processing
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Lecture - 3
Discrete Time Signal and System (Contd.)

Friends, excuse me, we shall be continuing with discrete time signals and the properties.

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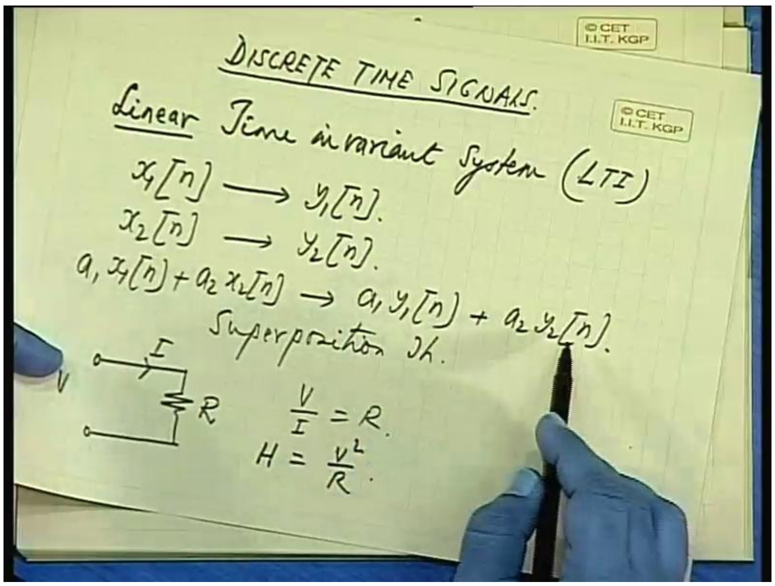

Today, we shall be discussing about linear systems, linear time invariant system; in short we call them LTI system. What is a linear system, the discrete domain we define a linear system as, the one where if I give an input x 1 n and you get a corresponding output as y 1 n, if I have x 2 n is the input, y 2 n is the corresponding output then for a combination of a 1 x 1 n plus a 2 x 2 n, these are scaled by factors one and two. If I get an output a 1 y 1 n plus a 2 y 2 n then it is a linear system that is, you can apply superposition theorem that is the acid test for any linear system.

Sometimes, you must have also studied in earlier classes; if you have a resistance R, you have a voltage V, there is a current I then V by I is equal to R, this a say heater coil of constant resistance, it is a linear system. If, I measure the heat generated then it will be V square by R, it is a non-linear system. So, the same resistance which is constant in magnitude, you can represent a linear system can represent a non-linear system also, it all depends on the input output pair that you are selecting.

In a dynamic system, if we have a differential equation; d square x by d t square plus 5 x is equal to log of t, okay, I should write small.

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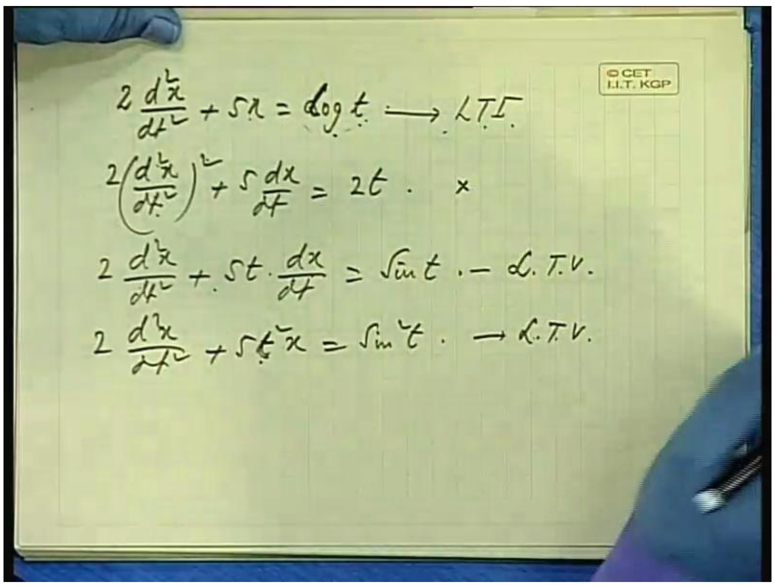

Or 2 into d square x by d t square whole square plus 5 d x by d t is equal to 2 t. 2 into d square x by d t square plus 5 t into d x by d t is equal to sin t, and 2 into d square x by d t square plus 5 t square x is equal to sin square t. Now, in this which one or which ones are linear system? First one, it is a linear system, any question? This is a non-linear function of time t, all right but this is a forcing function.

So, systems behaviour is independent, the systems nature is independent of the forcing function. This one, it is a non-linear function because in d x by d t or d square x by d t square; x and its derivatives are taken as the independent, the dependent variables and it is appearing in the non-linear form in the dependent variable, so it is non-linear.

What about this? This is linear but time varying all right, linear but time varying. What about this, this is also linear time varying. Now, we are trying to search for those functions which are linear and time invariant, so only the first one is linear and time invariant system; because on the left hand side you have, x x dot x double dot etcetera, without any coefficient involving the independent variable t, okay. So, we will be taking up now such system that is linear time invariant system, okay.

Now, linear time invariant system is represented by its impulse response h n, okay.

