Power System Operations and Control
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Module -2
Equipment and Stability Constraints in System Operation
Lecture 7

Welcome to lecture number seven of module two. In previous lecture, just we have seen that equal area criteria can be used to analyze the stability of the system, and that is only applicable for this single machine Infinite bus system or two machine system. And the last example we saw, that if we are increasing, the sudden increase in the power, how your system will be reacting, and what will be the stability of the system. Now, let us determine what will be the maximum power that can be increased, on any machine over on over what is loading, present loading, and that is basically governed by the stability of the system.

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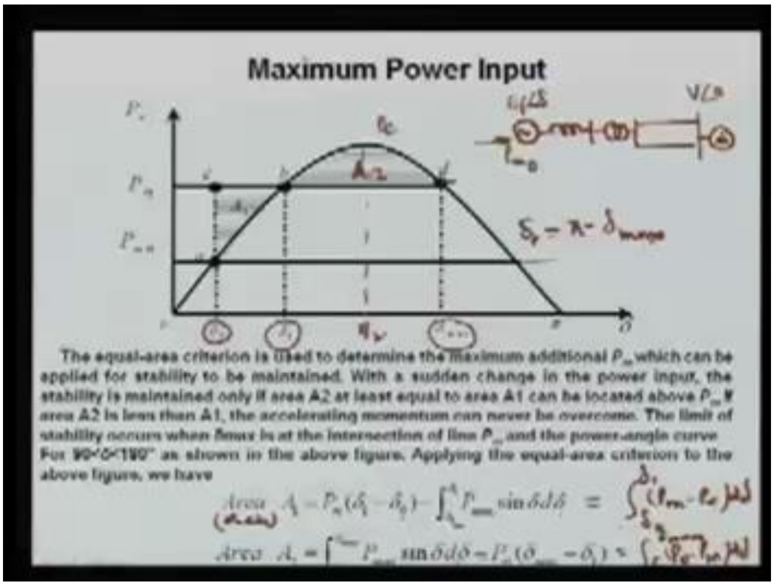

Let us take your machine is loaded again this machine, now your, I will draw. This is your synchronous machine. It is a here, it is a reactance, and this is the transformer, and then we have two lines, and then it is connected with the Infinite bus here, that is your Infinite here. So, this is your voltage angle zero, $\angle 0$; that is V infinite, V_∞. Here it is Ef angle δ, $E_f \angle \delta$. So, in this case, here your Pm that is your mechanical power; that is entering, and now I can say this is P_{m0}. For this case, your P_{m0} here where your machine is operating at angle δ_0, and this is your Pm and this A point; means A point will be the operation when your feeding, this mechanical power is P_{m0}. As again I explained that the operation point will be the Intersection of this mechanical power characteristic; that is Pm line here, and then that is your Pe line, this is your Pe curve, which is a function of sine delta. So, the Intersection here the point A now the question I want to know, how much power that we can increase that is your Pm you want to calculate, so that our machine should be stable without losing synchronous.

To know this we have seen that is equal area criteria, is used to determine the maximum additional Pm, which can be applied for stability to be maintained, with the sudden change in power Input. The stability is maintained only If the area A2. This area, this is your A2, at least equal to area A1. Means this area here, it will at least equal this, means this area may be larger, but it should not be less, but If this area is less than this A1, then your system is not stable. So, we want that maximum power that we can Inject, suddenly we can change on the loaded power system, or on this generator, that can be determined; that means, area A1 will be equal to area A , and then we can what will be the determine the Pm value. To determine this Pm we have to calculate this δ_1, or we can have calculate this δ_{max}, because this $\delta_{max}$is nothing, but here δ_{max}will be, or you can say is a $\pi - \delta_{max}$ will be your δ_1. So, to determine δ_1, we want to know δ_1, then we can get the point Pe, because the Pe characteristic we know. If we put this δ value in that one here; δ_1 in that Pe characteristic, we will get the point Pe, and thus we can determine the Pm. We will see in the following lines. So, here now this area one, here is accelerating power. Means this area as I said, so to maintain the stability area A2 at least equal to area A one, can be located above the Pm, If area A2 is less than area two. If this is less than area two the accelerating momentum can never be overcome.

The limit of stability occurs, when $\delta_{max}$is at the Intersection of the line Pm here, means this Pm we want to determine, and the Intersection of Pe and this Pm will give your δ_{max}, and this will be always here, will be δ will be more than ninety degree this side, means here in this zone, here it is your pi by 2 that is 90 degree. Applying equal area criteria to above figure, we can have this area A1. This area that is acb here you can say area acb, or you can say area one will be nothing, but integration from δ_0 to delta 1 Pm minus this Pe curve. So, means this is nothing, but here it is Integration of δ_0 to delta 1. Here this Pm minus Pe d delta. So, this Pm is constant. So, Integration of this we can take the Pm outside. So, it is a delta 1 minus delta 2 first term, and another term this Pe with the minus sign we are integrating from δ_0 to delta 1 that is P max sine delta d delta. Similarly, this area 2, it will be the Integration here from delta 1 to delta max. And now it is Pe minus Pm, because this curve is over, and then this minus this Integration from here to here, will be your area 2. And then we can know this Pe is a function of sine delta. So, it is integration from delta 1 to delta max P max sine delta d delta minus this Pm is constant, so delta minus delta 1.

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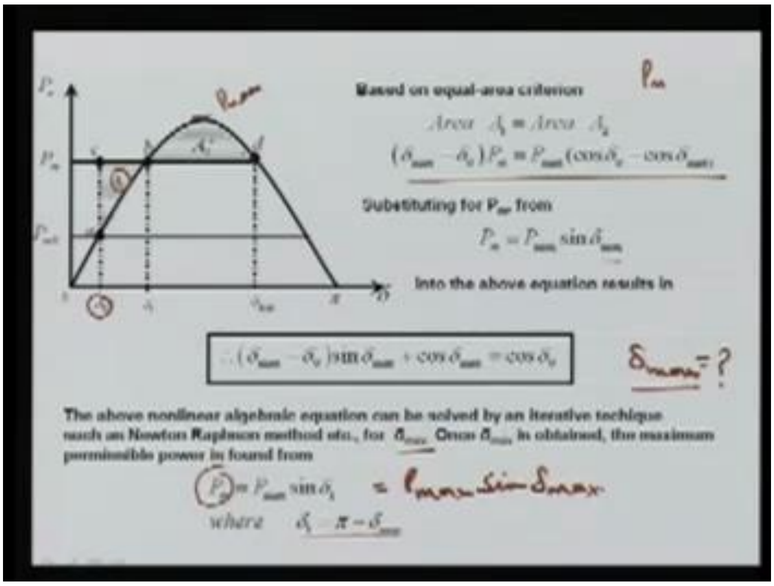

So, based on the equal area criteria to the system is stable, at least the area A2 should be equal to area A1. So, this area one for maximum Pm, the area A1 should be equal to the area A two, and then from the previous equation, If we will equate we will get this expression. Now, here the Pm we are getting here, and that Pm is nothing, but again we can get the Pm is equal to $P_{max} \sin \delta_{max}$. Means this value the Pm, will be again that will be your $P_{max} \sin \delta_{max}$ we can get this point, and then you can substitute this. So, we will get a expression of us a δ_{max}value, and this is a non-linear equation; means we have the sine delta max, we have another δ_{max}here, we have cosine of δ_{max}, and cos δ_0 is known to you. δ_0 is original, where your machine was operating, that can be calculated, because Pm is known to you, P_{m0} here. So, this equation is a non-linear algebraic equation, and can be solved by any Iterative techniques, including your Newton Raphson method, Gauss Seidal methods, Gauss Iteration method so on and so forth. So, that we can determine δ_{max}. Once this δ_{max}, means here our Intension is to find δ_{max}. Once you can determine the δ_{max}, then the maximum power Pmax will be $P_{max} \sin \delta$`here; this Pm, or you can get this one directly.

