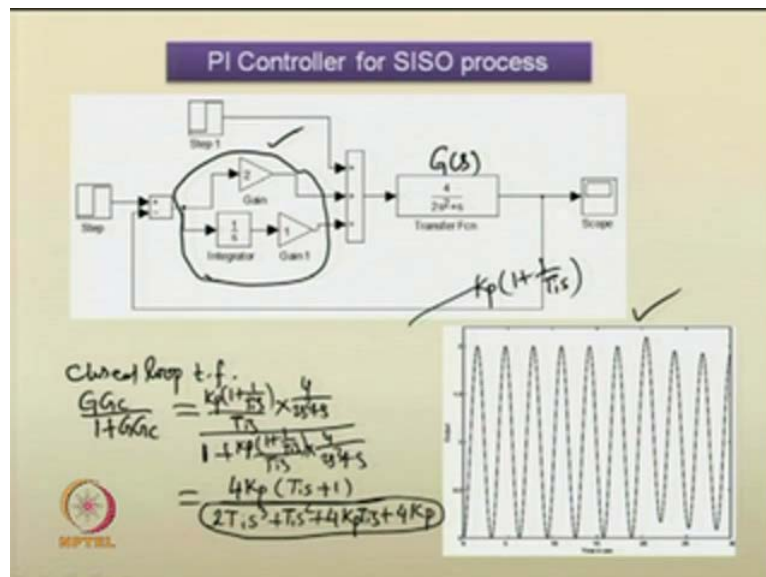


Advanced Control Systems
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Module No. # 01
Model Based Controller Design
Lecture No. # 05
Design of Controller for SISO System.

Welcome to this lecture, a wealth of techniques is available for analysis of linear time invariant systems and design of controller for SISO systems; what a SISO system, it is a system that has got single input and single output. In this lecture, we shall try to design a powerful technique for such SISO process, in spite of the wealth of techniques available still search **is on** to find suitable, and most acceptable technique for PID controller design. We shall see how a simple but powerful control design technique can be developed.

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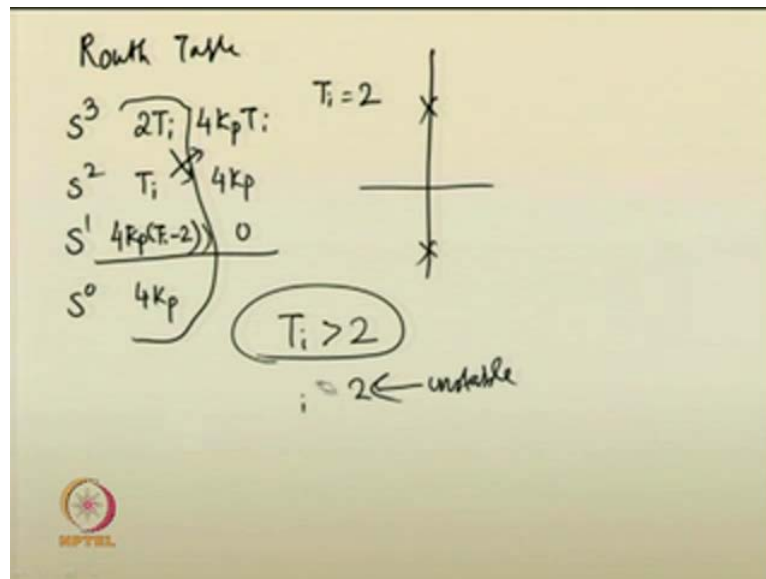
Coming to the PI controller for SISO process, here we have got a SISO process given by the transfer function $\frac{4}{2s^2 + s}$, and we have got a PI controller given by the proportional gain 2, and the integral gain 1, the PI controller can also be written in the

form of $G_c(s)$ equal to $2 + 1/s$, which in the standard form becomes $2 + 1/s$ upon $2s$; so, the integral time constant becomes 2 second.

Now, why this process is known as a SISO or single input single output process, the process gives us one output, and the process is subjected to one input, where we get such type of processes; let us consider this room, the room can be treated as one such process, if we try to control the room temperature, in that case the input to the room could be the full air that can be injected and the output from the room will be the room temperature. So, we can develop a controller for control of room temperature, and in this case, we have got the process $G(s)$ which dynamics is given by $4/(2s^2 + s)$, as we have said earlier, but when this controller is employed, we get an oscillatory output from the process, why do we get such oscillatory output to understand the logic behind that we need to analyze the systems dynamics.

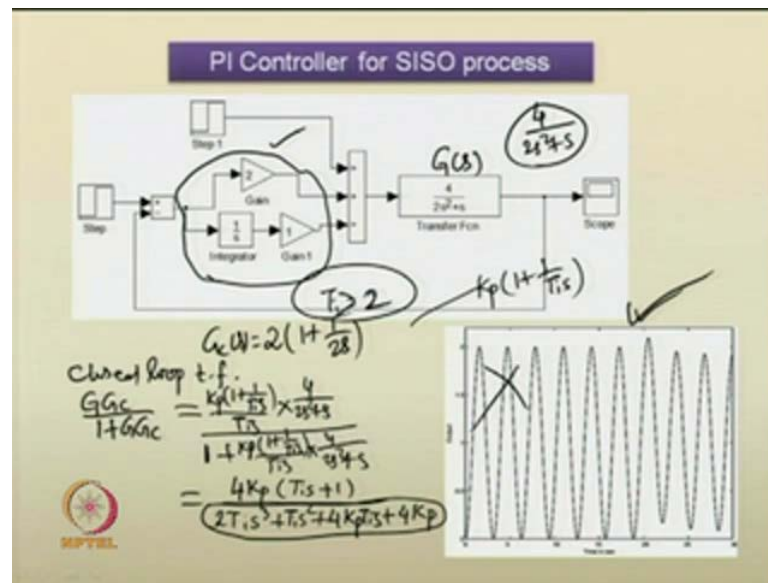
Let us consider the systems closed loop transfer function, closed loop transfer function which can be given in the form of $G_c(s)/G(s)$, for analysis let me assume the dynamics of this controller $G_c(s)$ to be in the standard PI controller form $K_p(1 + 1/T_i s)$, then that will give me the closed loop transfer function as $K_p(1 + 1/T_i s) \times 4/(2s^2 + s)$ divided by $1 + K_p(1 + 1/T_i s) \times 4/(2s^2 + s)$, which can ultimately be available in the form of $4K_p(T_i s + 1)$ in the numerator, and $2T_i^2 s^3 + T_i^2 s^2 + 4K_p T_i s + 4K_p$ in the denominator. So, the stability of the closed loop transfer function or the closed loop system can be ascertained from the denominator polynomial of the closed loop transfer function.