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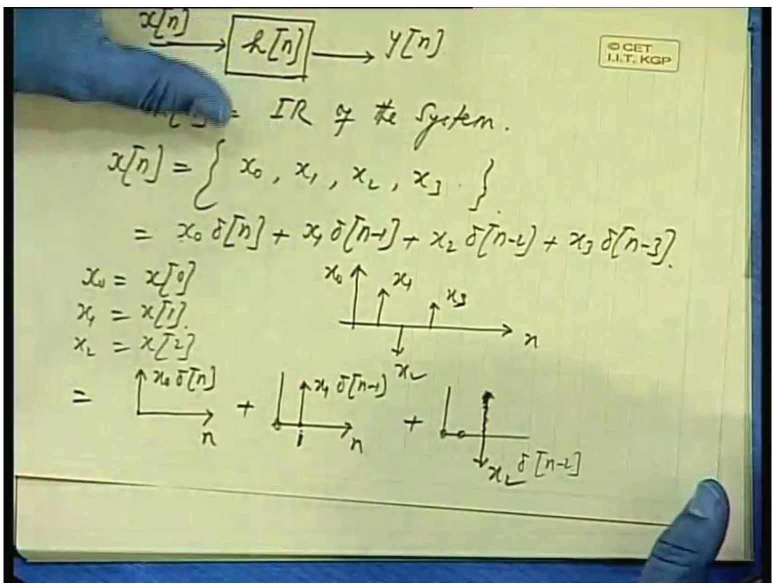

If I excite the system, the system is described by its impulse response all right. If I excite the system, given input of x n, what will be the corresponding output, y n? h n is given is the impulse response of the system, may be h zero, h one, h two, okay. We may give the values h zero, h one, h two. Then for any input sequence, what will be the output?

Now x n, I can write this as x 0, x 1, x 2, x 3 and so on. Suppose, it is a finite sequence like this, this can be conceived as x 0, occurring at t equal to zero; it is an impulse of magnitude x naught, okay and then it disappears. Plus, another impulse of magnitude x 1 but appearing after one, delay, plus x 2 delta n minus 2 plus x 3 delta n minus 3 where basically; x 0 is x at 0, x 1 is x at 1, x 2 is x, is it not? These are the values occurring at these points.

So, what you are doing is; suppose you are having values like this x 1, x 2, x 3, we are trying to, we are trying to represent this sequence as equal to x 0 multiplied by delta n plus x 1 delta n minus 1 that is of magnitude x 1 and an impulse occurring at n is equal to 1 plus, sorry x 2 delta n minus 2 and so on.

So, basically this sequence can be conceived as impulses, occurring at different instants, impulses of different magnitudes. Now, what will be, this is basically the sequence written in component forms. Now, if we apply superposition theorem, the impact of individual components can be summed up, we had already described a linear system as combination of a 1 x 1 n plus a 2 x 2 n is equal to a 1 y 1 n plus a 2 y 2 n.

So, whatever this can be considered as x 1, x 2, x t 3 and so on. So whatever is the response due to this plus response due to this, response due to this if all of them are added it together, all of them are added it together that will be the response, okay this is for a linear system.

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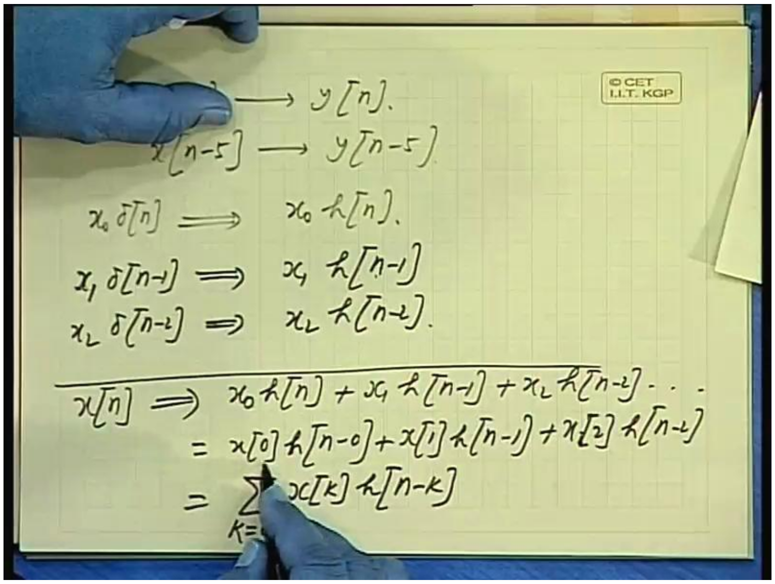

Now for timing variance system; if x n is the input and corresponding output is y n then if the same input is given instead of at t is equal to zero, I give it after say an interval of five steps, five sampling times. So, x n minus 5 will give me an identical output, y n minus 5 that is for a timing invariance system. It is response; nature of response will remain invariant okay, invariant of time.

So, if that is also applied then what will be the response due to this of this system, for delta n it is h n, so for x naught into delta n, it will be x naught times h n. So, the response for the first one will be x naught into h n, do you all agree? Then for the second component, excuse me for the second component which is x 1 delta n minus 1, what will be the corresponding output? x 1 h n minus 1 by invariance principle, I have shifted the input by one step, output is also shifted by one step.

x 2 delta n minus 2 will be giving me, x 2 h n minus 2, all right. So, I am using the principle of time invariance and now we are applying superposition theorem. So superposition theorem, says if I now have addition of these as the input, output will be addition of these; so corresponding to the sum is nothing but x n, original x n. The output will be correspondingly x 0 h n plus x 1 h n minus 1 plus x 2 h n minus 2 and so on.