So, you even though where δ_1 is your $\pi - \delta_{max}$, or here you can write this simply $P_{max} \sin \delta_{max}$. So, this will be the maximum power that you can load above this P_{m0}, without losing their stability. So, this is one way to determine that how much you can load suddenly. Again question here, if you are continuously increasing, then your maximum loading here is up to Pmax. But the sudden increase here for the sudden, means it will be accelerating and decelerating, because here is an acceleration power, here machine is a decelerating, here this zero, and here is again accelerating so on and so forth. So, since why this area A1 is called accelerating, because the Pm is more than Pe, so what will happen. Your machine will accelerate, and it will store the kinetic energy; however this area A2 is decelerating area, because your Pe is more means the power electrical power that you are taking from the system is more, means here, then your power that you are feeding to the turbine, so, this is a decelerating. So, accelerating power must be equal to you decelerating power, then the Pm can be determined.

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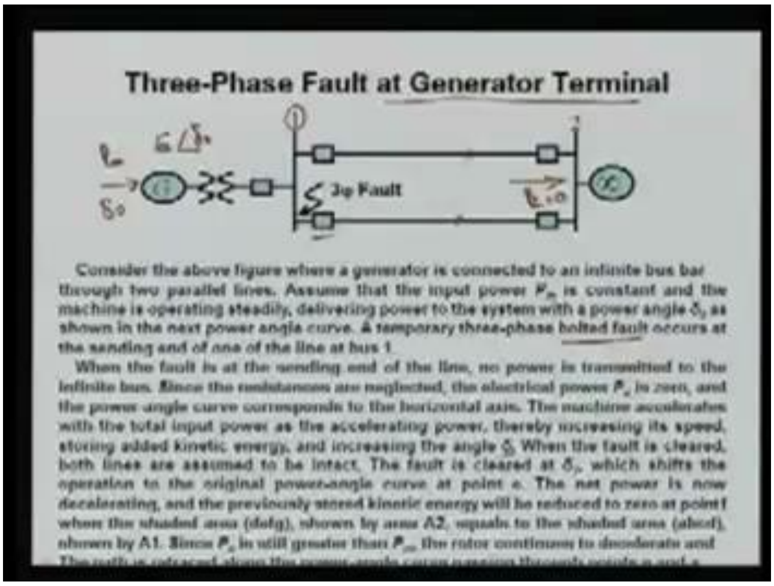

Now, let us see another consideration; means how this equal area criteria, can be useful for determining the other stability criteria. Let us suppose there is some fault. So, if we consider the three phase fault; the three phase fault can be a bus fault, it can be in the line fault, and it can be anywhere in the line either in sending or in receiving end or in between line. So, let us first consider the three phase fault at the generator terminal, here that is this terminal we are putting. Basically this is a generator, this is a GT generating transformer and this is a terminal, where we can say this is bus one. So, again this system is similar to our previous system; that here a generator is connected to infinite bus bar through two parallel lines; line one, and line two. Assume that the input power again here is Pm is your constant, and the machine is operating steadily delivering power to the system with an angle delta δ_0. Here means your δ_0 is Ef means, if you are here I can say E$\angle\delta_0$ during their steady state, and that will be shown in the next curve. Our temporary three phase fault that is bolted fault. Bolted fault means the fault without any up fault resistors, means directly three phase to ground fault, occurs at the sending end of one of the transmission line at the bus one here; three phase fault let us suppose has occur here in line two. When the fault is at the sending end of the line, no power is transmitted to the infinite bus. If the bus fault is here, what will happen. There will be power which will be flowing to Infinite bus. So, your Pe at that case, it will be your zero. Since the resistance is neglected of the line, the electrical power Pe is zero, and the power angle curve corresponding to the horizontal axis. Means you can see what will happen.

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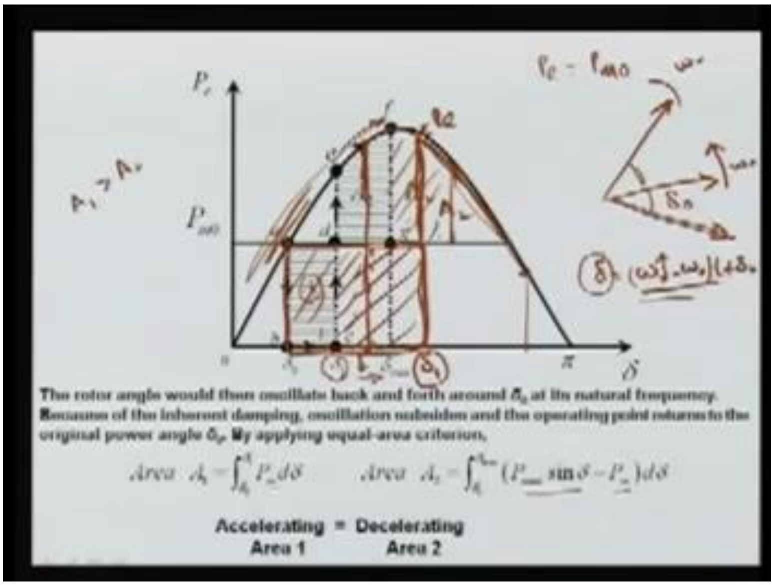

Your this Pm here A was your operating point, because your Pe here is equal to your P_{m0}; that is P_{m0} bus you are getting. So, suddenly there is a fault, what is happening your Pe becomes zero. So, this is suddenly coming to this point, and then till the fault is persisting here, it is going and at the δ_1, once the fault is cleared, and then again it is going back to the Pe. Again the Pe is going to be delivering to the Infinite bus. So, in this case the machine accelerates with the total input power, as the accelerating. Means the total Pm, because we are not taking Pe from that generator. So, this machine will get accelerated, means your input power is more than output power, so, whatever power you are feeding as you know the energy conservation law, then this energy will be stored in terms of kinetic energy. And if the kinetic energy of any system Increasing, means speed of that machine rotating mass is increasing. So, there by it increasing its speed, storing added kinetic energy, and increasing the δ. If machine is accelerating what is happening, now delta will be changing, the δ is angle if you see what is now question is delta.

δ is, let us suppose this your reference angle, any arbitrary reference angle. Here this is your δ_0, and your machine here this is a rotating two fluxes here. This is your synchronously rotating here, and it is actual rotor speed is ω_r; this is ω_0. So, this difference that angle; that means, your delta is $\omega_r - \omega_0$; that is t with δ_0. So, if though machine is accelerating this is increasing, this is your synchronous speed. So, what will happen this term is increasing with the time. So, your δ is increasing. Here δ_0 is any reference, so this δ_0 will be zero. So, once your machine is accelerating, means your δ is accelerating, increasing, and once it is decelerating means your δ; that is angle δ I have defined with this one, it is a decreasing. So, that is retained here the Increasing angle δ. When the fault is cleared, and it was assumed that the fault was temporary, and the fault it cleared without tripping the of the line, because it was the bus fault, and the bus fault is cleared. So, both line assumed to be Intact. Means your Impedance of the system is not going to be changed. So, again it will follow the Pe characteristic. The fault is cleared at δ_1, which the operation to the original power angle curve at point e; here this is your point d. Means your fault is cleared here, means it will suddenly follow this δ curve, because this is Pe here Pe is zero.