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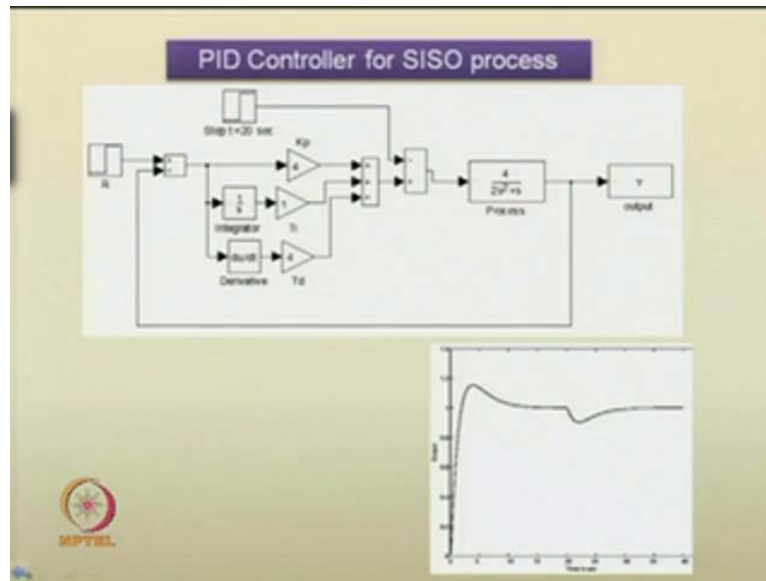
If we form the Routh table for the closed loop transfer function, in that case the Routh table assumes the form of s^3 , s^2 , s^1 , s^0 with the coefficient s of the row having s^3 as $2T_i$, $4K_p T_i$ for this T_i , and $4K_p$ are the coefficients, and next from this cross multiplication, and manipulation we get the coefficient of s^1 row as $4K_p$ times T_i minus $2 \cdot 0$, and then $4K_p$ in the last row. What we get from this Routh table, when the coefficients of the first column possess the same sign, in that case the system is stable the closed loop system must be stable. When T_i equal to 2, what happens at that time, both the coefficients of this row will be 0, when T_i equal to 2, I get the coefficients at 0, what that implies that implies that the closed loop transfer function will have 2 poles located on the imaginary axis; that means, the system response will be oscillatory, there must be a pair of complex conjugate poles in the s plane, and which implies that the system output, closed loop system, output might be oscillatory, exactly that is what is happening when T_i equal to 2 as we have seen.

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Since the integral time constant is T_i equal to 2 giving us, again of one here as we see, because we can express the PI controller in the form of 2 times 1 plus 1 upon 2, as we have we have seen. So, when T_i equal to 2, we get an oscillatory output from the system, but when T_i will be greater than 2 or T_i is very large compared to compared to 2, in that case we must obtain a stable response from the closed loop system, but when T_i is less than 2, the closed loop system will yield us an unstable response or the the response output response will explode, this is how one can design a PI controller for the process given as 4 upon 2 square plus s, the T_i Should be greater than 2 to obtain some stable response not like this.

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Next, let us attempt to design a PID controller for the same process; a PID controller can be designed with the same analysis, using the same analysis let me start with the closed loop transfer function assuming the form of the PID controller.

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$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right); G(s) = \frac{4}{2s^2 + s}$$

$$CL\ TF = \frac{G_c G}{1 + G_c G} = \frac{4K_p (T_i T_d s^2 + T_i s + 1)}{2T_i s^3 + (T_i + 4K_p T_i T_d) s^2 + 4K_p T_i s + 4K_p}$$

Routh Table

s^3	$2T_i$	$4K_p T_i$	
s^2	$T_i + 4K_p T_i T_d$	$4K_p$	
s^1	$4K_p T_i - \frac{8K_p}{1 + 4K_p T_d}$	0	
s^0	$4K_p$		

Let $T_i > 0; K_p > 0, T_d > 0$

$$T_i > \frac{2}{1 + 4K_p T_d} \Rightarrow T_i > \frac{2}{17}$$

$$T_i > 0.118$$

$K_p = 4; T_d = 1, T_i = 4$

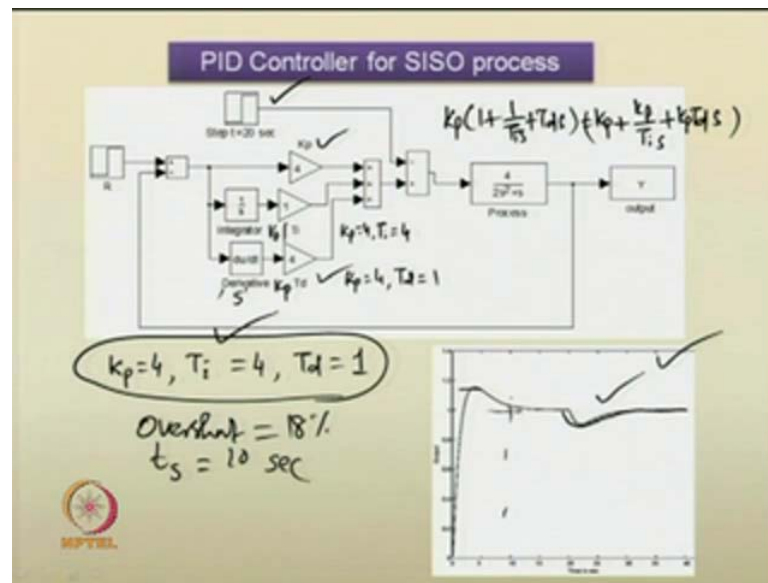
Let the PID controller be given as $G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$, and we know our process to be $G(s) = \frac{4}{2s^2 + s}$, thus that will give us the closed loop transfer function as $\frac{G_c G}{1 + G_c G}$, which can be obtained after simplification in the form of $\frac{4K_p (T_i T_d s^2 + T_i s + 1)}{2T_i s^3 + (T_i + 4K_p T_i T_d) s^2 + 4K_p T_i s + 4K_p}$ in the

