Advanced Control Systems
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Module No.# 01
Model Based Controller System
Lecture No. # 06
Controller Design to TITO Processes

Welcome to the lecture on controller design for TITO processes; TITO stands for two input two output; also we have got MIMO systems, where MIMO stands for multiple input multiple output. Initially, we shall design a SISO controller for SISO process, and then, extend that to design of controller for TITO process.

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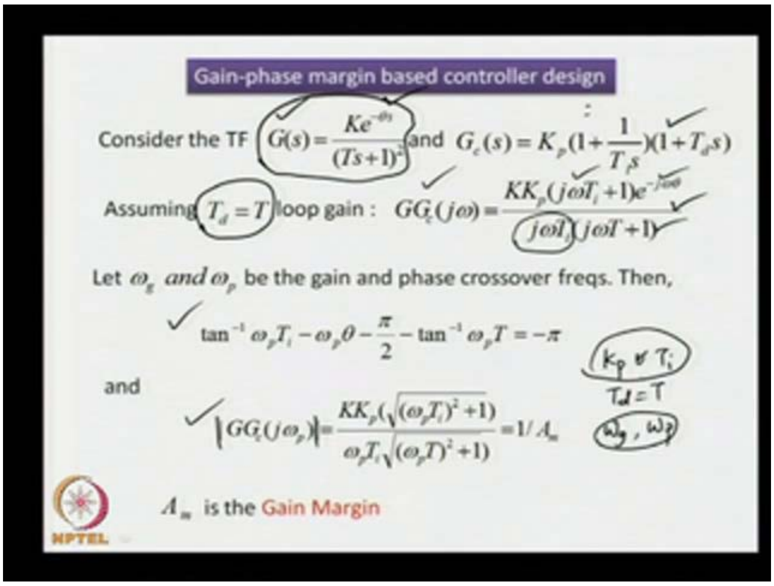

So, attempt will be made first to design a controller series PID controller based on gain and phase margins. The transfer function shown over here G c s shows us a series form of PID controller, where we have got the proportional gain given as K p, and integral time constant given as T i, and derivative time constant given as T d; this is a series form of PID controllers, unlike the parallel form of PID controller, which can be given in the form of G c s equal to K p 1 plus 1 upon T i s plus T d s, why do we select series form of PID controller, it has got some benefit that we shall see after some time.

Now, we shall assume the process, the SISO process to have the transfer function G s given in the form of k upon T s plus 1 square with a delay given by e to the power minus theta s. So, this gives us a second order transfer function with delay; assuming T d equal to t the loop gain of the closed loop system can now be given as G G c j omega equal to K K p times j omega T i plus 1 times e to the power minus j omega theta upon j omega T i times j omega T plus 1.

Now, this loop gain is nothing but the open loop gain, also sometimes we call this gain as the open loop gain of the closed loop system. The series form of controller enables us to make this choice T d equal to t, they are by cancelling 1, 0 of the controller with 1 pole of the process transfer function. Let omega g and omega p be the gain crossover and phase crossover frequency, then as we know at phase crossover frequency, the phase of the gain G G cj omega, the phase angle of gain has to be minus pi, now this loop gain can be used, and we can find the angles from the loop gain as 10 inverse omega p T i given by this term minus omega p theta given by the delay term minus pi by 2 given by the pole at the origin minus 10 inverse omega p t given by the pole j omega T plus 1 which is equal to minus 5 now; at this phase crossover frequency, the loop phase will be minus 180 degree or minus pi. At the same phase crossover frequency the gain of the loop G G cj omega p magnitude will be equal to K K p time square root of omega p T i square plus 1 upon omega p T i root of omega p T square plus 1, which is nothing but same as inverse of the Gain Margin. We have taken a m as the Gain Margin which is giving us the loop gain as 1 upon k m. So, these are the two equations, those are obtained using the phase crossover frequency point on the Nyquist diagram; suppose, we have got the Nyquist diagram, and this becomes our Nyquist plot, then we have got the phase crossover point over here at the frequency omega p, which is nothing but the phase crossover frequency; so, this point gives us these two equations.

Similarly, when the unity circle is considered, and thus giving us the gain crossover frequency omega g at this point giving us a gain crossover point, thus that will also result in two equations, and now solving the four equations, set of equations will enable us to estimate the two unknown parameters of the controller K p and T i, we have already assumed T d to be t and again omega g and omega p. So, the 4 equations will give us estimate for K p T i omega g omega p.

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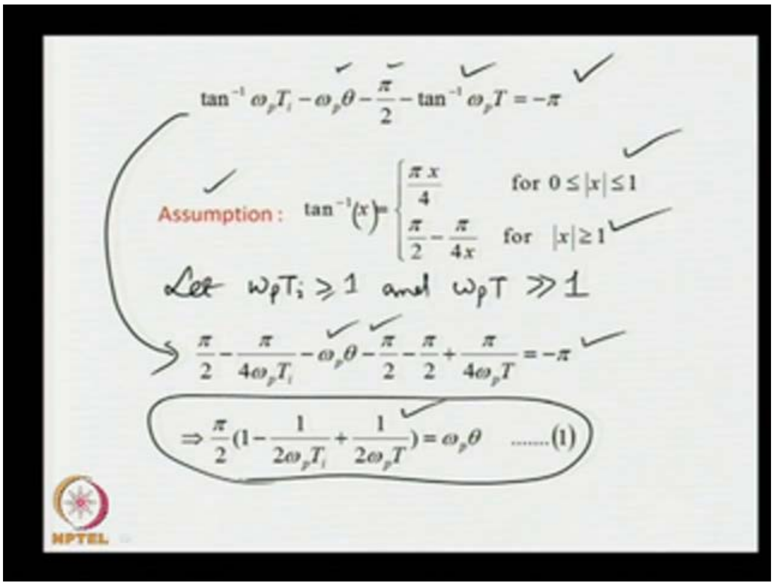

Let us see how we can simplify these expressions; the phase equation given in this form can be simplified further with the help of the assumption for arc tan function 10 inverse function. So, the 10 inverse x can be given as pi x upon 4 when x is less than equal to 1, and 10 inverse x becomes pi upon 2 minus pi upon 4 x for x is greater than 1; in our case, let us make some assumption, again let omega p T i is greater than equal to 1, and omega p T is greater than or larger than 1, in that case that helps us to write the equation in this form. The first term will give us pi upon 2 minus pi upon 4 omega p T i, the second term it is written over here, then minus pi by 2 is here, then 10 inverse omega p t can be written as minus pi upon 2 plus pi upon 4 omega p t which is equal to minus pi; collecting the terms, we get the simplified expression pi upon 2 times 1 minus 1 upon 2 omega p T i plus 1 upon 2 omega p t equal to omega p theta. So, we get one simplified expression using the information from the phase crossover point.