So, this Pe it was running at this point, suddenly due to the fault this Pe becomes zero; this is your zero Pe this is a Pe axis. And once fault is cleared, then it will again just it will follow the Pe curve here; that is here it is your Pe curve it is a function of $\sin\delta$. So, it is coming to your e point. The net power is now decelerating, and the previously stored kinetic energy will be reduced to zero at point f, when the shaded area is shown by the area two, equal to the area shaded in abcd shown by this. Now, you can see this area A one, as I said this one is your accelerating power. Means where in this area A1 your machine is accelerating, and you are storing kinetic energy. Now, once this kinetic energy is stored in the machine, what is happening at the e point, you can see this is now electrical power is more than your mechanical power. Means you are taking more power. So, whatever the energy is stored in the machine that will be taken out, and this area here it is called your decelerating. Means here it was accelerating, now from here it will decelerate till f, when again this your acceleration will become zero.

So, again you can see the δ is keep on increasing here and it is δ_{max}. Here again your decelerating power is zero, then again it will find that this is more than this it will try to retard, and again it will coming back. So, it is again there will be oscillation around point A, and finally, due to the damping of the system, it will be damp out, and your final steady state operate of the system it will be stable operation. Now, the question again here arise, If this area is less than this area. If this is area is more A1. Means your area A 1 is greater than area two, and till what point, till certain range here, then machine will be unstable. So, we can see the rotor angle would then oscillate back, and forth around here this not, means here it will be oscillating at its natural frequency, because of Inherent damping in the system. The oscillation subside, and the operating points return to the original power angle δ_0 here; that is at point A.

So, by applying the equal area criteria, this area A1 that is integration from δ_0 to delta one, here that is Integration of, here this completely Pm minus this zero,i.e, $\int_{\delta_0}^{\delta_1} P_m d\delta$. So, this Pm delta, and area A2 will be δ_0 integration from delta 1 to delta max. Here this Pe minus Pm d delta,$\int_{\delta_1}^{\delta_{max}} (P_e - P_m) d\delta$ means accelerating area one will be equal to accelerating area two. Now you can see, how much you can move ahead this is f. Let us suppose you have increased further, means it is not clear at this one. Let us suppose your machine, fault is clear at this point what will happen. Now this area here it is up to this area, and once fault is clear now it will follow here, and then it will go up to here area now area two will this much completely, and it will be your area two is this much. So, still your system is stable. Let us suppose further you have increased. Let us suppose your δ_1 your fault is cleared after longer time. So, this area now is registered here, and now you have come down here, and again you can see the area this is bounded here A2, is now less than your area A1 complete this area, this area here is larger than this. So, in this case your A1 is more than A2 and the system is unstable. So, what we normally do.

We try to determine what should be value of δ_1. Here what I am going to explain, that how much this δ_1 that can be maximum value means the fault can be cleared, so that our system will be stable. Why we are very concern about this this δ_1; that is maximum δ_1 at which we can clear the fault at which our system will be stable, this is known as the critical clearing time. Critical clearing angle If you are calculating this δ If you are calculating in the time then it is called the critical calculating time CCT, or here δ_1 is your critical clearing angle.

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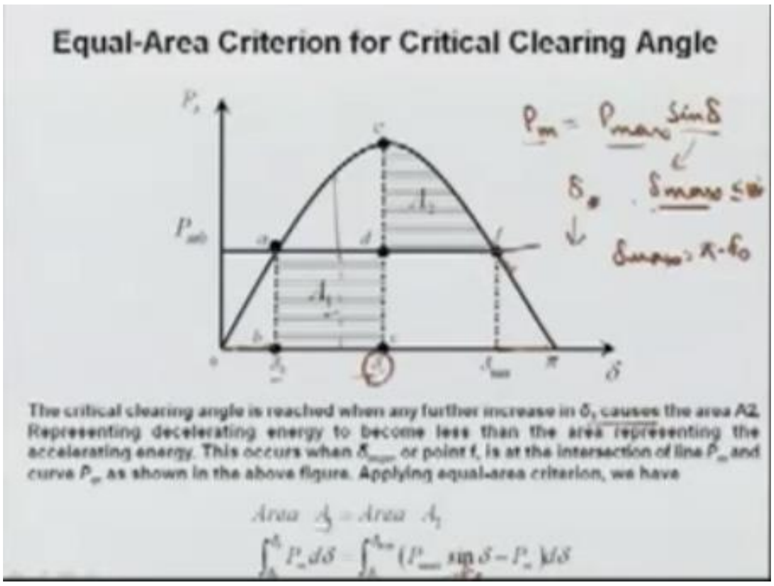

In the next slide we will see; that is δ_c is critical clearing time, that is what will be this value you want to determine this, so that you fault is cleared before that. So, in that case suppose your fault is cleared here. So, this condition will be always cleared here this area A2 will be always you can say more than your area A1, means your system will be stable. So, from here this complete area is always larger, and then A one. So, we can say our accelerating power will be always less than your decelerating power. So, the system will be stable. To determine this critical clearing time, that is very important. The critical clearing angle is reached when any further Increase in δ_1; that is δ_1 here, causes the area A2, representing decelerating energy to become less than the area representing the accelerating energy; that is your A one. This occurs when the δ_{max} here or the point f is at the Intersection of the line point Pm, here this is line Pm, and this is curve Pe then it is Intersection point f as shown in this figure.

Now for this critical value, here this point cannot come below, because this is decelerating up to this point Pe is more than Pm. So, this is decelerating. You cannot go beyond this because if you are coming here what will happen your Pm will be more than Pe and again it will accelerate. So, your system will be unstable if it is moving beyond this point. So, we have to this area A2 should be less than this δ_{max}, and then we have to determine this δ_c, so that we can know if your fault is cleared less than that angle, then we can say our system is stable. To be very critical for getting the maximum value of delta at which it is cleared for this stable operation. This area A1 should be equal to the area A2 here. So, area A1 that is your ABCD will be equal to your DEF be, this is area two. So, this area for this case it is you Pm Integration from delta naught to delta c, is equal to your Integration from delta c to delta max. Here Pe minus Pm, so it is Pm max here this is nothing, but pe minus Pm delta d delta. ,ie. $\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{max}} (P_e - P_m) d\delta$

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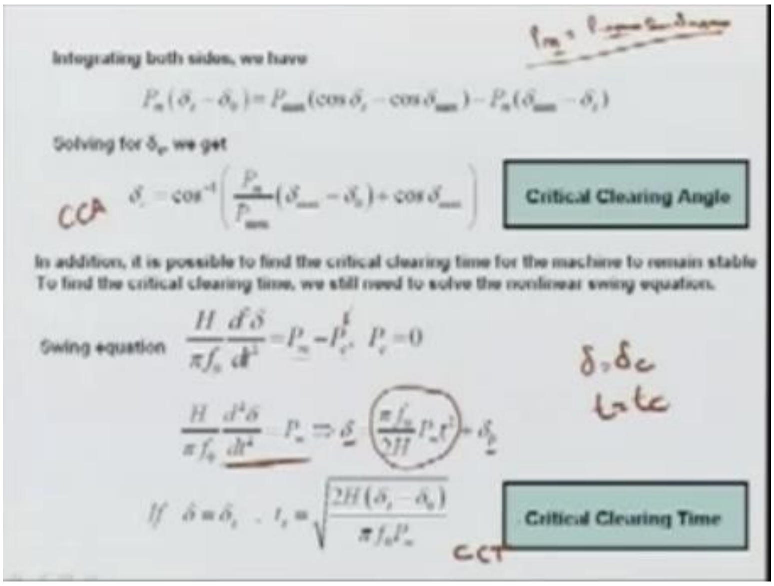

