Power System Operations and Control
Prof. S. N. Singh
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Module -2
Equipment and Stability Constraints in System Operation
Lecture 6

Now, let us see the lecture number 6 of module 2. In the previous lecture; that is lecture number five, we saw the various definitions of the various type of stabilities, and there difference of classification, again with respect to the disturbance means, magnitude of disturbance time frame, and again the quantities of interest. In this lecture I will discuss about transient stability of the power system, and then we will see what are the ways that we can analyze.

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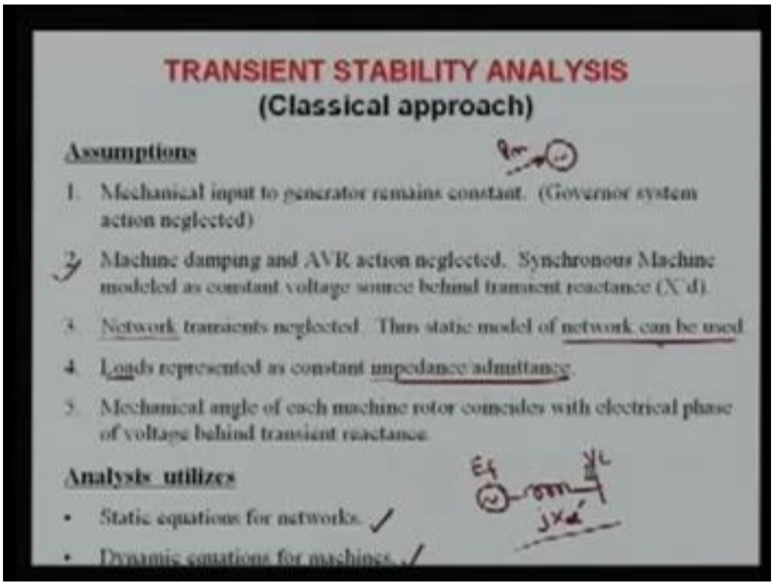

First one is a classical approach for the transient stability analysis, and the various assumptions are made in the classical approach. First one is the mechanical input to the generator remains constant. Means governing system action is neglected. Means if your system here this, whatever the generators here we are giving the mechanical power, and that is coming through your turbine system, again turbine system then governing system. So, this we are assuming for that analysis period, this pm is almost constant, and that assumption is valid. Because here the transient stability our analysis is concerned up to 5 to 10 seconds. So, by that here the governor action or prime over action is almost negligible, because the time constant of this is very high. So, we can ignore this, means your mechanical power input to the generators are remain constant. Second assumption here is the mechanical damping; that is machine damping, and the AVR action is also neglected means the damping of the machine that is D, and the automatic voltage regulator action s also neglected.

The synchronous machine modelled as constant voltage source, behind a transient reactance. Means here it is your generator. So, we model this generator with a some reactance here and this is your terminal voltage. So, this reactance is nothing, but your X_d'. Again as in the previous lectures we have seen the synchronous alternators may of two kind, two types; one is your cylindrical rotors, another is the salient type of rotor of machine, where Xd and Xq are different, however in the synchronous machine it is a Xs; that is constant and it is irrespective of the D and Q axis components. So, if it is a salient then it is X_d', and if it is your cylindrical rotor machine then it will be Xs. So, it is your some, the constant voltage source Ef behind this voltage behind this terminal voltage this t, and this is your sub transient reactance of this alternator than we can model in this way. So, the machine can be represented by the classical; that is also called the classical representation of the machine model. Also we know in the power system, all that components they have some dynamics, and they have some their equations and their dynamics are there. For your example your network dynamics, protection system dynamics, your governor dynamics, your mechanical inputs dynamics.

So, all are having the dynamics, but in this classical approach, we also ignore that network transients. Means those are neglected, thus the static model of network is considered. Only we have to go for the static model in this classical approach. Load also may have some dynamics; for example, your induction machine load, but also they are changing with the change of voltage, as well as changing in your reactive power requirement. But in this classical approach, we model load at the simple impedance or admittance form. The mechanical angle of each machine rotor coincides with electrical phase of voltage behind transient reactance. So, these are the five assumptions are made in the classical analysis of the transient stability. So, analysis utilizes the static equation of network, and the dynamic equation of machine. As I said, here the network, just we have to take static model, and others we have to take here the dynamics of the machine, here that is included.

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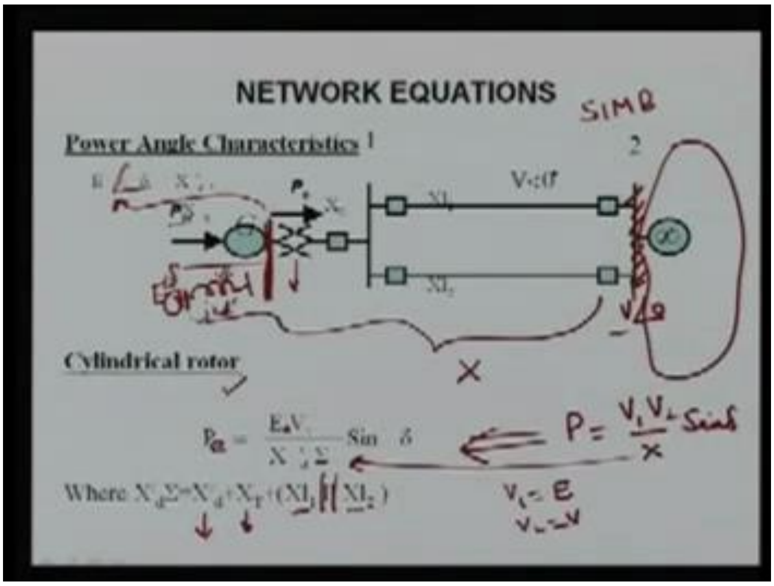

Let us see a generator G here that is connected with the transformer having impedance Xd, and here these blocks are basically the circuit breakers; means we can trip this portion, we can trip this transmission lines as well, and here we have the infinite system. Normally, most of the analysis people go for either single machine infinite bus system; that is called SIMB single machine infinite bus machine system. So, this is basically your Infinite bus, where normally this voltage angle is zero. Means again what is the infinite bus, why it is called infinite, whether infinite in terms of its location, infinite in terms of its, what is infinite term why. So, infinite bus is a bus, which is if you are injecting power, or you are extracting power from that bus. Normally they should not be any change in the voltage and the frequency of the system of that bus, so this is called infinite. Means you can draw effectively infinite power, which is basically not possible, but if you are drawing large power, and there is no change in the voltage and the frequency of that system, then we can say that bus where we are drawing the power, is known as your infinite bus. So, here basically this is another system, and where this bus is located, so you can draw more power here at this bus. Means here practically the voltage an angle is constant, whatever the power you are taking.

So, we had the system; one generator, the transformer, and then we have the two lines; line X is here that is impedance is X_{l1}, and another is here your X_{l2}. Now, this machine as I said in the classical model, we can represent this machine as E and then we can have a reactance here, and this is your voltage source; that is your X_d'. So, now, your this bus, this system we can represent here with this one. Now, this is a transformer. So, this Pm we have assumed constant, but this electrical power P is changing and it depends upon the system here the angle etcetera. Now, for the cylindrical rotor, we can write the P. If you remembered already we have defined is P for any machines here is $P = \frac{V_1 V_2}{X} \sin\delta$ and this δ is the difference between the angle of the two buses. And that bus is the element with Xs is considered. For this case we are just considering here one bus here, and another we are talking here. So, the Impedance in this bus, is let us suppose now here what will be this X.