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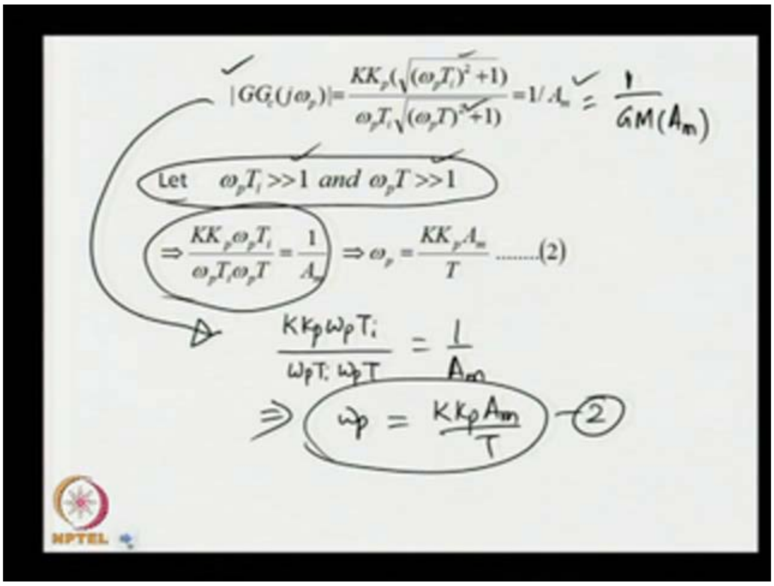

Similarly, the gain at the phase crossover frequency can be expressed as K K p times root of omega p T i square plus 1 divided by omega p T i root of omega p T square plus 1 equal to 1 upon a m the Gain Margin; this is same as 1 upon the Gain Margin which is nothing but a m in our case. Already, we have made this assumption that omega p T i is greater than 1, and omega p T is greater than 1, that assumption will help us to express the equation in a simplified form as K K p omega p T i upon omega p T i omega p t equal to 1 upon a m, that we have got here which upon simplification further gives us omega p equal to K K p a m upon T and this is the second equation we get. So, we have got an expression for the phase crossover frequency which is given as K K p a m upon T.

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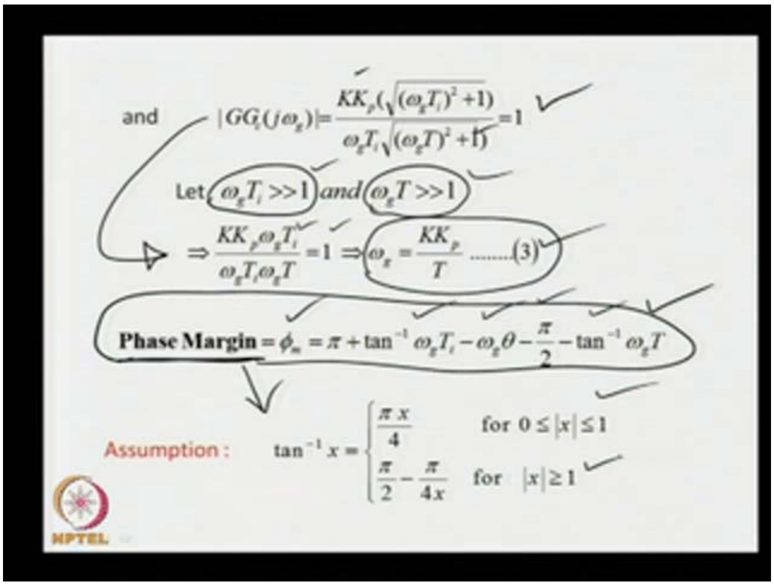

Let us make use of the other information; we get from the Nyquist diagram that is what happens at the gain crossover point. So, gain crossover point will result in two equations, one equation shown over here and the other one is the equation relating the phase margin. Now, at gain crossover frequency, the loop gain can be expressed as K K p times root of omega g T i square plus 1 divided by omega g T i root of omega g T square plus 1 equal to 1, because as we know at the gain crossover frequency, the loop gain is 1. Making the use of the assumption that omega g T i is larger than 1, and omega g T is larger than 1, this expression gives us now K K p omega g T i upon omega g T i omega g t equal to 1, which further gives us omega g equal to K K p upon t. So, let us give this equation number 3 for this equation; now, using the phase at the gain crossover frequency, the loop phase at the gain crossover frequency will result in the gain phase margin of the system. So, defining the phase margin of the system which is given by the symbol phi m equal to pi plus 10 inverse omega g T i minus omega g theta minus pi by 2 minus 10 inverse omega g t, this phase equation can easily be obtained using the expression loop gain for the system. Since arc tan functions are to be approximated to get simplified expression for these, we have to make use of the same assumption, we have made earlier, when 10 inverse x equal to pi x by 4 for x is less than 1 and equal to pi by 2 minus pi upon 4 x for x is greater than 1.