numerator, and $2T_i S^3 + T_i + 4K_p T_i T_d$ times s^2 in the denominator along with $4K_p T_i S + 4K_p$, now when a PID controller of this form is used in the series compensation scheme, we get the denominator polynomial of this form. Now, again, let me form the Routh table, so forming the Routh table as $S^3 \ S^2 \ S^1 \ S^0$ with coefficients as $2T_i \ 4K_p \ T_i$, then here we have got $T_i + 4K_p T_i T_d$ and here $4K_p$, now the next coefficient can be expressed as $4K_p T_i - 8K_p$ upon $1 - 4K_p T_d$, and this is 0, then the cross multiplication like this, and this divided by this will give us $4K_p$ as the coefficient of the last row. So, the closed loop system will give us a stable response, time response when all the coefficients of the first column have same sign. So, **let me assume**, let me assume T_i is greater than 0, then this becomes positive, when K_p plus greater than 0, then this becomes positive, and when $K_p T_i T_d$ is greater than 0, T_d is also greater than 0, we get this coefficient as positive; what about this coefficient, this will be positive provided T_i is greater than 2 upon $1 - 4K_p T_d$; so, these can be expressed in this form, when T_i is greater than 2 upon $1 - 4K_p T_d$, in that case the coefficients of the column will be positive, thus implying that the closed loop system will have a stable time response, it will not blow out.

Now, with the choice of K_p greater than 0, T_d greater than 0; obviously, one can get some constant on T_i , T_i is greater than some value, if that is made, if this condition is made certainly, we will get a stable response from the closed loop system, for the time being let us assume K_p equal to 4, let $T_d = 1$, when K_p equal to 4 and $T_d = 1$, then the condition gives us T_i has to be greater than 2 upon 17 , that is same as T_i is greater than 0.118 approximately. So, if we choose a T_i value which is greater than 0.118 with the choice of K_p equal to 4, and T_d equal to 1, certainly we will get a stable time response from the closed loop system.

Let us assume T_i to be 4, we are far off from this value. So, certainly it will ensure stability of the closed loop system. So, when T_i equal to 4 along with K_p equal to 4 and $T_d = 1$ what sort of response.

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We expect from the closed loop system, we expect such a time response from the closed loop system, the controller K_p is 4, now this is same as $K_p T_d$ which is equal to 4, in this case T_d is 1 K_p equal to 4, and T_d 1, therefore, $K_p T_d$ is 4, in this case, it is actually K_p by T_i equal to 1, since K_p equal to 4 and T_i equal to 4, thus we are getting K_p upon t of i as 1, because the form of the controller is $K_p (1 + \frac{1}{T_i s} + T_d s)$ which gives us upon multiplication $K_p + K_p$ by $T_i s$ plus $K_p T_d s$; therefore, we have written this K_p value K_p by T_i divided by s and $K_p T_d$ derivative term giving us S .

So, this PID controller is now giving us a satisfactory response with certain overshoot response time and so on; let us try to see how much we get for the choice of K_p equal to 4, T_i equal to 4 and T_d 1, when the controller is implemented, then the time response, we get is having some overshoot of 18 percent, overshoot of approximately 18 percent, and settling time of approximately 10 second, this is what we get, and similarly, when the disturbance load, disturbance is applied to the system, after time t equal to 20 second, we get satisfactory disturbance rejection as we see, because the disturbance is dying down after some time, and it is not having very much peaking. So, what we have got from these with the choice of the soon $K_p T_i T_d$ gains or values the PID controller is giving us a satisfactory time response.

Now, question may arise, why we choose K_p to be $4 T_i$ to be 4 or T_d to be 1, can we choose better than those values to give us still better time response of the closed loop, yes, it is possible, it all depends on how we are designing a controller for a given SISO process; now, how to choose appropriate $K_p T_i T_d$, such that one can meet the design specifications, that can be answered by designing the controllers by some appropriate techniques, one such technique powerful technique will be discussed in this lecture.

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$$G(s) = \frac{K}{s^2 + \alpha_1 s + \alpha_0} \quad \text{and} \quad G_c(s) = K_p \left(1 + \frac{1}{T_i s}\right)$$

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{K K_p (T_i s + 1)}{T_i s (s^2 + \alpha_1 s + \alpha_0) + K K_p (T_i s + 1)}$$

$$= \frac{T_i s + 1}{\frac{T_i}{K K_p} s^3 + \frac{\alpha_1 T_i}{K K_p} s^2 + \frac{(\alpha_0 + K K_p) T_i}{K K_p} s + 1}$$

$$s = \beta s_n, \quad \beta^3 = \frac{K K_p}{T_i}$$

$$T(s_n) = \frac{G_c(s_n)G(s_n)}{1 + G_c(s_n)G(s_n)} = \frac{T_i \beta s_n + 1}{s_n^3 + \frac{\alpha_1}{\beta} s_n^2 + \frac{(\alpha_0 + K K_p) T_i}{\beta^2} s_n + 1}$$

$$= \frac{C_1 s_n + 1}{s_n^3 + \alpha_2 s_n^2 + \alpha_3 s_n + 1} \quad T(\omega) = \frac{C_1 \omega + 1}{\omega^3 + \alpha_2 \omega^2 + \alpha_3 \omega + 1}$$

