

Advanced Control Systems
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Module No. # 01
Model Based Controller Design
Lecture No. # 10
Effects of Measurement Noise and Load Disturbances

Welcome to the lecture titled Effects of Measurement Noise and Load Disturbances. Unwanted responses in the output of the system are often known as noise responses and load disturbance responses. Noise responses are generally of high frequencies, whereas load disturbance responses are of low frequencies.

It is often very difficult to design a controller, which can reduce the **effects of** ill effects of noise and load disturbances simultaneously. We shall see in this lecture, how a noise canceller can be designed using standard form.

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Standard form based controller design

Consider a process $G(s) = \frac{K}{s^2 + \alpha_1 s + \alpha_0}$


To design a PI controller $G_c(s) = K_p + \frac{K_i}{s}$

Closed loop tf becomes

$$T(s) = \frac{(K_p / K_i)s + 1}{(s^2 / KK_i) + (\alpha_1 s^2 / KK_i) + (\alpha_0 + KK_p)s / KK_i + 1}$$

Assuming $KK_i = \beta^3$ and $s = \beta s_n$ $s_n = \text{normalized 's'}$
 $\alpha_1 \rightarrow d_2, d_1$

CLTF can be written as

$$T(s_n) = \frac{c_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1}$$


Consider a process, given as $G(s) = \frac{K}{s^2 + \alpha_1 s + \alpha_0}$. So, this all-pole second-order process has got poles located at, suppose at far off from the left

half of the s plane. Then in that case, it becomes very easy to design controllers whereas, we shall consider a typical case, where α_1 will be less than α_0 , which signifies that the process is of under damped type.

So, let us consider the process dynamics as shown over here (Refer Slide Time: 01:53), for this process when the PI controller dynamics is defined as $G_c(s)$ equal to K_p plus K_i upon s , then the closed loop transfer function $T(s)$ which is nothing but, $T(s)$ is equal to $G(s)G_c(s)$ upon $1 + G(s)G_c(s)$ can be retained and simplified and obtained in a form shown over here, where the closed loop transfer function numerator can have the terms K_p upon K_i times s plus 1 divided by s cubed upon K_i plus $\alpha_1 s$ square upon K_i plus α_0 plus $K_p s$ upon K_i plus 1.

So, when the transfer function has been obtained in this typical form, then assuming K_i is equal to β^3 and s equal to βs_n , where s_n stands for the normalized s Laplace on domain. Then, the closed loop transfer function can be written as, $T(s_n)$ is equal to $c_1 s_n$ plus 1 upon s_n cubed plus $d_2 s_n$ square plus $d_1 s_n$ plus 1.

Thus, it has been possible to obtain the closed loop transfer function in the standard third-order transfer function form. This standard transfer function, third-order transfer function has got three coefficients namely c_1 , d_2 and d_1 and we know that, for any given c_1 , it is not difficult to find d_2 and d_1 by **optimize** optimization or optimization of some standard criterion often known as integral square time error criterion.

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where $d_2 = \alpha_1 / \beta$; $d_1 = (\alpha_0 + KK_p) / \beta^2$; $c_1 = K_p \beta / K_i$

Next $d_1 = (\alpha_0 + KK_p) / \beta^2$ d_2, d_1, c_1

$\Rightarrow KK_p = \frac{\alpha_0 c_1}{d_1 - c_1}$

Similarly, $d_2 = \alpha_1 / \beta$

$\Rightarrow KK_i = \left(\frac{\alpha_1}{d_2}\right)^3$

$c_1 \rightarrow d_2, d_1$

$$KK_p = \frac{\alpha_0 c_1}{d_1 - c_1}$$

$$KK_i = \left(\frac{\alpha_1}{d_2}\right)^3$$

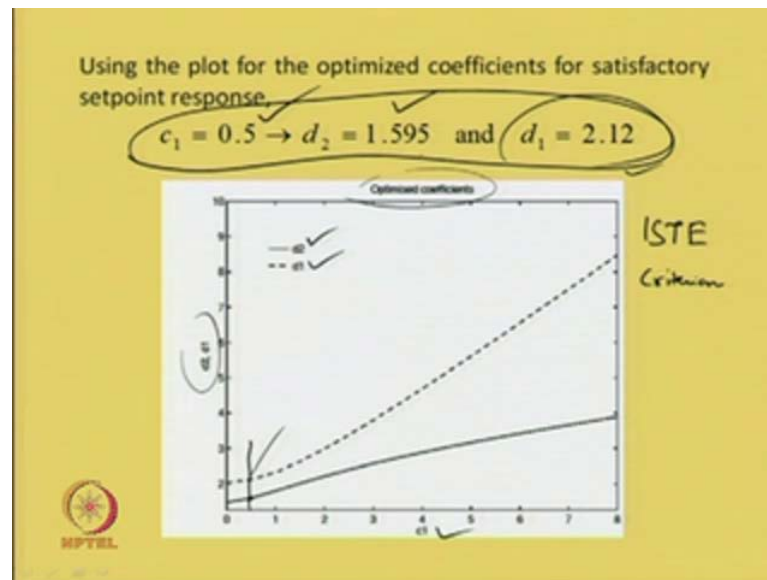
Now, we have got the d_2 expressed as α_1 upon β , d_1 expressed as α_0 plus K_p upon β^2 and c_1 is equal to $K_p \beta$ upon K_i . These are the three variables we have in the standard transfer function, now manipulating this three variables, using the three variables, one can write d_1 is equal to α_0 plus K_p upon β^2 is equal to α_0 upon β^2 plus K_p upon β^2 .

Again, this can be written as α_0 upon β^3 times β in the numerator plus K_p upon β^3 . Now, β we know is β^3 is equal to K_i . So, substitution of K_i will give you, K_i plus $K_p \beta$ upon K_i . So, again $K_p \beta$ is equal to $c_1 K_i$. So, $\alpha_0 \beta$ upon K_i plus here, one can write as K_i upon K_i . So, K_i cancellation will be there, this can be written as c_1 plus $\alpha_0 \beta$ upon K_i . So, manipulating this, it is not difficult to write the expression for K_p as K_p is equal to $\alpha_0 c_1$ upon d_1 minus c_1 .

Similarly, using the three expressions, the three expressions for d_2 , d_1 and c_1 it is not difficult to write the expressions for K_i as α_1 upon d_2 to the power 3. So, basically using the variables d_2 , d_1 and c_1 , we have been able to write the final expression, explicit expressions for the 2 unknowns K_p and K_i in the form of K_p is equal to $\alpha_0 c_1$ upon d_1 minus c_1 and K_i is equal to $\alpha_1 d_2$ upon to the power 3.