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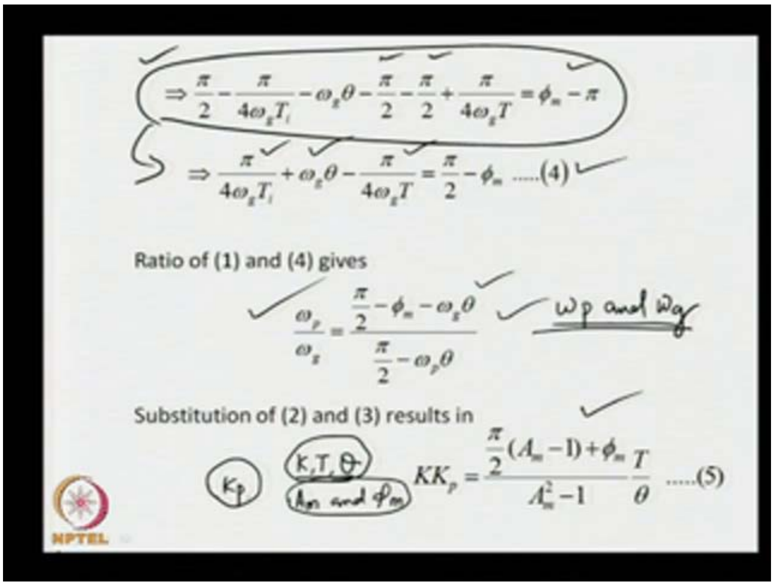

Then the phase margin expression gives us pi upon 2 minus pi upon 4 omega g T i minus omega G theta minus pi by 2 minus pi by 2 plus pi upon 4 omega g t equal to phi m minus pi, this is what we are getting from the expression, we have obtained earlier for the phase margin, considering the gain crossover point, now, this expression can further be simplified and written in the form of pi upon 4 omega g T i plus omega g theta minus pi upon 4 omega g t equal to pi by 2 minus phi m giving us equation number 4.

Now, ratio of equation 1 and equation 4 gives us omega p upon omega g equal to pi by 2 minus phi m minus omega G theta upon pi by 2 minus omega p theta; now, substituting omega p and omega G in this expression enables us to write an expression for K K phi in terms of the Gain Margin, phase margin, and the plant parameters t and theta. So, using this expression, along with the expression for omega g and omega p, we see omega g equal to K K p upon T and omega p equal to K K p a m upon t. So, using those two expressions, it is possible to find the expression for K K p, K K p as given over there, so, K K p equal to pi upon 2 times a m minus 1 plus phi m upon a square a m square minus 1 times T upon theta. So, what we get from this expression, this expression if you look at carefully has got the unknowns K p. So, K phi is the only unknown, since k, T, theta are known; k, T, theta are the parameters of the process model, thus those are known to us, and a m and phi m are the gain and phase margins defined by the user, thus a m and phi m are also known a priory this expression helps us to estimate the proportional gain K p.

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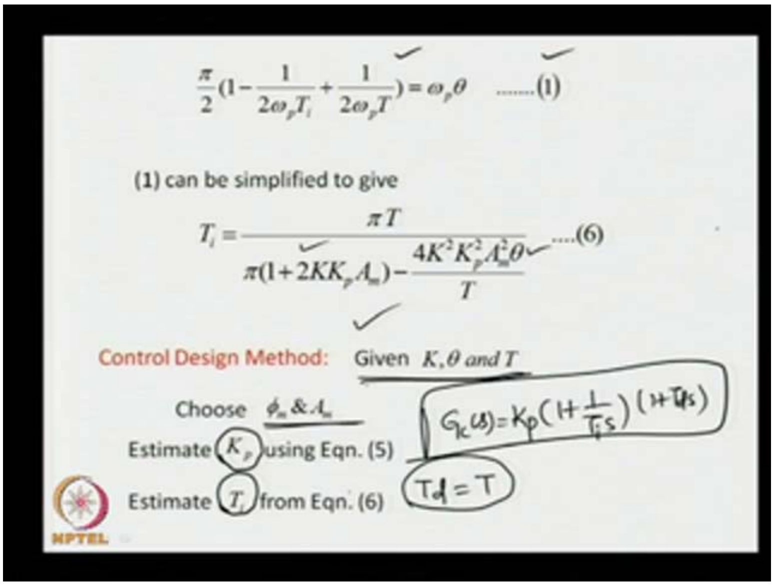

Similarly, attempt will be made to find one more expression to obtain the remaining parameter of the series PID controller. Expression 1, equation 1 is known to us which is nothing but pi upon 2 times 1 minus 1 upon 2 omega p T i plus 1 upon 2 omega p t equal to omega p theta, again substituting the expressions for omega p, one can simplify the above expression and obtain the expression for T i the unknown constant or unknown time constant associated with the series PID controller in the form of T i equal to pi t upon pi times 1 plus 2 K K p a m minus 4 k square K p square a m square theta upon T. So, this expression can be used to estimate the unknown parameter T i, let us look at the right hand side of equation 6, what are the known and unknowns over there in the right half, we can see that k t and theta, these are known to us, these are the parameters of the process model a m and phi m is not used here are, a m phi m are user defined; therefore, these are also known to us, then using this equation 6, expression 6, one can easily estimate the unknown parameter T I; thus one is able to estimate all the unknown parameters of the series PID controller using 5 and 6.

So, the control design method can be summarized as given below given k theta and T which are nothing but the process model parameters, we need to choose the phase margin and Gain Margins for the closed loop system, then the proportional gain of series PID controller can be estimated using equation 5; so, using equation 5 K p can be estimated. Similarly, equation 6 can be used for estimating the integral time constant of the series PID controller, thus the parameters of the series PID controllers K p T i and T d are estimated using the gain and phase margin; now, T d is already known to us which is nothing but equal to the time constant of the process model. We shall make use of this phase and Gain Margin method for designing controllers for a TITO system.