So, integrating both side, you can say left hand side here, your $P_m(\delta_c - \delta_0)$ will be there, your another side here it is a $P_m\,(\cos\delta_c - \cos\delta_{max}) - P_m\,(\delta_{max} - \delta_c)$. So, here this you can say this δ_c this δ_c will be cancelled, and we are getting a 1 equation of $\delta_c$very easily, and that we can say $\cos\delta_c$ will be this function or we can say δ_c will be $\delta_c = \cos^{-1}\left\{\frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}\right\}$, and δ_{max} can be determine again here we know this is Pm, is your Pmax, here $P_{max}\sin\delta_{max}$. So, if you have solve this knowing Pm Pm as well as Pe max, you will get the two values of δ_{max}. Means here in the previous case, If you will write this equation this Pm will be equal to this is Pm at this f point, it is your $P_{max} \sin\delta$If you will solve here, knowing this Pm, knowing this you will get the two values of delta, one you will get the δ_1, another δ_{max} and these value less than equal to 180.

So, since we are talking up to the pi degree, so you will get the two values. So, first one the small value will be your δ_0 here in this case; that is here, and this value δ_{max} you can get the another value that will be again the $\pi - \delta_0$ will be the δ_{max}, means here δ_{max} will be nothing, but your $\pi - \delta_0$. You can see from here it this angle will be equal to this angle. So, now we can get this by, because you are knowing this δ_{max}. We know this δ_0 and then Pm Pmax we know, and then we can determine the critical clearing angle and this is known CCA, and that is one of the major system stability. Means if your fault is cleared before this angle your system will be stable, and if it is cleared after this your system will be unstable. In addition it is possible to find the critical clearing time, because angle is very difficult to say what is angle and also measure, and the time is very important means at what time your fault is cleared, and that is called your critical clearing time and that can be calculated again by using the swing equation.

Your swing equation is nothing, but your $\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$, and this Pe is nothing, but your zero, because here in this curve you can see this is your Pe is zero. So, you want to Integrate from here to here, where the Pe is zero, or we can say this is Pe we can write the delta we can differentiate this equation ,we can solve this equation. Here we will get the $\frac{\pi f_0}{2H} P_m t^2 + \delta_0$, because it is starting from δ_0. So, δ_0 is added, and then this is with respect to time thus delta is increasing. So, δ is equal to now δ_c If you will put this value. So, at that time your t will become tc, means at δ is equal to your δ_c, your time will become your tc. So, the tc will be nothing, but here, it is delta c minus delta naught here, twice H is multiplied, divided by pi f Pm under root it will be your tc,$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_m}}$ and this is called your cct; that is a critical clearing time. So, this time can also be determined.

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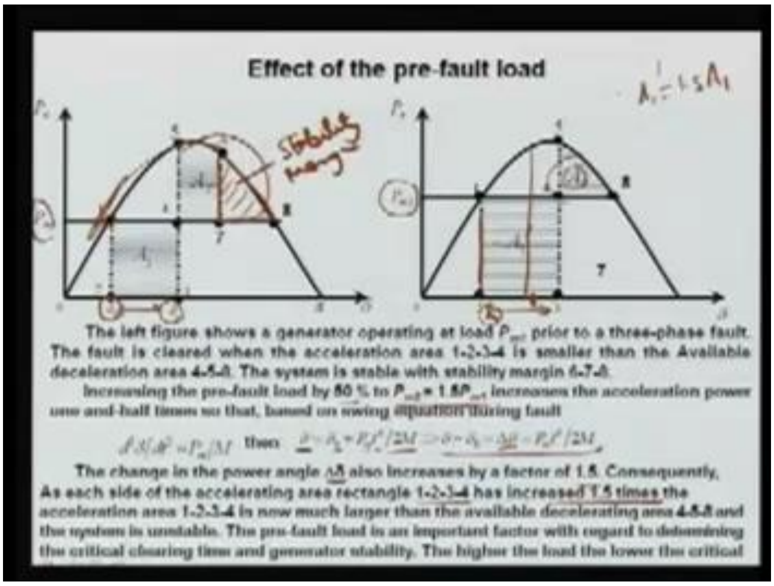

Now, let us see the effect of pre fault load, means earlier your system was loaded at the Pm one. Now in this what we are going to see let this loading. In this case again your fault same system; there is a three fault at the bus say, the curve is similar to the previous one. So, it was operating at the point 1, that is angle at δ_0 the fault was cleared after the delta c and this regard the A1 it will equal to A2 and system is stable. Now you want to your machine was loaded more than that your Pm 1 value. Let us suppose your value is Pm 2 and that your value is Pm 2 1.5 times more. Means this is more than what will happen. In this case also critical clearing time let us suppose the same fault is clear for the same duration. So, what happens you can say, here again the point 2 it was δ_0, and again this δ_0 is different from this δ_0. But this duration from two to three is same from here as well as here. So, this Pm 2 is 1.5 times more this I have already written Pm 2 is equal to 1.5 times of Pm 1 here.

Now, you can see this now area A 1; that this time is same, this area is increased by 1.5 times means this area was A 1. Now this area one A prime mean A 1 prime, will be 1.5 times of A 1, because this height was Increased by “one point five” times where this axis same. So, this area is increased. Now you can see, and at the same time your area 2 is in decrease. Means here what is happening, this area is now shorter, smaller. So, we can see this again, that the Increase in the pre fault load by 50 percent, the Pm2 Increases the acceleration power, one and half times here. It is increased by one and half times. So, based on the swing curve equation during the fault, here $\int \frac{d^2\delta}{dt^2}\frac{P_m}{M} = \frac{\delta_0 P_m t^2}{2M}$. Again it is written in terms of M your momentum. And therefore, we can see this is your angle. The change in the power angle δ here, this δ also increases by a factor 1.5 times consequently, at each side of accelerating area rectangle one to three has increased by 1.5 times this is Increased by 1.5 times the accelerating of one two three of here means this area is Increased than this one, is now much larger than allowable decelerating area 4 5 8, means this area is much less compared to this area.

So, what happens if your pre fault load is more, your system is again critical clearing time is lesser. Means you must clear your fault well before here so that we can have area A1 better more than your area A2, means we should have some margin we will see. Now question what is this area, because you A1 is equal to A2, your machine will here decelerating is equal to your accelerating power, and finally, your machine will oscillate around the point one, and finally, it will be stable, it will be stable. So, area 6 7 8 is called the margin, and that is known as stability margin, stability margin of the system. So, if this margin is larger, than we can say our system is more stable. So, this gives your relative stability. Means this margin is an Indication of stability of the system. in this case you can say the stability margin, If area is equal to this stability margin is zero, means in the case at the certain critical clearing time, as I said here in this critical clearing time, this area is equal to area two there is no margin here left out. So, if your system is cleared after this angle your system will become unstable, but If it is cleared here, then we will have some margin here, and we can say our system is stable.