Let us go to some analysis, now before going to the powerful technique, when the process, when the process is assumed to have the form $G(s)$ equal to k upon s^2 plus $\alpha_1 s$ plus α_0 , and when the controller $G_c(s)$ is assumed to have the transfer function $K_p \left(1 + \frac{1}{T_i s}\right)$ a PI controller for a all pole single input single output process, we get the closed loop transfer function $T(s)$ expressed as $G_c(s)G(s)$ upon $1 + G_c(s)G(s)$ as $K k_p T_i s + 1$ divided by $T_i s$ times s^2 plus $\alpha_1 s$ plus α_0 plus $K k_p T_i s + 1$; now, this can be simplified further, and expressed in the form of $T_i s + 1$ by T_i upon $K k_p s^3$ plus $\alpha_1 T_i$ by $K k_p s^2$ plus α_0 plus $K k_p T_i$ by $K k_p s$ plus 1, why are you doing like this, so, that with the assumption of s equal to βs_n and β^3 equal to $K k_p$ by T_i , it is easy to get the closed loop transfer function in the normalized form given as $T(s_n)$ as $G_c(s_n)G(s_n)$ upon $1 + G_c(s_n)G(s_n)$ in some convenient form, which can be written as $T_i \beta s_n + 1$ upon s_n^3 plus α_1 by βs_n^2 plus α_0 plus $K k_p$ by βs_n plus 1

$n + 1$, which can be written in the generalized form $C_1 s^{n+1}$ divided by $s^{n+3} + d_2 s^2 + d_1 s + 1$; so, why are you doing all this analysis.

When we have some all pole process transfer function and a PI controller of this form, the closed loop transfer function can always be expressed in this simple convenient generalized form, had there been a 4th order, all pole process dynamics, in that case what we will get the closed loop transfer function, in that case $T(s)$ will be simply of some $C_1 s^{n+1}$ in the numerator along with $s^{n+5} + d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + 1$; so, this is why I am calling we are getting some generalized transfer function, because irrespective of any order of the process dynamics, if the process dynamics is available in the all pole form, in that case it is always possible to get the standard transfer function which is expressed in the form of standard closed loop transfer function expressed in the form of having one numerator in the transfer function, and having denominator in this convenient form, and where we will have the denominator expressed as some s^n to the power n plus $d_{n-1} s^{n-1}$ and so on till 1.

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$$G(s) = \frac{K}{s^2 + d_1 s + d_0} \quad \text{and} \quad G_c(s) = K_p \left(1 + \frac{1}{T_i s} \right)$$

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{K K_p (T_i s + 1)}{T_i s (s^2 + d_1 s + d_0) + K K_p (T_i s + 1)}$$

$$T(s) = \frac{C_1 s + 1}{s^3 + d_2 s^2 + d_1 s + 1}$$

$$d_2, d_1 \rightarrow C_1$$

$$T(s_n) = \frac{G_c(s_n)G(s_n)}{1 + G_c(s_n)G(s_n)} = \frac{C_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1}$$

$$T(\omega) = \frac{C_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1}$$

So, this closed loop transfer function is assuming some typical form, if it is possible to find a convenient a satisfactory closed loop transfer function, which is of the form $C_1 s^{n+1}$ upon $s^{n+3} + d_2 s^2 + d_1 s + 1$, if it is possible to get said of d_2 and d_1 values for some chosen C_1 , by optimizing some performance index, in that case

what will happen, we may get a transfer function which will give us desired time response of a closed loop system that is the objective.

Then from back calculation, since d_2, d_1 are made of nothing but the coefficients or the parameters of the controller, one can easily estimate the controller parameters. So, what we have been doing, we are trying to find some standard transfer function which are known to us, and which are tested also that they are time response will be highly satisfactory, then the closed loop transfer function can be used by back calculation to calculate the unknown parameters of a controller, which will definitely yield very satisfactory closed loop performance of a closed loop system. So, we will see how to calculate some standard transfer function, how to find some suitable standard transfer functions of different order.

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PID Controller for SISO process

Consider 3rd order **Standard Transfer Function** $T(s) = \frac{Y}{R} = \frac{c_1 s + 1}{s^3 + d_2 s^2 + d_1 s + 1}$

The error function becomes: $\frac{R-Y}{R} = \frac{E}{R} = \frac{s^3 + d_2 s^2 + (d_1 - c_1)s}{s^3 + d_2 s^2 + d_1 s + 1}$

The error becomes $E(s) = \frac{s^2 + d_2 s + (d_1 - c_1)}{s^3 + d_2 s^2 + d_1 s + 1}$

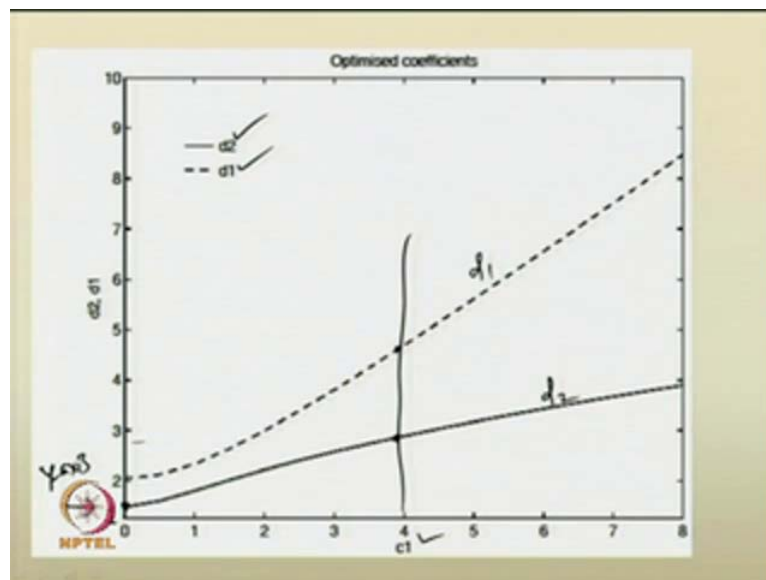
Use $J_{ISTF} = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(s)F(-s)ds$ where $F(s) = -\frac{\partial E(s)}{\partial s}$