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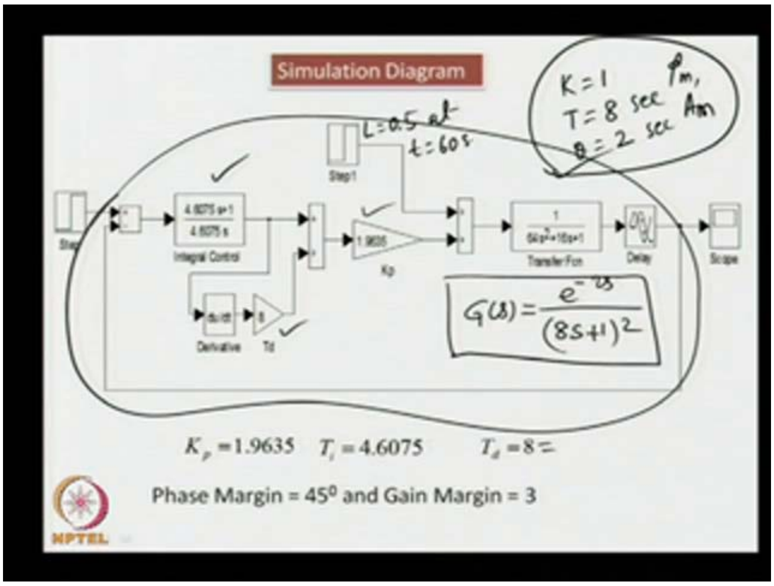

Before that let us see, how the control loop performs with the design values of the K p T i and T d, let us assume the process dynamics to have the form of G s equal to e to the power minus 2 s upon 8 s plus 1 square. So, we have got a process, second order process with repeated poles located at minus 1 upon 8 and with the delay of 2 seconds.

Now, using k equal to 1, t equal to 8 seconds, and theta equal to 2 seconds, the parameters of the series PID controller can be obtained with the assumption that with the choices that phase margin of the loop is 45 degree and Gain Margin is 3. So, using phase margin and Gain Margin values, and k t theta K p T i are estimated K phi s found to be 1.9635 using equation 5; so, using equation 5, K p can easily be obtained. Similarly, the integral time constant T i is estimated to be 4.6075 for this case for the choices of given phase and Gain Margins and T d equal to 8 equal to T. So, we have got series PID controller parameters given over here with the choices of phase and Gain Margins of 45 degree and 3 respectively; now, the integral controller is implemented over here, the derivative controller has a derivative time constant of 8 seconds and the proportional gain is given by a gain of 1.9635. When this simulation diagram is used simulation result gives us a closed loop response of this form. We have also assumed a step load disturbance of magnitude 0.5 occurring at time T equal to 60 seconds. So, a step load disturbance occurs over here, which is of magnitude 0.5 at time T equal to 60 seconds.

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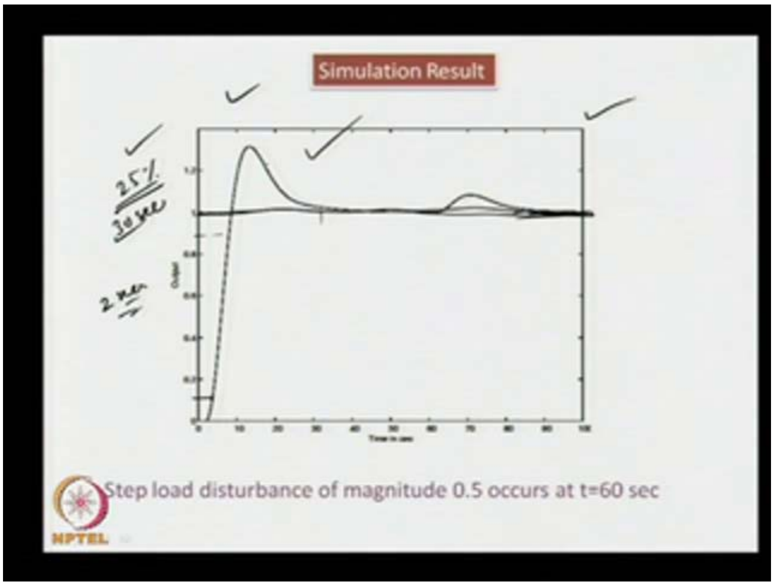

Then the response obtained is of this form, if we will carefully look the response, we have got a time response, which is giving about 25 percent overshoot, and a settling time of about 30 seconds, and we have got a disturbance rejection also, it ultimately settles down, if I draw the reference line correctly it goes in that form. So, although the overshoot is say bit higher, the settling time is higher, the rise time is very good, the rise time is approximately of the magnitude 2 seconds or so, we have got a very good rise time for the closed loop response. So, the closed loop response is found to be quite satisfactory as far as performance of the closed loop to plant or process parameter variations are concerned, because we are designing robust controller for the system based on gain and phase margins; therefore, we may have a bit higher overshoot and settling time, but the overall response is always satisfactory for variations in the process parameters.

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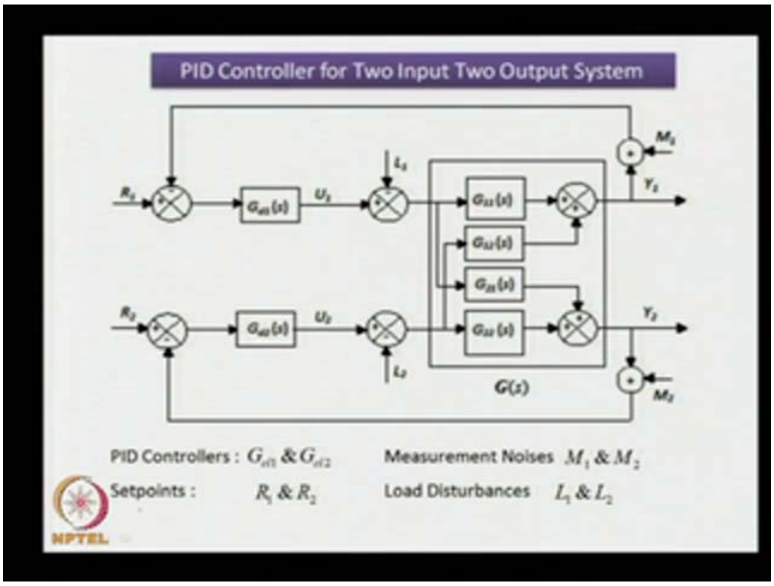

Now, attempt will be made to make use of this technique for designing PID controller for two input two output system, what a two input two output system or what a multiple input multiple output system is as, for example, let us consider a situation an office which has got two chambers, and we need to maintain different temperatures in the two chambers; suppose, one chamber has to have 30 degree centigrade, and the other has 25 degree centigrade, in that case one has to employ two sensors for the chambers; therefore, we will have two outputs for the system, the outputs are obtained from the sensors.