So, what is the beauty of a obtaining the explicit expressions for the two unknowns K_p and K_i in this explicit form? Since α_0 is known, α_1 is known then, for any given c_1 , using the standard form the standard coefficients d_2 and d_1 can be obtained. Thus c_1 , d_1 , d_2 all those quantities will be known to us, thus enabling us to estimate the unknowns K_p and K_i . So, with the powerful explicit expressions given over here, it is possible to find unique values for the unknowns K_p and K_i using the optimize coefficients d_2 and d_1 for any given c_1 .

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Now, using the plot which gives us optimum values for d_2 and d_1 for various c_1 by minimizing the ISTE criterion, **ISTE criterion** it is not difficult to obtain the values for d_2 and d_1 when c_1 is equal to 0.5. When c_1 is equal to 0.5, then your d_2 will be of 1.595 and d_1 the upper one, d_1 is of 2.12. So, for various c_1 it is possible to find various combinations of d_2 and d_1 using this plot, which is about the plot of d_2 d_1 versus c_1 optimizing the ISTE criterion. So, using this plot we have found for c_1 equal to 0.5, d_2 is equal to 1.595 and d_1 is equal to 2.12.

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$K = 1; \alpha_1 = 1$ and $\alpha_p = 4$

$c_1 = 0.5 \rightarrow d_2 = 1.595$ and $d_1 = 2.12$

$$K K_p = \frac{\alpha_p c_1}{d_1 - c_1} = \frac{4 \times 0.5}{2.12 - 0.5} = 1.2346$$

$\Rightarrow K_p = 1.2346$ Since $K = 1$

$$K_i = \left(\frac{\alpha_1}{d_2}\right)^3 / K = \left(\frac{1}{1.595}\right)^3 = 0.2464$$

Thus, the PI controller is designed as: *optimum PI controller for all pole unbalanced process*

$$G_c(s) = K_p + \frac{K_i}{s} = 1.2346 + \frac{0.2464}{s}$$

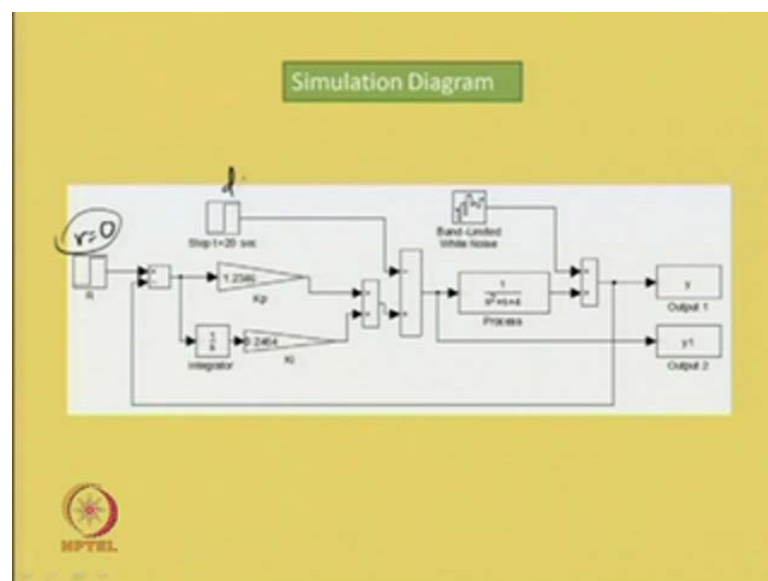
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So, using K equal to 1, let us assume the process model is having the dynamics given by $s^2 + s + 4$. So, we consider an all-pole transfer function of the form $G(s) = \frac{1}{s^2 + s + 4}$ and the damping ratio for this one is equal to $\frac{1}{2\sqrt{4}} = \frac{1}{4} = 0.25$. So, obviously, the damping ratio is 0.25 and we have got an under damped process for designing a PI controller for the system, for the second-order under damped system.

So, the second-order under damped process has got parameters $K = 1$, $\alpha = 1$ and $\omega_0 = 2$. And for $\zeta = 0.25$, we have $\omega_d = 1.96$ and $\sigma = 0.5$ then using the explicit expressions for K_p , which is $K_p = \frac{\omega_0^2}{\omega_d^2}$, which gives us $K_p = \frac{4}{1.96^2} = 1.2346$ thus, giving us K_p is equal to 1.2346 since K is equal to 1.

Similarly, using the next expression $K_i = \frac{\omega_0^2}{\omega_d^3}$ upon K , which gives us $K_i = 0.2464$. Thus the PI controller is designed as $G_c(s) = K_p + \frac{K_i}{s} = 1.2346 + \frac{0.2464}{s}$. So, this gives us optimum PI controller for the all-pole under damped process, **under damped process**. Let us try to simulate and see the responses we get from this PI controller.

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So, the simulation diagram can be met in this form, where we have a reference input which is nothing but, the set point input with value r is equal to 1. A unit step input we

have some disturbance input d , which is of step input occurs at time t equal to 20 seconds and this is of magnitude minus 0.5 and we have got also to the system band-limited white noise to see the effects of disturbances on the performance of the PI controller. Let us look at the controller dynamics of the process and so on.