Now this X component just we have to calculate. So, we have to write here now our V1 in this equation is nothing, but your E, because we have taken here E∠δ of this bus. So, your this V1 is E, and your V2 is nothing, but your V, and X is the total reactance of this whole system, including your this as well. So, this is your complete. So, we can write now this can be simplified this P; that is Pe, $P_e = \frac{EV}{X'_d} \sin\delta$ is equal to E multiplied by your voltage; that is infinite bus voltage divided by the X'_d sum. Means the total X here as I said here is X and the $\sin\delta$ basically angle here the zero and the internal here angle is your δ. Now, here this X'_d will be nothing, but the reactance of your generator; that is X'_d, the reactance of your transformer, here this is this. And these two lines basically this is in series with this, and the impedance parallel effect impedance is series with these three. So, these two are in parallel. So, it is a parallel combination of the reactance of these two lines this X_{l1} and X_{l2} thus it is a parallel. So, it is a total X'_d will be this, and then we can write electrical power which is flowing out from this system. Here Pe can be related here, and that is basically in terms of you cylindrical rotor machine.

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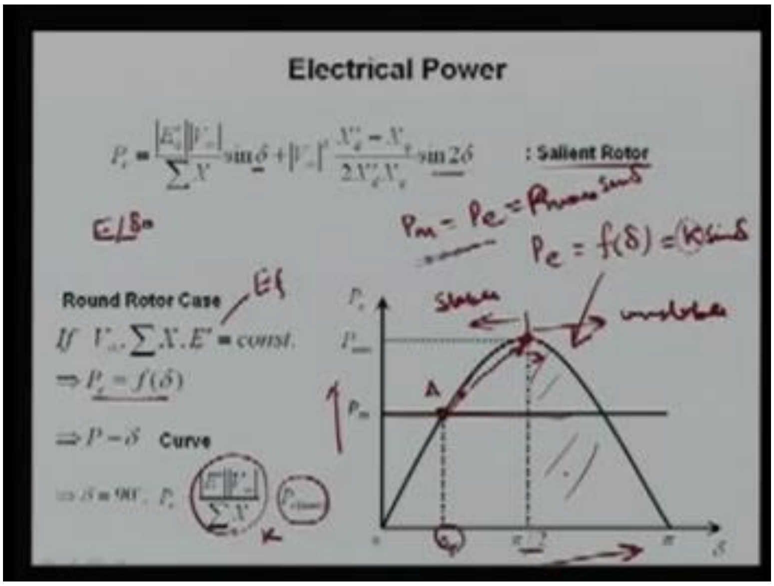

To see the same system, if your machine is a salient rotor type of synchronous machine, then we have also derived this. Here this is your E_q', here now here Eq and the` Vd both are the different one. Means here this E_q' multiplied by your V infinite plus some of the reactance's in that line, and again here we have another component here. So, here this is δ. It is a 2δ, and already in the previous lecture I have shown that. So, if your round rotor case, if your V_∞. This X and E' are constant. E' is nothing, but in the previous case it was E, if are constant, then we are write this Pe is nothing, but is a function of your simply δ, or you can say sineδ. And if you draw then you will find here a curve of your sinusoidal here. Means we are getting some constant k sineδ. Means if your δ is changing, the power that will be keep on changing, this Pe. So, this is called your P-δ curve, and at the 90 degree pi by 2. This k term will maximum and it is called Pe; that is Pe maximum. Means when it is your degree is 90 degree then we can have the P maximum, means maximum power that we can flow. So, the area which is just this side it is your stable zone, this and outside this is your unstable. Now, always what will be your operating point. Let us suppose your mechanical power that's Pm is this axis. So, your electrical always we know for a steady operation, this Pm should be equal to your Pe. Then here we can have your k; that is k term is here. This is nothing, but your k; that is Pmax we can say $P_{max} sine\delta$. So, this is a curve of sine δ.

So, this will satisfy at a point, and that is basically the intersection of this curve, this and this curve. So, this point A is your operation of point, and the angle that is a torque angle or machine angle that will be your δ_0 here. So, your machine which was here this I wrote E angle here, now it will be δ_0, if your system is without any disturbance and this is steady state it is operating with this, your previous case, this your transmission like this. Now, and this is your δ angle where it is operating. So, the we can go up to slowly if you are slowly and slowly we can increase, without disturbing the system slowly power is increased, then we can go up to this point, and this is normally called your steady state stability limit, but always we operate our power system, must be low this steady state that is Pmax value, because always you know there is a dynamics, always there is a change in the system. So, if you are operating here, there may be possibility your system may be your unstable zone, because this area is unstable and this side beyond this pi by 2, and then your system will be collapse, and it is not possible to operate

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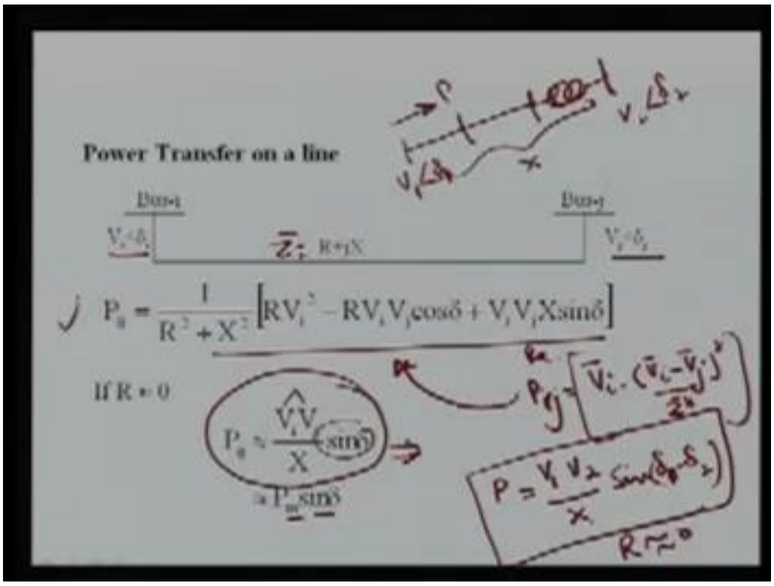

Let us see the power transfer on a transmission line. So, far we had seen the power transfer including your machine, transformer, and the transmission line. Here let us we have a bus i and the bus j, and we have the angle here $V_j \delta_{ij}$ and here the voltage magnitude is V_i and angle is δ_i , and this Z of this line is $R + jX$. So, we can write here the P_{ij} in this line with this formula. This is nothing, here this P_{ij} is nothing, but your Vi; that is complex into your I_{ij}that is your $\left(\frac{V_i - V_j}{Z'}\right)^*$ and here basically the real of this, and if you simply you will get this expression. So, basically it is a Z^2. If your R is zero, means your line is loss less, means normally R is very small. So, we can ignore compared to the

X then we can get whole expression R is zero we will get this and this is zero. So, we can get this X X will be cancelled and we will get the Pij, is approximately $P_{ij} = \frac{V_i V_j}{X} \sin \delta$, or we can say this Pmsineδ. So, for the line as of full system this formula is equally valid. Only you have to remember in this case, that this δ is the angle between the two buses, where these two voltages.