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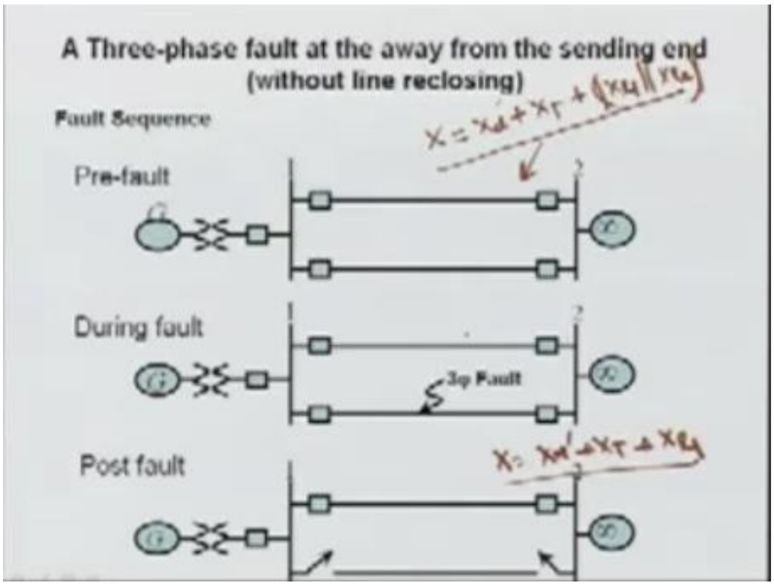

Let us say the three phase fault in the different scenarios. So, far just we have discussed the three phase fault; that is occurring at the bus one, but let us it is a fault is occurring in the line, and followed by the tripping of the line. So, during the pre fault again your system as the previous, just we have consider all the slides, same here your generator, generating transformer two lines and connected to the Infinite bus system. So, during fault what is happening, now fault earlier it was at bus one, now I am taking fault as line two, and that is your three phase fault. And this fault is cleared only after tripping of this line. So, this is your post fault scenario, means your line here is tripped fault is cleared. Now what is happening, the system impedance here, is a different here due to the fault Impedance is different, and now after tripping this Impedance is a different. Now in this case what will be your X, and using this X here it your X_d'; that is of your generator, your Xt of your transformer, and plus hereX_{l1} parallel to X_{l2}, If you remember this value with I was using in lecture number six. So, this is your X in this case. Now in this case what will be happening, your X will be your X_d' your Xt plus your Xl 1, only one line is there. So, now you can say this Impedance is lesser than this X one, because the parallel Impedance of the two systems is always less than Impedance of its own. So, here this value is now more compared to this. Now during the fault again the Impedance of the system is reduced, whether the reduce it is less than this because this is a faulty, and less power is flowing into the system.

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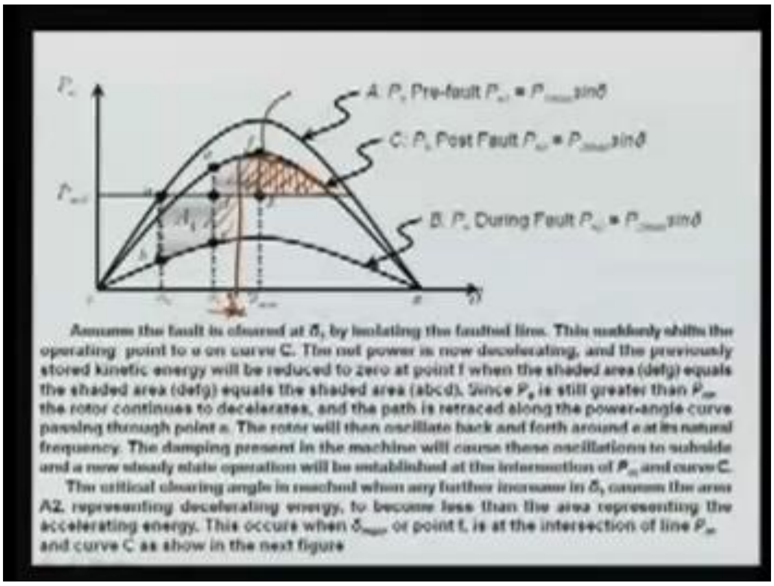

To understand this now you can see these curves, we have now three curves; one is your pre fault, another is your post fault, and then another is during the fault. So, let your curve is A, outer curve from 0 to pi, and that is denoted by your a and it is called the pre fault curve. Means we are all the Impedances are intact, where X is your X_d' plus Xd plus the parallel Impedance of the two lines. Since Impedance is less in that case, so what is happening this, this curve will be higher why. Again you remember this is your VE upon Xd prime, means X prime simply sine delta, $\frac{VE}{X_d'}\sin\delta$, If this value is small then Pe will be larger value. So, it is outer where it is follow. Now after the fault one line is tripped. So, what is happening your Impedance is Increased, that let us suppose your point your c curve that is curve here, that is following this, means you can see this curve.

During the fault what is happening, again there is a bolted fault, Impedance is again increased tremendously and the power which is showing Inside the Infinite bus is reduced, and let us suppose this one curve. There is a curve, and where it is denoted at the during fault and it is denoted at the Pe2, that is during post and one is pre fault. Now, you can see the loading of the system was P_{m0}, before fault, your operating point was A, because it is Intersection of pre fault curve A and your P_{m0} curve, and this is your operating, that is pre fault operating point, and the angle of the excitation voltage your δ_0 is this one. Now, let us fault we have to apply. Once fault has come what will happen, during the fault that which is occurring here, this is suddenly it is shifting to another curve, because it will always follow the Pe curve here. Now the Pe curve it is now during the fault we have this curve, that is curve B, and it will be following this curve.

Now, what will happen, now in this case your Pe which is fed to the Infinite bus is reduced, because here it is Pm is more and Pe is less, so machine will accelerate. Accelerate means here your delta will Increase. Let us suppose at δ_1, your fault is strictly cleared and this fault clearing is associated with this tripping of the line, means suddenly we have to reach the post fault condition; that is point e of this curve that is the post fault curve Pe. And then what will happen, this is your accelerating energy, this curve, then here it is going to decelerate till f, till the point when the A1 will be equal to A2, and then again it will go back, and then it will be again oscillating, and your final point of operation will be now here. It will be another, it is your k point will be here the final point, because it will be oscillating around this point, and your system is stable, if damping is there and the system will subside.

So, this is the condition when it is the fault, means during fault, curve means this during fault, this is post fault, this is a pre fault. So, this is retaining in this with the fault location away from the sending end, the equivalent transfer reactance between bus bare is Increased, lowering the power transfer capability. And the power angle curve is represented by the curve b during the fault, because less power is fed. Finally, the curve c represent the post fault power angle curve, assuming the faulted line is removed. When the three fault phase occurs, the operating point shift Immediately to point B on curve B. here that is coming during the fault, and the axis of mechanical Input power of this machine accelerate the rotor, there by storing kinetic energy, and the angle this is Increased here up to this point. So, this is your point of movement, here it is coming back, and then finally, attained it would be attaining now, later that it will come follow this one. It will come here and then go back. So, in this curve we will see, that this area again is here again, I can say in this vertical hashed line.