Now, since we have to maintained different temperatures in the chambers; therefore, cool yards are to be pumped in with the help of docks, then we have got 2 docks for the 2 chambers, thus giving us two inputs to the system; therefore, the office room dynamics can be regarded as a two input two output system, where we have got two inputs and two output.

Similarly, we have got multiple inputs and multiple outputs in real time industrial environments. Now, how we can make use of the phase and Gain Margin based controller design for two input two output system that we shall see now. So, a two input two output system is different from a single input single output system, in the sense that there is no coupling present in the SISO process, whereas there are couplings interactions between the loops of a TITO system that is the only difference we have between the

TITO system or TITO process and a SISO process. So, the MIMO or SISO processes have got spatial coupling. Now, let us see the block diagram representation of a two input, two output system; so, we have got one input over here, another input over here, thus giving us two inputs for the system and we have got two outputs. Now, the two input, two output system has got interactions also given in this session, that is why there are 4 blocks representing the dynamics of a TITO system which has got interactions present between the loops.

Now, when two PID controllers are employed for controlling the TITO process, we have got two loops, suppose this is loop 1 and this is the loop 2. So, the two loops will give us dynamics in such a manner, that the dynamics are affecting each other; there is enough loop interaction between the individual closed loop systems. So, the controller can be designed provided the dynamics of the TITO processes obtained, is obtained in some convenient form; is it possible to obtain the dynamics of a TITO or MIMO system in some convenient form, yes, it is possible; so, employing model identification techniques, it is possible to obtain the dynamics of TITO system in some convenient form in the block diagram, what else we see apart from the 2 series controllers G c 1 s and G c 2 s we have got load disturbances given by l1 and l 2, and noise disturbances given by M 1 and M 2.

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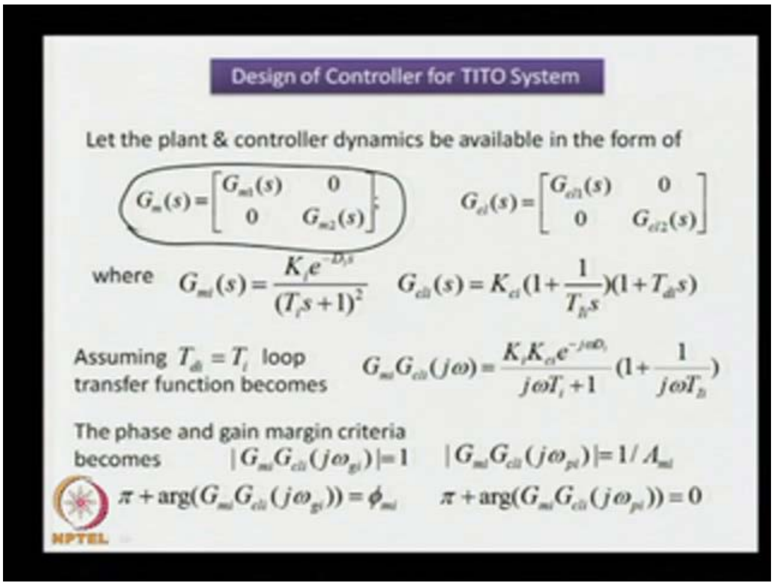

Now, the dynamics of the TITO process is assume to have the form G m s equal 2 G m 1 s and G m 2 s in diagonal of this transfer metrics. So, the TITO process dynamics is given by a transfer function metrics often known as transfer metrics given as G m s equal to G m 1 s 0 0 G m 2 s, we, thus we get a diagonal transfer function metrics for the TITO system. This form of transfer function representation helps us to design conveniently controllers for the system.

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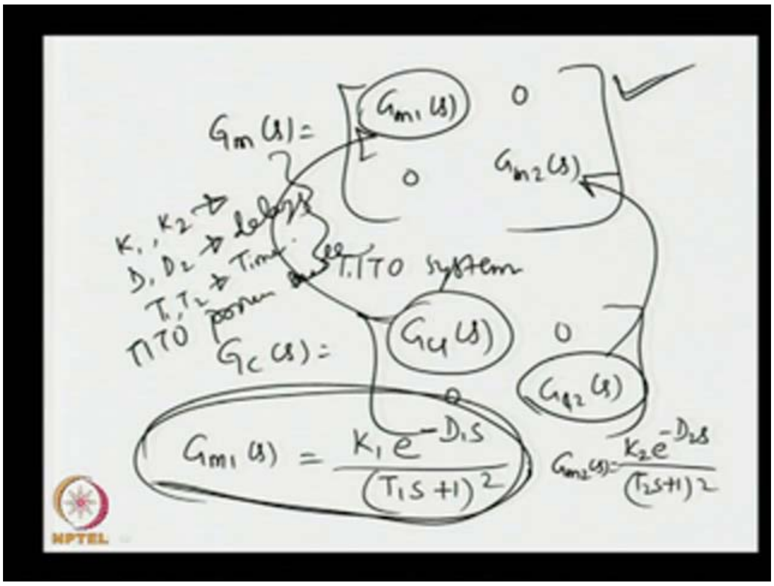

Now, the controller dynamics is also given in a similar fashion, we have got controller 1 and controller 2 present in this fashion, what benefit one gets from here, this sort of representation of the plant dynamics, and the controller dynamics help us to design 2 controllers in a convenient manner, in the sense that as if there is no loop interaction between the two loops of the TITO system, one has to tune the parameters of the series series PID controller, based on the dynamics we have obtained in this form. Now, the controller G c 1 s s s will be designed based on the dynamics of G m 1 s only; similarly, the controller parameters of G c 2, G c l, 2 or G c 2 s will be obtained purely, based on the parameters of G m 2 s, thus giving us freedom to design the 2 series PID controllers in a way we have designed for SISO processes.