So, that is why here I write this P between two nodes. Node means two buses. Here this voltage of one bus, voltage of another bus divided by X between these two buses. Means there may be several for example, here one is this, there may be another line here there may be some transformer here and then another bus here. So, if you are talking this is V 1 and V two. So, that this Whole will be the total of this line one line going transformer. And the angle here is the difference between angles of these two buses. So, here X is the total X and then it is your sine basically $\delta_2 - \delta_1$, or basically or basically here $\delta_1 - \delta_2$ what is the direction. Means if your power is your calculating the power showing P in this one. So, it is $\delta_1 - \delta_2$ as I explained in the previous lectures, the real power will flow from higher angle to lower angle. So, this expression is equally valid, and this is the case when R is very small and that can be taken as zero, then this full expression is valid. If R is very high, then we can write this one, especially for the distributed system you can write this. You have to take this expression which is the original expression.

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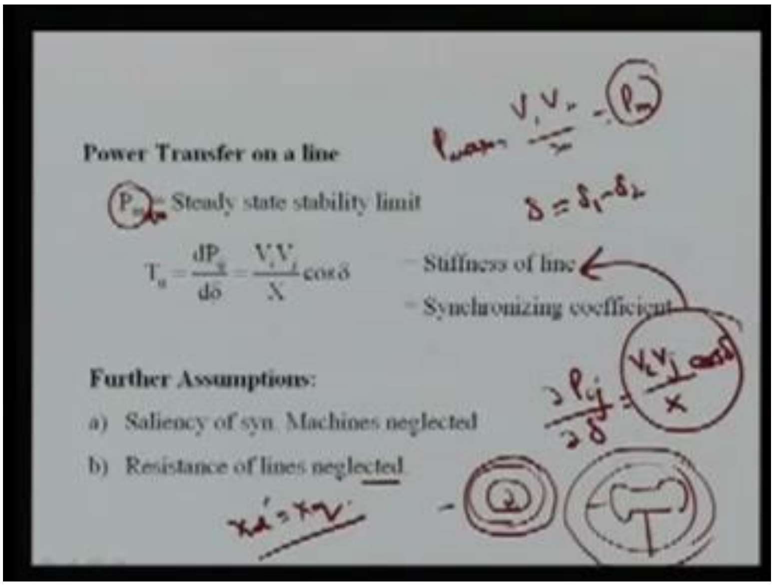

Now, let us see another one. This steady state stability limit that is Pm; that is your $\frac{V_1 V_2}{X}$. It is sometimes called Vm sometimes Pmax, so it is given in the different book here. Normally this Pm I will denote the mechanical power. So, it is better to write the Pmax. So, it is the steady state stability limit. If you differentiate the Pij in the previous case, if you differentiating this, that we have if you are differentiating that the Pij with reference to the δ here that is again here δ, I have just written it is nothing the δ between the bus 1 and bus 2 is your δ. So, always you must be very much careful what is this delta. So, if you are differentiating this, then the voltage here is a Vi Vj and here you X, it is your cosδ. And this coefficient is called the stiffness of line or it is called the synchronizing coefficient of the line. So, this term gets your idea that what is the stiffness, constant of the line that is very much important, how much your system is stable.

Also in the classical approach, we may need more assumptions, and that is basically the saliencies of synchronous machines are neglected, and resistance of lines are also neglected. In the classical approach, as I said your two types of machines may be there, here it is a stator, and here is your rotor which is rotating. So, if it is there the constant air gap, then it is called wound rotor, and if you are having some poles, and we have the different air gap; that is rotating in the machine here. So, here we have the different air gap, here we have different air gap and this called the saliency of machine. So, we can ignore means here we can write this X'_d will equal Xd then we have to ignore saliency we have to assume, but both quantities are very close, but normally it is not so easy, it is very different. And also the resistances of the lines it is not only lines, even though of the transformers we have to ignore, and the resistance of your synchronous machine is also ignore.

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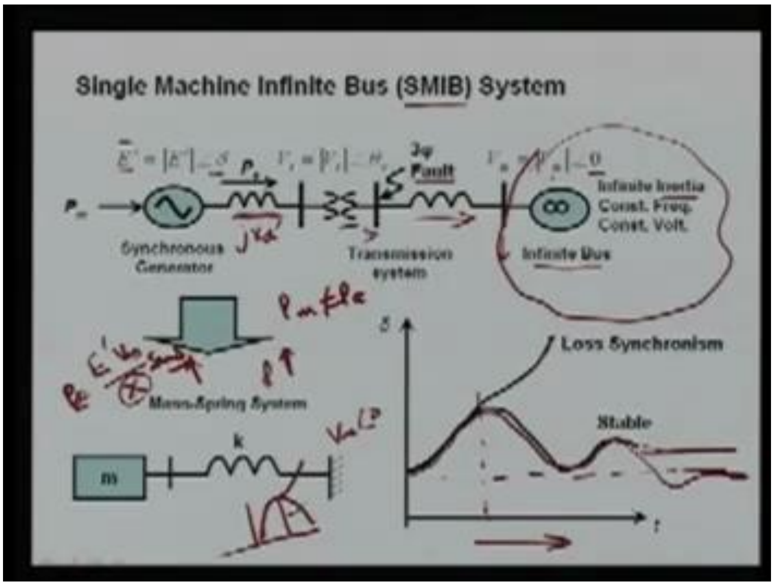

Now, let us see the single machine infinite bus; this SMIB the single machine infinite bus system, where again the previous case we use here a generator G. This is your machine is represented by its equivalent impedance; that is nothing, but our jxd or X'_d here. You have to take the sub synchronous transient reactance, and this machine is the internal here the E' that is equal to magnitude this is complex quantity. So, the magnitude with the angle δ with reference to the infinite bus. So, the Infinite bus basically denoted by, it has the infinite inertia, system inertia; whole this system basically. This is a system where this bus is there, so it has the infinite inertia, it has a constant frequency, and it has a constant voltage. So, these three criteria basically gives you the criteria for the infinite bus, and the voltage here as I said the voltage is V_∞, here I have written angle zero, and with the reference to this angle we have taken this angle δ. This is your transformer this is your terminal voltage of generator.

This is another bus, where this is a line there and it has some impedance. Now, this machine here is nothing, but your spring mass system. Here there is some inductance it was K the total this we can denote whole this machine. This is your infinite bus where this $V_\infty\angle 0$, and this is another mass. So, this is just like a masses spring system, and then it will have some, and then we can analyze dynamics of the system, as you know the dynamics of the system can be retained and solve the differential equation. Now, there is a two type of concept, if there is a three phase fault. Let us suppose at this bus as we shown here. If there is three phase fault, there will be this machine δ will be changing, why. This electrical power due to this fault, more power will be flown as i said the power here is your nothing, but your $E' \; V_{\infty}$ that is infinite the total X completely X sineδ. What will happen now this is in steady state, and this Pm is almost constant. We have assumed this now this three phase fault is there, what will happen. This X will be change, and this X will be reducing.

So, means more power is flowing. So, your Pm is not equal to Pe during this three phases fault. So, this to make it what will happen this Pe this δ will keep on increasing, because this value is larger, if it maintain the Pe constant this δ, you can say this δ will increase, and now you can say δ is increasing. So, over time period during the fault, this δ again can follow the different trajectory. One condition you can see if δ is falling like this, and finally, it is stable. No doubt, there is a possibility at this point, this fault is cleared. If the fault is persisting, and this fault is not severe, so δ will increase and then it will be stable, but if this fault is cleared. So, then δ again may come to its initial value, may be, again depends upon the, whether fault is removed or not. If fault is removed, then it may come to its original value, and the system is called to be stable, but this δ can increase like this, till this fault is again cleared, the system is this δ is keep on increasing and it losing their stability, because here if you see, if this is your δ and this δ is keep on increasing, then system is unstable and system is unstable then it is called there is a loss of synchronism. So, it is losing synchronism operation, and then we can say system is unstable.