This is basically your margin that the system had. Means your system is stable no doubt, and this area if this area is larger, we can say our system is more stable. is very small, then we can say our system is less stable. So, what happens there is a possibility, that to fault even though if it is not cleared then here $\delta_1$less than that or at delta. So, this value will be kept on Increasing. So, there is a possibility, let us suppose our δ is increased here. So, this machine will go up to accelerating here, and this complete area, will be equal to If area A2. Then this angle again theta δ_c is called as a critical clearing. Means the critical clearing angle is the angle that beyond that fault cleared that system will be unstable, and if it is cleared before that angle, or you can say that time then your system will be stable, and there will no margin means that is a critical clearing time.

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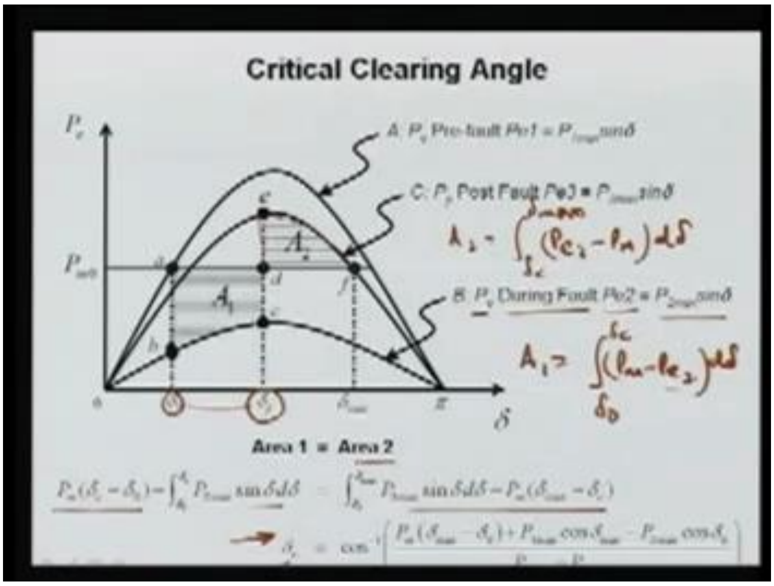

So, to calculate this, the critical clearing angle, means I want to calculate this δ_c at which this area up to this point; means that is your Pm line. This area A2 will be equal to your area A one, then we can say our system is margin less stable, means angles beyond this If fault is cleared system is unstable. To know this critical clearing time, because we have to plan our system; means your protective device, Including your relays, as well as circuit breakers that must clear the fault, if any fault occurs before this angle, and we can also calculate the time. So, this δ_c is the dead line, less than that fault must be removed otherwise your system, this generator will be out of the unstable. So, this area A1 will be equal to your area A2 and then we can determine again your δ_c as the previous example. Again this area is nothing, but now (Pm – Pe) again the Pe here of the during fault that is Pe 2; means I can say this area A 1 will be your Integration, $\int_{\delta_0}^{\delta_c}(P_m - P_{e2})d\delta$ from here. So, this is your area A 1.

Similarly your area A2 will be your Integration from delta c to delta max, $\int_{\delta_c}^{\delta_{max}}(P_{e3} - P_m)d\delta$ So, if we equating this I am integrating for this case you can say integrating this Pm is constant. So, we will get this $P_m(\delta_c - \delta_0)$; this term. Another term here is with the negative sign, integrating from delta naught to delta c of this Pe2 and Pe2 is your P2 max, Pe2 max we are integrating here. Similarly, for another area two we can write in this fashion and again you can see this Pm delta c here Pm δ_c it is this side it is plus plus it will cancelled out. So, we can determine this δ_c that is a critical clearing angle, it is a $\delta_c = \cos^{-1}\left\{\frac{P_m(\delta_{max}-\delta_0)+P_{3max}\cos\delta_{max}-P_{3max}\cos\delta_0}{(P_{3max}-P_2\text{max})}\right\}$. So, we can determine the critical clearing angle for the different decision.

Similarly, we can go for the different one let us see in that case just fault is cleared, by the tripping of the line, but there may be the possibility that the circuit breaker those are in the line, they may have some reclosing facility. Why we go for the re closing, because whenever there is temporary fault occurs. There may be possibility, that this circuit breaker is opening and fault is automatically cleared, and after the closing, it was found that there is no fault. For example, let us suppose your there is a flux over on the Insulator, due to the fogging, due to the motion on Insulator, there may be possibility that is line, due to the higher line voltage, and there is a flash over on the line Insulator, and there will be the LG fault. So, your circuit breaker will be clear, simply that it will be opening, and then it will try after few cycles. Again depends upon the circuit breaker consideration type and make etcetera. So, it will try to reclose the fault to see, whether fault is cleared or not for example, let us suppose some bird, has come to the wires all three wire. So, there is a dead three phase fault, three phase short circuit. So, once this any live animal or bird there is on that phase here. Once that will die automatically that is again all these phrases are operating in Isolation. So, the fault is automatically cleared after one short. So, the circuit breaker normally for EHV transmission line, they will have reclosing facility.

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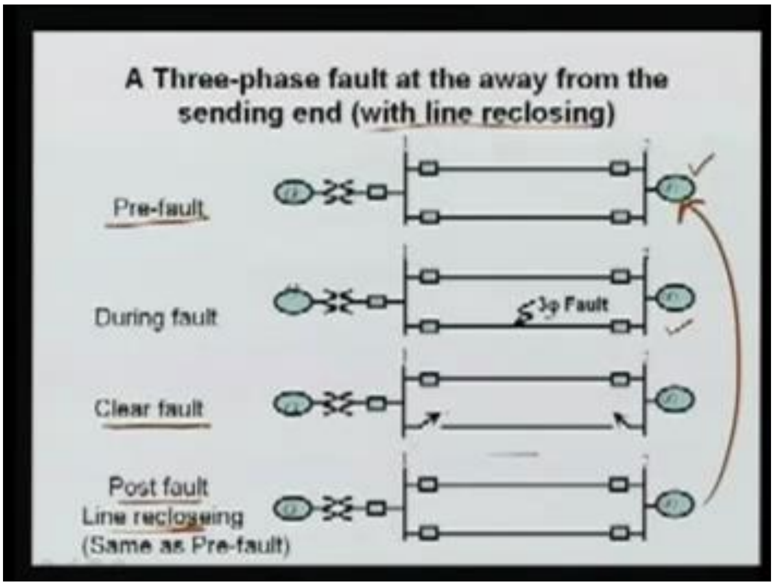

So, with that we can see the line reclosing as well. Means here our pre fault system as usual in the previous case, during fault we have the three phase fault. Now we hear the fault is cleared by that tripping, and again it is the line is re closed with the help of re closing facility of the circuit breaker after few cycle. And again is your system configuration here is similar to the pre fault condition. So, now, what will be the scenario to see this. Again we have now three you can see three categories; here this is equal to this, means your pre fault, and your post fault with the line reclosing is your same Impedance. So, same curve, and that curve is here, it is written that Pe of Pre fault. Now, during the fault, again it is similar to this. And the post fault that the fault which has occurred here sorry. So, this fault here, you can see this Impedance is same. Here this line is cleared. So, this is fault clear. So, this Impedance is just like in the previous case post fault.