One may write, what is x zero; it is nothing but this h, I may write n minus 0, n can be written as n minus 0 plus x 1 is this one into h n minus 1 plus x 2 into h n minus 2 and so on. So, this can be written as a summation x K h n minus K, submitted over K, K is equal to 0 to infinity, it is an infinite sequence, all right.

So, this is known as convulsion sum of the two sequences x n and h n. Let us see, how this can be computed in a very effective manner. Suppose, you have the sequence x n as 1, minus 1, 2 okay and then it is all 0. h n minus 2, 1 minus 1 all right, what will be y 0?

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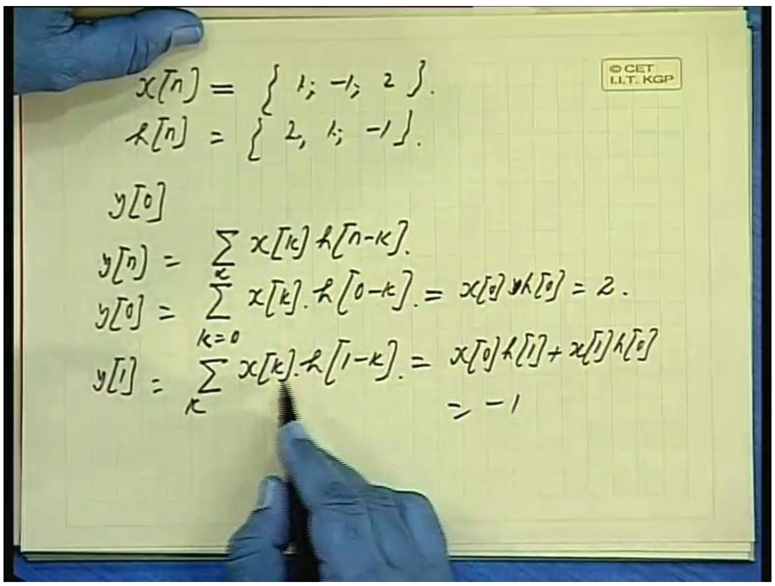

And this is my y n, this corresponding t input, this is an arrow not equal to corresponding to input x n the output is this which is y, so what is y n? y n you have written as summation x K h n minus K. So, y 0 should be, n is equal to zero it should be x K h 0 minus K; you keep on varying K all right. Take K is equal to 0, it will be x 0 h 0, okay nothing else can be taken.

So, that gives me 1 into 2, two. What will be y 1 x K h 1 minus K, vary K; so if I take K is equal to zero, this is x 0 h1 plus x 1 h 0, only two terms and that is x 0 h 1 plus x 1 h 0; 1 into 1 minus 1 into 2, so that gives me minus 1 okay, and so on. y 2 will be similarly, you have to make other combinations, is it all right? Now this can be made, this computation for each step you have to make such expanded terms; you have to compute and you have to make such calculations, can there be a shortcut, let us see.

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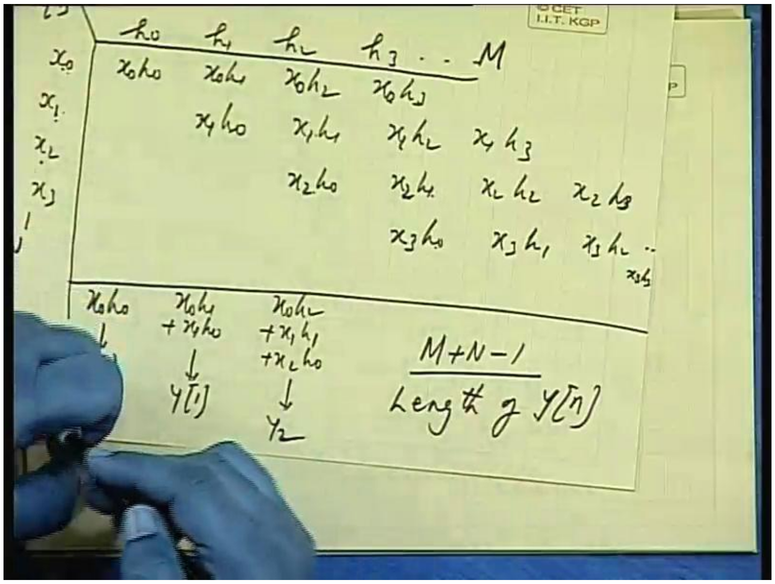

Suppose we make a table, on this side we arrange h, h 0, h 1, h 2, h 3 and so on. This side we write x 0, x 1, x 2, x 3 and so on, okay; whatever be the length of the sequence? So, this is the response due to a delta function, delta n if I excited by an impulse of magnitude x naught, what will be the response? This whole thing gets multiplied by x naught. So, this will be x naught h naught, x naught h 1, x naught h 2, x naught h 3, and so on. x 1 here applying after one sampling time after one interval, so that the corresponding response which will be x 1 h 0, x 1 h 1, x 1, h 2, x 1 h 3 and so on, will start after one interval, all right.

So, it is start from this point x 1 h 0, x 1 h 1, x1 h 2, x 1 h 3. Similarly, the impulse of magnitude, x two will start after two intervals, so it will be x 2 h 0, x 2 h 1, x 2 h 2, x 2 h 3 and so on. Similarly, x 3 will have its effect felt from this point onward and x 3, h 3, now, you apply superposition theorem. So, what is the response in the first interval? At the first instance, it will be only x 0, h 0, so this is y 0.

At the second interval, that is at n is equal to 1, the impact of the first one is still there and the new one has come, so it will be this plus this, this will be y 1. Similarly, three terms will be added here, that will be y 2; x 0 h 2 plus x 1 h 1 plus x 2 h 0, this will be y 2, you can write either this or in bracket it is one in the same thing.