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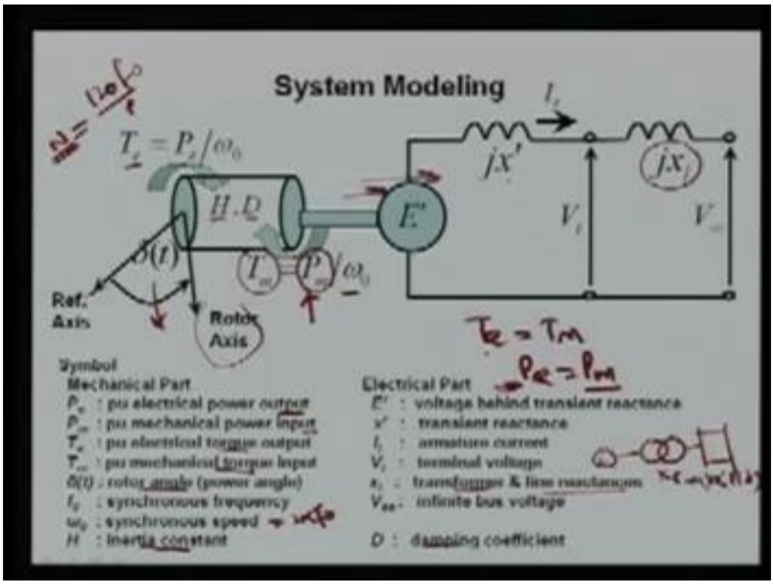

To see the system modeling, again this whole the rotating mass, here it has some inertia constant. This rotating mass we have the synchronous generators. The systems also we are have some damping. Damping means we have some resistances, means there is some damping factor as well, and let us say it is D, and here this your Tm this mechanical torque that is mechanical power divided by this normal frequency that is ω_0. And this ω_0 is nothing, but it is equal to your $2\pi f_0$, and f_0 is your synchronous frequency. This synchronous frequency is decided as I said; it is $\frac{120f}{P}$; that is how much you are rotating. This rotating is p this is giving your frequency based on the rotation. So, number of poles of the machine. So, this is a rotating mass, and again the two forces; one is Tm, another is your Te. So, your electrical power which is coming through this one going that side, it is an electric power and the input we are giving in terms of mechanical power. So, at the balance as I said here again this Te must be equal to your Tm, or we can say this Pe will be equal to your Pm. Means your electrical power will be equal to your mechanical power, and then it is called steady state. But during the steady state it may not be equal it will try to equalize this Te.

As I said this Pm we have achieving the constant in this transient stability analysis, so only parameter which is changing your Pe. So, various in this system modeling that will be used, this Pe in per unit electrical power output that is power which is coming out, Pm is mechanical power input that Te is electrical torque, Tm is mechanical torque input again, δ is the rotor angle. And this angle is the angle between the two here your reference axis, and here your rotor axis. So, this is the angle between the reference where you are measuring all these angles, and this is the angle between your, and this is called your torque angle, all load, angle between the two axes here. Your h is the inertia constant of machine; whole system basically here we are talking. If all machines are coming into 1 1, one combined machine then it is inertia constant of complete system, or if you are analyzing one machine with one machine, then it is inertia constant of one machine.

So, D is your damping coefficient as I said. Here this is your V_∞ is here infinite voltage. This is terminal of this generator. This is the X'; that is a transient reactance of synchronous machine, and here is E is behind the transient reactance. So, what about the power that is here coming out, again this will be equal to your Pm in the steady state. This Xl this include your the transformer and the line reactance’s, as I said we had one transformer and then we had the two lines, again the same example and here it was Ef. So, this is your $X_t + X_{l1} \,||\, X_{l2}$. So, it is X is the combine of the transformer as well as the two transmission lines, those are in parallel.

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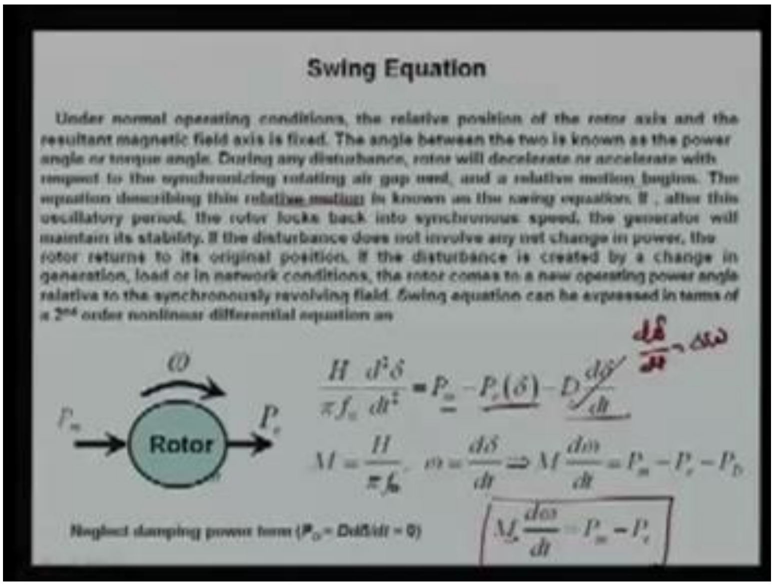

So, for the transient analysis, normally we use the swing equation. Let us what is the swing equation. Under the normal operating condition, the relative position of rotor axis, and the resultant magnetic field axis is fixed. So, the rotating axis of the rotor, and the magnetic axis is field, is fixed. The angle between these two is known as the power angle or the torque angle. During any disturbance, rotor will accelerate or decelerate. There by what will happen, there will be change in the rotor angle. So, with acceleration and the deceleration, with respect to the synchronizing rotor air gap mmf and a relative motion begin basically. The equation describing this relative motion is known as swing equation. Means during any disturbance, rotor will decelerate or accelerate. Again depending upon the Pm and Tm balance; means whether the Tm is more. Means input is more than the electrical output, then machine will accelerate .If your input is less than your output then it will be decelerate. Then accelerate with respect to the synchronizing rotating air gap mf, and our relative motion begins, and the equation describing this relative motion is known as swing equation.

If after this oscillatory period the rotor locks back into the synchronous speed, the generator will maintain its stability. If the disturbance. does not involve any net change in power. The rotor returns to its original position. If the disturbance is created by a change in generation, load, or network condition, or network topology. The rotor comes to a new operating angle, relatively synchronously revolving field. Swing equation can be expressed in terms of the second ordered non-linear differential equation as here $\frac{H}{P\,f_0}\frac{d^2\delta}{dt}$; that will be equal to $Pm - Pe(\delta)$ and that is a function of your δ minus this damping constant and then we have $\frac{d\delta}{dt}$. Basically this $\frac{d\delta}{dt}$ is nothing, but your changing speed. So, if we neglect this term this speed E; the damping effect as I said in the classical analysis we always neglect this. So, this can be neglected. So, we will have the equation here like this. Now you can see earlier I was using this H, and I now I am using you know. So, this M and H they are defined they are related, as this M angular momentum is equal to $\frac{H}{P\,f_0}$, and this ω is nothing, but this del $\frac{d\delta}{dt}$. So, whole this equation can be replaced and can be retained in terms of ω.