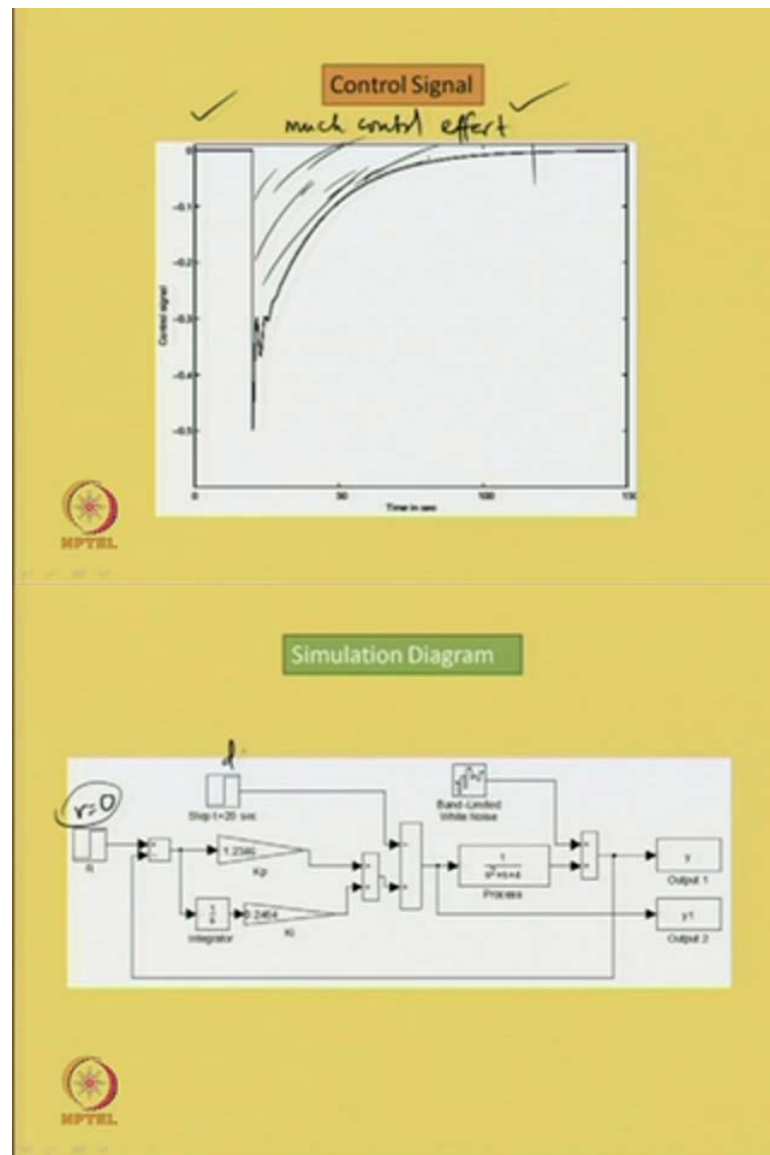
Now, the process dynamics is shown over here, which is given as 1 upon s square plus s plus 4 , this is obtained when we assume this specified values for K is equal to 1 and α_1 is equal to 1 and α_0 is equal to 4 because the standard form of the all-pole second-order transfer function is K upon s square plus $\alpha_1 s$ plus α_0 .

Now, the process dynamics is shown over here, now the PI controller is employed with K_p as 1.2346 and K_i as 0.2464 . So, this gives us the PI controller dynamics. So, we have got the PI controller over here, in the feed forward path for the process. What else we have to see the impact of load and measurement noise disturbances.

Let us set r equal to 0 that is no need for any set point input because we have interest in the disturbance responses. Therefore, let us set the disturbance d of magnitude 0 minus 0.5 which occurs at time t equal to 20 seconds. Similarly, the band-limited white noise, let this occur from the beginning of the simulation; that means, at time t equal to 0 , we have band-limited white noise for the system.

Now to clearly see the effects of these disturbances, initially what we shall do? We shall not apply any sort of disturbances or measurement noise to the system and see the impacts of these disturbances on the system. So, for that when the system is not subjected to any disturbances, what sort of output response we get, that we shall see first.

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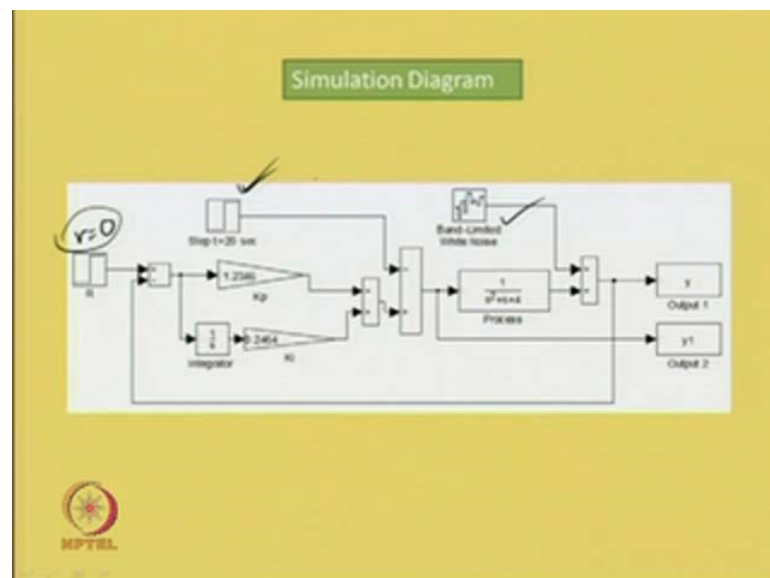
So, when the system is not subjected to band-limited white noise, but it is subjected to a static load disturbance of magnitude minus 0.5, which occurs at time t equal to minus t equal to 20 seconds is shown over here. So, when a static load disturbance of magnitude minus 0.5 occurs at time t equal to 20 second, the response is shown over here. As it is evident from the response, we have got a very sluggish response because the settling time the disturbance occurs here and it takes almost more than 100 seconds to go to the steady state.

Therefore, the response takes almost 80 seconds to go to the steady state, although the response is fast as is apparent from here and the response magnitude load disturbance rejection is not instantaneous, but still it has a faster load disturbance excursion means

response whereas, the response is very sluggish, it takes almost 80 seconds to go to the steady state. This is the type of load disturbance response we get from the system, closed loop system in the absence of measurement noise.

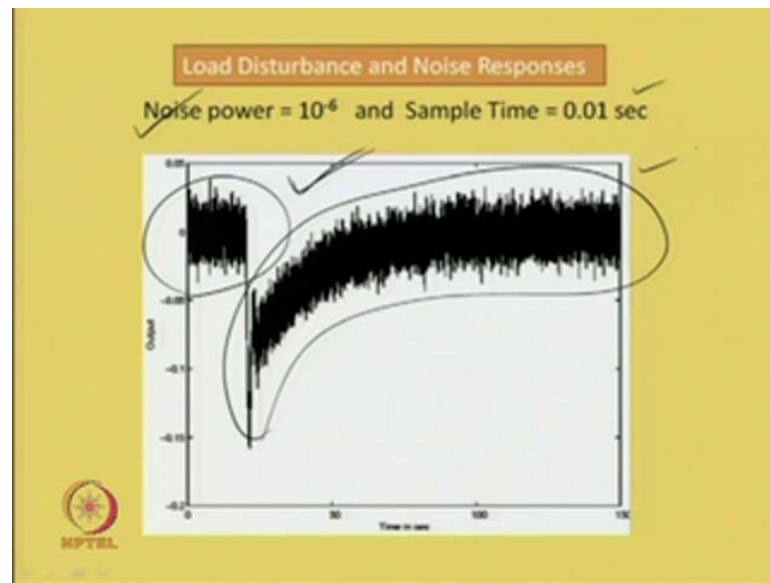
What type of control signal we will get, we get a control signal of this form, what information we get from here? It shows that much control effort much or energy spent to give closed loop performance of the system. So, much control effort is here because the time taken is very large, you see to go to the steady state, we take almost more than 100 seconds. So, then this is the amount of energy we spent, this is the control effort we spent for the closed loop dynamics. So, next we shall see the impact of the measurement noise on the system.