Now, after assuming the form of the controllers, and the process dynamics in the convenient forms, let us assume the dynamics of the processes to be available in the second order plus delay transfer function form; what is this i stands for, i stands for i varies from 1 to 2, so when i equal to 1, we get a process transfer function which is given as G m 1 s is equal to k 1 e to the power minus D1 s upon T 1 s plus 1 square, that means, G m 1 s is given as k 1 e to the power minus D 1 s upon T 1 s plus 1 square; so, this is the dynamics of the process transfer function.

Similarly, G m 2 s can be expressed as k 2 e to the power minus D 2 s upon T 2 s plus 1 square, what are those K 1 K 2, T 1 T 2, D 1 D 2, those are the steady state gains time delays and time constants of the process model. So, K 1 K 2 are the static gains D 1 D 2 are the time delays and T 1 T2 are the time constants of the TITO process model.

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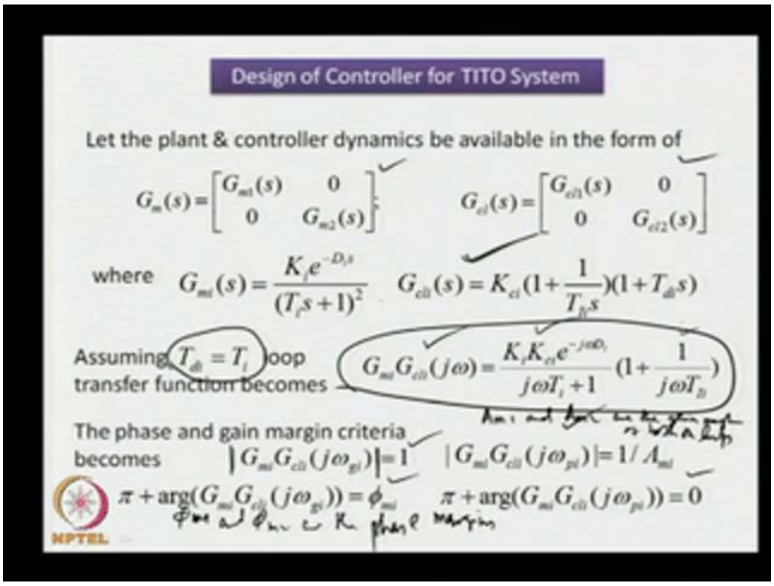

Similarly, the controller dynamics now can be obtain in the form of G c l i s is equal to K c i times 1 plus 1 upon T i s times 1 plus T d i s thus giving us 2 series PID controllers, when I assumes values like 1 and 2. So, when i equal to 1, i get the series PID controller with parameters k c 1 T i 1 and T D 1, whereas when i equal to 2, the controller parameters become k c 2 T i 2 and T D 2; now, attempt will be made to estimate the unknowns estimate the controller parameters K c 1 K c 2, T i 1 T i 2, T D 1 and T D 2.

So, once these values are estimated using the phase and Gain Margin techniques, then the controller has been designed for the TITO process, like the earlier case let us make this assumption with the choice of T d i equal to T i, then the loop transfer functions can be given as G m i G c l i j omega is equal to K i ,K c i e to the power minus j omega D i upon j omega T i plus 1 times 1 plus 1 upon j omega T i I; again, we shall make use of the phase crossover point and the gain crossover point and I am showing information from those points to design the controller parameters. The phase crossover point gives us equations like the magnitude of the loop gain at phase crossover frequencies omega phi will be equal to 1 upon a m i, where a M 1 and a M 2 are the Gain Margins of the two loops Gain Margins of both the loops. At the phase crossover frequency, the net phase angle contributed by the loops will be equal to minus pi which can be put in the form of pi plus angle of G m i G c l i j omega phi equal to 0; similarly, the gain crossover point enables us to obtain two more equations in the form of magnitude of loop gain G m i G c l i j omega G i is equal to 1, and pi plus argument of G m i G c l i j omega G i is equal to phi m i, where phi M 1 and phi M 2 are the phase margins, phase margins of the loop thus what are known, and what are unknown in these equations, we have obtained a set of 4 equations; now, the 4 equations can be solved to estimate 4 unknowns. What are those 4 unknowns, definitely unless the number of equations and number of unknowns are equal, we may not get unique solutions from the equations.

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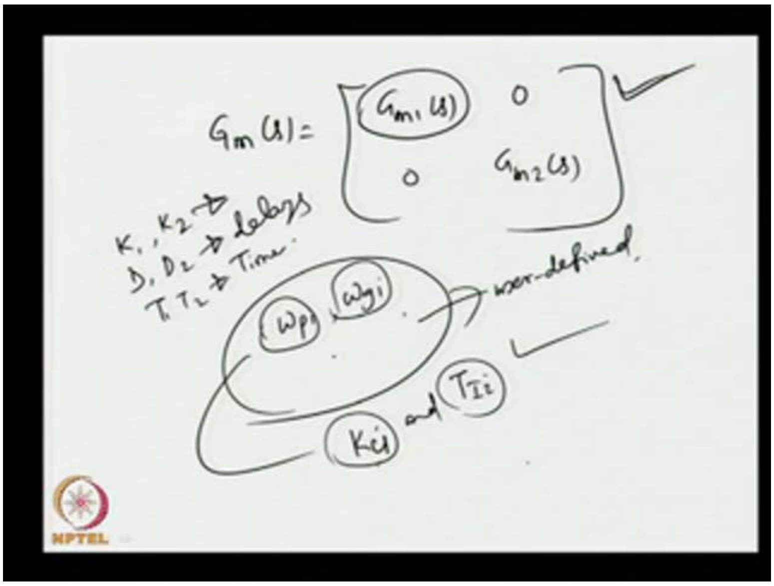

So, the 4 unknowns are nothing but omega p 1, omega p 2 or I can say omega phi, rather omega phi omega G i and phi m i a m i. So, we have got two unknowns like this, and these two variables are known to us in the sense that these are user defined, then the unknowns, two more unknowns like K c i know we have put the gains as k c. So, thus K c i and T i i are the 2 more unknown; so, the unknowns are omega phi omega G i K c i and T i i, these are the four unknowns and we have got four equations enabling us to estimate unique values for the unknowns K c i and T i i, the unknowns of the series PID controllers.