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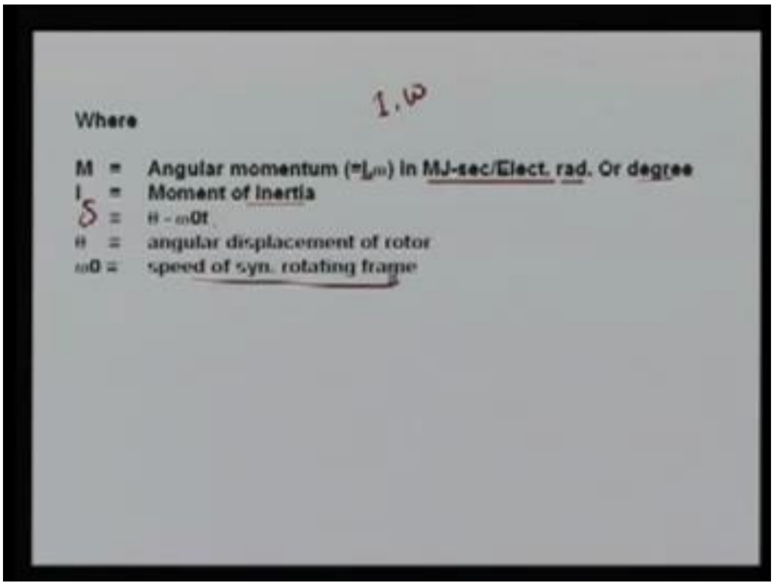

So, this M is angular momentum, and this is defined as inertia i of whole system multiplied by angular speed, and its unit is your mega joule second per electrical radian or degree. i is your moment of inertia, here you $\delta = \theta - \omega_0 t$, and theta is angular displacement of rotor, and ω_0 is the speed of the synchronous rotating frame.

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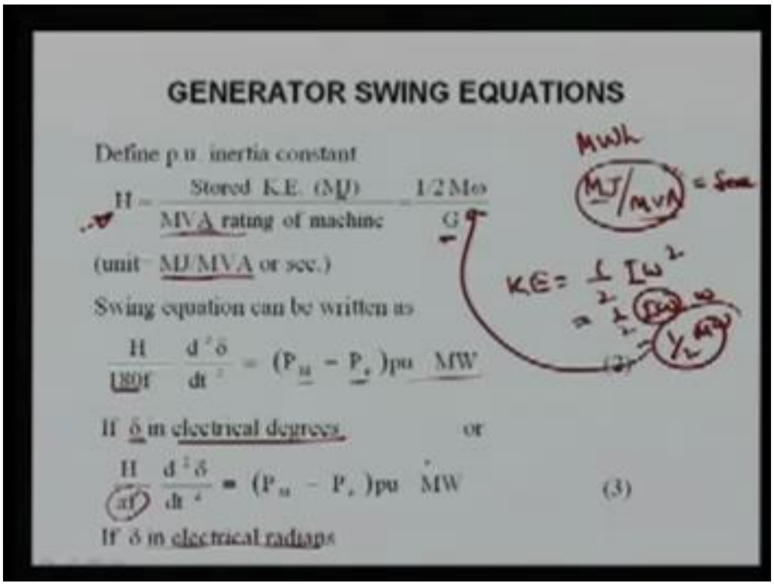

Now, let us again I use this H and how we have to get the information of this H. Now let us define the per unit inertia constant H; that is H is defined at the stored kinetic energy in mega joule divided by the MVA rating of the machine. If you will see the unit of this here the unit is nothing doubt here the unit of the kinetic energy is mega joule. So, I can write here mega joule and over your MVA. So, this your power this is energy, and you know the energy here it is nothing, but I can say MWh. So, here the total, this is nothing, but your second. So, the unit here mega joule per MVA, or it is second, and H is very widely used, because H gives what is the inertia constant, and this kinetic energy is stored, it is nothing, but I can write here $\frac{1}{2}I\omega^2$; this KE. And this will be equal to i can write $\frac{1}{2}I\omega\omega$, and this is nothing, but your M, and here I can say $\frac{1}{2}M\omega$, and this is nothing, but your G, and G is your rating or installed capacity of your synchronous machine.

So, knowing this M, knowing your G and ω; that is a synchronous speed we can get H, or knowing H we can get M vice versa. So, the swing equation can be written as in terms of H. Again if you are using δ in electrical degrees, then it is 180f here double differentiation of δ with respect to time, and here you power minus, this mechanical power minus your electrical power in per unit. So, here we are talking just about the per unit, it is divided by the base unit. So, it is per unit always, because this is H in per unit, this unit at all in per unit. So, it is not in megawatt. So, you have to write the base, but you have to used, and base is of course, is a MVA rating of the machine. So, simply we have divided here. So, if you are using the δ in electrical radians, then here it is a pi f and then we can write this one.

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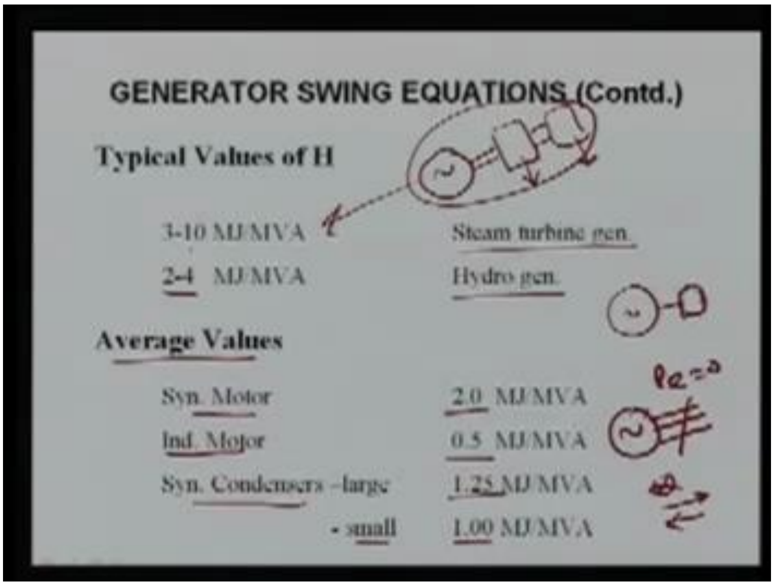

Now, the various types of machines are used; synchronous machines I am talking, and then the inertia constant is different for the different type of machine, and this inertia constant H here it is basically 3 to 10 second for steam turbine generator system. It is from two to four for a hydro generator system. Normally if H is large then we say this machine is large, if it is a small, then machine is small. You can see the Induction motor, this inertia constant is 0.5 second means its size is very small. So, this inertia constant gives information about the size of the machine. Why this steam turbine generator system is having high inertia, because you know here we have this generator, that generator is coupled with the various types of turbines, different stages of turbines. So, they also high speed and rotating mass are there. So, the total H of the system is more compared to the hydro, where one the hydro turbines are there, and then it is coupled with your turbine system here. So, it has a lesser time constant H, means inertia constant.

So, it is 2 to 4 second however the steam turbine depending upon the size, if it is very large then H very high, and if it is small then it is again small. Again, in terms of synchronous motor, because we can use the synchronous motors as well. So, the average value of H, it may two again 2 second. Synchronous condensers, if it is large then it may be average value is 1.25, and if it small then it is one. What is the synchronous condenser. To understand here the synchronous condenser are nothing, but they are some sought of alternators, but mechanical power output, this electrical power output is zero. Means if you are using here this generator, and there is no here Pe is zero. You are rotating to provide the reactive power only Q, then it is known as synchronous condenser. So, you can either generate, or absorb reactive power by this synchronous alternator. We know this by changing the excitation system we can generate or we can absorb the reactive power. So, it is acting as some variable capacitor along with the inductor. See one zone it is giving, another it is also absorbing. So, it can do, and we are not taking any real power load then it is called synchronous condenser.