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Now, when both the static load disturbance of magnitude minus 0.5 and band-limited white noise is present in the system, then the output of the system is obtained as shown over here.

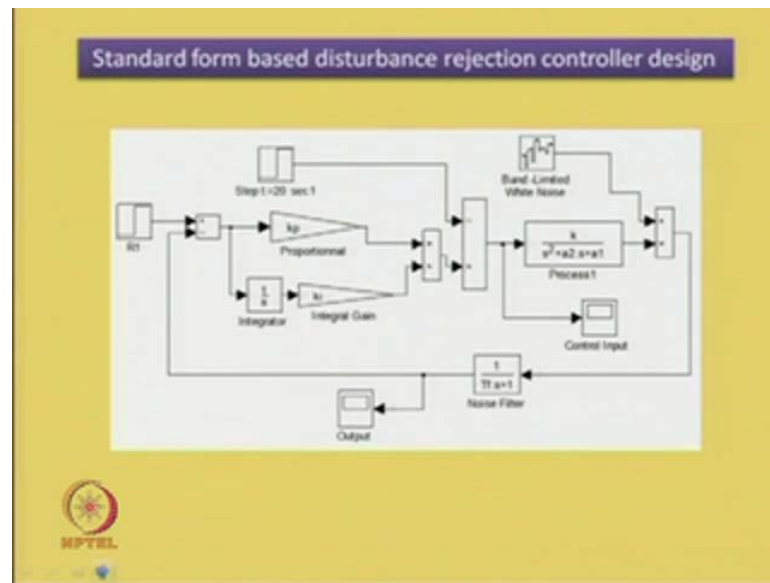
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So, we get the load disturbance and noise responses, keep in mind that we have set the reference input or set point input to be 0, because we have interest in the noise responses or load disturbance responses. So, when the noise power is of 10 to the power minus 6 and the sample time is 0.01 second, then the **output of the** disturbance output of the system can be obtained of in this form. Now, it is very much noisy, not only the response is sluggish, the output is very much noisy and it is very difficult to find out the exact output from the system unless some filtering is used.

Similarly, the control effort also can be, control signal can also be plotted and seen to be of this form, where we have got the effect of noise and the control effort is very high. So, the measurement noise is quite evident from the control signal as well as the response, noise responses of the system.

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Next, effort will be made to design a disturbance rejection controller. Where is that controller in this loop? We have got a controller placed in the feedback path. So, the controller now is known as a noise filter, this noise filter the primary job of this noise filter is to filter out the measurement noise. This process the measurement noise can be shown in the form when some sensor is put over here; the noise introduced by the sensor is shown by the band-limited white noise injected over this point. Now if some measurement noise filter is put in the loop in this form, then it can give some improved performance than the earlier PI controller. So, the effort will be made now to design a noise filter for the closed loop system; apart from the PI controller, the PI will be there we shall have noise filter in the closed loop.

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Consider the process $G(s) = \frac{K}{s^2 + \alpha_1 s + \alpha_0}$

The PI controller is $G_c(s) = K_p \left(1 + \frac{1}{T_i s}\right)$


Let the Noise Filter dynamics be $G_f(s) = \frac{1}{T_f s + 1}$

Closed loop tf becomes

$$T(s) = \frac{T_f s + 1}{s^3 T_i / (K K_p) + \alpha_1 s^2 T_i / (K K_p) + \alpha_0 s T_i / (K K_p) + 1}$$

Assuming $K K_p / T_i = \beta^3$ and $s = \beta s_n$

CLTF can be written as

$$T(s_n) = \frac{c_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1}$$


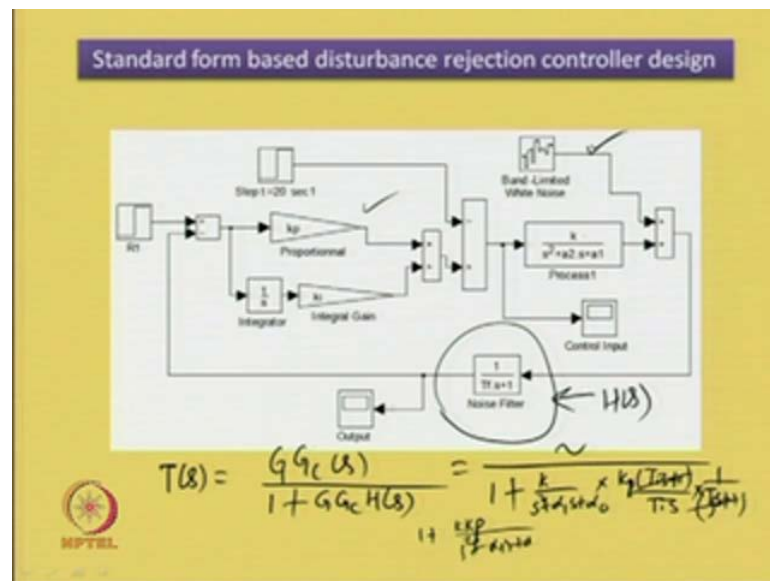
Then **all the process we have** all the steps we have made used earlier can be repeated with the changes that, a noise filter dynamics of the form $G_f(s)$ is equal to $1/(T_f s + 1)$ is injected, this is new. So, when the noise filter dynamics is injected is inserted then, the analysis gives a closed loop transfer function of the form $T(s)$ is equal to $(T_f s + 1)$ upon $s^3 T_i / (K K_p) + \alpha_1 s^2 T_i / (K K_p) + \alpha_0 s T_i / (K K_p) + 1$. How do we get this form of closed loop transfer function? This we get with the help of a new type of PI controller.

Please keep in mind, the type of PI controller we had considered earlier was your $G_c(s)$ equal to $K_p + K_i / s$ whereas, we have consider a parallel PI controller of the form $G_c(s)$ is equal to $K_p \left(1 + \frac{1}{T_i s}\right)$. So, we have got different parameter here T_i in place of K_i . So, T_i has been introduced in place of K_i , this has been done intentionally for is in analysis of the closed loop transfer function, one can make use of the earlier form of course with some difficulties.

Now, again consider the all-pole second-order transfer function of the process given by $G(s)$ is equal to K upon $s^2 + \alpha_1 s + \alpha_0$ and the PI controller of the form $G_c(s)$ equal to $K_p \left(1 + \frac{1}{T_i s}\right)$. Let the noise filter dynamics be given by $G_f(s)$ is equal to $1/(T_f s + 1)$. Why this has been, **why we have** we are using we have interest in this particular type of PI controller form? The reason is that, one pole 0 cancellation can be initiated and we can get a simpler closed loop transfer function.