Compute optimum values d_2, d_1 that results in step responses for $T(s)$ for various c_1

Consider a third order transfer function, which can be given in the form of $T(s)$ equal to y upon r equal to $C_1 s + 1$ divided by $s^3 + d_2 s^2 + d_1 s + 1$, this is called a standard transfer function, because we have seen that with appropriate d_2 and d_1 with a choice of C_1 , this transfer function closed loop transfer function gives us quite satisfactory time responses, what we wish to have from a closed loop control system, then the error function can be obtained as $R - Y$ upon R which is same as E upon R equal to $s^3 + d_2 s^2 + d_1 s + 1$ minus $C_1 s + 1$ divided by $s^3 + d_2 s^2 + d_1 s + 1$, then the error function we obtain in that form.

Now, this error can be minimized using the performance indices, in this case how using some ISTE performance index, one can minimize and find the optimum values of C_1 , d_2 and d_1 is explained the ISTE criterion is expressed as one upon 2π J integration from minus J infinity to J infinity $F(s) \times F(-s) ds$, where $F(s)$ equal to partial differentiation of $E(s)$, how to compute the coefficients C_1 , d_2 , d_1 using some minimization routine, one can easily use lustrums refer ship algorithm writing some simple met lab core, it is easy to minimize these performance index minimization of this performance index yields a said of C_1 , d_2 and C_1 . So, a said of d_2 means all the d_2 coefficients d_2 , d_1 and so on, and C_1 can be obtained, and that ensures us the minimum value of the ISTE criterion, one can use the ISE criterion also, but often it is found that the IST criterion, minimization of IST criterion often leads to very satisfactory closed loop performances of a closed loop system, how such values are obtained, we can see. So, using the ISTE optimization or minimization, one can obtain d_2 and d_1 coefficient for various C_1 ; this gives the X axis goes from 0 to 8, and the Y axis goes from 0 to 10; so, this is the beginning point for the Y axis.

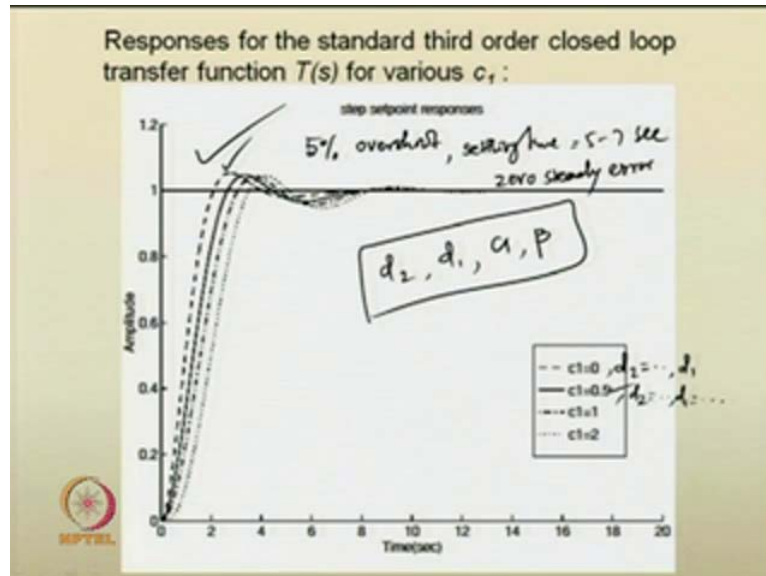
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So, this is what we have got d_2 and d_1 . So, for any C_1 , suppose C_1 equal to 4, I can find the Optimum values for d_2 and d_1 , from here Optimum values for d_2 and d_1 , when C_1 equal to 0, the Optimum values of d_2 is this much, and the Optimum values for d_1 is this much. So, this set of Optimum C_1 , d_2 , d_1 gives us a standard third order transfer function, and when the closed loop transfer function with the use of a controller

becomes a standard third order transfer function, then in that case we get definitely a quite satisfactory time response from the system.

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What sort of time responses is expected from the standard transfer functions? Let us see, when C_1 equal to 0, the closed loop time Response will be of this form, when C_1 equal to 0.5, we get, you see when C_1 is equal to 0, we have got finite d_2 and d_1 value; similarly, for each C_1 values, we have got the said of these values are d_2 is something and d_1 equal to something, all those values can be obtained from this plot. So, for any given C_1 , always we get some d_2 and d_1 , these Optimum values of d_2 and d_1 for any given C_1 always yields a quite satisfactory time response of the closed loop system, why do I call this quite satisfactory time response, if we look at the responses, we see that the responses are having at most 5 percent of over shoot.

Similarly, the responses have got settling time, settling time from 5 to 7 second, and the responses has got 0 steady state error, these are quite desirable while designing a closed loop control system or while designing a controller, so the standard transfer functions are giving us quite satisfactory time responses of the closed loop system, then from the back calculation, we can find out the parameters of a controller using the expressions d_2, d_1, C_1 and expressions for beta.

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$$T(s) = \frac{Y}{R} = \frac{c_1 s + 1}{s^3 + d_2 s^2 + d_1 s + 1}$$

Loop Gain

$$GG_c(s) = \frac{c_1 s + 1}{s(s^2 + d_2 s + d_1 - c_1)}$$

$$T(s) = \frac{GG_c(s)}{1 + GG_c(s)} \Rightarrow GG_c(s) = \frac{T(s) - 1}{T(s)}$$

GM, PM c_1, d_2, d_1

We shall see in the form of some example, **in some**, in the simulation example, how convenient controller can be designed, before going to that, we have seen that the standard transfer function can be expressed in this form, where $T(s)$ is the closed loop transfer function having the numerator $C_1 s + 1$ upon $s^3 + d_2 s^2 + d_1 s + 1$, it is loop gain can be obtained, how do we get loop gain, because we know that $T(s)$ is nothing but $GG_c(s)$ upon $1 + GG_c(s)$. So, which can be used to get $GG_c(s)$ as $T(s) - 1$ upon $T(s)$ with little manipulation, it is easy to see that, one can obtain $GG_c(s)$ as $T(s) - 1$ upon $T(s)$.