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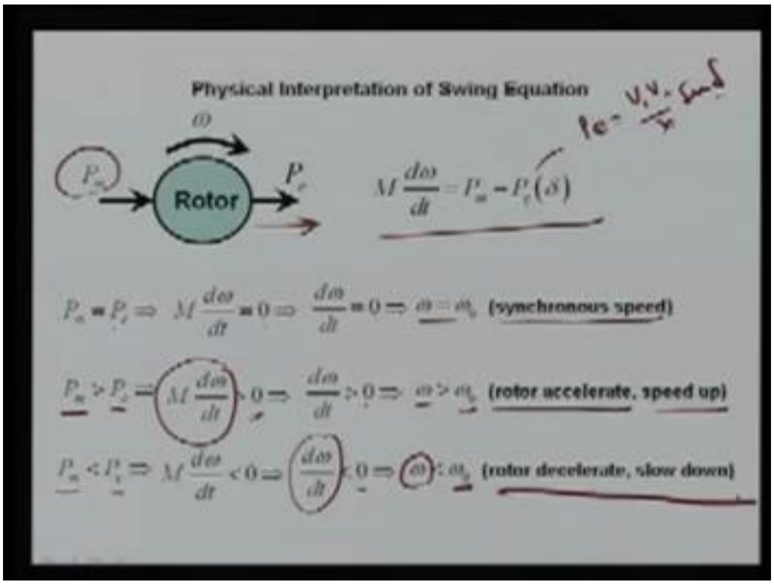

Now, let us see the physical interpretation of swing equation; that is here. Means we can derive this swing equation in terms of change in the speed with respect to time, the derivative of speed with respect to time this M value this Pm minus Pe, and that Pe is a function of δ. Again this Pe is nothing, but Pe; that is approximate the $P = \frac{V_1 V_2}{X} \sin\delta$. Normally these voltages are constant, we assume that constant, so it is a function of sineδ, and it is a non-linear function. So, this differential equation is a non-linear differential equation. So, here it is your rotor, and rotor is rotated by the mechanical input, and then we are taking the electrical output, and this machine is rotating with a speed of ω. Now, if this Pm is equal to Pe what will happen. This component will be equal to zero, means if Pm is equal to Pe, then this component here $M\frac{d\omega}{dt}$ will be zero and this. So, that $\frac{d\omega}{dt}$ is zero; means your synchronous machine is running at the synchronous speed; neither it is accelerating nor it is decelerating, so, it is a steady state operation.

Let us take if mechanical power is more, means here this more than this output, what will happen. This machine will rotate, and accelerate, why it is. So, because always we know this energy is conserved, when this power is coming here, it is going less. So, that power will be stored in this machine itself. And how an alternating, or you can say rotating mass will store the energy, and that is no doubt in terms of kinetic energy means that energy will be stored in the kinetic energy, so the kinetic energy will increase. Therefore, the speed of the machine will increase, so this is same concept. So, if the Pm is more than Pe, than this component will be more than zero. What does it mean. Your rate of change of speed is more, means your this ω will be now more than your synchronous speed, and then this rotor will accelerate, or you can say it will be speeding up. The reverse is also true. Means your mechanical power is less than electrical power, means you are taking more power, than your input, what will happen. The energy which is here in this mass will be taken out, and then means kinetic energy will be reducing; therefore, the speed of system will be reducing, and then we can say this change in ω with respect to t will be less than zero, or the rotating here speed will be less than its normal or can say frequent synchronous speed of the system, and the rotor will decelerate, or it will be slowing down.

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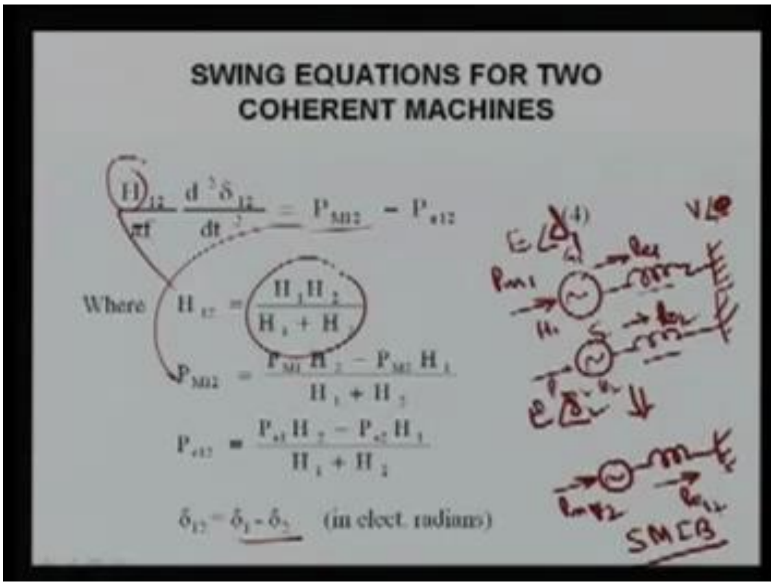

So, the swing equation of the two coherent machines. So, far we have used H. here it is H or M whatever you are talking about. So, if it is one machine here, and then it is connected with your Infinite bus here. Now, if there are two machines; means here you having two machines, and then you want to include and combine together, then we can say this two coherent machine can be combined, and their inertia constant can be also added together. So, here it is your P_{m1}, another machine here it is your P_{m2}, and this another here, this is synchronous machine. So, if we are having two coherent machines; like here machine one, I can say G_1. Here it is your G_2, and it is connected by a, its equivalent here internal impedance. Here again internal of this machine, and this is your infinite bus; means voltage and angle is constant. So, these two machines can be combined together, and then again we can basically this, we can write a single machine here, with the some here equivalent and this machine is like this. So, here this is your, now this Pm equivalent, and again the electrical power which is going the P equivalent here. So, I can say P_{e12}, here P_{m12} I can write.

So, we can represent these two machines here like this one here, if both are in coherent means they are oscillating and rotating in the same speed. So, if inertia constant this here H_1, here is H_2, and the electrical power which is feeding is P_{e1} and here is P_{e2}o, then we can write this H_{12}, will be nothing, but $\frac{H_1 + H_2}{H_1 + H_2}$ we can have this one expression for here. And this your P_{m12} can be related at the Pm 1 H 2 minus Pm two, means Pm 2 here multiplied by this H one. So, this $\frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2}$ Similarly the P_{e12} just combine here, will be the P1 here multiplied by this H2 minus here P_{e2} multiplied by H1 and then divided by here and the angle here. If this is the angle $E_1\angle\delta_1$, here this is $E_2\angle\delta_1$o. So, the angle here the difference will be your $\delta_1 - \delta_2$ with reference to zero, and then we can go for equivalent. So, this is again why we are going for if we are having two machines, then you can again go for, this is nothing, but your single machine infinite bus system SMIB, and then you can go for classical approach and then you can analyze it. So, for analysis for the transient stability for this single machine infinite bus system, we can use the equal area criteria, and that is very popularly used.