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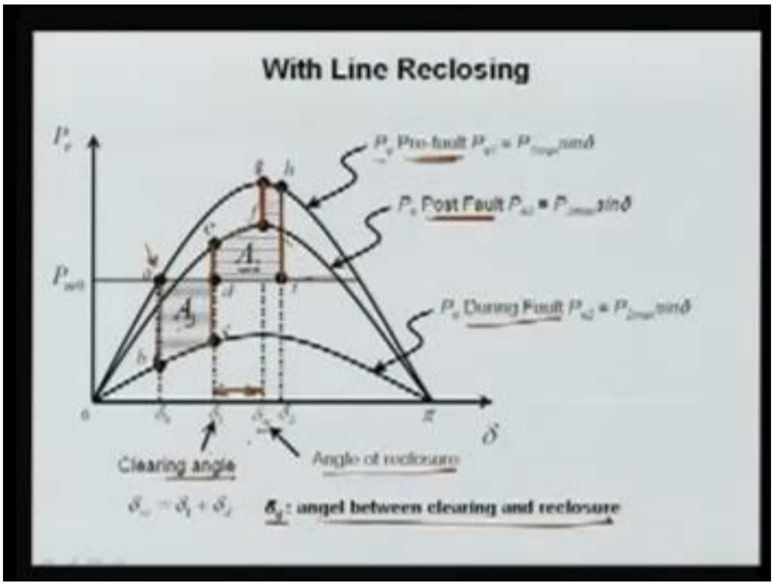

So, just we have here curve this curve here, is a basically the fault clear at that time fault is clear, but it is not a post fault. Post fault is again here If this condition where is again it follows the same Pe. So, in this case here your operation is at point A. Again it is a Intersection of P_{m0}, and you are the Pe angle; that is pre fault, and this, after this there is some fault your system is coming here, that is your during the fault condition, it will be try to follow here. And then fault is clear, there is a clearing angle here, it will try to go at this point, because the Pe during the once fault is cleared it is following this, now till it is accelerating this line was opened it follows this and at the same time circuit breaker try to close It. Means it is suddenly here it is Increase here before that here it is a decelerating completely, it is your after few cycle it Is. What is happening now, If this is coming and now again following this, and then it is coming back, because this area is now more. So, it is more accelerating, because we are taking more power here. So, now, area again for stable, here this area one will be equal to area two. And in this case you can see area of margin here it is very high, because line is not tripped. So, you can feed more power. So, your system is more stable. So, the outage of line, again create some criticality of the line. If you saw in the previous case it was this area only, now we have this area. So, your system is more stable. So, outage of lines sometimes creates some Instability and the system is very near to the stability margin. So, we want this Rc is called angle of reclosure. We want to know, and this your delta d angle between clearing and the re closure this angle. We can determine, and we can know that it is cleared at this point, and then we can see what should be the critical clearing time in this case.

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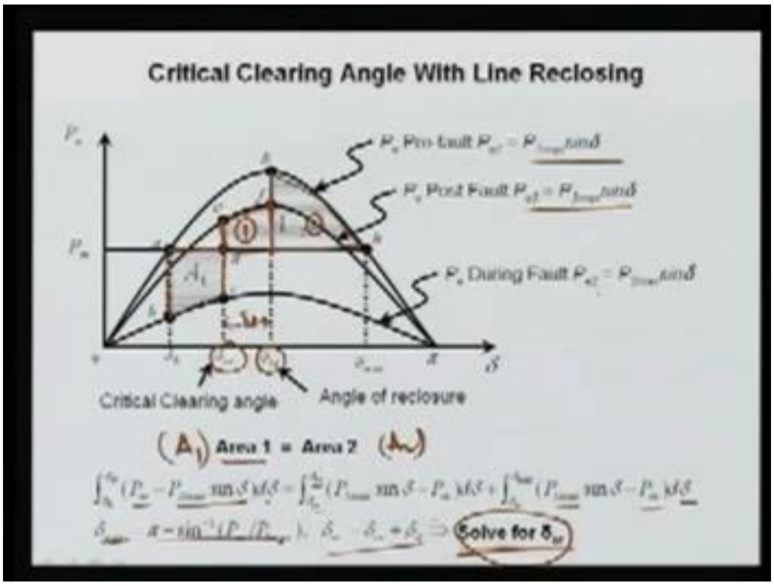

We can again similarly determine. Now again you want to get the what should be the critical clearing time here δ_{cr}, having the re closing facility this, this angle is given to you that is delta dt it is to note, because it depends upon the circuit breaker configuration automatically, it may re close may be one cycle re closing may be one and half or two cycle re closing. Means one cycle it will wait and then again it will re-close. And again the fault is persisting what will happen, this will open, circuit breaker will open and it will not re close again and again, it is only once. So, in this case we want to determine what will be the critical clearing angle this one. So, that knowing this facility in your circuit breaker. You want to determine that your circuit breaker fault must be clear before this angle, otherwise your system become unstable. So, in this case your area again using equal area criteria this area, should be equal to area this now. Your area defghd. So, this is the complete area now you want to know.

So, area one or you can say A 1 should be equal to area A two. Now this area A 2 will be nothing, but the $\int(P_m - P_{e2})$ of integration, that will be equal to from here to here. Now we have the different curves and different area. Now we can divide this area in two parts; one is here one, here is another is two. So, one is your nothing, but your either Pe 3, because it is a Pe 3 here, Pe 3 max minus Pe, and it is from δ_{cr} to δ_{rc}` re closing angle plus this area this area is from $\int_{\delta_{rc}}^{\delta_{max}}(P_{m1} - P_m)d\delta$. And now you have to solve, because this Angle you know you can determine this knowing the Pm and Pm 1 max, you can calculate δ_{cr} knowing all this value. So, here we can get the value, we can substitute rc cr here and we can get this value δ_{cr}; that is your critical clearing angle, and then we have to design our system based on that one.

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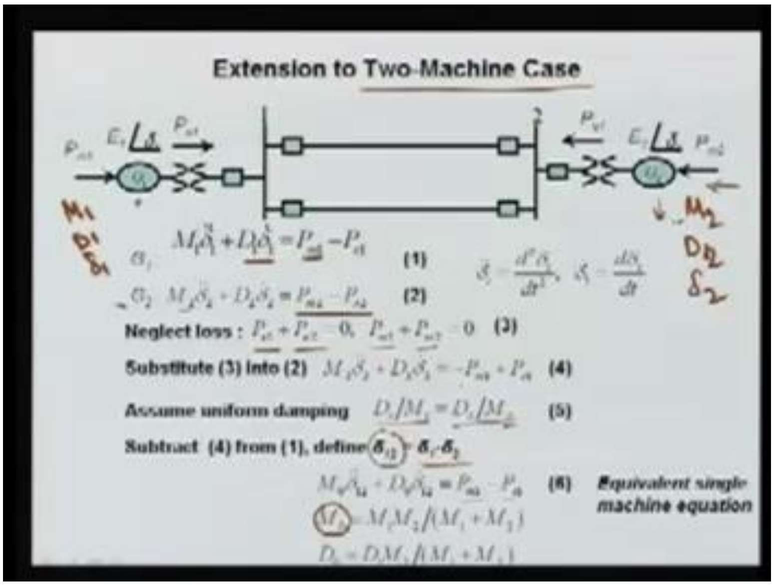

Now, same theory can be extended, can be applied to the two machine case, in which here this two machine having the different Inertia constant. Here it has a different Inertia constant, and then we can add, and we can apply together. So, in this case we want to apply the equal area criteria for the two machine case, is a special case, and this two machine case can be represented by equivalent a single machine connected to Infinite bus. Here we have the generator one, and another is generator two. This generator two is having all the parameter; that is denoted by the two here suffix. So, here it is having M one, it is having D1 damping constant, it is having delta on side it two, so it is two. This case it is your M1 it is your D1 and δ_1. So, we can write the dynamic equation; that is a swing equation, even then including damping. So, here the damping term is Included. Normally till now we have ignored the damping, because it is related with the speed of the system.