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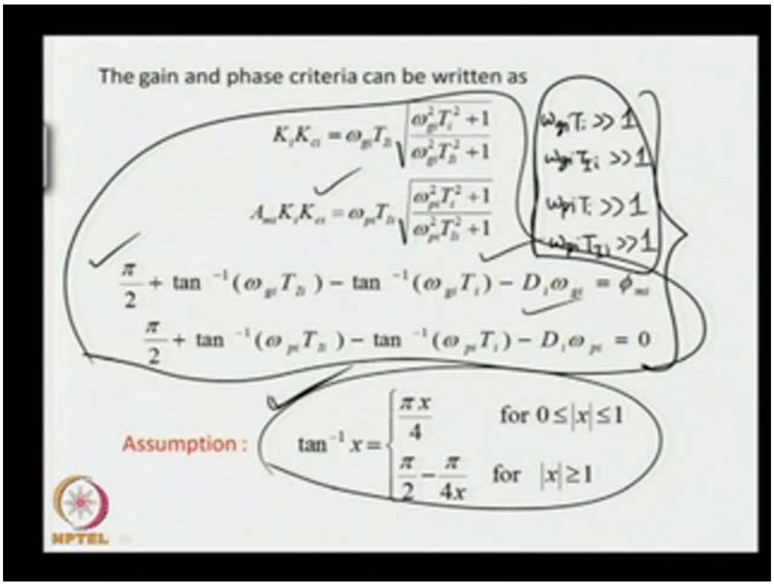

Using the phase and Gain Margin criteria, the equations further can be simplified in the form of equations giving us k i K c i equal to omega G i T i i square root of omega G i square T i square plus 1 upon omega G i square T i i square plus 1 a m i k i K c i equal to omega phi T i i square root of omega phi square T i square plus 1 upon omega phi square T i i square plus 1, and the phase equations can be written in the form of pi upon 2 plus 10 inverse omega G i T i i minus 10 inverse omega G i T i minus G i omega G i equal to phi m i, and phi by 2 plus 10 inverse omega phi T i i minus 10 inverse omega phi T i minus d i omega phi equal to 0.

So, these are the set of the equations we have obtained from the four inequalities are equations, one can obtain using the phase and gain crossover points; those four equations are to be simplified, now with the assumption of the arc tan function 10 inverse x given in this form, for that lets assume that omega G i T i is greater than 1, omega G i T i i is very large compared to 1, and omega phi T i is very large then 1, and finally, omega phi T i i is large compared to 1.

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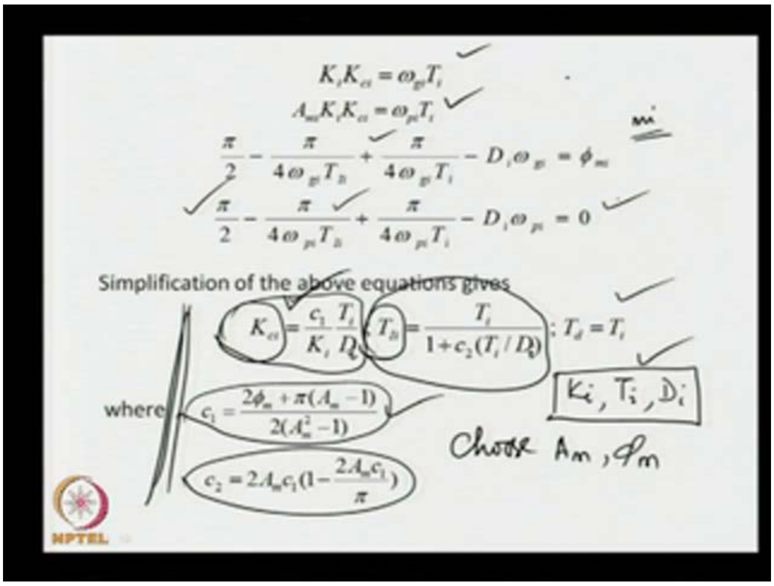

So, when these four assumptions are made, and the arc tan function is used, then the four equations, we have obtained from the phase and gain crossover points can further be written in the form of k i K c i is equal to omega G i T i a m i k i K c i equal to omega phi T i, keep in mind a m i is nothing but the Gain Margins of the loop and pi upon 2 minus pi upon 4 omega G i T i i plus pi upon 4 omega G i T i minus G i omega G i equal to phi m i, where m, phi m stands for the phase margins of the gain phase margins of the loop, sorry phase margins of the loop.

Now, the last equation can be put in the form of pi upon 2 minus pi upon 4 omega phi T i i plus pi upon 4 omega phi T i minus G i omega phi equal to 0; simplification of the four equations give us expressions for the unknowns K c i T i i, as K c i equal to K c i equal to c 1 upon k i times T i upon d, and T i i equal to T i upon 1 plus c 2 times T i upon d, and we have already made this assumption at the beginning that T d equal to T i. So, we have got the expressions for the unknown K c i and T i i, in terms of all the known parameters, what are those known parameters, we know k i we know T i and we know d i, actually this will be d i d i. So, these are nothing but the known parameters, since we know the transfer function models of individual loops; now, the c 1 and c 2 in the expressions are given, these are the two constants given in the form of c 1 equal 2 phi m plus pi times at m minus 1 upon 2 times a m square minus 1, and the second constant c 2 is given as 2 a m c 1 times 1 minus 2 a m c 1 upon pi. So, choose the phase and Gain Margins for your loops a m and phi m, once the phase and Gain Margins are chosen for the loop, then c 1 and c 2 constants are obtain, using c 1 and c 2, and the known k i T i d I, it is not difficult to estimate the unknowns K c i T i i; thus we obtain the plant thus we have estimate the unknown controller parameters using the final expressions given here.