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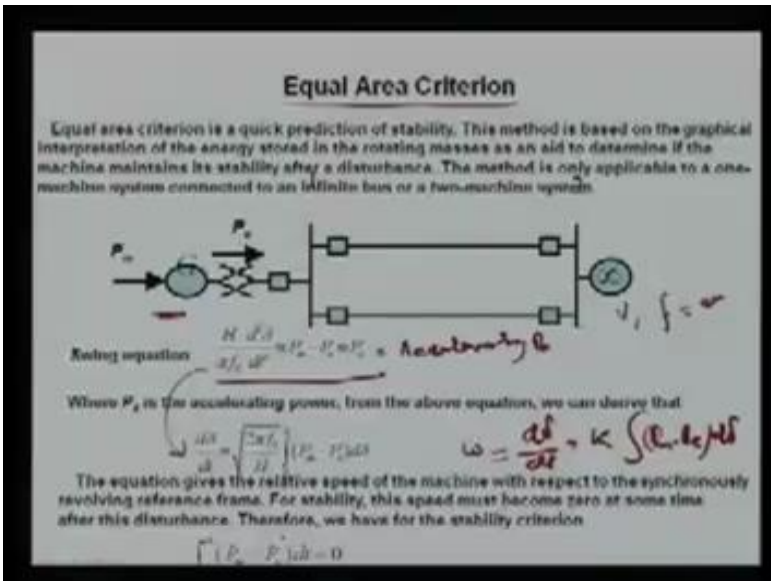

So, the equal area criteria, is a quick prediction of stability. It is a very quickly you can analyze, whether your system is stable or not, following the fault. This method is based on the graphical interpretation of the energy stored in the rotating mass, as an aid to determine if the system machine maintains its stability after a disturbance. The method is only applicable to one machine system connected to an infinite bus or two machines system. Means if the two machines basically what is one machine infinite bus system, you can see again the same example which I explained earlier one. Here, one generating system Ef. Here we have the infinite bus, means this is also a machine, which have the high inertia. Means large inertia, infinite inertia, and where the voltage and the frequency are the constant. So, it is only applicable for the two machines systems means this is two machines, or other words you can say it is the single machine infinite bus system. So, it is only limited to that, it is not possible to go for the multiple machine system. If the machines are several and connected by the different transmission lines, then we have to go for another analysis.

We have to solve the equations the non-linear equations by help of several differential equation solution methods. So, the swing equation here as again I can write; that is written by this one, and this $P_m - P_e$ is nothing, but it is Pa it is called the accelerating power. So, whether Pa is zero or it is positive or negative, it decides whether your machine accelerating, your machine is decelerating, or it is in steady state operation. So, this equation, now you can see, this equation can be written as in this form. Means here from this equation you can see I can write this $\frac{d\omega}{dt}$, is some constant integration of $P_m - P_e\ (\delta)$, we can difference. We can derive this equation no doubt very easily. We have to write the two differential equation and finally, or you can differentiate this equation. Here you will find this equation without any problem. So, this equation gives the relative speed, what is this. This is your relative speed of the machine, with respect to synchronously revolving frame reference, for stability this speed must be zero, at some time after this disturbance, therefore, we have for the stability criteria this must be zero, means we can have this δ from δ_0 where it was operating to the δ, if we are integrating for that period then it will be zero. To understand this it is very easy.

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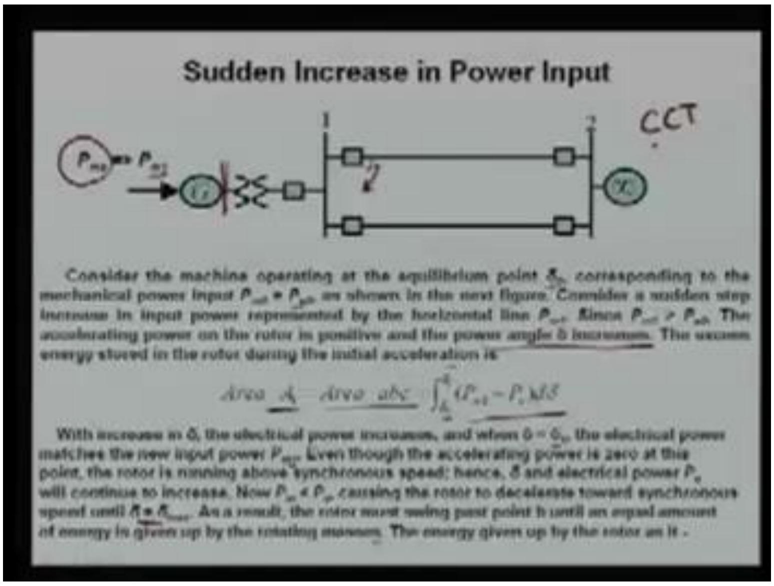

Let us consider the sudden increase in power input, means here this is your system, this is infinite bus, where we have suddenly. Earlier it was operating at the P_{m0}, and then we have increase this P_{m1}. Suddenly we have increase in power input. Consider the machine operating at the equilibrium point delta not, what is the δ_0.

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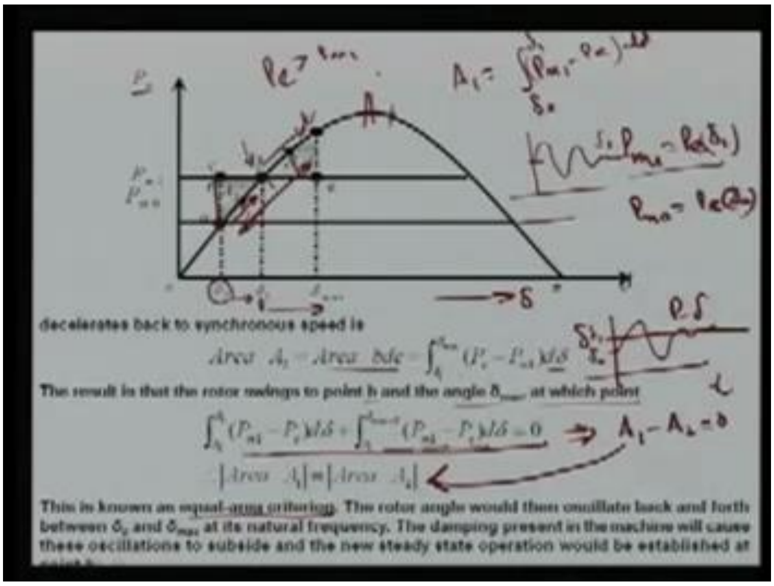

You can see here, this is your delta not. This is your δ variation, this is your Pe, this is nothing, but your Pe δ curve of the combined component system, including you are here this infinite bus, here another terminal, here your generating terminal, here your generator, and this is your transformer included everything. So, it is operating here as I said, here it is operating, because mechanical power P_{m0} is this axis. So, the operating point will be your A, where it is intersection of this P δ curve with this your P_{m0} curve that is constant. So, it is a constant we have assumed. So, this δ_0 is your initial operating angle, and corresponding to this mechanical power P_{m0} that will be not equal to your P_{e0}; means your electrical power is equal to your mechanical power in the steady state, as shown in the figure, consider a sudden step. Sudden step here, you can see suddenly it is increased here, to P_{m1}. Now, what will happen, now its stable, this is Pm, so your B point is the post increased stable point, this B point, because at that point here this P_{m1} will be equal to Pe, now it is I can say δ one, where it is equal.

So, at the Initial it was P_{m0}, it was equal to your Pe δ_0. So, this B point is your another post increased power stable. So, just we have seen the sudden change; since P_{m1} is greater than P_{e1}. The accelerating power on the rotor is positive, because we have increased the Pm, means at that point your Pe was P_{e0}t, so once you are increasing your machine will accelerate, because if Pm is more than Pe as in previous slide I showed you that it will be increasing, and it will be accelerating, so your power δ angle δ will increasing. Means here your this δ, here it will be increasing like this. So, it is increasing. The excess energy is stored in the rotor during the initial accelerating is here area A1; that is that is area ABCD; means here this area, this energy is that is ABC, that area you can say area A1 is the stored energy of this machine; that is ABC and it is basically integration of δ_0 to δ_1, means here is a δ_0, means this curve minus this curve if you are integrating from here to here, means your area A1 will be nothing, but integrating from zero naught here, to δ_1. Here this is a constant curve. So, this is your P_{m1} minus this curve; that is Pe, and again it is a δ.