So, this closed loop transfer function one obtains provided T_f is equal to T_i . Please keep in mind, unless you make this assumption you do not get a closed loop transfer function of this form. Why I say so, because if you carefully observe look at the closed loop transfer function, we do not get any term with T_f . So, term with T_f is missing here; that means, certainly some cancellation has been made, some approximation has been made otherwise, T_f must appear in the closed loop transfer function.

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Now, the closed loop transfer function **for the system** for the closed loop system, for this system has to be given in the form of $T(s)$ is equal to $G_c(s)$ upon one plus $G_c(s)H(s)$. Then here in the denominator, do not mind about the numerator; in the denominator, it can be written as $G(s)$ is your K upon s^2 plus α $1/s$ plus α 0 , then $G_c(s)$ is $K_p T_i$ $1/s$ plus 1 upon $T_i s$, then $H(s)$ is this noise filter dynamics we give it by $H(s)$.

Here we have got one upon $T_f s$ plus 1 . So, this when we approximate T_f is equal to T_i then this approximation enables us to cancel one pole with one 0 . Thus giving us in the denominator terms like 1 plus $K K_p$ upon s^2 plus α $1/s$ plus α 0 times $T_i s$ only. So, **that is the** that is why, we have got a closed loop transfer function which is divide of the T_f term.

So, after getting the closed loop transfer function, again making use of the approximation that, $K K_p$ upon T_i is equal to β^3 and s is equal to β times s_n (Refer Slide Time: 25:49). The same closed loop transfer function can be written in the form of c/s

n plus 1 in the numerator, having a denominator of s n cubed plus d 2 s n square plus d 1 s n plus 1, thus we get a standard third-order transfer function with the coefficient c 1, d 2 and d 1.

Now, like the earlier case with the help of simplification, T i is not difficult to find explicit expressions for the two unknowns of the controller K p and T i. And since we have T f is equal to T i therefore, there is need for estimating two unknowns for the controller, K p and T i are the two unknowns then all other things are known and the closed loop system will have a controller in place. Now, we have d 2 is now alpha 1 upon beta, if you look at carefully **if you look at carefully** then, this d 2 can be written as alpha 1 upon beta and d 1 is equal to alpha 0 upon beta square and c 1 is equal to beta T i.

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where $d_2 = \alpha_1 / \beta$; $d_1 = \alpha_0 / \beta^2$; $c_1 = \beta T_i$

Next $d_1 = \alpha_0 / \beta^2$
 $\Rightarrow K K_p = \frac{\alpha_0 c_1}{d_1}$

Similarly, $d_2 = \alpha_1 / \beta =$
 $\Rightarrow T_i = \frac{d_2 c_1}{d_1}$

$c_1 \rightarrow d_2, d_1$

K_p, T_i

Now, since d 1 is equal to alpha 0 upon beta square T i is same as alpha 0 beta upon beta cubed and beta cubed is nothing but, beta cubed is K Kp upon T i. So, you can write alpha 0 beta T i upon K Kp again beta T i is c 1. So, we get in the numerator alpha 0, in the numerator alpha 0 beta T i is c 1 alpha 0 c 1 upon K K p. So, d 1 is equal to **K K** alpha 0 c 1 upon K Kp implies that K Kp is equal to alpha 0 c 1 upon d 1. So, this is how we obtain explicit expressions for the unknown K p.

So, K Kp is equal to alpha 0 c 1 upon d 1; similarly, making use of the second expression d 2 expression for d 2, which is given as d 2 is equal to alpha 1 upon beta. Again you can

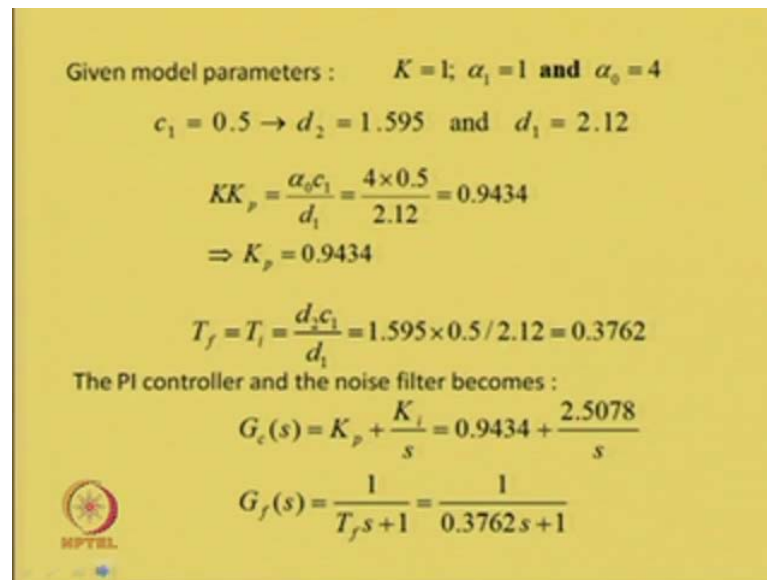
write down this as $\frac{\alpha_1 \beta^2}{K K_p}$ upon t times T_i in the numerator, then you will get here α_1 . So, if I substitute now βT_i , so I have got $\alpha_1 \beta$ time's c_1 upon $K K_p$. So, $K K_p$ is nothing but, now α_0, c_1 and d_1 , so this type of manipulation will give you $c_1 c_1$ cancellation.

So, therefore, we will get this is equal to d_2 then yes, now, d_2 is equal to $\alpha_1 \beta d_1$ upon α_0 , with further manipulation will get $\alpha_1 \beta d_1$ upon α_0 . So, with little manipulation it will not be difficult to obtain T_i in the form of $d_2 c_1$ upon d_1 . So, ultimately with substitution of d_1 now here and c_1 will further d_1 , substitution of d_1 rather will enable because there will be cancellation.

So, that will give us T_i ultimately in the form of T_i is equal to d_2, c_1 upon d_1 . So, the final expressions we have got from the analysis of the coefficients of the third-order standard transfer function is that, $K K_p$ can be obtained in the form of $\alpha_0 c_1$ upon d_1 and T_i in the form of $d_2 c_1$ upon d_1 . The main reason for obtaining in this convenient form is that, since for any c_1 , it will not be difficult to get d_2 and d_1 therefore, assume any c_1 and obtain optimum d_2 and d_1 .