Have you observed one thing? In any sequence the sum of these terms, will be same as this index, all right. So while adding up you knows, what are the terms to be taken for calculating the products and then final addition, okay? Suppose, you have taken a four point sequence here and a four point sequence here, all right.

Then you see, what will be the length, length of this? Length of this, four; there are two trains of length, say four compartments and five compartments all right, when they meet then only it start computing, this value into this value, Say, okay, this is h zero, h one, h two, h three, h four and so on and this side is x one, x one the sequences are like this, so we have reversed it all right; in a time we have made a time reversal, so this is x zero, x one, x two, x three and then you bring them together, all right.

So the first point, it will be x zero into h zero, next it is x zero h one and x one h zero and at the end when they just touching inside, then only you have an interaction after that there is no effect, is it no? So, when do the tails of these two meet; so, how many steps are there? So, there are four steps on this and there are five steps on this. So, if you take from this point and this point, how many steps you have made?...eight five plus four minus one, all right, five plus five plus four minus one.

So, here we are having four points sequence, four point’s sequence it is including zero mind it. So, it will be four plus four eight minus one. So, the length of the sequence will be seven, is it all right? So, length of the sequence, if this is of length M, this is of length N then the length of the output sequence will be M plus N minus 1, this is the length of y n, okay. The sometime back, the last lecture where discussed about, even and odd functions.

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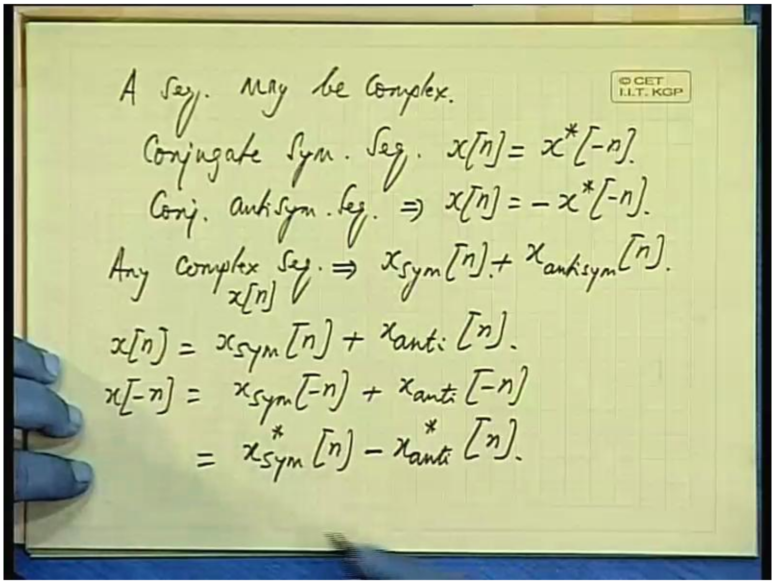

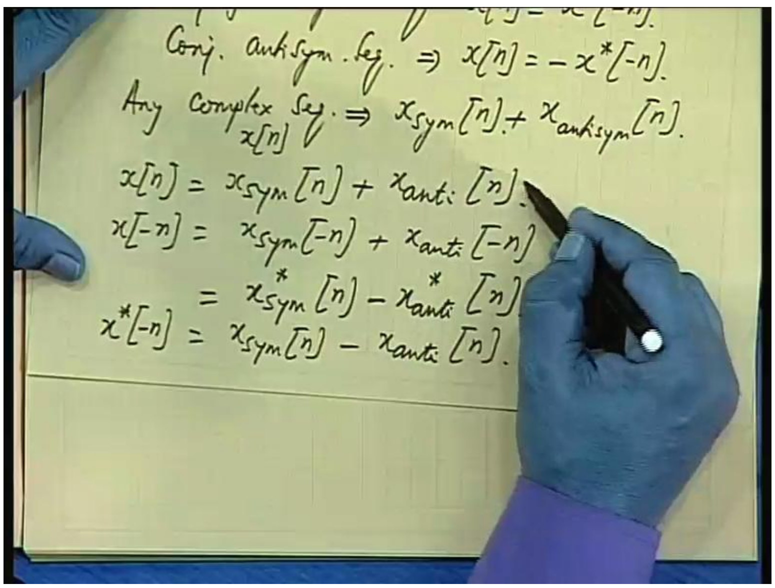

If a sequence is complex, sequence may be complex. If you have a complex sequence then we define another kind of symmetry, conjugate symmetry sequences, x n; if it is equal to x star minus n then you call it conjugate symmetry, okay. And similarly conjugate anti-symmetric sequences, we defined as this, okay.

Once second, any sequence any complex sequence, can be written as the sum of symmetric part, conjugate symmetric sequence and conjugate anti-symmetric sequence, all right. So, any complex sequence x n can be written as this. What will be these components? x n is equal to x symmetry n plus x, I just write anti n. What will be x minus n? x symmetry minus n plus x anti minus n, okay. And what is x symmetric minus n? For a symmetric function, x minus n it will x star; so this is nothing but x symmetric star n, is it not? And, this one; it will be minus x anti-star n, is it all right?

So from here, what will be x star minus n? It will be x symmetric n minus x anti symmetric n, I just again taken conjugate of that on both sides, so that gives me this. Now, add this this this with this, this will get cancelled, okay.