So, when $T(s)$ is given in this form, then $GG_c(s)$ will be available, it can be obtained in the form of $C_1 s + 1$ divided by $s^3 + d_2 s^2 + d_1 s - C_1$, where you getting these, loop gain from the loop gain, it is easy to get the gain margin phase margin of the closed loop system. So, this gain and phase margins will give us information about frequency response of the system. So, when the stand up C_1, d_2 and d_1 which are optimum as far as the standard transfer function is concerned are good in this expression, and the gain and 5 phase margins are found, definitely one will get very satisfactory phase and gain margins.

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Standard form based controller design

Consider a process $G(s) = \frac{0.5}{s^2 + 1.595s + 1.62} = \frac{K}{s^2 + \alpha_1 s + \alpha_0}$

To design a PI controller $G_c(s) = K_p + \frac{K_i}{s} = K_p \left(1 + \frac{K_i}{K_p s}\right)$

Closed loop tf becomes

$$T(s) = \frac{(K_p/K_i)s + 1}{(s^3/KK_i) + (\alpha_1 s^2/KK_i) + (\alpha_0 + KK_p)s/KK_i + 1}$$

Assuming $KK_i = \beta$ and $s = \beta s_n$

The tf can be written as

$$T(s_n) = \frac{c_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1}$$

where $d_2 = \alpha_1 / \beta$; $d_1 = (\alpha_0 + KK_p) / \beta^2$; $c_1 = K_p \beta / K_i$

$d_2, d_1, c_1 \rightarrow \text{coeffs } T_i$

Let us go to the simulation study, where we will make use of the standard form based controller design, a third order standard transfer function will be used to design a PI controller for a all pole, for an all pole SISO process. So, Let us assume that the process dynamics is having all poles, means, having poles only with a static gain of k, and its denominator polynomial expressed as s square plus alpha 1 s plus alpha 0, then we will go back to some specific form of, form of transfer function, and design the PI controller. Let the PI controller be expressed in the transfer function as K p plus K i upon s, which can again be expressed in the standard form as K p 1 K i by K p s.

So, for each in analysis we have expressed the transfer function of the controller in the form of K p plus K i s. So, with this process dynamics, and PI controller dynamics, the closed loop transfer function becomes T s equal to K p upon K i s plus 1 divided by s cubed by K K i plus alpha 1 s square by K K i plus alpha 0 plus K K p times s by K K i 1; so, the closed loop transfer function can be obtained in this form.

Now, assuming K K i as beta cubed and setting s as beta s n, the above transfer function can be expressed as T s n equal to C 1 s n 1 divided by s n cube plus d 2 n square plus d 1 s n 1, where d 2 equal to alpha 1 by beta, if you see d 2 equal to this alpha 1 by beta, then d 1 equal to alpha 0 plus K k p by beta square and C 1 equal to K p beta upon K i. So, all these comparing this transfer function, with this one definitely one can write the expressions for d 2 d 1 and C 1, in this fashion; why we are writing in this fashion, now

you see the beauty of this method, now the d_2 parameter is having the plant parameter or process parameter α_1 and β d_1 is continuing the unknown controller parameter K_p and C_1 is controlling the unknown controller parameter K_p and K_i both.

So, using the Optimum values of d_2 , d_1 and C_1 , it is possible to use back calculation, and get the design values for K_p and T_i , this is how one designs a controller using standard form. Now, let us go to the particular case, for our case, as we have seen when the process dynamics is given as 0.5 upon s square $1.595 s$ plus 1.62 at that time k becomes 0.5 α_1 becomes 1.595 and α_0 becomes 1.62 .

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Given model parameters : $K = 0.5$; $\alpha_1 = 1.595$ and $\alpha_0 = 1.62$

Using plot for the optimized coefficients,

$c_1 = 0.5 \rightarrow d_2 = 1.595$ and $d_1 = 2.12$

But $d_2 = \alpha_1 / \beta \Rightarrow \beta = 1$

Then, $KK_i = \beta^3 \Rightarrow K_i = \beta^3 / K = 1/0.5 = 2$

and $c_1 = K_p \beta / K_i \Rightarrow K_p = c_1 K_i / \beta = 0.5 * 2 / 1 = 1$

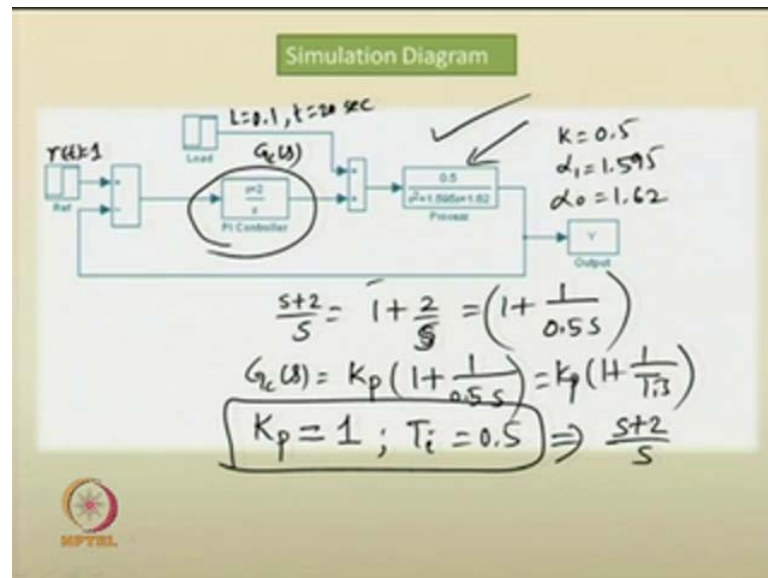
Thus, the PI controller is designed as :