So, known quantities will be α_0, α_1 is not there, now for any c_1 , we get d_1, c_1 we get d_1 and d_2 ; therefore, all the quantities in the right half of the two expressions are known to us. Thus it will be possible to estimate K_p and T_i values making use of the value c_1, d_2 and d_1 for a given process transfer function model. So, if the transfer function model of a process is known and if T_i is available in the all-pole form, then it becomes very easy to design PI controller with a noise filter in the loop for the closed loop system.

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Given model parameters : $K = 1$; $\alpha_1 = 1$ and $\alpha_0 = 4$

$c_1 = 0.5 \rightarrow d_2 = 1.595$ and $d_1 = 2.12$

$$KK_p = \frac{\alpha_0 c_1}{d_1} = \frac{4 \times 0.5}{2.12} = 0.9434$$
$$\Rightarrow K_p = 0.9434$$
$$T_f = T_i = \frac{d_2 c_1}{d_1} = 1.595 \times 0.5 / 2.12 = 0.3762$$

The PI controller and the noise filter becomes :

$$G_c(s) = K_p + \frac{K_i}{s} = 0.9434 + \frac{2.5078}{s}$$
$$G_f(s) = \frac{1}{T_f s + 1} = \frac{1}{0.3762 s + 1}$$

Now, the closed loop system is having K equal to 1, we consider the same process model as we have considered in the earlier case, where K is equal to 1, alpha 1 is equal to 1 and alpha 0 equal to 4. That means the process G s is now given in the form of 1 upon s square plus s plus 4. So, we consider the same second-order under damped process with the steady state gain K is equal to 1.

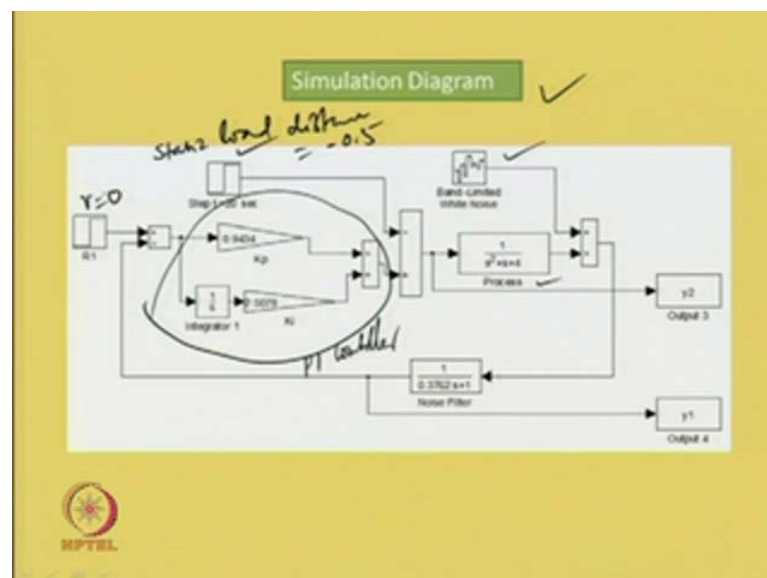
Now, c 1 equal to 0.5 is d 2 1.595 and d 1 is of 2.12, this we get from the minimization of ISTE criterion. So, please do not forget from the minimization of ISTE criterion, we get the standard values d 2 and d 1, for any given c 1. Now, putting those values the explicit expressions for K Kp which is nothing but, K Kp is equal to alpha 0 c 1 upon d 1 is equal to 4 times 0.5 c 1 is 0.5 alpha 0 is 4; therefore, 4 times 0.5 upon 2.12 gives us K Kp as 0.9434, where K p becomes 0,934. Now, T f is equal to T i is given as d 2 c 1 upon d 1, now which is nothing but, d 2 is 1.595 times 0.5 upon 2.12, which gives us T f is equal to T i is equal to 0.3762.

So, thus we design the PI controller as well as the first-order noise filter for the closed loop system. For the noise controller, dynamics is given by G f s is equal to 1 upon T f s plus 1 is equal to 1 upon 0.3762 s plus 1 and G c s obtained in the earlier form for the sake of comparison is G c s is equal to K p plus K i upon s is equal to 0.9434 plus 2.5078 upon s. How do you get that one? It is not difficult to obtain that value for the K I,

making use of the comparison that, when the $G_c(s)$ is written in different form, $K_p + \frac{K_i}{s}$ this gives us K_p plus $\frac{K_i}{s}$.

So, this is again expressed in the form of $K_p + \frac{K_i}{s}$ therefore, this value 2.5018 has been obtained from the ratio of K_p and T_i . So, if you take the ratio of K_p and T_i ; that means, $\frac{0.9434}{0.3762}$ will be equal to 2.5078. So, thus we get a PI controller of the earlier form for the sake of comparison or I can say that, to make use of the sense simulation model, I have obtained the K_i in that earlier form, then let us go to the simulation diagram.

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So, this simulation diagram has got a noise filter and rest of the things remains as it is. Earlier, we have made use of this simulation diagram, now this simulation diagram has got an additional transfer function in the loop which is nothing but, the noise filter. So, this is our PI controller, **PI controller** process band-limited white noise and the step load disturbance, which occurs at time t equal to 20 second and the magnitude of the static load disturbance or static load disturbance magnitude is equal to minus 0.5 like the earlier case, for the sake of comparison. Again we set r is equal to 0, because we have interest in the disturbance responses of the closed loop system.

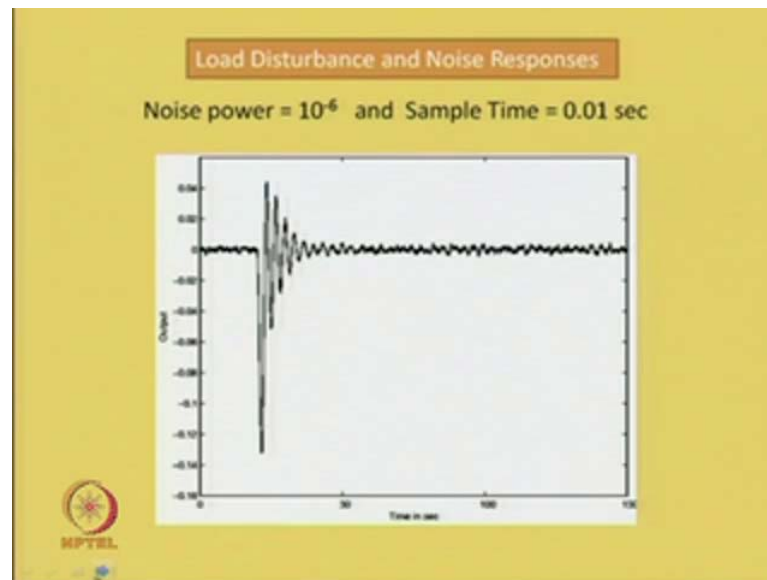
Then we get the load disturbance responses of this form, please keep in mind the static load disturbance is of magnitude 1 is equal to minus 0.5 occurring at time t equal to 20 seconds (Refer Slide Time: 37:06). Now, this is what we get from the current scheme.