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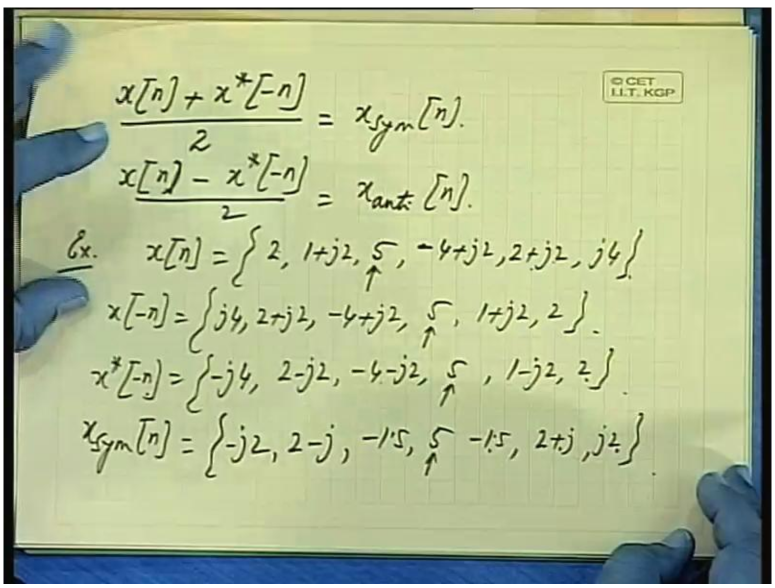

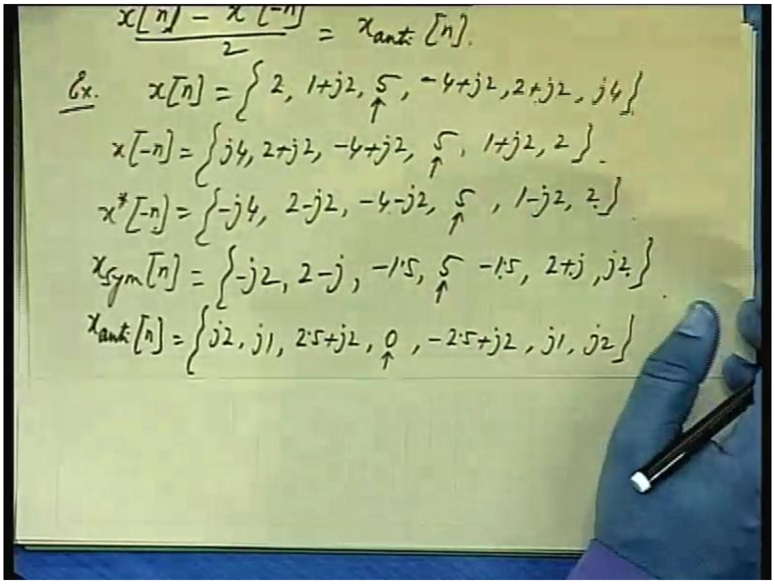

Therefore, if we take x n plus x star minus n by 2, will get the symmetric component and similarly the conjugate anti-symmetric component will be, x star minus n by 2. Let us take an example; x n is given as 2, 1 plus j 2, 5 minus 4 plus j 2, 2 plus j 2 and j 4. Therefore, x minus n will be from the other side, if you take the sequence; 2 plus j 2 minus 4 plus j 2, 5, 1 plus j 2 and 2, okay.

So, x star minus n would be complex conjugate of this sequence, minus j 4, 2 minus j 2 minus 4 minus j 2, 5, 1 minus j 2 and 2, again. So, what will be the symmetric component? Symmetric component, half of x n and x star minus n divided by 2; so this plus this by 2. Now where should I, this plus this divided by 2 keeping the origin intact, so this does not have any counterpart here, so this divided by two, so minus j 2, okay.

Then, 2 minus j 2 plus 2 divided by 2, so 4 minus j 2 by 2. So 2 minus j, correct me if I am wrong. Then this and this divided by 2, so minus 4 plus 1 minus 3 by 2 minus 1.5, and j 2 and minus j 2 will get cancelled. Then 5 plus 5 by 2 is 5 itself. Then 1 minus j 2 and minus 4 plus j 2, again minus 1.5; and then this one and this one 2 plus 1, sorry 2 plus j 2 so it will be 2 plus j and j 4 it will be j 2.

So, symmetric component you can see for yourself on the right hand side and left hand side, the imaginary parts are complex conjugate, they just of opposite signs, all right. This is j 2, this minus j 2, this 2 plus j, this 2 minus j, this is minus 1.5, minus 1.5, so this a symmetric component.

Similarly the anti-symmetry component, you can see for yourself; j 2, j 1, 2.5 plus j 2 then 0 then minus 2.5 plus j 2 then j 1 j 2. In the anti-symmetry component, you will find the imaginary parts; they remain as it is is, the real part which will be having opposite signs, okay.