So, this area is just this area here it is written as this one. With the increase in the δ the electrical power will increase, and when this δ is equal to δ_1, the electrical power of machine matches the new power that is P_{m1}, even though the accelerating power is zero at this point, the rotor is running above the synchronous speed. Hence the δ and the electrical power Pe will continue to increase, and now this Pm will be less than Pe and causing the rotor to decelerate the synchronous speed until again this δ is δ_{max}. As a result the rotor must swing past point B, until an equal amount of energy is given by the rotating mass. The energy given by the rotor as it is. Now what happens, now this from here it has to go here, and your rotor is now accelerating. Once it is accelerating here, it has reached, but your rotor is accelerating at this point B, it is satisfied this, but it is your machine is accelerating. So, what will happen it will try to decelerate and it may go up to this point D. And here, again now speed of the machine will be synchronous speed.

Here it was synchronous speed it started accelerating, due to this mechanical power is more, and here it is, it saw that Pm is Pe, then it will be retarding, but the δ will be keep on increasing and it will be increasing up to D, and again it will be, at this point it will see that your Pe is more than P_{m1}. So, it will again retard it back. So, this is basically it will be going back and the system damping will try to stabilize at certain point B. So, the deceleration here back to the at the synchronous here area A two; that is the area BDE will be nothing, but the integration from the delta 1 to δ_{max}. Here at this δ_{max}what is happening, the machine is not accelerating, machine speed has. It is rotating at the synchronous speed, but their electrical power is more. So, it is integration between this and this. So, it is Pe - P_{m1}. It is this curve minus this curve, if you are integrating from here to here. This delta 1 to δ_{max}; that is Pe - P_{m1}.. If you are integrating with δ then you can see the total.

The resultant is the total swing is to point B; the angle δ_{max}at which the point, you can add these two areas; that is delta 2 to delta 1; that is P_{m1} - Pe(δ), the pervious area plus here from this area, if you are, means from the previous equation here basically. From this equation if you can write this equation in here. Means, it 0 naught to 0 1 P_{m1} - Pe. Here again delta1 to δ_{max}, here the same equation, what is happening. now you can see this is nothing, but your minus area one. So, I can write this is nothing, but area A1 minus area A2 is equal to 0, this is the area which we have taken. So, now what is happening from here we can say, the area A1 is equal to the area A2, and this is the case of the stability, if the system is stable that must be satisfied. So, if the area here that is accelerating energy, kinetic energy, here and is equal to area A2 then it is known as system is stable, if area 1 is equal to area two, and this is known as equal area criteria. The rotor angle would done oscillate back and forth between angle here.

So, basically your rotor will be oscillating between these two points. It is not going beyond that range it is here again less than this, and its natural frequency. The damping frequency in the machine will cause these oscillations to subside, and the new steady state oscillations would be established at the point B after certain time. So, the machine is here basically if you see it is oscillating your δ here, from here it is oscillating and finally, it is new delta 1 it will be stabilized. So, if you are plotting with respect to time what will happen. Here this is let us suppose your time, this is δ. Means you are machine here operating initially at delta not, when the disturbance has taken up. Means just you have to increase of the sudden input of the generating system, what will happen, here it will be oscillating, and the finally, it will be steady state at the δ_0. So, this is basically, and this damping is system damping, because there is some damping in the system, and it will be stabilized and the system will be stable. But there is possibility if there is again excessive change.

This point D has gone beyond this then machine will not come back, and then it is set the machine is system is unstable. So, in this lecture we saw that transient stability analysis, using the classical approach we made certain assumptions, but it is very advantageous if you can form the single machine infinite bus system, and then you can see the behavior of that machine with reference to certain fault in the system. It is quickly giving the idea with your system is stable or not without integrating the system. Means without solving the differential equation here, with the help of the two areas we can know, whether system is stable or not.

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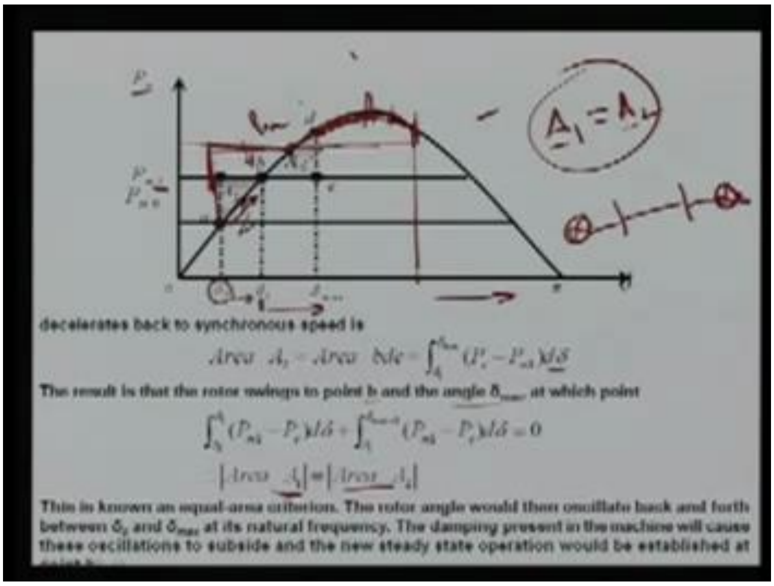

In this case we saw that area one, here is equal to your area 2 for the stable system, but if this area is more than your area two, the system is again this area kinetic, this area basically always gives information that whether you are at the verge of the stability, or whether your system is very strong in the sense of the stability. So, this equal area criteria is used to know that whether how much you can load your system; for example, in this case you can see your system is operating here. Now, it is always advantageous to know that how much you can load in the system. Means in this case it is a P_{m1}, there is a possibility you can go up to P_{m2}. Here it is you P_{m2} this point, and then there is a sudden change here your what happen, your system may go somewhere here, and then you are somewhere angle here. So, what now what will happen now this area is the different area, then what you have area here. So, with the help of we can know what is the maximum loading, that suddenly we can put on the generator, without losing the stability of the system.

And this classical approach as I said it is limited to the single machine infinite bus, or the two machines here, this is another machine, and this is another machine then we can analyze, and then we can know using your equal area; means your area one; that is your energy storage here; that is accelerating. So, energy that you are giving that is increase, that energy in terms of accelerating and here is a decelerating. So, in this lecture we have seen, that only just I have taken only one case, when there is a sudden change in the input to the power. In the next lecture, we will see that how much we can load, and at the same time we can also see if there will be any fault in that system here. There may be some fault here at this line at the end, and if it is tripped out, then your whole your reactance the topology of the system will change.

There may possibility to do this fault this circuit breaker is tripped, and then we will see what will happen during the fault, what will happen the after the fault, and what will be the system stability criteria. We can also determine what will be the critical clearing angle. Means at what time you will clear your fault; otherwise you system will unstable, and then we will see with the help of equal area criteria, you can determine your critical clearing time; normally it is called CCT. So, to know their stability of the system; one approach here is the CCT, means what is your critical clearing time. This also gives the stability measures, at the same time some another approached is used; that is called energy margin functions, how energy margin you have, and that is also used to assess system stability in event of the transient faults; transient disturbances, or I can say large disturbances and for that you have to analyze your transient stability analysis.

Thank you.