So, this is M 1 δ_1 double dot that is double differentiation of δ_1, $\delta_1$of here it is δ_1 written; that is feeding power Pm one. Here the mechanical power that is coming to this generator is Pm 1 minus Pe, $M_1\ddot{\delta}_1 + D_1\dot{\delta}_1 = P_{m1} - P_{e1}$. Similarly for generator two we can write this, $M_2\ddot{\delta}_2 + D_2\dot{\delta}_2 = P_{m2} - P_{e2}$; that is coming from this side If losses are neglected what will happen. This Pe 1 plus Pe 2 in this system the total Pe here that is injecting that must balance that must conserve, energy cannot be going anywhere. So, losses are zero means the energy which is feeding that will be the balance out, means whatever you are feeding it must be taken by some machine. So, this,$P_{e1} + P_{e2} = 0$. Similarly the energy, mechanical energy that is coming into the system the $P_{m1} + P_{m2} = 0$. Now, substituting equation three into two here, even two substitute the equation three here.

So, substitute the equation three into two, we will get here your M 2 delta 2 double dot plus D 2 here this side, and this we are just replacing, complete this 2 equations; that is Pm 2 and Pe 1 in terms of P 1 we can write this minus Pm 1 this, and it will be plus Pe one. Assuming the uniform damping, means damping here is a D 1 upon M 1 that will be equal to D 2 upon M two, or you can Ignore the damping. If you are also Ignoring that is no problem, then it is very simple, but if your damping is uniform. So, D 1 upon M 1 and D 2 upon M 2 is equal. Also we define the δ_1 minus δ_2 that is a δ_{12}. So, we can write a equation M naught delta 12 double dot plus delta 2 delta naught δ_1 single dot is equal to Pm 1 minus Pe 1, and this P_{m0} will be the basically the parallel combination of this angular momentum that is M1 and M two. So, $M_0 = \frac{M_1 M_2}{M_1 + M_2}$. Similarly $D_0 = \frac{D_1 M_2}{M_1 + M_2}$.

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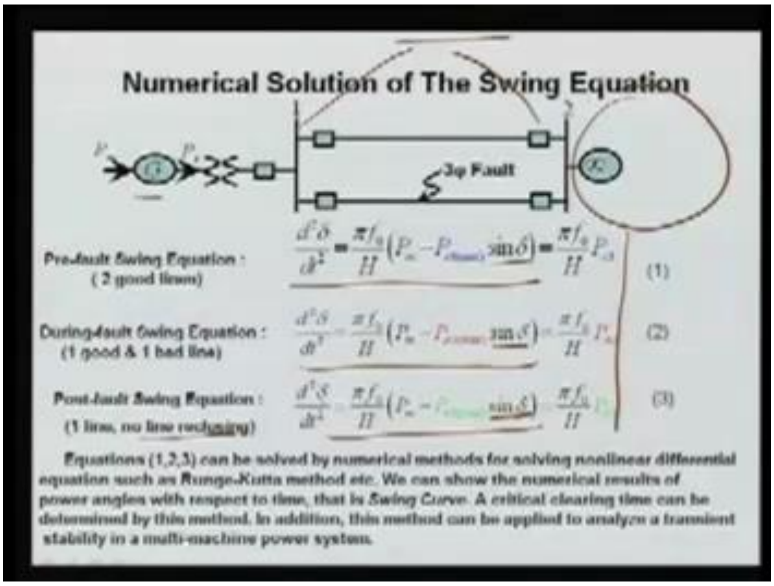

So, this is your equal length system, and we can again apply our equal area criteria. Now, for the single machine, it is very easy, no doubt to know, whether system is stable or not. We can analyze and we can calculate your critical clearing time, we can see the stability of the system, and we can only know the relatively stable system. We want to get what is the δ variation, it is not a time domain simulation at all. It is only giving Information about the stability of the system, means your system is stable or it is not a stable. But that it is not only our concept. We want to know what is the system behavior during the period, means what is the speed deviation, how speed is changing, what is your δ is changing that is also require, and for that we have to go for solving differential equation for different Intervals. So, this is the numerical solutions to the swing equations. We have written the swing equation with this during the pre fault condition.

During the faulted condition this equation, and post fault here If know line re closing we are not considering means fault is cleared, by the outage of the line means this line is tripped, after certain time only post fault is this only one line is existing along with GT generator transformer, and this is your this equation. So, the equation one two three here, can be sold by the numerical methods, for solving non-linear differential equation, why this δ here this is also δ this non-linear function here, this non-linear non-linear. So, this differential equation is a non-linear differential equation of second order, because of D two. And that can be solved by the various methods. Methods may be your Eulers method, Trapezoidal methods Runga kutta methods. We can show numerical results of the power angles with respect to time; that is a swing curve. Our critical clearing time can be determined by this method. In addition this method can be applied to the analyzer transient stability in a multi machine system.

For the single machine system, they as said the equal area can be easily determined, and we can say our system is stable or not stable. We can also assess what is the margin available with you, if fault is cleared at particular time so we can know the relative stability, but we want to know the performance that is a time domain simulation is required. So, we have to solve these three differential equations. More over whole this swing equation here, we have modeled a simple classical approach, and that is not valid. Means we have to go for the detailed modeling of the machine. Here we have to Ignore we have made seven six seven assumptions, that can be Incorporated and then there will be several hundreds of machines in a big power system. So, all these will be solved for the system, then we can see the performance that the δ etcetera variation of each Individual generators, and then we can say, whether the system is stable or not.

So, in this lecture we have seen the equal area criteria for the application for the various, configuration of various fault with re closing, without re closing, fault at bus, fault at line, and then we can saw this equal area criteria can give you the relative stability. It will give Information about without, whether it means whether your system is stable or not at the same time. It can also with the margin that is stability margin available with the system, and it is very useful and very fast no doubt, but only the limitation of this is that, we have made several assumptions; those may not be valid for the power system. And also it is not possible to formulate to form the system into single machine Infinite bus, equivalent and then we have to go for the complete and the real time practical system for the simulation. And for that we have to solve these differential equations for all the machines, not only one machine, this is the case of one machine. So, we have to go in this Infinite bus system there will be several generators, and we have to include.

Again this generator is not a classical generator. Classical generator means is a generator which we have model in the classical form, means we have taken only this generator with the some Inertia constant, then we can go for several order of modeling of synchronous machine. It will equip with your excitation as well, it will be equipped with the governing system. So, all dynamics will be, dynamical equation will be clubbed together along with algebraic equation, because here the power flow of the different transmission lines, they are not connected with the transmission lines generating stations, the different transmission lines that they will be there. So, we have to form this algebraic equation, and then we can solve. Some times what we do, we this all the buses, those are not connected we try to eliminate. We will see in the later sections. Here this they will be eliminated only the generator terminal buses are kept Intact, where there is a fault is occurring, and then we can analyze the system behavior, means we can see the relative motion of the Individual generator with reference to any particular generation, and then we can say whether your system is stable or not stable. So, with this, now this chapter seven, lecture seven is closed.

Thank you.