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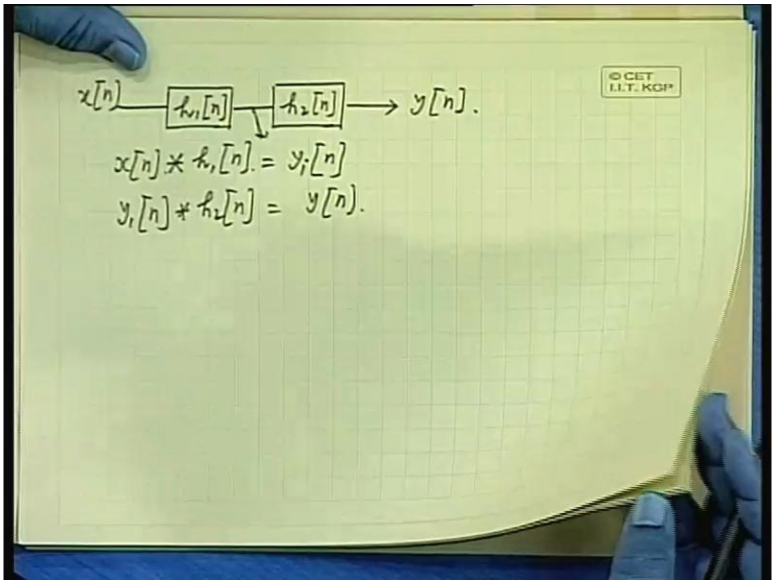

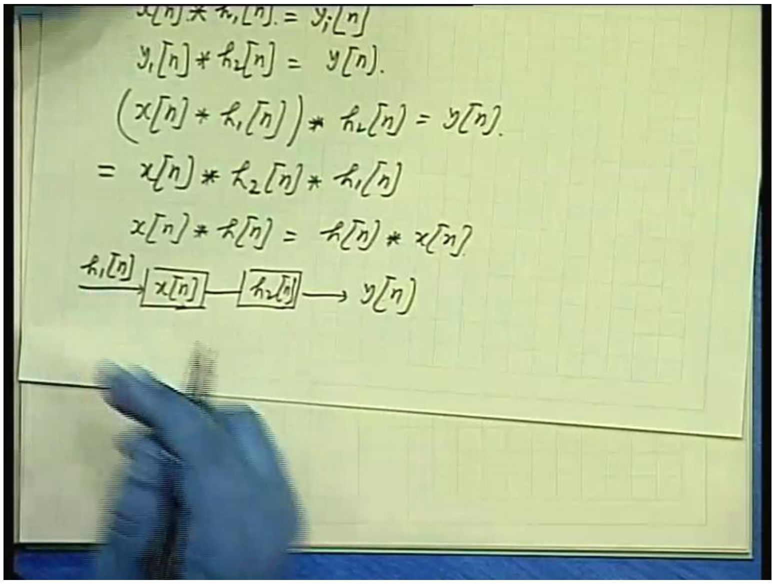

Yes please, is it all right? Now, we have h 1 n as one systems impulse response, there is another system, h 2 n. This is the input x n, this is the output y n. So, x n the convolution that we performed earlier is represented in a compact form; like this x n star h 1 n is a output from this block okay, let us call it as some intermediate output y i n y, intermediate n, this one. y i n convolved with h 2, n gives me y n, okay.

So, basically x n star h 1 n, the whole thing again convolved with h 2 n is equal to y n. Now, we will show that order of appearance of these two blocks will not change the final output; that means this is x n star h 2 n star h 1 n, it will be same as this. You can also, sorry you can also see x n start any h n can be written as h n star x n because; you must have observed when we are discussing about the terms, see it was x 0, h 0, x 0, h 1, x 1, h 0, so you could have interchange the positions of h and x, all right.

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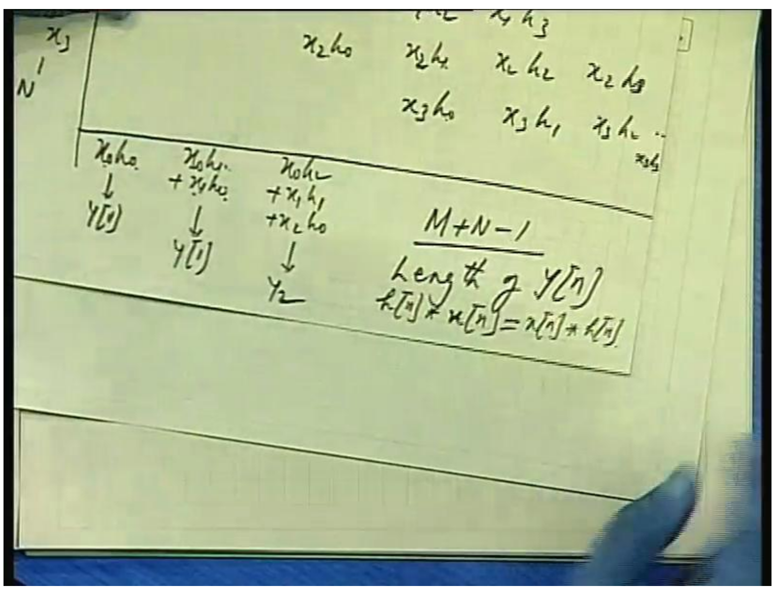

So, h convolved with x n is same as x convolved with h n, okay. So, we will be showing that; if there are two such blocks, you can interchange them or you can also interchange with input. I could have put a sequence; corresponding to h 1 n as the input and if a system response is equivalent to a sequence x n, then also I will get the same output, the same as this, okay. Again you can interchange these; you could have put h 2 as the input.