$$G_c(s) = K_p + \frac{K_i}{s} = 1 + \frac{2}{s}$$

That is what we have got, then substituting these values and using the plot for the optimized coefficients, when C_1 equal to point 5 d_2 can be obtained as 1.595 , and d_1 equal to 2.12 , where from we get when C_1 equal to 0.5 , what are the Optimum values for d_1 and d_2 , when C_1 equal to 0.5 , I can find the d_2 to be of this 1.595 , when C_1 equal to 0.5 , d_2 , 1.595 and d_1 equal to 2.12 ; so, this what we get from the plot; next, but d_2 can also be expressed as α_1 by β , α_1 is known to us, thus giving us β as 1 α_1 is 1.595 , whereas d_2 is 1.595 , therefore, $\beta = 1$. Next expression we have got that, we have got one more expression KK_i equal to β^3 , that we have assumed KK_i equal to β^3 , then since $\beta = 1$; therefore, K_i can be obtained as β^3 by KK_i is given, that is one of the process parameter 1 upon 0.5 , so k become 2 .

Similarly, using the expression $C_1 = \frac{K_p \beta}{K_i}$, K_p unknown, since we know $C_1 \beta$, and K_i are known the only unknown K_p can be estimated as 1, thus we have been able to design a PI controller, which is given as $G_c(s) = K_p + \frac{K_i}{s}$; what is expected from this controller? This controller must give us a time response which is as we have seen for the standard transfer function.

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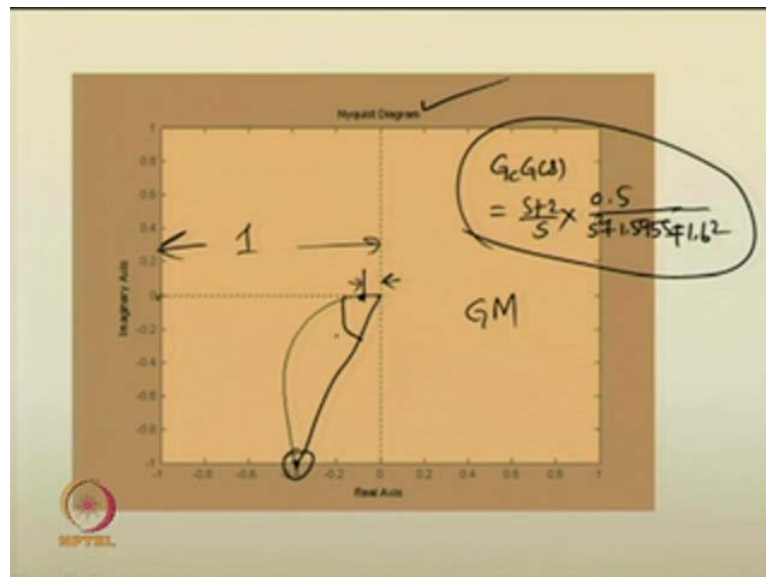
Now, going to the simulation diagram in the simulation diagram, what we have, we will apply unit step input. So, $r(t)$ the reference equal to some unit step input 1, then let the load be also given by some magnitude L equal to 0.1 with the time at which it occurs at time t equal to 20 second; now, the PI controller, we have designed for the given process is $s + 2$ upon s which is nothing but $s + 2$ upon s can be expressed as one time 1, one upon 0.5 s, no, in this case, it will be different, let us go back to the exact form we have got, I have got $1 + \frac{2}{s}$, so, in this case since I am making use of this one. So, I will have one 0.5 over here, than in that case, it is giving us $0.5s + 1$, and we are not getting that way how can we express, this in the standard form the form, I have written is not correct actually. So, to get it in the standard PI transfer function form $s + 2$ by s can be written as $1 + \frac{2}{s}$ which is nothing but $1 + \frac{1}{0.5s}$.

So, it is in the form of $G_c(s) = K_p \left(1 + \frac{1}{0.5s}\right)$, where we have got 0.5, in the form of again $K_p \left(1 + \frac{1}{T_i s}\right)$. So, I can say that I have designed a PI controller,

where the proportional gain is one and the integral time constant equal to 0.5 giving us a PI controller in the form of $s + 2$ by s .

So, this is how we have realized the PI controller $G_c(s)$ for the process having parameters k equal to 0.5 and $\alpha = 1.595$ and $\alpha = 0.162$. So, what sort of output we will expect from this one, when this is stimulated, definitely we will expect a quite satisfactory time response from the closed loop system. As we expect the output here is having almost settling time of 5 percent and of **overshoot of, sorry overshoot of** overshoot of 5 percent settling time is of 8 second as expected. So, what we get basically, we get a closed loop transfer function response, that is much similar to that we get from a standard closed loop transfer function.

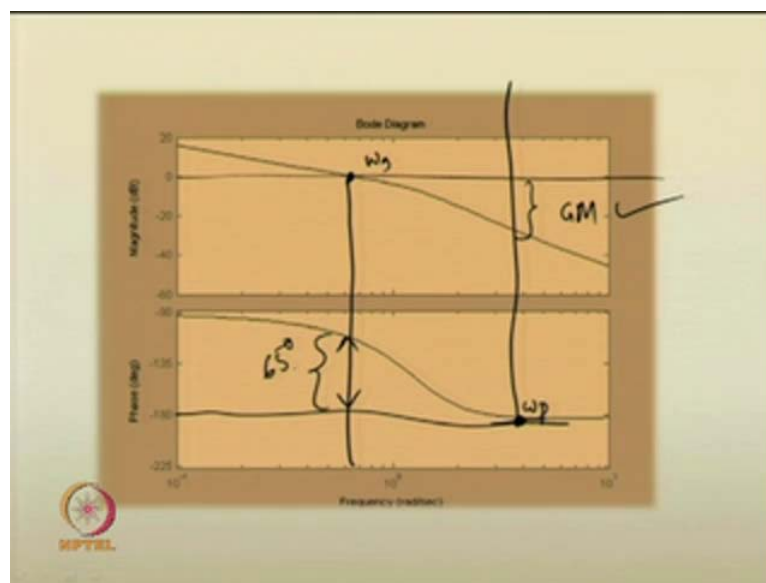
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Coming to the other analysis, frequency analysis or frequency response of the closed loop system, the Nyquist diagram of the closed loop system can be drawn in this case, for which we need to get the loop gain $G_c(s)G(s)$ the loop gain for this will be $s + 2$ upon s time, we have got 0.5 by $s^2 + 1.595s + 2.1162$, we have got 1.62 , 1.62 . So, using this loop gain or loop transfer function, one can obtain the polar plotter Nyquist diagram what information, we get from Nyquist diagram, the phase crossover point is somewhere around here, and the gain cross over point will be somewhere around here, as we see measure this span, and inverse of this will give us the gain margin, this span is very small, because this span total span from here to here one.