Now, with the inclusion of a noise filter in the loop, the static load response has improved significantly, it is easily observable from this plot that, the response is not only fast, it settles down **within some 40 no, 20 means 30** within some 30 second . So, it take almost 30 seconds, in place of 80 seconds for the earlier case to settle down to the steady state and the magnitude of the load disturbance response is also not higher compared to the earlier case. You see the magnitude is not changing there is little bit of over shoot or under shoot, but those are insignificant. So, overall the static load disturbance response for the second scheme is quite satisfactory. Now, I can say this response is quite satisfactory from the point of speed of response as well as settling time.

Let us see the control signal, the control effort one has to provide with the inclusion of the static field with the noise filter in the loop is very less, if you see the excursion the amount of energy you spent is very less compared to the energy you spent, you take the area of the lower curve and area of the upper curve, then below the line, zero line then that gives a significant improvement in the second case.

So, with the inclusion of a noise filter in the loop, one **the unit provides,** the unit requires much control effort and there will not be actuator saturation also for the second case. So, actuator saturation is a very big problem when the control effort is very higher and the control signal is of high magnitude, then control saturation occurs and actuator or valves may get saturated. In that case, there are problems to overcome, those problems often it is desirable to provide suitable noise disturbance rejecters in a closed loop system. Now, let us investigate the effects of measurement noise on the system.

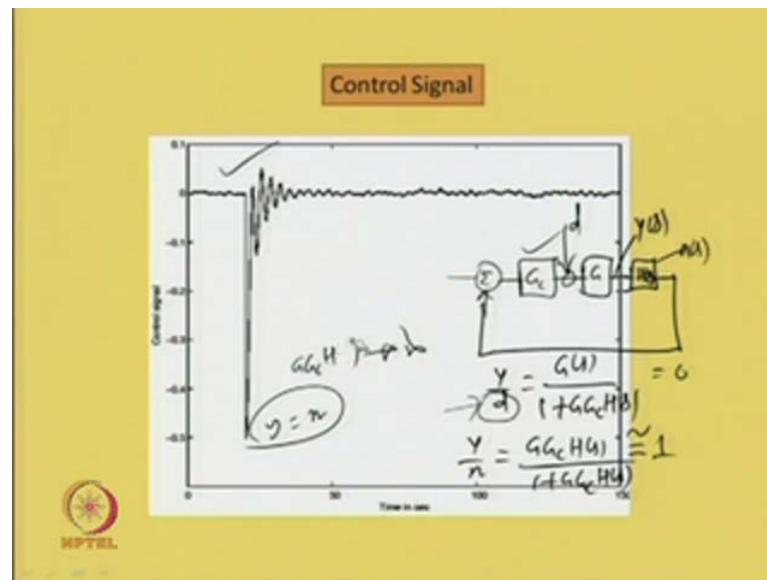
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So, when a noise with power 10^{-6} and a sampling time of 0.01 second is used, then the load disturbance response and the noise responses can be obtained of this form. You see we have got quite satisfactory load disturbance and noise responses compared to the earlier case. Let us go back to the first case where, the response of the system is shown in this form. This is what we get, when we do not have a noise filter in the loop, this is the output response we get (Refer Slide Time: 40:42).

So, please observe it, the magnitude is from minus 0.2 to 0.05 with high value of excursion of measurement noise. Now, compared to that, we have got a response of this form. So, excursion is not only less, earlier it was minus 0.2, we have come up to minus 0.13 and here also, it was very high. So, both ways not only the improvement in the magnitude of the load responses has taken place, there is significant reduction in the noise level as well.

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So, the noise filter has given us satisfactory load disturbance and noise measurement noise responses, the control signal is also quite satisfactory. So, with much less control effort, one is possible to obtain satisfactory closed loop performances from the closed loop system, of course with a noise filter in the loop.

So, before going to summary, let us do little bit of analysis over here, why that happens so. Suppose the closed loop system is given like this, we have got a controller G_c here and we have got G , now this is the measurement noise, when we have a sensor over here as we have been doing earlier H s. So, definitely there will be measurement noise. Now, without putting a noise filter, what is the transfer function we get for different type of disturbance inputs?

Suppose the load disturbance d is occurring is here, then y upon d , y upon d is given as G in the absence of H s. Now, actually the output will be not here (Refer Slide Time: 43:00), here this is the sensor because the output is the output of the system Y s. Then Y upon d is given as G upon $1 + G G_c H$ s whereas, as far as the noise input is concerned. Suppose, here the noise input of noise power n s is occurring, then y upon n is given as $G G_c H$ s upon $1 + G G_c H$ s.

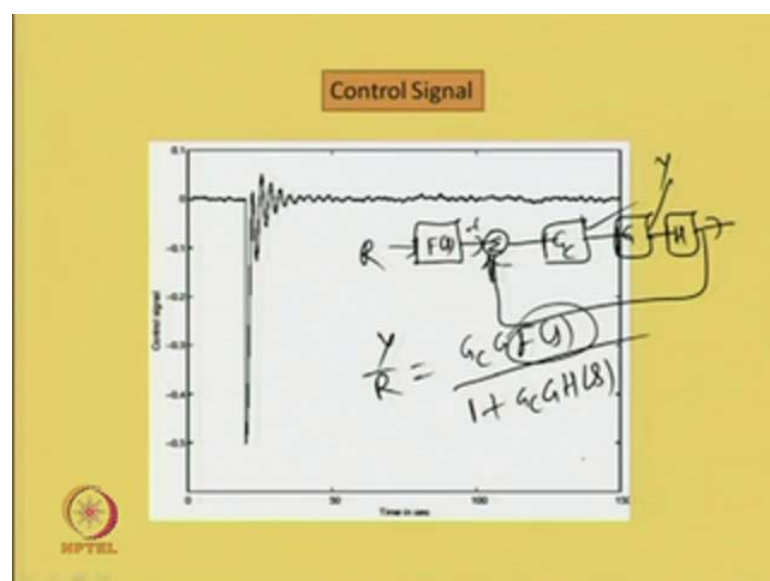
So, if I look at the two transfer functions, what happens? To minimize the effects of load disturbance responses, effects of load, effects of static load inputs, what has to be done if $G G_c H$ s is very high a large number? In that case, this will be 0 that is our aim because

we do not have to have any effect of static load disturbances d , but when $G_c H s$ is very high, look at the second transfer function, then in that case it will be equal to 1, it will be approximately 1. That means, the output y will be equal to n therefore, the effects of measurement noise will be very much present in the output of the closed loop system. Thus, it is not possible to design a controller G_c , which will give us satisfactory noise rejection as well as static load disturbance rejection. So, one has to have some compromise while designing a controller for rejecting disturbance in the system.