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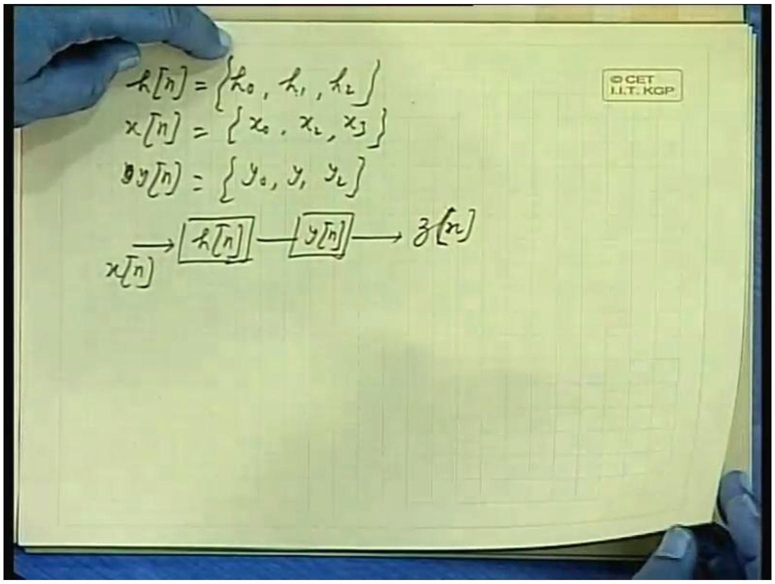

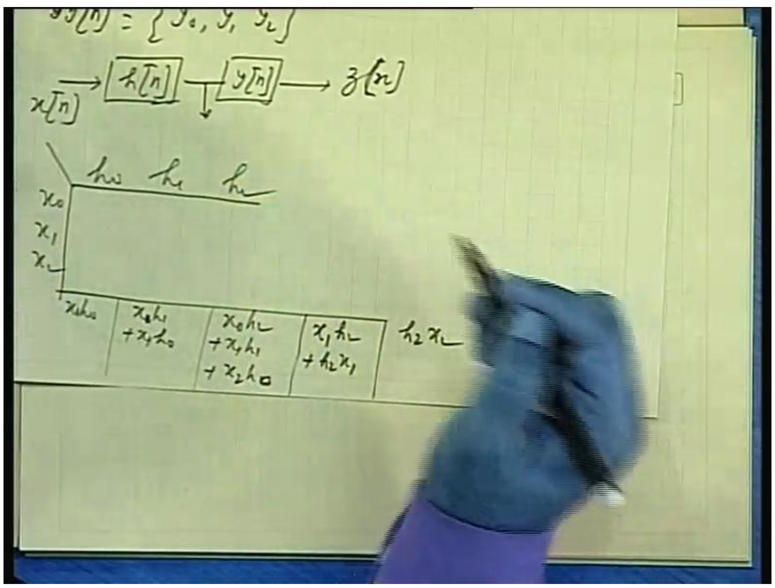

So, let us take a very simple example, h n given as h 0, h 1, h 2; I am taking small sequences for convenience, you can try with any other sequence. Similarly, some okay some y n as y 0, y 1, y 2. I have taken x h and y instead of h 1, h 2 because the individual terms are also having h 1, h 2; so to avoid confusion, I have taken three three sequences x n, h n, y n, I could have chosen anything else and say the output is z n, okay.

What is the output corresponding to this? First two, if you remember h 0, h 1, h 2, x 0, x 1, x 2; the output sequence, that we got was x 0, h 0 then x 0, h 1 plus x 1, h 0. Then the next output was, x 0, h 2 plus x 1, h 1 plus x 2, h 0, okay. Next, this one; x 0 h 2, there is nothing like h 3. So, x 1 h 2 plus h 2 x 1, last one was h 2 x 2, okay. So, these are the five outputs. Now, let us feed that there is a output, here the one that we have got here is a output here, all right? Now, this is to be convolved with y n, so let us take it in the next step.

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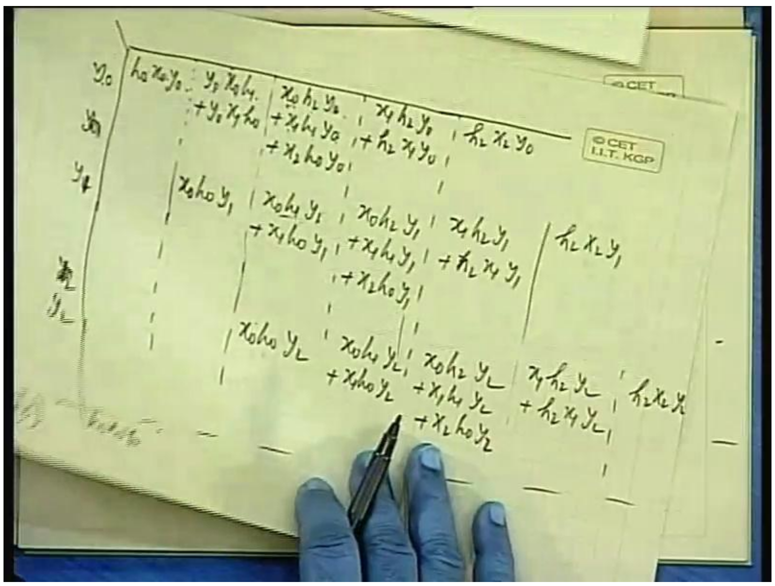

This is my input and this is y 0, y 1, y 2. This is sitting over here, so what will be the output here? It will be y 0, x 0, h 0, all right? Next y 0 into x 0 h 1 plus y 0, x 1, h 0, all right; next x 0, h 2, y 0 plus x 1, h 1, y 0 plus x 2 h 0 y 0, agreed? Then x 1 h 2 y 0 plus h 2 x 1 y 0, next h 2 x 2 y 0, okay.