So, once the controller parameters has been obtained, then only u have got the two series PID controller for the TITO process; now, this is how we have extended the controller design method technique to TITO process.

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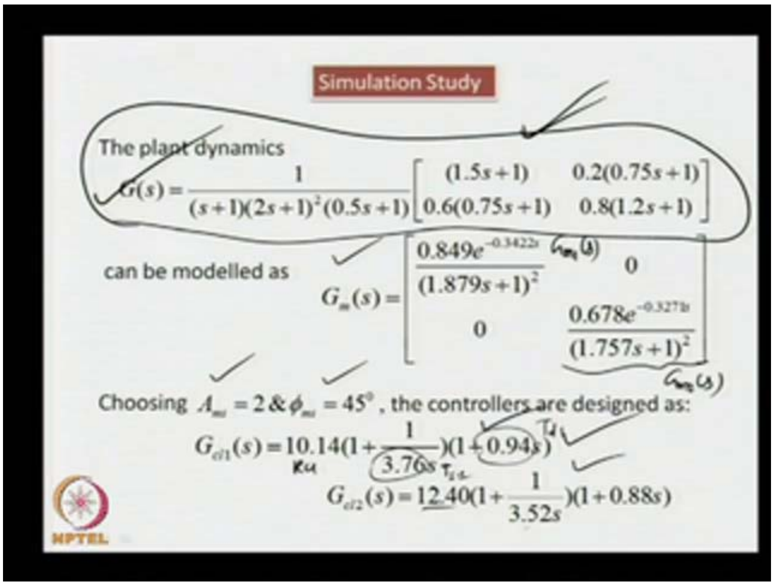

Now, we shall see how the control loops are performing, for the stimulation study let us assumed the plant dynamics to have the transfer function 1 upon s plus 1 2 s plus 1 square point 5 s plus 1 with interactions also the original plant dynamics can be modeled in the convenient form given over here. So, the original plant dynamics has been modeled as G m s equal to given by 1 G m 1 s 0 0 G m 2 s; next choosing the phase margins of 2 and Gain Margins of 45 degree for both the loops, the controller parameters are designed in such a way, that we have got 2 series PID controller for the TITO process given in the form of G c l 1 s equal to 10.14 times 1 plus 1 upon 3 point 7 6 s times 1 plus 9 1 plus 0.94 s and g c l 2 s equal to 12.4 times 1 plus 1 upon 3.52 s times 1 plus 0 point 8.8 s. So, we have got here k c 1, this value is the T i 1, and this is our T D 1, similarly we have got k c 2 T i 2 and T D 2.

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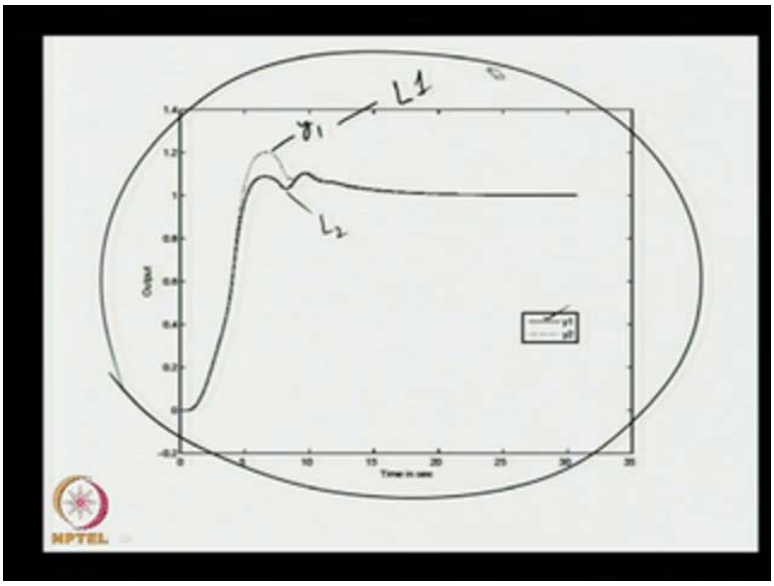

So, employing the controllers, we get a time response of this form, where y 1 gives us the output of the first loop l 1; so, this is the output of loop 1, and this is the output of loop 2, we can say that since we have got a TITO process, we have got two outputs for the process given as y 1 and y 2 t. So, the two outputs are found to be satisfactory, in spite of interaction among the loops or interaction between the loops in the T i to process, the output response is found to be quite satisfactory. The lesson can be summarized in this fashion, we have designed gain and phase margin based series PID controller for a SISO process initially, and the design technique the controller design technique can be extended to TITO process which is provided, the TITO process dynamics is available in diagonal transfer metrics form simulation diagrams, and simulation results show us that it is possible to obtain satisfactory closed loop performances from, the TITO process using control design using phase and Gain Margins.

Points to ponder, first point, if somebody asks where do we use approximation for the 10 inverse function, the arc tan function has been approximated for each in analysis, if the approximation is not used, then the set of 4 equations are to be solved simultaneously, since those equations are non-linear equations, therefore, solution may or may not convert or with proper choice of initial conditions only the solutions may convert. So, there are ways in solving the set of non-linear equations to avoid that to get explicit expressions for the plant for the given plant to obtain explicit expressions for the controller parameters, it is better to make assumptions.

If the second question goes like this, whether transfer function metrics is given in diagonal form, not necessarily the dynamics has to be made available in the diagonal transfer function form, if it is not so, in that case, some coupling technique has to be made use to design effective controller for the TITO process.

Lastly the question is like this, that are, there any method available to design SISO controllers for a TITO system, yes, one can use the couplers, one can use some other technique to reduce loop interactions and design controllers for a TITO system that is all in this lecture.