Now, is there any other way we can handle this situation? Yes of course, if one designs other type of controller, let us say a 2 degree of freedom controller, if I put a some filter over here, $F s$ yes, it is possible then the G_c , G_c can be purely designed for rejection of the disturbances and $F s$ later on can be designed for satisfactory closed loop performances. In place of that, what we have done now? In place of injecting or putting a reference filter in the reference path, one can put a filter, noise filter in the feedback path that is what we have done.

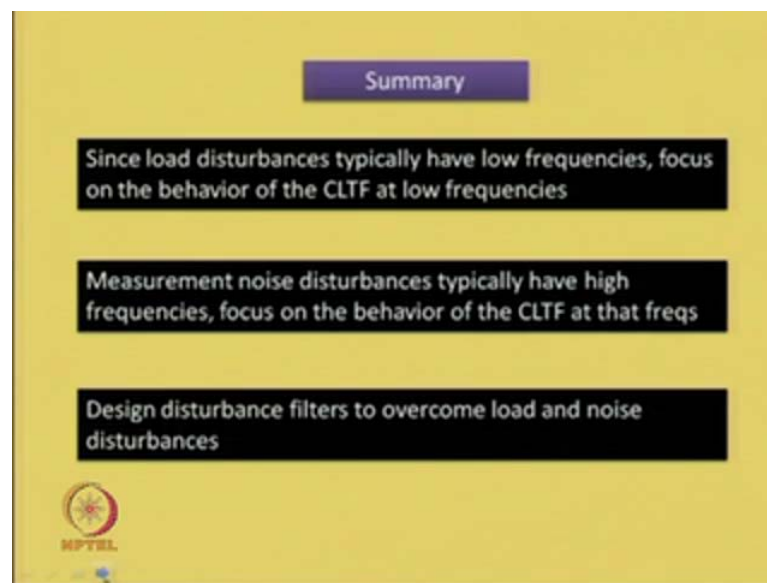
So, putting a noise filter of the form $T f s + 1$ in the feedback path, still we are able to provide a 2 degree of freedom controller to the structure somehow. Now, the job of this is purely to reject disturbances whereas, the job G_c will be to provide overall satisfactory closed loop responses.

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Now, so, the impact of the disturbance rejection filter has been investigated here. But, let us try to analyze the system when you have got a 2 degree of freedom controller. So, when we have got F_s in the loop, in that case $G_c G_H$ the closed loop transfer function, where we have got Y upon r will be equal to $G_c G_f s$ upon $1 + G_c G_h s$. So, this F_s can be designed in a suitable way to provide satisfactory set point responses and G_c can be designed in a way to provide satisfactory disturbance rejections in the system.

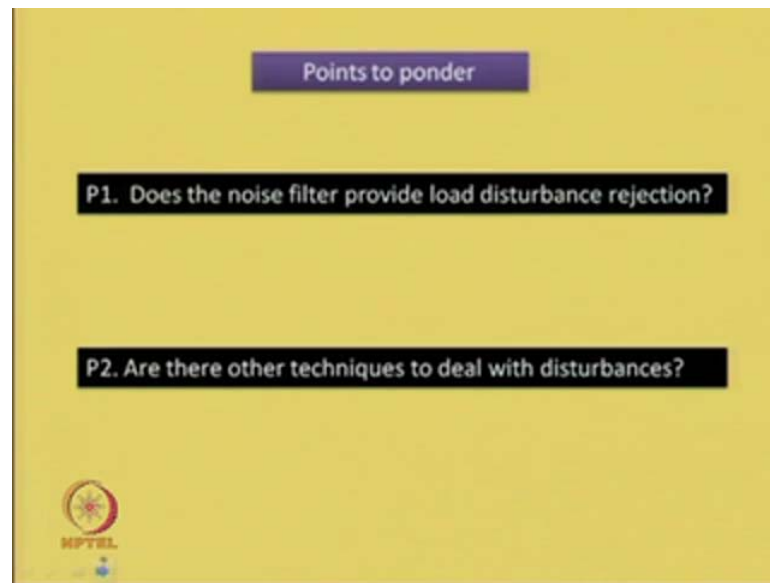
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Now, in summary we can say, load disturbances are typically of low frequencies. Therefore, focus on the behavior of the closed loop system at low frequencies should be made to design suitable compensator for rejecting load disturbances whereas, measurement noise disturbances are typically of very high frequencies. Therefore, focus should be on to design controllers that would reject high frequency inputs or excursions in the closed loop system.

Now, disturbance rejection filters can overcome both load and noise disturbances if design suitably, but often it is desirable to design 2 degree of freedom controllers for many closed loop system to overcome the ill effects of disturbances, both sort of disturbances not only measurement noise, input to the system, rather the static load input to the systems can be overcome by designing suitable filters.

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Now, coming to the points to ponder, we have two important points to discuss about related to this topic. First is does the noise filter provide load disturbance rejection? Yes, the noise filter which primary job is to reject noise inputs to the system. The ill effects of noise inputs to the system can also provide satisfactory load disturbance rejection, but that is not true that may not be true always. Therefore, always it is necessary to design some load estimator and controller in a closed loop system.

So, noise filter has got its own limitation, it can be of any order when we employ higher order noise filter, in that case the design method will change. And we may not be able to straightforward we may not be able to find explicit expression for the unknowns of the filter in a straightforward manner. So, those are the limitations of designing higher order filter for a closed loop system for disturbance rejection. But of course, with a higher order filter, often it is possible to design filters which can provide not only control action for measurement noise inputs, rather for static load disturbances as well.

The second point is, are there other techniques to deal with disturbances? Obviously, as we have seen towards the end of this lecture, one can go for a 2 degree of freedom control structure for dealing with the disturbances, disturbance inputs to a system. The beauty of the 2 degree of freedom control structures are that, one controller can purely we designed for rejection of disturbances, disturbance inputs to the systems whereas, the

other controller can be designed for overall satisfactory performances, time and frequency domain performances of a system.

So, we have got many more design techniques for dealing with disturbances in a system. Now, one can design high gain, low gain, one can concentrate on high gain, low gain zones of the bode magnitude and phase plots and Nyquist plots to design suitable controllers for disturbance rejections, that is all in this lecture.