Now the same sequence; let me write with a little distance, now this gets multiplied by y 1, this sequence gets multiplied by y y 1 and shifted by one step forward. So here it be, x 0, h 0, y 1, x 0, h 1, y 1 plus x 1, h 0, y 1, all of them are multiplied just by y 1. x 0, h 2, y 1 plus x 1, h 1, y 1 plus x 2, h 0, y 1 and here it will be x 1, h 2, y 1 plus h 2, x 1, y 1, the next, h 2, x 2, y 1. And lastly, y 2, it will start form here; it will be x 0, h 0, y 2, x 0, h 1, y 2 plus x 1, h 0, y 2 then x 0, h 2, y 2 plus x 1, h 1, y 2 plus x 2, h 0, y 1, sorry y 2.

Next x 1, h 2, y 2 plus h 2, x 1, y 2 and the last term will be h 2, x 2, y 2. Now, what will be the oral response Z n? First one will be h 0, x 0, y 0 all right. Next, you add up these and these shortage of space, but anyway if you add up column wise, you can see the terms; the first one will be all zeros then x 0, y 0, h 1 sum is 1 and all possible combinations of x h and y. The sum is two; starting from 0 to 2; all these indices, all possible combinations that you can think of, okay. x 0, h 2, y 0, keeping y 0 fixed, you vary the other to get a combination of total sum of two.

Then again vary y, y 1 and then get a combination as one, so that the total sum is two, and so on similarly the next column. So, you find, I can from these general terms, since all possible terms at present you can always have any combinations, of appearing appearance of h n and y n or for that matter x n, is it all right?

So, output of such sequences which are interchangeable, will remain intact; this all for a linear system.

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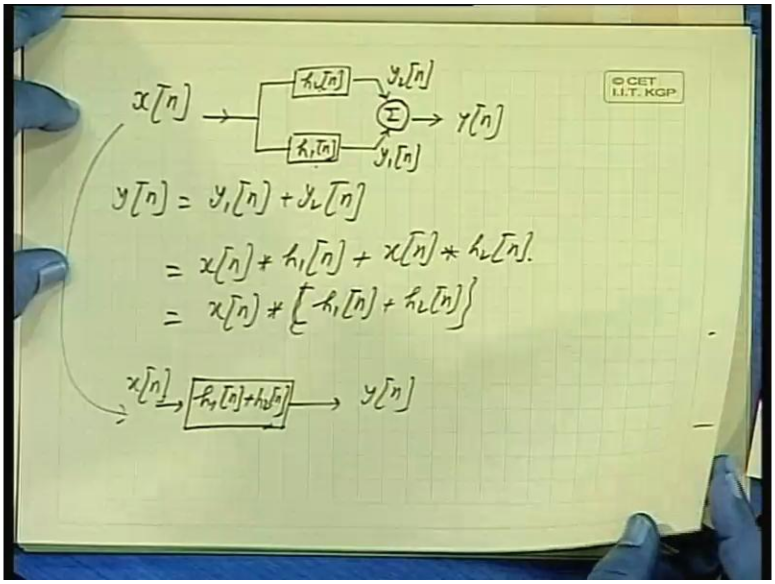

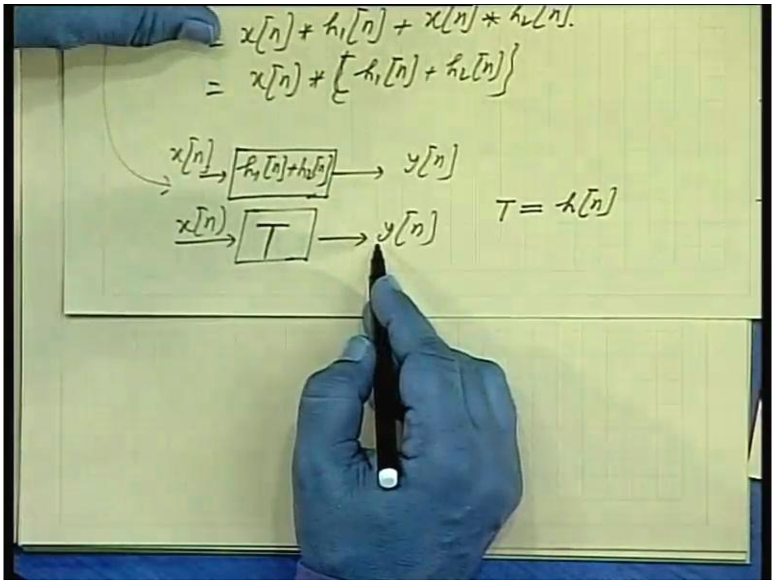

If you have, h 1 n, h 2 n, y n; in this case is h 2 n convolved with x n plus h 1 n convolved with x n. This is y 1 n, y 1 and y 2; if I write this as, this is y 1, this is y 2 then this is x n star h 1 plus x n star h 2. So, that is same as x n star, if you take term by term we will find the sequences h 1 and h 2 are basically added and convolved with x star n. You can try with one example again, very similar to this kind of table, you can make. So, this can be written as h 1 n plus h 2 n, okay.

So, basically we write x n, sometimes we write T for transformation not for delay, capital T for transformation; that is x n gets transformed to y n, through this system, system function. So, T is nothing but this sequence, h n is one representation, T is the system all right; call it in general a fitter, anything any system is a filter. Need not be one of those standard ones, but anything any system is trying to alter the input and give you a corresponding output y n. So, sometimes call it general transform, transform of the system or a filter. Sinusoidal sequence, well today the time is over, we will take it up in the next class. Thank you very much.