So, we will have a very good gain margin for the system; similarly, if I draw a line from here to here, and measure this angle, I will get a very good phase margin for the system. What are those values, let us see using the analysis, one can obtain those values as some gain margin of 12 in absolute value term, and the phase margin of some 65 degree, we know that a gain margin of greater than 2, and a phase margin of greater than 30 degree ensures a robust control system or a quite stable closed loop system. So, as far as stability of closed loop transfer function is concerned, we are far off from those values, and therefore, we have got a quite robust closed loop control system.

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Now, same information can be derived using the bode diagram. So, using the bode diagram, we can track the phase and gain cross over frequencies, and then the phase and gain margin. So, phase cross over frequency, if I draw a line over here at 180 degree, I will get somewhere like this, and that will give me from the phase cross over frequency, we can draw a line vertically, and find how much you are away from 0 degree, this much will give you the gain margin. So, minus of this value will give us the gain margin; so, we get a quite high gain margin for the systems; similarly, let us track the gain cross over frequency. So, this is the gain cross over frequency, the frequency at which the gain of the system is 1 or in dB the gain of the system is 0 dB.

So, we will get the gain cross over frequency of this magnitude, for which let us find what is the phase angle at that time, how far you are from the minus 180 degree, we are

far of which gives us a phase margin of approximately some 60 to 65 degree, thus what we see, that the closed loop system is not only giving us a quite satisfactory time response, also we are getting a quite satisfactory frequency response from the system.

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PID Controller for SISO process

Assume the PID controller is

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s}\right) (1 + T_d s)$$

Let the process model be of the form

$$G(s) = \frac{K(s + c_1) \dots \dots \dots}{(s + d_1)(s + d_2) \dots \dots \dots}$$

Choose $T_d = 1/d_1$ and design the remaining controller parameters

Handwritten notes: 'system' with a plot, 'd1 -> small value', and a checkmark.

Now, there is one problem associated with this method that one has to make use of series form of PID controller in place of the parallel form of PID controller. What a parallel form of PID controller is we write a controller given by the transfer function $G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right)$ as the transfer function; for a parallel form of PID controller, when a parallel form of PID controller, then the controller itself is introducing 20 dB/dec , if you simplify this one, it is giving us an expression in the form of $K_p \left(T_i^2 s^2 + T_i s + 1\right) / T_i s$. So, from this expression we say that the transfer function of the PID controller, which is in parallel PID controller form gives us 20 dB/dec introduces 20 dB/dec in the closed loop system and one pole at the origin.

So, when we will have 20 dB/dec given by the controller, and the plant is all pole system in that case, what happens the closed loop transfer function will be having 20 dB/dec in the numerator; that means, the standard transfer functions will have 20 dB/dec , for which it is very difficult to find the coefficients done d_2 and so on, the point is that we have got standard transfer functions expressed in the form of $C_1 s + 1$ upon $T_i s$, then here we can have any order s to the power n plus $d_n s^{n-1}$ to the power $n-1$ and so on, but in the numerator we have got 10 only, so, how to handle those situations, to handle those

situations, what we have to have, we can always have a parallel form a series form of the PID controller, in which case one of the zero of the PID controller is to be cancelled with one of the pole of the all pole transfer function. How to select, how to make cancellation, we should choose the T_d of the controller as one upon d , and this d should be as small as possible d should assume some small value, in that case, we are assuming that a pole is cancelled a poled, which a pole which is located far off from the left half from the origin of s plane is cancelled, it is getting cancelled with the derivative term of the series PID controller, then we can make use of a series form of PID controller, and a all pole process model to design controllers using standard form.

Now, what have we learnt from the lesson, we have seen how a standard transfer function results in satisfactory time and frequency responses, and how a standard transfer function can be used to design the parameters of a p PI or PID controller using back calculation, but there is one limitation with the method the process dynamics has to be available in the form of all pole transfer function for, and the form of controller, one can use, when we have a PID controller is that a series PID controller.

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Points to ponder

P-1 : What happens when a process dynamics is $G(s) = \frac{Ke^{-\theta s}}{(T_1 s + 1)}$?

$G(s) = \frac{K}{(1 + \theta s)(T_1 s + 1)}$ where $\theta \rightarrow$ small value
plant order reduction

P-2 : Why do we design series PID controller?

$d's$ & c_2, G

Now, we will go to the points to ponder, what are some limitations of the method, what happens when a process dynamics is available in the first order plus dead time transfer function form, when the process dynamics is available in this form, one can always get the process dynamics expressed in the form of k upon $1 - \theta s + T_1 s + 1$, of course,

when θ is a small number small value, when that is not so, make use of plant order reduction technique or process order reduction technique, order reduction technique to get the plant dynamics available in the form of all pole transfer function, second point to ponder is that, why do we design series PID controller, as we have discussed that a series PID controller enables us to cancel one of the zero of the controller with one pole of the all pole transfer function of the process, then thus giving us $1, 0$ in the standard transfer function, if that is not the case, in that case, when we use one parallel PID controller $2, 0$ Es appears in the standard transfer function, and it is very difficult to find the Optimum values of d s and c 2 C 1 , in the case of parallel PID controller, that is why we need to design series PID controller, that is all in the lecture .